



# Stagnation point flow and heat transfer behavior of Cu–water nanofluid towards horizontal and exponentially stretching/shrinking cylinders

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**Abstract** In this study we analyzed the stagnation point flow and heat transfer behavior of Cu–water nanofluid towards horizontal and exponentially permeable stretching/shrinking cylinders in presence of suction/injection, heat source and shape of nanoparticles. The governing boundary layer equations are transformed to nonlinear ordinary differential equations using similarity transformation which are then solved numerically using *bvp4c* Matlab package. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables. The study indicates that the solutions for the horizontal and exponential cylinders are non-unique and shape of nanoparticles also influences the rate of heat transfer. Comparisons of the present results with existed studies are presented. Present study has an excellent agreement with the existed studies under some special conditions.

**Keywords** Stagnation point flow · Stretching/shrinking · Suction/injection · Heat source · Nanofluid

## Introduction

Nanofluids are the suspension of metallic, nonmetallic or polymeric nano-sized powders in base liquid which are employed to increase the heat transfer rate in various applications. The study of stagnation point flow has various applications like transpiration, oil recovery, nuclear

reactors and production. The revolution of stagnation point flow has started by Massoudi and Rameza (1990) and analyzed the heat transfer characteristics of a boundary layer flow of viscoelastic fluid towards a stagnation point. Sami Akoz and Salih Kirkzgoz (2009) presented the numerical and experimental analysis of the flow around a horizontal wall-mounted circular cylinder. Singh and Mittal (2005) discussed the flow over a circular cylinder experimentally. Experimental and numerical investigation of the separation angle for flow around a circular cylinder at low Reynolds number was analyzed by Wu et al. (2004). Monalisa and Kumar (2014) illustrated the experimental investigation of flow past a rough-surfaced cylinder.

The stagnation point flow of two-dimensional steady nanofluid flow in the presence of stretching or shrinking sheet was analyzed by Bachok et al. (2011). Najib et al. (2014) discussed the stagnation point flow over a stretching or shrinking cylinder with chemical reaction effect. In this study they highlighted the solutions for shrinking cylinder. Ali et al. (2014) studied heat transfer effects of unsteady stagnation flow over a shrinking surface in presence of radiation. Abel and Nandeppanavar (2009) analyzed the MHD and non-uniform heat source or sink effects of viscoelastic fluid flow existence of linear stretching sheet. Ishak et al. (2011) illustrated MHD stagnation point flow in a stretching sheet by considering heat flux. Unsteady stagnation point flow of boundary layer flow past a stretching or shrinking surface was illustrated by Bhat-tacharyya (2013). He concluded that hike in the value of unsteadiness parameter enhances the heat transfer rate. Nazar et al. (2004) discussed the behavior of steady-state stagnation point flow of a micropolar fluid over a stretching sheet. Stagnation point flow of mixed convective and radiated nanofluid past a stretching or shrinking surface with viscous dissipation and heat generation in a porous medium

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was discussed numerically by Pal and Mandal (2015). The researchers Julie et al. (2007) and Ferrouillat et al. (2011) discussed the influence of particle shape on drug delivery and convective heat transfer cases.

Heat transfer characteristics of MHD viscoelastic fluid flow over nonlinear stretching sheet in the presence of radiation and heat generation/absorption was presented by Rafael Cortell (2014). Rana and Bhargava (2012) presented finite element and finite difference methods for nonlinear stretching sheet problem. Zaimi et al. (2014) extended the work of Rana and Bhargava and they studied the heat transfer and steady boundary layer flow of a nanofluid over a stretching/shrinking sheet. Mishra and Singh (2014) investigated axe-symmetric flow of a viscous incompressible fluid flow over a permeable shrinking cylinder. Convective boundary layer flow of nanofluid past a linear stretching sheet was studied numerically by Makinde and Aziz (2011). Mohankrishna et al. (2014) discussed unsteady natural convective flow of a nanofluid over a vertical flat plate by considering heat source. A boundary layer analysis of non-Newtonian and mixed convective flow of nanofluid past a nonlinearly stretching surface was discussed by Subbareddy and Mahesh (2012). Makinde et al. (2013) discussed buoyancy effects of MHD stagnation point flow and heat transfer of a nanofluid over stretching/shrinking surface. Mustafaa et al. (2013) analyzed the heat transfer characteristics of a nanofluid past an exponentially stretching sheet with convective boundary conditions. An unsteady stretching sheet analysis of a nanofluid flow with convective boundary conditions was investigated by Makinde and Oluwole (2013), they concluded that copper-water has high heat transfer rate at the surface.

A homotopy analysis of nanofluid flow past a non-linear isothermal permeable stretching surface with transpiration was discussed by Rashidi et al. (2014). Vajravelu et al. (2011) discussed heat transfer analysis of convective nanofluid through a stretching surface with Ag–water and Cu–water. They concluded that increase in volume fraction of nanoparticles depreciates the velocity and thermal boundary layers. Sandeep et al. (2013) discussed influence of radiation on unsteady convective flow of a nanofluid over a vertical plate. Mansur and Ishak (2013) analyzed the flow and heat transfer characteristics of a nanofluid over a stretching/shrinking sheet. The enhanced thermal conductivity of nanofluid was discussed by Singh (2008). Khan and Pop (2010) discussed the boundary layer flow of a nanofluid over stretching surface. Abu-Nada and Chamkha (2010) studied CuO–EG–water nanofluid flow over a stretching surface with variable flow properties. Very recently Sandeep and Sulochana (2015) presented dual solutions for MHD nanofluid flow over exponentially stretching sheet by considering suction and injection effects.

Majority of the above studies focused on flow through either stretching or shrinking surfaces. In this study we are investigating the stagnation point flow and heat transfer behavior of Cu–water nanofluid at different shapes of nanoparticles towards horizontal and exponentially permeable stretching/shrinking cylinders with suction/injection in presence of heat source. The governing boundary layer equations are transformed to nonlinear ordinary differential equations using similarity transformation which are then solved numerically. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables.

## Mathematical formulation

Consider a steady stagnation point flow (Fig. 1) and heat transfer of Cu–water nanofluid towards horizontal and exponential stretching/shrinking cylinders with radius  $R$  placed in an incompressible viscous nanofluid of constant temperature  $T_w$ . It is assumed that the free stream and stretching/shrinking velocities for horizontal and exponential cylinders are, respectively,  $u_e = ax/L$ ,  $u_w = cx/L$  and  $u_e = ae^{Nx/L}$ ,  $u_w = ce^{Nx/L}$ , where  $a, c$  are constants,  $L$  is the characteristics length and  $N$  is the exponential parameter. A uniform heat source  $Q$  is considered in this study.

The boundary layer equations as per above assumptions are given by

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \quad (1)$$

$$\rho_{nf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = \mu_{nf} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + u_e \frac{\partial u_e}{\partial x}, \quad (2)$$

$$(\rho c_p)_{nf} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right) = k_{nf} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + Q(T - T_\infty), \quad (3)$$

where  $r$  is the coordinate measured in the radial direction,  $u$  and  $v$  are the velocity components in the  $x$

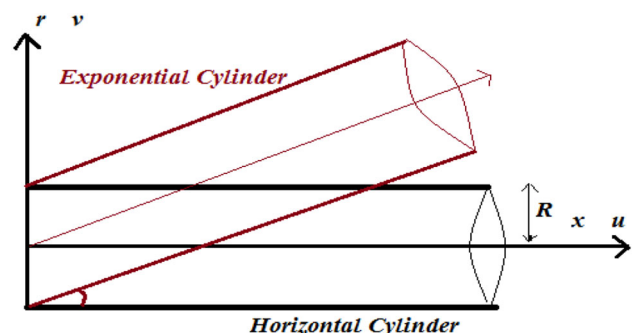


Fig. 1 Physical model of the problem

and  $r$  directions, respectively. Further,  $T$  is the temperature in the boundary layer,  $\rho_{nf}$  and  $\mu_{nf}$  are the density and the dynamic viscosity of the nanofluid, respectively,  $T_\infty$  is the free stream temperature,  $(\rho c_p)_{nf}$  is the heat capacitance of nano-fluid,  $k_{nf}$  is the effective thermal conductivity of nanofluid,  $Q$  is the heat source parameter, here  $Q = Q_0$  for horizontal cylinder and  $Q = Q_0 e^{Nx/L}$  for exponential cylinder.

Boundary conditions for horizontal cylinder  $u = u_w$   $v = v_w$ ,  $T = T_w$ , at  $r = R$   
 $u \rightarrow u_e$ ,  $T \rightarrow T_\infty$  as  $r \rightarrow \infty$  (4)

Boundary conditions for exponential cylinder  $u = u_w$ ,  $v = v_w$ ,  $T = T_w = T_\infty + T_0 e^{Nx/2L}$  at  $r = R$   
 $u \rightarrow u_e$ ,  $T \rightarrow T_\infty$  as  $r \rightarrow \infty$  (5)

where  $v_w$  is the suction ( $v_w < 0$ ) or injection ( $v_w > 0$ ) velocity.

The nanofluid constants are given by

$$\left. \begin{aligned} \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s, (\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s, \\ \frac{k_{nf}}{k_f} &= \left( \frac{k_s + (n - 1)k_f - \phi(n - 1)(k_f - k_s)}{k_s + (n - 1)k_f + \phi(k_f - k_s)} \right), \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \end{aligned} \right\} \quad (6)$$

where  $\phi$  is the volume fraction of the nanoparticles,  $n$  is the nanoparticle shape,  $n = 3/2$  for cylindrical-shaped nanoparticles and  $n = 3$  for spherical-shaped nanoparticles (Hamilton and Crosser 1962). The subscripts  $f$  and  $s$  refer to fluid and solid properties, respectively.

For getting the similarity solutions of Eqs. (1)–(3) with respect to the boundary conditions (4), we are setting the following similarity transformation

$$\eta = \frac{r^2 - R^2}{2R} \sqrt{\frac{a}{v_f L}}, \psi = \sqrt{\frac{v_f a}{L}} x R f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (7)$$

Similarly, for obtaining the similarity solutions of Eqs. (1)–(3) with respect to the boundary conditions (5), we are setting the following similarity transformation

$$\eta = \frac{r^2 - R^2}{2R} \sqrt{\frac{a}{2v_f L}} e^{Nx/2L}, \psi = \sqrt{2v_f L a} R f(\eta) e^{Nx/2L}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (8)$$

where  $\eta$  is the similarity variable,  $\psi$  is the stream function defined as  $u = r^{-1} \partial \psi / \partial r$  and  $v = -r^{-1} \partial \psi / \partial x$ , which identically satisfied the continuity Eq. (1). By defining  $\eta$  in this form, the boundary conditions at  $r = R$  reduce to the boundary conditions at  $\eta = 0$  which is more convenient for numerical computations.

Substituting (7) into Eqs. (2) and (3) we get the following nonlinear ordinary differential equations for horizontal cylinder

$$\frac{1}{(1 - \phi)^{2.5}} [(1 + 2\eta K) f''' + 2K f''] + \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) (ff'' - f'^2) + 1 = 0, \quad (9)$$

$$\frac{1}{\text{Pr} \left( 1 - \phi + \phi \left( \frac{\rho c_p}{\rho c_p} \right) \right)} [(1 + 2\eta K) \theta'' + 2K \theta'] + (f\theta' - f'\theta) + Q_H \theta = 0, \quad (10)$$

Substituting (8) into Eqs. (2) and (3) we get the following nonlinear ordinary differential equations for exponential cylinder

$$\frac{1}{(1 - \phi)^{2.5}} [(1 + 2\eta K) f''' + 2K f''] + N \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) \times (ff'' - 2f'^2) + 2N = 0, \quad (11)$$

$$\frac{1}{\text{Pr} \left( 1 - \phi + \phi \left( \frac{\rho c_p}{\rho c_p} \right) \right)} [(1 + 2\eta K) \theta'' + 2K \theta'] + N(f\theta' - f'\theta) + Q_H \theta = 0, \quad (12)$$

Subject to the boundary conditions are:

$$\left. \begin{aligned} f(0) &= S, f'(0) = \lambda, \theta(0) = 1, \\ f'(\infty) &\rightarrow 1, \theta(\infty) \rightarrow 0, \end{aligned} \right\} \quad (13)$$

where  $K$  is the curvature parameter, Pr is the Prandtl number,  $Q_H$  is the heat source parameter,  $S$  is the suction/injection parameter, here  $S > 0$  for suction and  $S < 0$  for injection and  $\lambda$  is the stretching/shrinking parameter, here  $\lambda > 0$  for stretching and  $\lambda < 0$  for shrinking, these are given by

$$\left. \begin{aligned} Q_H &= Q_0 L / a (\rho c_p)_f, K = (v_f L / a R^2)^{1/2}, \\ \text{Pr} &= v_f / \alpha_f, S = -v_w (2v_f / u_w)^{1/2}, \lambda = c/a, \end{aligned} \right\} \quad (14)$$

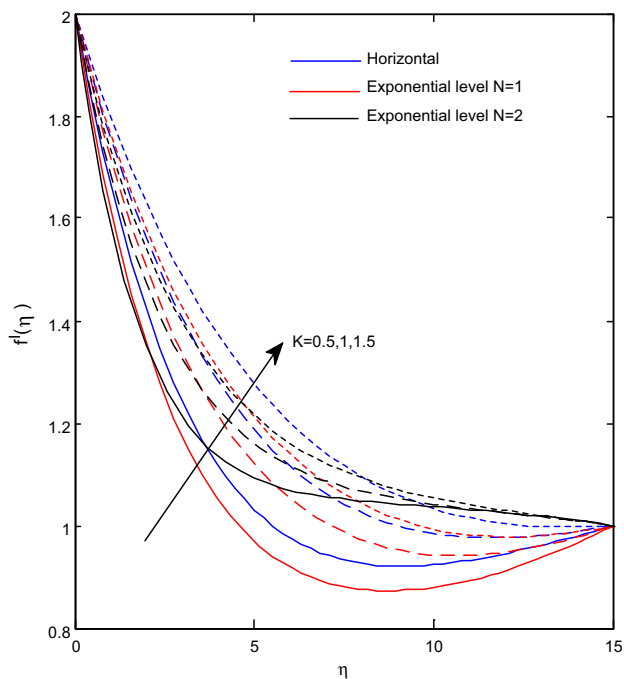
The main physical quantities are interest of  $f''(0)$  being a measure of the skin friction and the temperature gradient  $-\theta'(0)$ . Our aim is to find how the values of  $f''(0)$  and  $-\theta'(0)$  vary with the non-dimensional governing parameters for horizontal and exponential cylinders.

## Results and discussion

Equations (9)–(12), subject to the boundary conditions (13) are solved numerically. For numerical results we considered Pr = 6.2,  $\eta = 50$ ,  $K = 0.5$ ,  $Q_H = 1$ ,  $\lambda = 2$ ,  $n = 3$  and

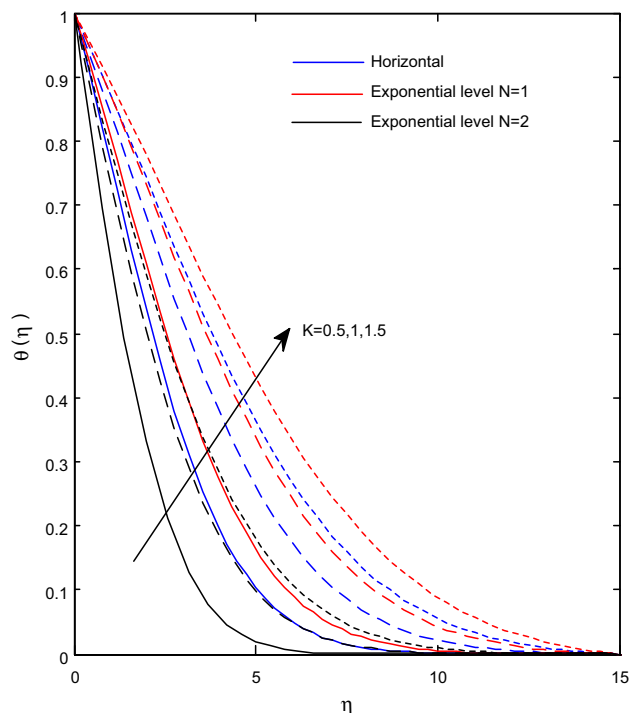
**Table 1** Thermophysical properties of water and Cu nanoparticles

	$\rho$ (kg m <sup>-3</sup> )	$c_p$ (J kg <sup>-1</sup> K <sup>-1</sup> )	$k$ (W m <sup>-1</sup> K <sup>-1</sup> )
H <sub>2</sub> O	997.1	4179	0.613
Cu	8933	385	401

**Fig. 2** Velocity profiles for different values of curvature parameter  $K$ 

$S = 1$ . These values are kept as common in entire study except the varied values as displayed in respective figures and tables. Results show the influence of non-dimensional governing parameters like curvature parameter  $K$ , heat source parameter  $Q_H$ , suction/injection parameter  $S$ , nanoparticles volume fraction  $\phi$ , shape of the nanoparticles  $n$  and stretching/shrinking parameter  $\lambda$  on velocity and temperature profiles along with skin friction coefficient and Nusselt number. In this study we are highlighting the exponential parameter  $N$ . That is an increase in exponential parameter depreciates the characteristic length  $L$  of the cylinder. Here  $L$  is the volume of the body divided by surface area. In mathematical analysis till now, no studies have been focused on this. Table 1 depicts the thermophysical properties of the base fluid (water) and the copper nanoparticles.

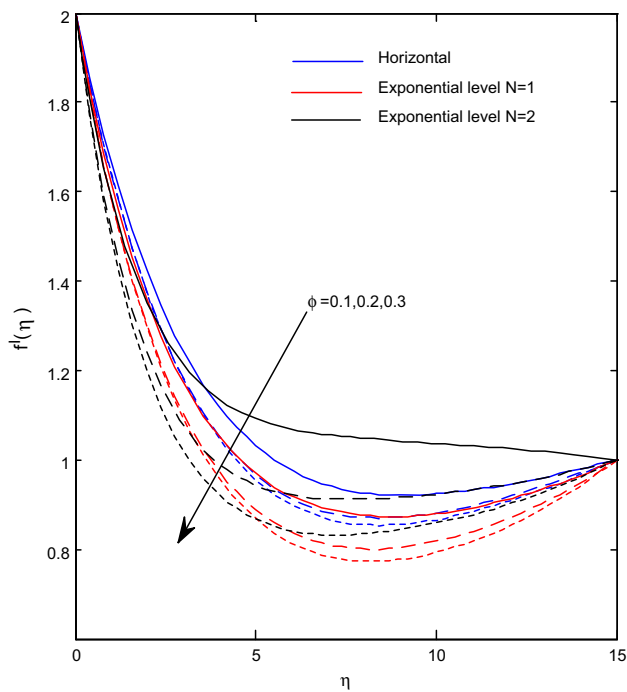
Figures 2 and 3 illustrate the influence of curvature parameter on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking cylinders. It is clear from figures that increases in curvature parameter enhance the velocity as well as temperature profiles. Generally increase in  $K$  enhances the radius of the

**Fig. 3** Temperature profiles for different values of curvature parameter  $K$ 

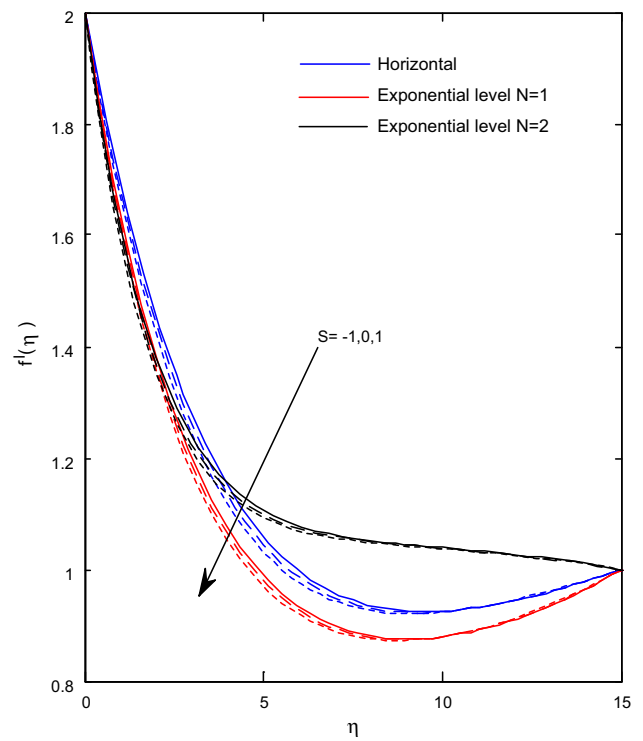
cylinder, this helps to reduce the contact area of the cylinder with the fluid and hence improves the velocity boundary layer thickness along with boundary layer thickness of temperature. It is also observed that increase in exponential parameter enhances the velocity profiles of Cu–water nanofluid and depreciates the temperature profiles. It is evident from Fig. 3 that due to increase in exponential parameter temperature profiles fall down at  $\eta_\infty = 5$  level afterwards which is equal to free stream temperature.

Figures 4 and 5 show the influence of volume fraction of nanoparticles on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking cylinders. It is evident from figures that enhancement in volume fraction of nanoparticles depreciates the velocity profiles and improves the temperature profiles of the flow. This is due to the fact that enhancement in volume fraction of nanoparticles reduces the velocity boundary layer thickness due to the friction near the walls. At the same time which helps to enhance the thermal conductivity of the flow. Due to this reason we noticed a raise in the temperature profiles. This hike in temperature profiles is high in exponentially stretching/shrinking cylinder at the exponential parameter  $N = 1$ .

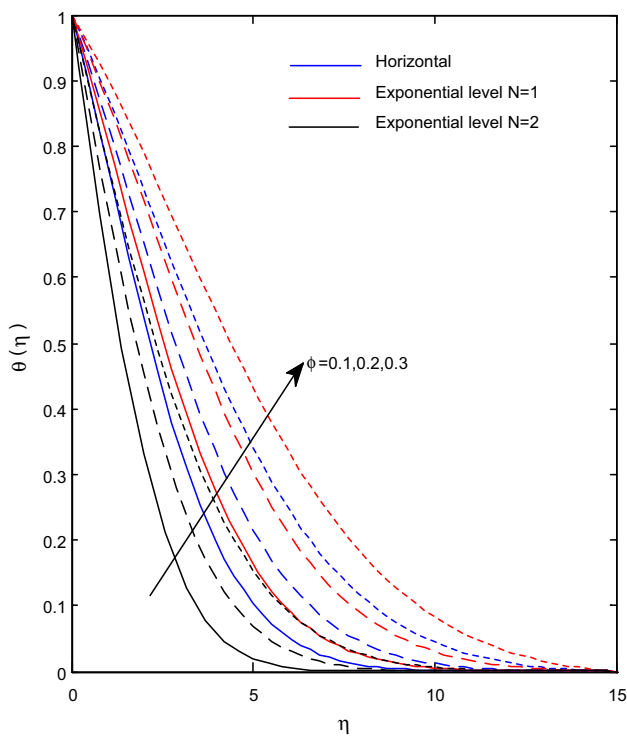
Figures 6 and 7 depict the influence of suction/injection parameter on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking cylinders. It is observed from the figures that increase in suction/



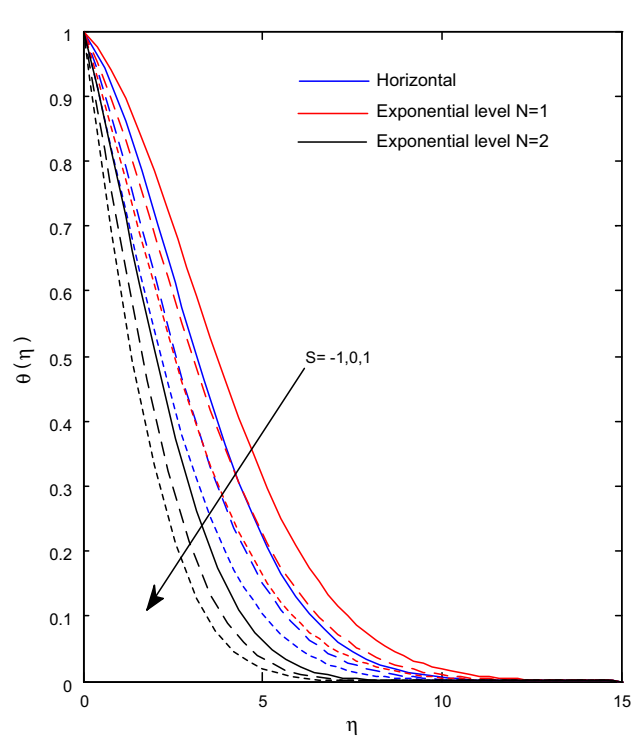
**Fig. 4** Velocity profiles for different values of nanoparticle volume fraction  $\phi$



**Fig. 6** Velocity profiles for different values of suction/injection parameter  $S$



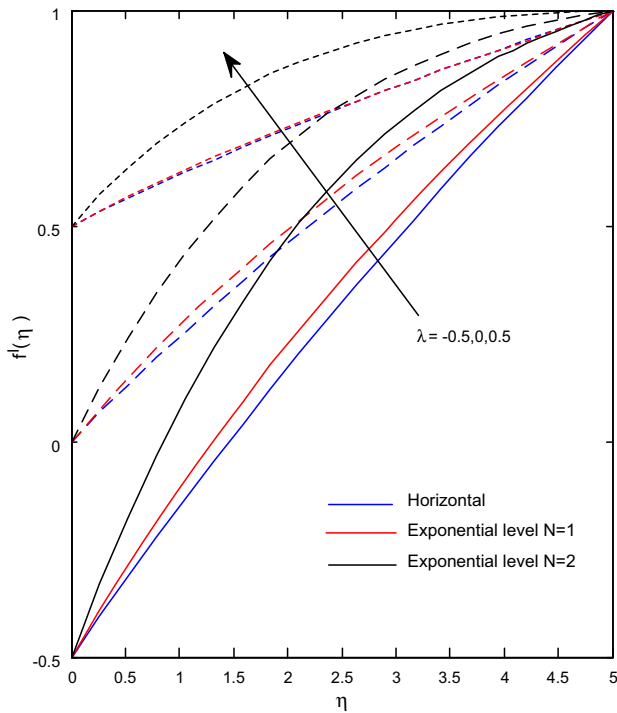
**Fig. 5** Temperature profiles for different values of nanoparticle volume fraction  $\phi$



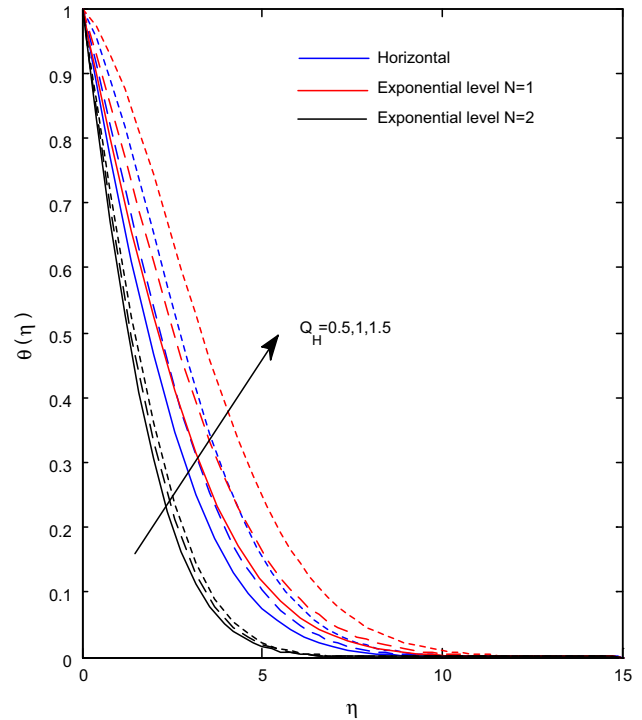
**Fig. 7** Temperature profiles for different values of suction/injection parameter  $S$

injection parameter decreases the velocity and temperature. Generally increase in suction/injection parameter reduces the velocity and thermal boundary layer thickness. This

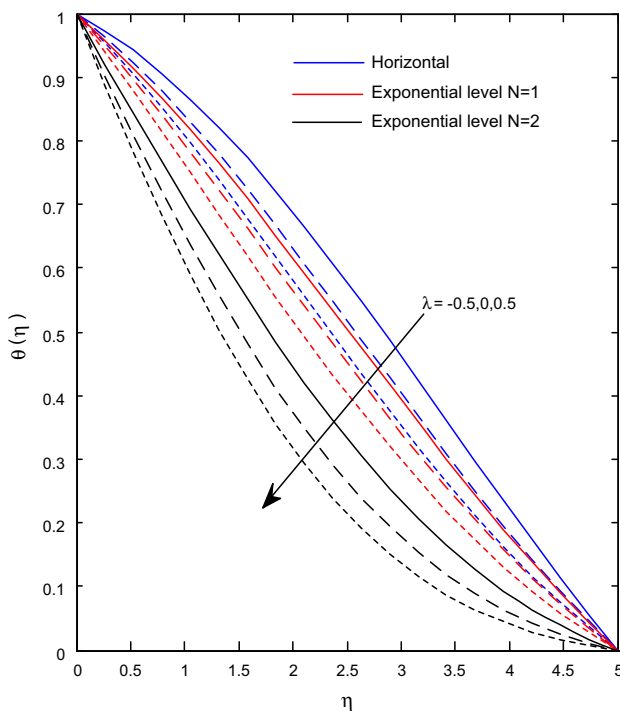
agrees the physical behavior of the suction/injection parameter. Here negative value of  $S$  indicates suction while positive indicates injection.



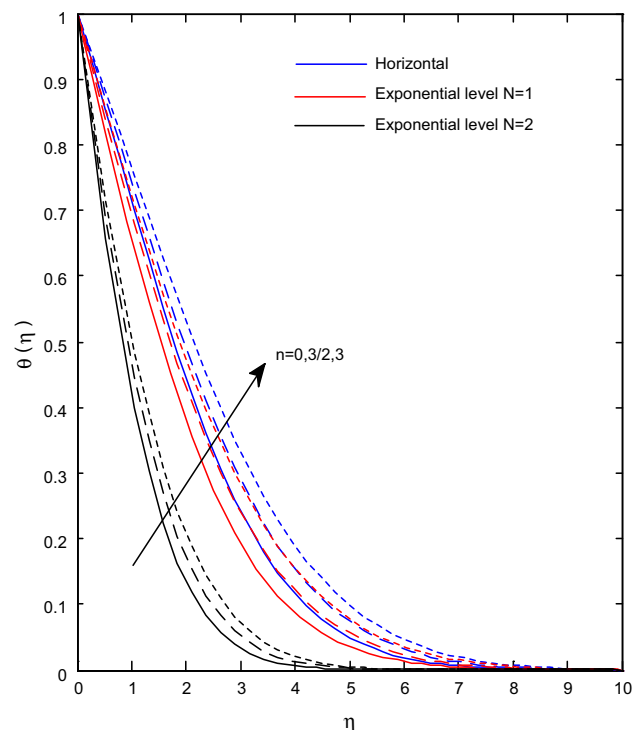
**Fig. 8** Velocity profiles for different values of stretching/shrinking parameter  $\lambda$



**Fig. 10** Temperature profiles for different values of heat source parameter  $Q_H$



**Fig. 9** Temperature profiles for different values of stretching/shrinking parameter  $\lambda$



**Fig. 11** Temperature profiles for different shapes of nanoparticles  $n$

Figures 8 and 9 display the influence of stretching/shrinking parameter on velocity and temperature profiles for both horizontal and exponentially stretching/shrinking

cylinders. It is noticed from figures that the enhancement in stretching/shrinking parameter increases the velocity profiles and declines the temperature of the flow. In this case

**Table 2** Comparison of the values of  $f''(0)$  for different values of  $\lambda$  and  $K$ , when  $\phi = 0$

$\lambda$	Najib et al. (2014) when $K = 0.2$	Present study	Najib et al. (2014) when $K = 0.4$	Present study
-0.25	1.5396153	1.5396152	1.6672783	1.6672782
-0.50	1.6705695	1.6705694	1.8307527	1.8307526
-0.75	1.7125346	1.7125345	1.9119385	1.9119385
-1.00	1.6297678	1.6297677	1.8836199	1.8836199

**Table 3** Variation in  $f''(0)$  and  $-\theta'(0)$  for different values of  $K, Q_H, S, n, N$  and  $\lambda$

Cylinder type	$K$	$\phi$	$Q_H$	$S$	$N$	$\lambda$	$n$	$f''(0)$	$-\theta'(0)$
Horizontal	0.5	0.1	1	1	-	2	3	-0.385753	0.209761
	1.0	0.1	1	1	-	2	3	-0.270187	0.140924
	1.5	0.1	1	1	-	2	3	-0.220590	0.115190
Exponential	0.5	0.1	1	1	1	2	3	-0.460126	0.170589
	1.0	0.1	1	1	1	2	3	-0.323070	0.115798
	1.5	0.1	1	1	1	2	3	-0.263030	0.097429
Horizontal	0.5	0.1	1	1	-	2	3	-0.485753	0.109761
	0.5	0.2	1	1	-	2	3	-0.421464	0.150192
	0.5	0.3	1	1	-	2	3	-0.317395	0.211178
Exponential	0.5	0.1	1	1	2	2	3	-0.637770	0.299158
	0.5	0.2	1	1	2	2	3	-0.634668	0.299957
	0.5	0.3	1	1	2	2	3	-0.554184	0.327404
Horizontal	0.5	0.1	0.5	1	-	2	3	-0.385754	0.292972
	0.5	0.1	1.0	1	-	2	3	-0.385753	0.209761
	0.5	0.1	1.5	1	-	2	3	-0.385754	0.097032
Exponential	0.5	0.1	0.5	1	2	2	3	-0.537770	0.443222
	0.5	0.1	1.0	1	2	2	3	-0.537770	0.399158
	0.5	0.1	1.5	1	2	2	3	-0.537770	0.350649
Horizontal	0.5	0.1	1	-1	-	2	3	-0.352069	0.070408
	0.5	0.1	1	0	-	2	3	-0.368614	0.138896
	0.5	0.1	1	1	-	2	3	-0.385753	0.209761
Exponential	0.5	0.1	0.5	-1	2	2	3	-0.493039	0.201889
	0.5	0.1	0.5	0	2	2	3	-0.514945	0.296295
	0.5	0.1	0.5	1	2	2	3	-0.537770	0.399158
Horizontal	0.5	0.1	0.5	1	-	-0.5	3	0.362467	0.091264
	0.5	0.1	0.5	1	-	0	3	0.255436	0.128529
	0.5	0.1	0.5	1	-	0.5	3	0.124568	0.164446
Exponential	0.5	0.1	0.5	1	2	-0.5	3	0.664528	0.305361
	0.5	0.1	0.5	1	2	0	3	0.505587	0.374973
	0.5	0.1	0.5	1	2	0.5	3	0.288998	0.435864
Horizontal	0.5	0.1	0.5	1	-	2	0	-0.379031	0.263224
	0.5	0.1	0.5	1	-	2	1.5	-0.379031	0.234754
	0.5	0.1	0.5	1	-	2	3	-0.379030	0.213021
Exponential	0.5	0.1	0.5	1	2	2	0	-0.621059	0.722499
	0.5	0.1	0.5	1	2	2	1.5	-0.621059	0.642116
	0.5	0.1	0.5	1	2	2	3	-0.621059	0.580628

hike in exponential parameter also helps to improve the velocity profiles. This may happen due to the reason that increase in exponential parameter causes to get the volumetric change in the cylinder. Here negative value of  $\lambda$  represents shrinking of cylinder and positive value is for stretching cylinder. It is also observed that for higher values of  $\lambda$ , boundary layer vanishes and the velocities of the cylinders and fluid are same.

Figures 10 and 11 show the effect of heat source parameter and nanoparticle shape on the temperature profiles of horizontal and exponentially stretching/shrinking cylinders. It is clear from Fig. 10 that increase in heat source parameter enhances the temperature profiles. From Fig. 11 it is observed that spherical-shaped nanoparticles showed better heat transfer enhancement compared with cylindrical-shaped nanoparticles. But in both cases increase in exponential parameter falls down the temperature profiles near the boundary.

Table 2 shows the comparison of the present results with the existed results of Najib et al. (2014). Present results showed an excellent agreement with the existed results under some special conditions. This proves the validity of the present results. Table 3 displays the influence of non-dimensional governing parameters on friction factor and Nusselt number. It is evident from the table that increase in curvature parameter increases the skin friction coefficient and reduces the Nusselt number. Enhancement in volume fraction of nanoparticles increases the friction factor and heat transfer rate. Friction factor and Sherwood number are not influenced by heat source parameter. But a raise in heat source parameter depreciates the Nusselt number. Enhance in suction/injection parameter, stretching/shrinking parameter declines the friction factor and improves the heat transfer rate. It is interesting to mention here that the spherical-shaped nanoparticles reduced the small amount of friction factor compared with cylindrical-shaped nanoparticles. But increase in both shapes of nanoparticles reduces the heat transfer rate.

## Conclusions

In this study we investigated the stagnation point flow and heat transfer behavior of Cu–water nanofluid at different shapes of nanoparticles towards horizontal and exponentially permeable stretching/shrinking cylinders with suction/injection in presence of heat source. The governing boundary layer equations are transformed to nonlinear ordinary differential equations using similarity transformation which are then solved numerically. The influence of non-dimensional governing parameters on the flow field

and heat transfer characteristics are discussed and presented through graphs and tables. The conclusions are made as follows:

- Curvature parameter has tendency to increase the friction factor and reduce the heat transfer rate.
- Increase in heat source parameter does not influence the friction factor but depreciates the Nusselt number.
- Spherical-shaped nanoparticles effectively enhance the thermal boundary layer thickness compared with cylindrical-shaped nanoparticles.
- A rise in the nanoparticle volume fraction improves the temperature profiles of the flow.
- Increase in suction/injection parameter and stretching/shrinking parameters helps to enhance the heat transfer rate.
- Solutions exist only for certain range of stretching/shrinking parameter, for higher values of stretching/shrinking parameter velocity of the fluid is equal to free stream velocity.
- Flow and heat transfer behavior of Cu–water nanofluid through the horizontal and exponential stretching/shrinking cylinders are non-unique

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