Finding and optimising the key factors for the multiple-response manufacturing process

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With the advent of modern technology, manufacturing processes became so sophisticated that a single quality characteristic cannot reflect the true product quality. Thus, it is essential to perform the key factor analysis for the manufacturing process with multiple-input (factors) and multiple-output (responses). In this paper, an integrated approach of using the desirability function in conjunction with the Mahalanobis-Taguchi-Gram Schmit (MTGS) system is proposed in order to find and optimise the key factors for a multiple-response manufacturing process. The aim of using the MTGS method is to standardise and orthogonalise the multiple responses so that the Mahalanobis distance for each run can be calculated and the multi-normal assumption for the correlated responses can be relaxed. A realistic example of the solder paste stencil printing process is then used to demonstrate the usefulness of our proposed approach in a practical application.

Keywords: multiple-response process; solder paste stencil printing process; Mahalonobis distance; desirability function; Taguchi method

1. Introduction

With the advent of modern technology, manufacturing processes have become so sophisticated that a single quality characteristic cannot reflect the true product quality. Thus, it is essential to perform the key factor analysis for the manufacturing process with multiple-input (factors) and multiple-output (responses).

A traditional experimental design and analysis method is typically used in finding and optimising the key factors for one response variable. Simultaneous consideration of multiple responses are difficult when (1) these multiple responses are highly correlated, and (2) if one response belongs to 'the bigger the better', while the other belongs to 'the smaller the better' quality characteristic in a multiple-response process. In this paper, we propose a multivariate technique based on the Mahalanobis-Taguchi-Gram Schmit (MTGS) system as well as the desirability function to explore and optimise the key factors for a multiple-response manufacturing process.

2. Literature review

Various multi-response methods have been proposed, for example, the dual response approach proposed by Myers and Carter (1973), the distance function introduced by Khuri and Conlon (1981), the desirability function proposed by Harrington (1965) and modified by Derringer and Such (1980). Logical solutions to multiple response problems are to optimise overall quality characteristics by compromising between important responses (Myers and Montgomery 1995).

The desirability function is used to reduce the dimension of a multi-response problem into a single-response problem. Derringer and Such (1980) modified Harrington's (1965) approach and extended to the following three types of quality characteristics:

(1) For the larger-the-better (LTB) type:

$$
d_i = \begin{cases} 0, & Y_i \leq Y_{i*} \\ \left[\dfrac{Y_i - Y_{i*}}{Y'_i - Y_{i*}}\right]^t, & Y_{i*} \leq Y_i \leq Y'_i \\ 1, & Y_i \geq Y'_i \end{cases} \tag{1}
$$

where Y_{i*} is the minimum acceptable value of Y_i, Y'_i is the highest value of Y_i.

(2) For the smaller-the-better (STB) type:

$$
d_i = \begin{cases} 1, & Y_i \leq Y''_i \\ \left[\dfrac{Y^*_i - Y_i}{Y^*_i - Y''_i}\right]^r, & Y''_i \leq Y_i \leq Y^*_i \\ 0, & Y_i \geq Y^*_i \end{cases} \tag{2}
$$

where Y''_i is the lowest value of Y_i, Y^*_i is the maximum acceptable value of Y_i.

(3) For the nominal-the-best (NTB) type:

$$
d_i = \begin{cases} \left[\dfrac{Y_i - Y_{i*}}{c_i - Y_{i*}}\right]^s, & Y_{i*} \leq Y_i \leq c_i \\ \left[\dfrac{Y_i - Y^*_i}{c_i - Y^*_i}\right]^t, & c_i < Y_i \leq Y^*_i \\ 0, & Y_i > Y^*_i \text{ or } Y_{i*} > Y \end{cases} \tag{3}
$$

where s and t are the exponential parameters that determine the shape of a desirability function, and c_i is the most likely acceptable value, $Y_{i*} \leq c_i \leq Y^*_i$, i.e. the target value of the NTB type response. The user has a greater flexibility in the setting up an appropriate desirability function by selecting a suitable r, s, t.

After finding the individual desirability function for each response, an overall desirability function can be obtained by

$$
D = \sqrt[k]{d_1 \times d_2 \times \cdots \times d_k} \tag{4}
$$

where $d_i, i = 1, 2, \ldots, k$ is the individual desirability function for each response. Chiao and Hamada (2001) mentioned that the overall desirability $D = (d_1(Y_1), d_2(Y_2), \ldots, d_k(Y_k))^{1/k}$

is difficult to construct if its desirability is complex functions of the k responses, where $(Y_1, Y_2, \ldots, Y_k)$ are k responses and $(d_1, d_2, \ldots, d_k)$ are the individual desirability functions. They proposed a quality measure to be the probability that the k responses are simultaneously meeting a specification region for searching the key factors.

Since a traditional multi-dimensional or multivariate system only considers response variables that are mutually independent, it is easy to make mistakes in determining the key factors for the dependent response variables in a multi-dimensional system. Taguchi and Jugulum (2002) proposed a Mahalanobis-Taguchi system (MTS) as shown in Figure 1 to modify the traditional multi-dimensional system.

MTS can be used to detect the abnormality of observations. If the Mahalanobis distance of a data set is too large, then the observation is considered to be abnormal. It should be pointed out that there have been debates on the MTS vs. classical multivariate analysis approaches recently. (For details please refer to Woodall *et al.* 2003, Jugulum *et al.* 2003, Abraham and Variyath 2003 and Hawkins 2003).

3. Research methodology and data analysis procedures

The data analysis procedures for determining and optimising the key factors of a multi-response manufacturing process are shown in Figure 2. The determination of key factors for a multi-response manufacturing process is discussed in Section 3.1. The main idea in the data analysis procedures is to use the concept of Mahalonobis distance to convert multi-response variables into a single dimensional performance index in analysis of variance (ANOVA). The optimisation of a multi-response manufacturing process using a desirability function is discussed in Section 3.2.

3.1 *The key factor analysis for a multi-response manufacturing process*

Denote n observations of the k responses by $W_1, W_2, \ldots, W_k$, where $W_i = (w_{i1}, w_{i2}, \ldots, w_{in})'$, $i = 1, 2, \ldots, k$. The original data structure for a multi-response process is shown in Table 1.

Step 1: *Perform the standardisation.*
The original experimental data (w_{ij}) need to be standardised as below

$$z_{ij} = \frac{w_{ij} - \bar{w}_i}{\sqrt{S_i^2}} \begin{cases} i = 1, 2, \ldots, k \\ j = 1, 2, \ldots, n \end{cases} \tag{5}$$
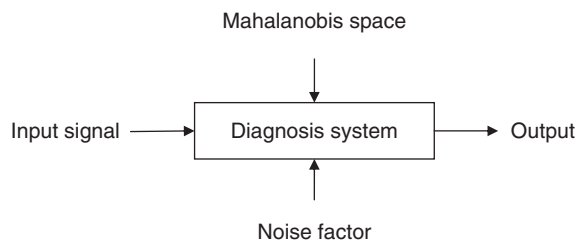

Figure 1.   The Mahalanobis-Taguchi multidimensional system.

Determination of the key factors

Optimisation of the
multi-response process

(1) Collect n observations W_i for the k-responses, where
$W_i = (w_{i1}, w_{i2}, \ldots w_{in})', i = 1, 2, \ldots k$

(2) Standardise each w_{ij} into z_{ij},
$i = 1, 2, \ldots, k \quad j = 1, 2, \ldots, n$

(3) Use GSP to convert $(Z_1, Z_2, \ldots, Z_k)$ into an orthogonal basis.

(4) Calculate Mahalonobis distance for each run using $MD_j = \dfrac{1}{k} V_j' D^{-1} V_j$, where $V_j = (u_{1j}, u_{2j}, \ldots, u_{kj})'$ is the jth run after standardisation and orthogonalisation.

(5) Determine the initial key factors for the response $\sqrt{MD_j}$ by effect normal probability and Pareto plots.

(6) Perform analysis of variance to confirm the significant main and interaction effects.

(7) Perform regression and residual analyses to construct a fitted regression model.

(8) Calculate the desirability function for each response under different combinations of key factors

(9) Calculate the overall desirability function under different combinations of key factor levels, i.e.
$D = \sqrt[k]{d_1 \times d_2 \times \quad \times d_k}$.

(10) Find the maximum overall desirability function that is the optimal combination of key factor levels.

(11) Perform confirmatory tests using the fitted regression model and calculate Mean Absolute Percent Error (MAPE). A MAPE less than 15% is preferred.
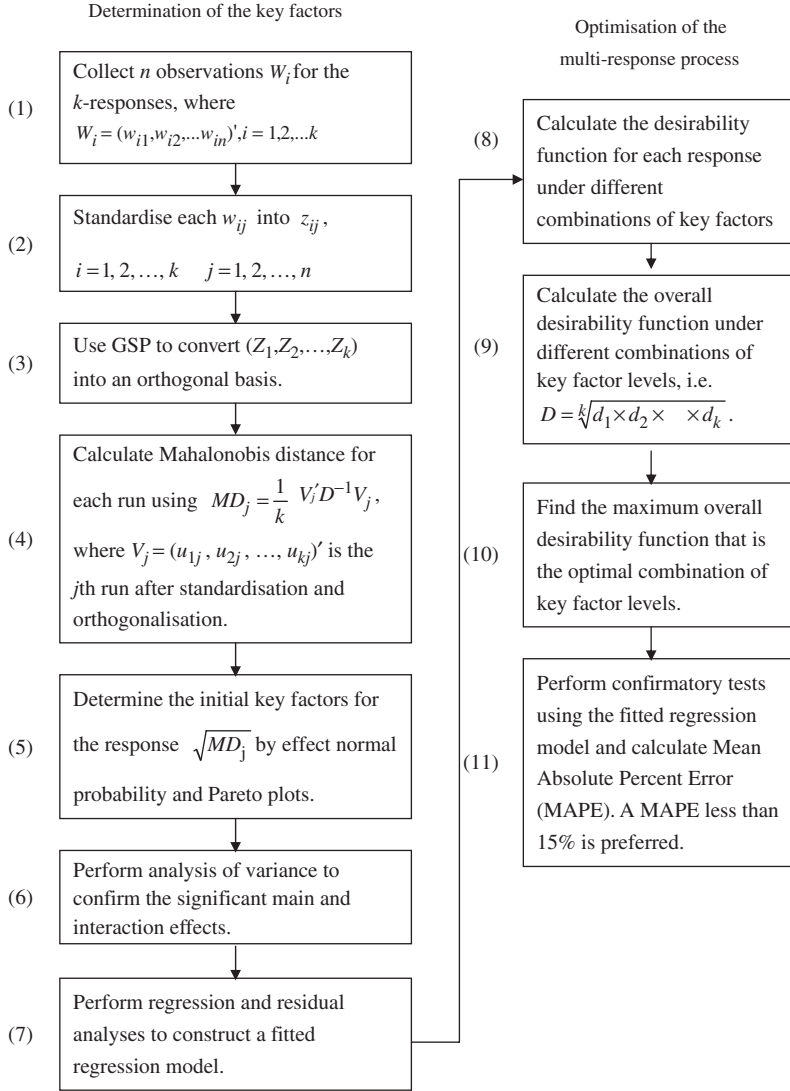
Figure 2. The data analysis flowchart for determining and optimising the key factors of a multi-response process.

Table 1. The original data structure for a multi-response process.

Run j \ Response i	W_1	W_2	$\ldots$	W_k
1	w_{11}	w_{21}	$\ldots$	w_{k1}
2	w_{12}	w_{22}	$\ldots$	w_{k2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	w_{1n}	w_{2n}	$\ldots$	w_{kn}

where

$$\bar{w}_i = \frac{\sum_{j=1}^n w_{ij}}{n} \quad \text{and} \quad S_i^2 = \frac{\sum_{j=1}^n (w_{ij} - \bar{w}_i)}{n-1}$$

Step 2: *Perform the orthogonalisation.*
According to the orthogonal procedure of Gram-Schmidt process (GSP) described below, the standardised response $Z_1, Z_2, \ldots, Z_k$ can be converted to an orthogonalised response $U_1, U_2, \ldots, U_k$. The orthogonal data for a multi-response process is shown in Table 2.

$$\begin{aligned}
U_1 &= Z_1 = (u_{11}, u_{12}, \ldots, u_{1n})' \\
U_2 &= Z_2 - c_{2,1} U_1 = (u_{21}, u_{22}, \ldots, u_{2n})' \\
U_3 &= Z_3 - c_{3,1} U_1 - c_{3,2} U_2 = (u_{31}, u_{32}, \ldots, u_{3n})' \\
&\vdots \\
U_k &= Z_k - c_{k,1} U_1 - c_{k,2} U_2 - \cdots - c_{k,k-1} U_{k-1} = (u_{k1}, u_{k2}, \ldots, u_{kn})'
\end{aligned} \tag{6}$$

where $c_{i,j} = (Z'_i U_j / U'_j U_j)$ are the coefficients of vectors in the Gram-Schmidt process.

Step 3: *Compute the Mahalonobis distance.*
Let $V_j = (u_{1j}, u_{2j}, \ldots, u_{kj})'$, $j = 1, 2, \ldots, n$ be the jth run data after standardisation and orthogonalisation, then the Mahalonobis distance for the jth run data can be calculated by

$$MD_j = V'_j D^{-1} V_j$$

where D is a variance-covariance matrix of $U_1, U_2, \ldots, U_k$. Since $U_1, U_2, \ldots, U_k$ are orthogonalised data and they are uncorrelated, D is denoted by:

$$D = \begin{pmatrix} \sigma_{U_1}^2 0 & \cdots & & 0 \\ 0 & & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \sigma_{U_k}^2 \end{pmatrix}. \tag{7}$$

Table 2. The standardised and orthogonal data for a multi-response process.

Response i / Run j	U_1	U_2	$\cdots$	U_k
1	u_{11}	u_{21}	$\cdots$	u_{k1}
2	u_{12}	u_{22}	$\cdots$	u_{k2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	u_{1n}	u_{2n}	$\cdots$	U_{kn}

Thus, the Mahalonobis distance for the jth run data in MTGS can be rewritten as:

$$MD_j = \mathbf{V}'_j \mathbf{D}^{-1} \mathbf{V}_j$$

$$= \left(\frac{u_{1j}^2}{\sigma_{U_1}^2} + \frac{u_{2j}^2}{\sigma_{U_2}^2} + \cdots + \frac{u_{kj}^2}{\sigma_{U_k}^2} \right), \text{ when } \sigma \text{ are known.}$$

$$= \left(\frac{u_{1j}^2}{S_{U_1}^2} + \frac{u_{2j}^2}{S_{U_2}^2} + \cdots + \frac{u_{kj}^2}{S_{U_k}^2} \right), \text{ when } \sigma \text{ are unknown.} \tag{8}$$

According to Taguchi and Jugulum (2002), one can use $y_j = \sqrt{MD_j}$ as a single response variable. Following the above three steps the multi-response variables can be reduced into a single dimensional performance index, i.e. Mahalonobis distance.

Step 4: *Perform ANOVA and regression analysis.*
If one only considers the main and two-factor interaction effects in an experimental design, then the effect model for a multi-factor experiment is

$$Y_{ij\ldots ml} = \mu + \alpha_i + \beta_j + \cdots + \tau_m + \gamma_l + (\alpha\beta)_{ij} + \cdots + (\tau\gamma)_{ml} + \varepsilon_{ij\ldots ml}, \quad \begin{cases} i = 1, 2, \ldots, k \\ j = 1, 2, \ldots, n \\ \vdots \\ m = 1, 2, \ldots, o \\ l = 1, 2, \ldots, p \end{cases} \tag{9}$$

where μ is the overall mean, α_i is the ith treatment effect of the first factor, β_j is the jth treatment effect of the second factor, $(\alpha\beta)_{ij}$ is the ith and the jth interaction effect of the first and second factors. Similarly, τ_m and γ_l are the mth and lth treatment effects, $(\tau\gamma)_{ml}$ is the mth and lth interaction effect. The three assumptions for the experimental errors in ANOVA are:

(1) Normality, i.e. the error terms ε_i are normally distributed.
(2) Constant variance, i.e. $\text{Var}(\varepsilon_i) = \sigma^2$.
(3) Independence, i.e. the error terms ε_i are mutually independent.

Since we are interested in testing the treatment and interaction effects against zero, the null hypotheses of the treatment and interaction effects can be expressed respectively as:

$$\begin{cases} H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0 \\ H_0 : \beta_1 = \beta_2 = \cdots = \beta_n = 0 \\ \vdots \\ H_0 : \gamma_1 = \gamma_2 = \cdots = \gamma_p = 0 \end{cases}$$

and

$$\begin{cases} H_0 : (\alpha\beta)_{ij} = 0 \quad \text{for all } i, j \\ \vdots \\ H_0 : (\tau\gamma)_{lm} = 0 \quad \text{for all } l, m \end{cases}.$$

The main and interaction effects are considered to be significant if the null hypotheses are rejected. Normally, data analysis procedures for finding the key factors include the following five steps:

(1) Determine the initial key factors by using effect normal probability and Pareto plots.
(2) Confirm the key factors using ANOVA, and highlight the significant main and interaction effects.
(3) Construct a fitted regression model.
(4) Perform ANOVA and residual analyses including checking the three assumptions for errors, i.e. normality, independence and constant variance.
(5) Perform confirmatory tests by using the fitted regression model, and calculate mean absolute percent error (MAPE) by

$$\frac{1}{m} \sum_{i=1}^{m} \frac{|y_i - \hat{y}_i|}{y_i}$$

where y_i are the actual observations and $\hat{y}_i$ are the predicted values. It is assumed that no important factors were omitted from the experiment if the percent contribution due to error is less than 15% (Ross 1988). Thus, a MAPE less than 15% is recommended.

3.2 Searching the optimal combination of key factor levels

After finding the key factors, the multi-response manufacturing process can be optimised using the desirability functions. Assume that

$$B_{ij...ml} = \mu + \alpha_i + \beta_j + \cdots + \tau_m + \gamma_l + (\alpha\beta)_{ij} + \cdots + (\tau\gamma)_{ml}$$

where $B_{ij...ml}$ is a function of $\mu, \alpha_i, \beta_j, \ldots, \tau_m, \gamma_l, (\alpha\beta)_{ij}, \ldots, (\tau\gamma)_{ml}$. The least squares estimators of the parameters in a multi-factor effect model can be found by minimising the sum of squares of the errors from Equation (9), i.e.

$$\min_{\mu, \alpha_i, \beta_j, ..., \tau_m, \gamma_l, (\alpha\beta)_{ij}, ..., (\tau\gamma)_{ml}} \sum_{i=1}^{k} \sum_{j=1}^{n} \cdots \sum_{l}^{p} \varepsilon_{ij...ml}^2$$
$$= \min_{\mu, \alpha_i, \beta_j, ..., \tau_m, \gamma_l, (\alpha\beta)_{ij}, ..., (\tau\gamma)_{ml}} (Y - B)'(Y - B) \qquad (10)$$

The solution to Equation (10) is not unique and depends on the following constraints:

$$\sum_{i=1}^{k} \alpha_i = \sum_{j=1}^{n} \beta_j = \cdots = \sum_{l=1}^{p} \gamma_l = \sum_{i=1}^{k} (\alpha\beta)_{ij} = \sum_{j=1}^{n} (\alpha\beta)_{ij} = \cdots = \sum_{m=1}^{o} (\tau\gamma)_{ml} = \sum_{l=1}^{p} (\tau\gamma)_{ml} = 0$$

Using the above constraints, the least square estimators

$$\hat{\mu}, \hat{\alpha}_i, \hat{\beta}_j, \ldots, \hat{\tau}_m, \hat{\gamma}_l, \hat{(\alpha\beta)}_{ij}, \ldots, \hat{(\tau\gamma)}_{ml}$$

to the normal equations can be found. Hence, the predicted values $Y''_{ij...ml}$ is obtained by plugging the above least square estimators into Equation (9). Once the predicted values for different combinations of key factor levels are determined, the individual desirability function proposed by Derringer and Such (1980) can be

calculated for each of the k responses. The given engineering specification may be either the larger the better, the smaller the better, or the nominal the best. Finally, the overall desirability function $D = \sqrt[k]{d_1 \times d_2 \times \cdots \times d_k}$ for k responses is calculated for each different combination of the key factor levels and an optimal combination of the key factor levels can be determined by maximising the overall desirability function.

4. A comparative analysis using simulation

The Taguchi method using signal-to-noise ratios (S/N) is commonly used in finding the key factors for a multi-response process. However, the key factor levels obtained by Taguchi method may not be an optimal one. To compare the two methods, an example of advertising styles (Sharma 1996) is used for illustration. There are three advertising styles in his study, i.e. the humour, sentiment and contrast-oriented. Since the evaluation of an advertising style is also affected by the gender, 12 males and 12 females are randomly selected and divided into three groups. Each group consists of four males and four females. Under the influence of each advertising style, every group member is asked to give two scores on the categories of reliability and degree of information for evaluating the advertising impact. The given scores were ranging from 1 to 10 and the correlation coefficient between the reliability and degree of information for each advertising style were found in a range from 0.6 to 1.0, i.e. the two response values are highly correlated. Performing a multivariate ANOVA (MNOVA), the main effect of advertising style and the interaction effect of advertising style and gender are significant at a 90% significance level ($\alpha = 0.1$). The observations can be expressed in a matrix form $\mathbf{Y} = \mathbf{XB} + \mathbf{E}$ as shown below, where Y and $\mathbf{E}$ are 24×2 matrices, X is a 24×12 matrix, and $\mathbf{B}$ is a 12×2 coefficient vector of regression.

$$
\begin{bmatrix} \mathbf{y}_{111} & \mathbf{y}_{211} \\ \mathbf{y}_{112} & \mathbf{y}_{212} \\ \mathbf{y}_{113} & \mathbf{y}_{213} \\ \mathbf{y}_{121} & \mathbf{y}_{221} \\ \mathbf{y}_{122} & \mathbf{y}_{222} \\ \mathbf{y}_{123} & \mathbf{y}_{223} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_1 & \mu_2 \\ \alpha_1 & \alpha_2 \\ \boldsymbol{\beta}_1 & \boldsymbol{\beta}_2 \\ \alpha\boldsymbol{\beta}_1 & \alpha\boldsymbol{\beta}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{111} & \boldsymbol{\varepsilon}_{211} \\ \boldsymbol{\varepsilon}_{112} & \boldsymbol{\varepsilon}_{212} \\ \boldsymbol{\varepsilon}_{113} & \boldsymbol{\varepsilon}_{213} \\ \boldsymbol{\varepsilon}_{121} & \boldsymbol{\varepsilon}_{221} \\ \boldsymbol{\varepsilon}_{122} & \boldsymbol{\varepsilon}_{222} \\ \boldsymbol{\varepsilon}_{123} & \boldsymbol{\varepsilon}_{223} \end{bmatrix}
$$

where $\mathbf{y}_{lij} = (y_{lij1}, y_{lij2}, y_{lij3}, y_{lij4})'$, $\boldsymbol{\alpha}_l = (\alpha_{l1}, \alpha_{l2})'$, $\boldsymbol{\beta}_l = (\beta_{l1}, \beta_{l2}, \beta_{l3})'$, $\alpha\boldsymbol{\beta}_l = (\alpha\beta_{l11}, \alpha\beta_{l12}, \alpha\beta_{l13}, \alpha\beta_{l21}, \alpha\beta_{l22}, \alpha\beta_{l23})'$, $\boldsymbol{\varepsilon}_{lij} = (\varepsilon_{lij1}, \varepsilon_{lij2}, \varepsilon_{lij3}, \varepsilon_{lij4})'$, $(\varepsilon_1, \varepsilon_2)'_{ijk} \sim N_2(\mathbf{0}, \boldsymbol{\Sigma})$, $l = 1, 2$, $i = 1, 2$, $j = 1, 2, 3$, $k = 1, 2, 3, 4$, and

$$
\sum = \begin{bmatrix} \sigma^2_{adv} & \text{cov(adv, gender)} \\ \text{cov(adv, gender)} & \sigma^2_{gender} \end{bmatrix}.
$$

Since $\mathbf{B}$ and $\mathbf{E}$ could be estimated from data, 10,000 times of simulation run were repeated using this model $f(\mathbf{Y}|\hat{\mathbf{B}}, \hat{\mathbf{E}})$. We then performed the key factors analysis based on the MTGS and the Taguchi (S/N) methods. The simulation results are summarised in Table 3. It shows that percent accuracies for advertising style and gender using the

Table 3. Comparison of the simulation results for MTGS and Taguchi (S/N) methods.

Factors	MTGS method		Taguchi (S/N) method	
	Times*	Percent accuracy	Times*	Percent accuracy
Advertising style	8916	89.16%	3080	30.80%
Gender	597	94.03%	2119	78.81%
		(100%–5.97%)		(100%–21.19%)
Interaction	9123	91.23%	9266	92.66%

Note: *Number of times a factor has been classified to be significant at a 90% confidence level. Percent accuracy for the gender = 1−false alarm rate since it is not a significant factor by MANOVA.

MTGS method are equal to 89.16% and 94.03% respectively, which are more accurate than using the Taguchi method (30.8% and 78.81%). Note that both the MTGS and Taguchi methods can achieve more than 90% accuracy for the interaction effect. Table 3 shows that the MTGS method is more accurate than the Taguchi method in finding the key factors. Therefore, the MTGS method is suggested for finding the key factors for a multi-response process.

5. A numerical example of the solder paste stencil printing process

In this section, we perform the statistical analysis of solder paste stencil printing data (Pan *et al.* 2004) using an integrated MTGS and desirability function approach.

5.1 *Introduction to the solder paste stencil printing process*

The experiment conducted by Pan *et al.* (2004) has four quality characteristics, i.e. the solder paste deposited volume, area, height, and transfer ratio. Six input variables/ factors are:

- stencil thickness,
- solder paste type,
- board finish,
- print speed,
- aperture size, and
- aperture shape.

Aperture size and shape are considered to be the blocks. The specifications for the experimental design are listed in Table 4. There are $2^4 \times 5 \times 4 = 320$ levels of combination in this experimental design. The experimental data contained 21,106 observations. For the sake of convenience 'aperture size', 'stencil thickness', 'aperture shape', 'board finish', 'solder paste type', 'print speed', and 'transfer ratio' are abbreviated thus: Pitch, ST, AS, BF, SPT, PS and TR respectively.

Pan *et al.* (2004) found the key factors affecting solder deposited volume are aperture size (pitch), stencil thickness (ST), aperture shape (AS), board finish (BF), print speed (PS); Pitch*ST, pitch*BF, ST*AS, ST*SPT, ST*PS, AS*PS, BE*SPT, SPT*PS.

Table 4. Input variables and their levels for a QFP solder paste stencil printing process.

Stencil thickness	Thickness (mil)	4			6	
	Level	1			2	
Solder paste type	Type	Type 3			Type 4	
	Level	1			2	
Board finish	Method	Immersion Ag			HASL	
	Level	1			2	
Print speed	Speed (in/sec)	2			8	
	Level	1			2	
Aperture size	Pitch (mil)	30	25	20	16	12
	Level	1	2	3	4	5
Aperture shape (see Figure 3)	Shape	BSQFP	SQFP	QFP1	QFP2	
	Level	1	2	3	4	

Note: QFP1 represents vertical rectangle and QFP2 represents horizontal rectangle.
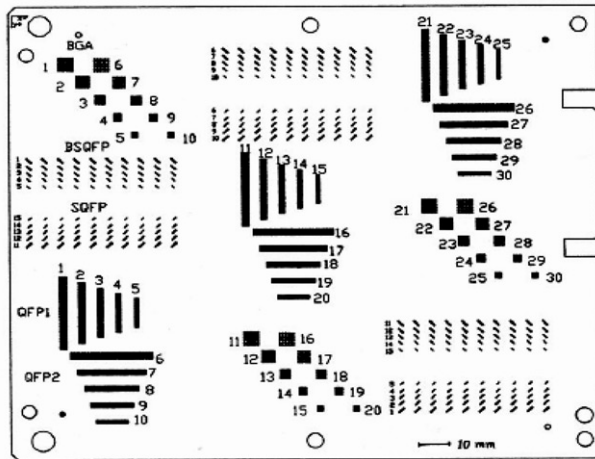

Figure 3. Test pattern.

5.2 The key factor analysis for the solder paste stencil process

According to the key factors analysis procedure illustrated in Figure 2, the original multi-response data were first standardised. Then, the GSP was employed to obtain an orthogonal basis for the standardised data, which is used for calculating the Mahalonobis distance of k responses for each run. Finally, $y_j = \sqrt{MD_j}$ was used as a performance index. It was found that the residuals did not follow a normal distribution. After a Box-Cox transformation with $\lambda = 0.15$, the normal assumption is tested to be valid. The ANOVA results for the transformed data are summarised in Table 5. The main effect plot and the two-factor interaction effect plot are shown in Figures 4 and 5, respectively.

Both Figure 4 and Table 5 show that the main factors affecting the QFP manufacturing process are aperture size (Pitch), board finish (BF), solder paste type (SPT). Both Figure 5

Table 5. ANOVA for QFP data after Box-Cox transformation.

Source	d.f.	Seq. SS	Adj. SS	Adj. MS	F	P
Pitch	4	24.03401	24.03401	6.00850	149.53	0.000
ST	1	0.06130	0.06130	0.06130	1.53	0.218
AS	3	0.17222	0.17222	0.05741	1.43	0.235
BF	1	1.11963	1.11963	1.11963	27.86	0.000
SPT	1	0.38961	0.38961	0.38961	9.70	0.002
PS	1	0.02913	0.02913	0.02913	0.73	0.395
Pitch*ST	4	0.44115	0.44115	0.11029	2.74	0.029
Pitch*AS	12	0.73673	0.73673	0.06139	1.53	0.114
Pitch*BF	4	0.41793	0.41793	0.10448	2.60	0.037
Pitch*SPT	4	0.24493	0.24493	0.06123	1.52	0.196
Pitch*PS	4	0.47021	0.47021	0.11755	2.93	0.022
ST*AS	3	5.78590	5.78590	1.92863	48.00	0.000
ST*BF	1	0.85804	0.85804	0.85804	21.35	0.000
ST*SPT	1	0.01328	0.01328	0.01328	0.33	0.566
ST*PS	1	0.08762	0.08762	0.08762	2.18	0.141
AS*BF	3	0.42955	0.42955	0.14318	3.56	0.015
AS*SPT	3	0.23467	0.23467	0.07822	1.95	0.122
AS*PS	3	0.42388	0.42388	0.14129	3.52	0.016
BF*SPT	1	0.41331	0.41331	0.41331	10.29	0.002
BF*PS	1	0.11319	0.11319	0.11319	2.82	0.094
SPT*PS	1	0.01422	0.01422	0.01422	0.35	0.552
Error	262	10.52805	10.52805	0.04018		
Total	319	47.01856				

Note: The bold, underlined abbreviation represents significant at $\alpha = 0.05$.
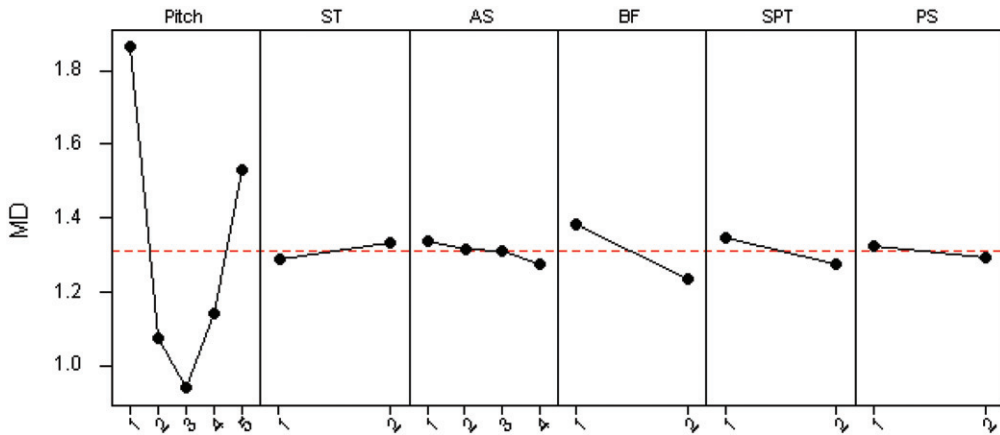

Figure 4. The main effect plots for QFP data after Box-Cox transformation.

and Table 5 show that the significant two-factor interactions are aperture size and stencil thickness (Pitch*ST), aperture size and board finish (Pitch*BF), aperture size and print speed (Pitch*PS), stencil thickness and aperture shape (ST*AS), stencil thickness and board finish (ST*BF), aperture shape and board finish (AS*BF), aperture shape and print speed (AS*PS), and board finish and solder paste type (BF*SPT). Adding three

factors ST, AS, PS, into the above 11 significant factors, a fitted regression equation can be constructed as below:

$$Y_t = \mu + \sum_{i=1}^{4} \alpha_i \, \text{pitch}_i + \beta \times \text{ST} + \sum_{k=1}^{3} \gamma_k \text{AS}_k + \tau \times \text{BF} + \lambda \times \text{SPT} + \nu \times \text{PS}$$

$$+ \sum_{i=1}^{4} (\alpha\beta)_i \, \text{pitch}_i \times \text{ST} + \sum_{i=1}^{4} (\alpha\tau)_i \, \text{pitch}_i \times \text{BF} + \sum_{i=1}^{4} (\alpha\nu)_i \, \text{pitch}_i \times \text{PS}$$

$$+ \sum_{k=1}^{3} (\beta\gamma)_k \text{ST} \times \text{AS}_k + (\beta\tau)\text{ST} \times \text{BF} + \sum_{k=1}^{3} (\gamma\tau)_k \text{AS}_k \times \text{BF}$$

$$+ \sum_{k=1}^{3} (\gamma\nu)_k \text{AS}_k \times \text{PS} + (\tau\lambda)\text{BF} \times \text{SPT} + \varepsilon_t,$$

$$t = 1, 2, \ldots, 320 \tag{11}$$

where $\mu, \alpha_i, \beta, \gamma_k, \tau, \lambda, \nu, (\alpha\nu)_i, (\beta\gamma)_k, (\beta\tau), (\gamma\tau)_k, (\gamma\nu)_k,$ and $(\tau\lambda)$, $i = 1, 2, 3, 4$, $k = 1, 2, 3$ are coefficients of the fitted regression model. Table 6 shows that both the main and interaction effects of the regression model are significant. If one wants to estimate the response value for the combination of factors at Pitch $= 2$, ST $= 2$, AS $= 2$, BF $= 1$, SPT $= 1$, PS $= 2$ level, then the predicted value can be obtained from

$$\hat{Y} = \hat{\mu} + \hat{\alpha}_1 + \hat{\beta} + \hat{\gamma}_1 + \hat{\nu} + \widehat{(\alpha\beta)}_1 + \widehat{(\alpha\nu)}_1 + \left(\widehat{\beta\gamma}\right)_1 + \widehat{(\gamma\nu)}_1$$
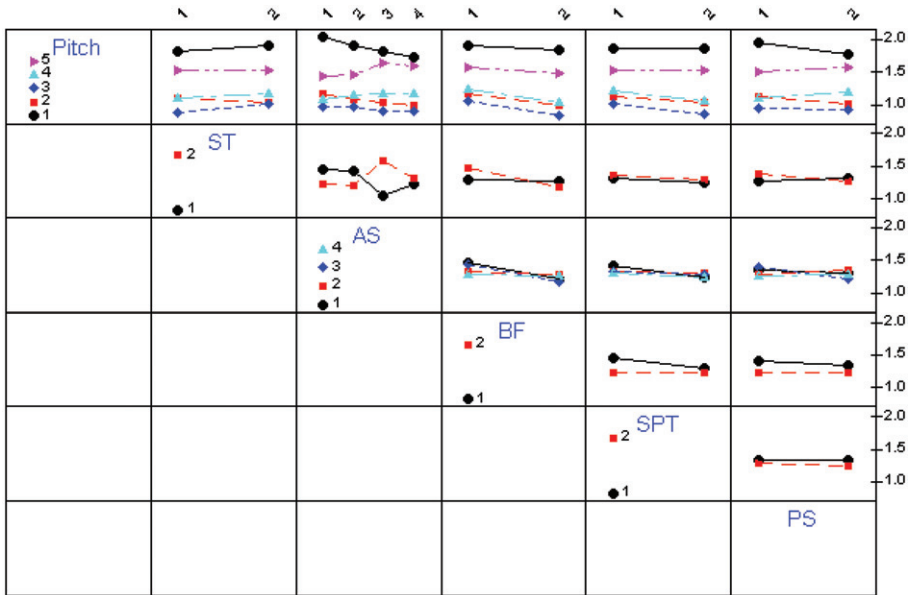
based on Equation (11).

Figure 5.   The 2-factor interaction effect plots for QFP data after Box-Cox transformation.

After checking the validity of the three assumptions of error terms for QFP data after a Box-Cox transformation, the percent contribution for each factor is calculated by:

$$\rho_{factor} = \frac{SS'_{factor}}{SST} \times 100\% = \frac{SS_{factor} - df_{factor} \times MSE_{pooled}}{SST} \times 100\%$$

where SS'_{factor} is the pure sum of squares for factors, SST is the total sum of squares, SS_{factor} is the sum of squares for factors, df_{factor} is a degree of freedom (d.f.) for factor, MSE_{pooled} is the mean of squares for pooled errors. The significant factors are sorted by the percent contribution and their results are shown in Table 7.

Table 7 indicates that the contributing factors of the solder deposited volume and the transfer ratio for a QFP manufacturing process are aperture size (Pitch) and the interaction of stencil thickness and aperture shape (ST*AS).

5.3 *Using desirability function to find the optimal level of key factors*

To find the optimal combination of key factor levels, we need to specify the quality characteristics target values. In the solder paste stencil printing process, the target values of solder paste deposited volume, area, and height are the volume, area, and height of stencil apertures. The target value of transfer ratio is 1 or 100%. All the four quality

Table 6. The result of ANOVA.

Source	d.f.	Adj. MS	F	P
Main effect	11	2.345991	58.387031	0.000
Interaction effect	46	0.232274	5.780839	0.000
Error	262	0.040180		
Total	319			

Table 7. The percent contribution of the key factors after a Box-Cox transformation.

Source	d.f.	Seq. SS	Adj. MS	Percent contribution (%)
Pitch	4	24.0340	6.0085	50.76
ST*AS	3	5.7859	1.9286	12.04
BF	1	1.1196	1.1196	2.29
ST*BF	1	0.8580	0.8580	1.74
BF*SPT	1	0.4133	0.4133	0.79
SPT	1	0.3896	0.3896	0.74
AS*BF	3	0.4295	0.1432	0.64
Pitch*PS	4	0.4702	0.1176	0.64
AS*PS	3	0.4239	0.1413	0.63
Pitch*ST	4	0.4412	0.1103	0.58
Pitch*BF	4	0.4179	0.1045	0.53
Pooled Error	290	12.2353	0.0422	
Total	319	47.0186		

characteristics belong to the nominal-the-best type. The specification limits are required for setting up the desirability function proposed by Derringer and Such (1980). In this example, we use five-sigma specification limits for the volume, the area, and the height of solder paste deposited and the data are listed in Table 8. The specification limit for the transfer ratio is set to [0.8, 1.2] as it is commonly used in the industry. By plugging different key factor levels into the fitted regression Equation (11), one can obtain the predicted values for solder paste deposited volume, area, height, and TR for each combination of key factor levels. Let t and s in Equation (3) equal to 1, the individual desirability functions (d_1, d_2, d_3, d_4) of four responses (i.e. solder paste deposited volume, area, height, and TR) can be calculated for different combinations of key factor levels. The overall desirability function $D = (d_1 \times d_2 \times d_3 \times d_4)^{1/4}$ can be achieved by locating the maximum overall desirability function for all the combinations of key factor levels. The optimum combinations of key factor levels for different stencil thicknesses and aperture sizes are shown in Table 9. Note that the levels of each input variables are listed in Table 4. It should be pointed out that the effect of ST and AS cannot be ignored in the optimisation stage although Table 5 shows that ST and AS are not significant main factor. The reason is that their interaction ST*AS and ST*BF, AS*PS, AS*BF are significant as shown in Table 5.

Table 8. The specification limits of quality characteristics under different combinations of stencil thicknesses and aperture sizes.

Pitch (mil)	ST (mil)	Volume (mm^3)		Area (mm^2)		Height (mm)	
		LSL	USL	LSL	USL	LSL	USL
30	4	0.039	0.119	0.476	1.104	0.060	0.140
	6	0.070	0.165	0.513	1.047	0.103	0.097
25	4	0.011	0.099	0.252	0.847	0.048	0.152
	6	0.043	0.123	0.312	0.788	0.090	0.210
20	4	0.009	0.069	0.112	0.668	0.052	0.148
	6	0.024	0.094	0.162	0.618	0.089	0.211
16	4	0.004	0.048	0.057	0.463	0.049	0.151
	6	0.013	0.065	0.000	0.555	0.101	0.199
12	4	0.000	0.038	0.000	0.339	0.000	0.210
	6	0.005	0.429	0.000	0.444	0.095	0.205

Table 9. The optimal combinations of key factor levels under different combinations of stencil thicknesses and aperture sizes.

Pitch	ST (4 mil)				ST (6 mil)			
	AS	BF	SPT	PS	AS	BF	SPT	PS
30	4	1	1	2	3	1	2	2
25	4	1	1	2	3	1	2	2
20	4	1	1	2	3	1	2	2
16	4	2	2	2	3	1	2	1
12	2	1	1	2	4	2	1	1

The equation

$$\frac{1}{m}\sum_{i=1}^{m}\frac{|y_i - \hat{y}_i|}{y_i} < 0.15$$

is used for calculating the mean absolute percent error (MAPE) after confirmatory tests, where y_i are actual observations and $\hat{y}_i$ are predicted values. Normally, a MAPE less than 15% is preferred to ensure that the accuracy of key factors found is greater than 85%. The results in Table 10 show that all the MAPE's of solder paste deposited volume, area, height and transfer ratio (TR) are smaller than 0.15, which indicate that key factors for the QFP manufacturing process are confirmed.

6. Discussions

Most multivariate analysis methods are based on the assumption that multi-responses are mutually independent and follow multi-normal distribution. However, the four responses in our stencil printing process are mutually dependent. To relieve the assumptions of multi-normality and mutual independence among responses, an integrated approach of

Table 10.   Summary of the confirmatory tests for the optimal combinations of key factor levels.

Level of the factors						$\hat{y}_i\left(\dfrac{|y_i - \hat{y}_i|}{y_i}\right)$			
Pitch	ST	AS	BF	SPT	PS	Volume	Area	Height	TR
5	1	4	1	1	2	0.005	0.202	0.059	0.853
						(0.084)	(0.126)	(0.011)	(0.106)
4	1	4	1	1	2	0.019	0.325	0.076	0.997
						(0.127)	(0.016)	(0.097)	(0.113)
3	1	4	1	1	2	0.034	0.459	0.081	1.016
						(0.154)	(0.019)	(0.152)	(0.076)
2	1	4	2	2	2	0.055	0.649	0.083	0.906
						(0.092)	(0.029)	(0.049)	(0.003)
1	1	2	1	1	2	0.083	0.893	0.086	1.051
						(0.148)	(0.091)	(0.118)	(0.144)
5	2	3	1	2	2	0.025	0.122	0.110	0.729
						(0.091)	(0.033)	(0.042)	(0.069)
4	2	3	1	2	2	0.039	0.245	0.126	0.873
						(0.002)	(0.023)	(0.004)	(0.043)
3	2	3	1	2	2	0.054	0.379	0.131	0.893
						(0.037)	(0.010)	(0.047)	(0.007)
2	2	3	1	2	1	0.072	0.569	0.128	0.932
						(0.122)	(0.035)	(0.094)	(0.061)
1	2	4	2	1	1	0.093	0.786	0.127	0.832
						(0.120)	(0.043)	(0.047)	(0.120)
MAPE $\dfrac{1}{10}\sum_{i=1}^{10}\dfrac{|y_i - \hat{y}_i|}{y_i}$						0.0977	0.0425	0.0661	0.0742

Note: y_i are the actual observations, $\hat{y}_i$ are the predicted values, and the values in parentheses represent a MAPE.

Table 11. ANOVA results using the Taguchi S/N method.

Source	d.f.	Seq. SS	Adj. SS	Adj. MS	F	P
Pitch	4	292,989.5	292,989.5	73,247.4	164.10	0.000
ST	1	36.1	36.1	36.1	0.08	0.776
AS	3	19,579.1	19,579.1	6526.4	14.62	0.000
BF	1	6.0	6.0	6	0.01	0.908
SPT	1	4402.8	4402.8	4402.8	9.86	0.002
PS	1	90.7	90.7	90.7	0.20	0.653
Pitch*ST	4	93,320.0	93320	23330	52.27	0.000
Pitch*AS	12	63,296.1	63,296.1	5274.7	11.82	0.000
Pitch*BF	4	813.4	813.4	203.3	0.46	0.768
Pitch*SPT	4	2443.1	2443.1	610.8	1.37	0.245
Pitch*PS	4	767.9	767.9	192	0.43	0.787
ST*AS	3	15711	15711	5237	11.73	0.000
ST*BF	1	1714.3	1714.3	1714.3	3.84	0.051
ST*SPT	1	758.3	758.3	758.3	1.70	0.194
ST*PS	1	627.1	627.1	627.1	1.41	0.237
AS*BF	3	7041.7	7041.7	2347.2	5.26	0.002
AS*SPT	3	8382.5	8382.5	2794.2	6.26	0.000
AS*PS	3	5247.4	5247.4	1749.1	3.92	0.009
BF*SPT	1	1304.2	1304.2	1304.2	2.92	0.089
BF*PS	1	202.9	202.9	202.9	0.45	0.501
SPT*PS	1	3446.9	3446.9	3446.9	7.72	0.006
Error	262	11,6945.6	116,945.6	446.4		
Total	319	63,9126.5				

using the desirability function in conjunction with the MTGS method is proposed to find and optimise the key factors.

We performed another analysis for the same example using the Taguchi S/N (signal-to-noise) method. Note that the four responses or quality characteristics in our stencil printing process, i.e. the solder paste deposited volume, area, height and transfer ratio belong to 'the nominal-the-best' type. Thus, one can apply the following formula of Taguchi's S/N ratio for 'the nominal-the-best' type:

$$S/N = 10^* \log_{10}\left(\frac{\bar{Y}^2}{S^2}\right),$$

where $\bar{Y} = \sum y_i/n$, $S^2 = \sum (y_i - \bar{Y})^2/(n-1)$, y_i is the quality characteristic. After transforming the four response values into S/N ratios under each combination of factor levels, the sum of S/N ratios for the four quality characteristics becomes the response value in our ANOVA using the Taguchi method. The ANOVA results using the Taguchi method are shown in Table 11. The ANOVA results from the MTGS method for main factors are compared with the ANOVA results from the Taguchi S/N method and the comparisons are listed in Table 12. It shows that the conclusions drawn from both methods are similar except for AS and BF. Further study is needed to test which method is better in search of the key factors. It should be pointed out that the optimal combinations of key factor levels described in Section 5.3 have not been validated by experiments.

Table 12. Comparison of the ANOVA results from the MTGS method and the Taguchi method.

Source	Using the MTGS method	Using the Taguchi S/N method
Pitch	Significant	Significant
ST	Not significant	Not significant
AS	Not significant	Significant
BF	Significant	Not significant
SPT	Significant	Significant
PS	Not significant	Not significant

Note: Significant at 95% confidence level.

7. Summary

A MTGS method was presented to find and optimise the key factors for a multiple-response manufacturing process. Detailed procedures of finding and optimising the key factors were described and illustrated with the example of a solder paste stencil printing process. The advantage of this method is that it does not require the assumption of multi-normality and mutual independence among multiple responses.

A comparison of ANOVA results from the MTGS method and the Taguchi S/N method was made. The conclusions drawn from both methods are similar though there are some inconsistencies. Further study is needed to explore both the benefits and limitations of the MTGS method.

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