PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 105, Number 1, January 1989

ALEXANDER'S SUBBASE LEMMA

S. SALBANY AND T. D. TODOROV

(Communicated by James E. West)

This note provides a very simple proof of Alexander's subbase lemma, from the point of view of Nonstandard Analysis. There is no direct appeal to Zorn's lemma or equivalent principle (as in [1]). This set theoretic principle is, of course, embodied in the construction of the nonstandard extension ${}^*X$.

Notation and terminology is that of A. Hurd and P. Loeb [2].

Lemma (Alexander). *Let* (X, T) *be a topological space and* $\mathscr{S}$ *a subbase of closed sets. If every family of closed sets in* $\mathscr{S}$ *with the finite intersection property has nonempty intersection, then* (X, T) *is compact.*

Proof. Recall that (X, T) is compact iff ${}^*X = \bigcup_{x\in X} \mu(x)$ [2, Theorem (2.9), Chapter III] and that the monad of x is $\mu(x) = \bigcap\{{}^*G | x \in G,\ X - G \in \mathscr{S}\}$ [2, Proposition (1.4) of Chapter III]. Let $\alpha \in {}^*X$. Consider $\mathscr{F} = \{F | F \in \mathscr{S},\ \alpha \in {}^*F\}$. Then $\mathscr{F}$ has the finite intersection property and, by assumption, there is a point x such that $x \in \bigcap\{F | F \in \mathscr{F}\}$. We show that $\alpha \in \mu(x)$: if $x \in G$ and $X - G \in \mathscr{S}$, then $\alpha \notin {}^*(X - G)$, by our choice of x, hence $\alpha \in {}^*G$, as required.

REFERENCES

1. J. L. Kelley, *General topology*, Van Nostrand, New York, 1955.
2. A. E. Hurd and P. A. Loeb, *An introduction to nonstandard real analysis*, Academic Press, 1985.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ZIMBABWE, HARARE, ZIMBABWE

INTERNATIONAL SCHOOL FOR ADVANCED STUDIES (SISSA/ISAS), 34014 TRIESTE, ITALY

Received by the editors June 13, 1988.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 03H05, 54J05, 54D30.
Key words and phrases. Nonstandard extension, Alexander's subbase lemma.