u-Singularity and t-Topos Theoretic Entropy

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Abstract We will give descriptions of u-singularities as we introduce the notion of t-topos theoretic entropies. The unifying methodology for a u-singularity is the universal mapping property of an inverse or direct limit. The qualitative, conceptual, and structural analyses of u-singularities are given in terms of inverse and direct limits of micro decompositions of a presheaf and coverings of an object in t-site in the theory of temporal topos.

Prologue

One of the main reasons for introducing a categorical approach to quantum field theory is to avoid divergent expressions, e.g., for the total amplitude of a quantum process. One may also take categorical and sheaf theoretic methods as avoidance of the Dedekind-Cantor continuum approach to physical entities. We should mention here the possibility that sheaf theory is relevant to some non-perturbative approaches to quantum gravity, e.g., loop quantum gravity and non-perturbative superstring theory. The concept of a sheaf has been effectively used for the foundations of quantum physics and quantum gravity, especially among people in the C. Isham school at Imperial College as in [1–3], and Mallios' school as in [4, 5], and Penrose as twistor cohomology of sheaves in [6], and even though direct connections to our temporal topos method are not known, a few names also should be mentioned: Mulvey, Van Oystaeyen, Heller, and Sasin. In particular, the noncommutative geometry approach, called virtual topology of F. Van Oystaeyen, seems to be quite relevant to our work (see the treatise *Virtual Topology and Functor Category*, Tayler and Francis Group, 2007). See [7–9] for the developments and the history of sheaf theory in the theory of holomorphic functions in several complex variables, algebraic analysis, and algebraic geometry. This is the third paper in the series on the fundamentals of the theory of temporal topos, (t-topos) following [10, 11]. Our method of temporal topos, referred to as t-topos for short, differs from Isham' s and Mallios' schools, and also from the Russian school directed by A.K. Guts and E.B. Grinkevich. However, we should acknowledge the motivational influence coming especially from [1]. In a way, compared with other approaches to quantum gravity via topos, our method of t-topos is a more direct and straightforward application of commonly used familiar algebraic geometric (cohomological-algebraic) methods. That is, in order to express the changing state of a particle over a time period, the associated presheaf representing the particle is "parameterized" by an object in a (t-)site. We call such an object in t-site a generalized time period. Namely, we introduce such a state controlling parameter as a generalized time period-object in the t-site to keep track of varying states of a particle. See below for more on t-site. One of our goals is to study the topos of presheaves (t-topos) defined on a t-site and to study applications to quantum gravity. However, in t-topos theory, a presheaf is not always defined on every object in a t-site. When it is defined, a presheaf in t-topos satisfies the usual properties of a contravariant functor. This is one of the issues relevant to the Kochen-Specker theorem in [2] and [3]. For such a connection of t-topos to the approach taken in [2] and [3], the reader is referred to [10]. The theory of t-topos is a background independent and scale independent theory* (see (*) below) in the following sense: all the concepts (e.g., particle-wave duality, quantum entanglement, light cones) discussed are defined in terms of presheaves associated with either a macro or micro particle together with the associated space and time presheaves. For a particle, we associate a presheaf m so that each (particle) state of the particle is represented by a pair of the presheaf m and an object V (which is called a generalized time period V) in a site S. [At the Second International Conference on Theoretical Physics and Topos, held at Imperial College, London, 2003, (*)C. Isham at the Conference said "(In the definitions in t-topos theory) a particle can be replaced by an elephant."] Such a site as used in t-topos theory is called the t-site. Recall that a site in general is a category with a Grothendieck topology as defined in [9, 12, 13]. A particle ur-state of the presheaf m associated with a particle is expressed as $m(V)$ as an object in a product category

$$\prod_{\alpha\in\Lambda} C_\alpha. \tag{0.0}$$

(See [10, 11, 14].) One of the reasons for introducing the product category indexed by a finite set is that for each physical quantity possibly measured, we need a category where such a measurement (a morphism) can take place. Following the terminology used among topos theorists, the category $\hat{S}$ of presheaves on S (with a restricted sense as follows) is said to be a *temporal topos* or simply *t-topos*. Namely, $\hat{S}$ is the category of contravariant functors from the t-site S to $\prod_{\alpha\in\Lambda} C_\alpha$. However, such a t-topos theoretic presheaf is more restricted than the usual definition of a presheaf. That is, $m(V)$ may not be defined for every pair of an object m of $\hat{S}$ and an object V of S. Hence, an object of the t-topos $\hat{S}$ may be more appropriately called an *ur-presheaf* rather than just a presheaf. Let m and P be presheaves. We say that m is *observable* (*measurable*) by P over a generalized time period V (i.e. an object of the t-site S), when there exists a morphism from $m(V)$ to $P(V)$. For a presheaf m associated with a particle, there are the space and time presheaves κ_m and τ_m associated with m. As a consequence of an entanglement, those associated space and time presheves depend upon a particle presheaf m. This dependency means that space and time are locally determined by the particle m, and that the space and time presheaves do not exist without the particle. (See [10].) Also recall that a presheaf m is said to be in a *particle ur-state* if there exists an object V in S such that $m(V)$ is defined. Otherwise, m is said to be in a *wave ur-state*. That is, for example, when such an object V in the t-site cannot be specified as in the case of double slit experiment, m is said to be in a wave ur-state. (See [14] for the application of t-topos to double slit experiment.) Recall also that m and m' are *ur-entangled* when presheaves m and m' are defined always on the same objects of S. (See [10, 15] for connections to EPR type non-locality.)

In this paper, for a presheaf m representing a particle and for an object V in the t-site, a decomposition of m and a covering of V play major roles to define a notion of entropy. Let

$$m = \prod_{j \in J} m_j \tag{0.1}$$

be a (micro) decomposition of m into a product of subpresheaves m_j of m, indexed by a finite set J, and let

$$\{V \overset{\varphi_k}{\longleftarrow} V_k\}_{k \in K} \tag{0.2}$$

be a covering of V by the family of objects V_k, indexed by a finite set K, in the sense of [12, 13], or [9]. For a covering in (0.2), $\varphi_k : V \longleftarrow V_k$ induces the morphism $\tau_m(\varphi_k) : \tau_m(V) \longrightarrow \tau_m(V_k)$ on time presheaf τ_m. This morphism $\tau_m(\varphi_k)$ is regarded as the restriction of the time period from the longer time period $\tau_m(V)$ to the shorter period $\tau_m(V_k)$. Notice that even though $m(V)$ is defined, $m_j(V_k)$ may not be defined, which is connected to the Kochen-Specker theorem discussed in [2, 3]. See [10] for a connection to the notion of t-topos.

Definition 0.1 For an object m in $\hat{S}$ and an object V in S, the pair (m, V) in $\hat{S} \times S$ is said to be *compatible* when $m(V)$ is defined.

Namely, for a (micro) decomposition of m and a covering of V as in (0.1) and (0.2), respectively, among all the possible pairs $\{m_j(V_k)\}_{(j,k) \in J \times K}$, only for a subset of $J \times K$ we have compatible pairs $m_j(V_k)$. We will define a notion of entropy of the state $m(V)$ as a number of such compatible pairs in the next section. For a detail discussion of a microdecomposition, see [11].

1 Methods of t-Topos

Since the first two papers of this trilogy have been published by a different journal, we will give a concise description of the results in the earlier two papers [10] and [11], which is relevant to the current paper. We have introduced notions of a microdecomposition and a micromorphism. For example, the concept of a t-topos theoretic light cone is viewed like a light cone with holes similar to 'Swiss cheese'. This is because the notion of a micromorphism gives the impossibility of factorization between two states given by two objects in the t-site. See Epilogue for more on micromorphisms. Together with a microdecomposition and a further refinement of a covering in what will follow, we get similar "unreified" pairs of particle-decomposed presheaves and covering-decomposed objects in a t-site. Such a situation where there exist "floating" abundant *unmatched* pairs of particle presheaves and objects in t-site is an *ultra microcosm* and also closer to "singularity" condition.

Even though the method of t-topos is a more kinematical and qualitative theory, the dynamical aspect is embedded in the space and time presheaves. Namely, space presheaf κ_m and time presheaf τ_m are associated with a particle. Hence, for example, when the curvature of $\kappa_m$is measured (specifying a category among the product category in (0.0)), the fundamental composition principle (see what will follow) can be used to assign a (real) value. Another view of a dynamical aspect of t-topos is the following. Suppose two particles are close enough to influence spacetime in the common "region" of two spacetime presheaves $(\kappa_{m'}, \tau_{m'})$ and (κ_m, τ_m). Then one can associate with the two gravitationally interacting particles the "product spacetime," of the associated spacetime presheaves $(\kappa_{m'}, \tau_{m'})$ and (κ_m, τ_m) (see p. 176 of [11]).

Let a presheaf m associated with a particle be observed twice over V and U. Namely, we consider the case when m is observed over V first and then over U. That is, time $\tau_m(V)$ precedes time $\tau_m(U)$ in the usual classical linearly ordered sense. Then there exists a morphism g from V to U in the t-site S. Note that not every morphism from V to U in S represents such a linear temporal order in the above sense. This is one of the reasons for introducing the notion of a site rather than just a topological space for our sheaf theory. From the contravariantness of m, there exists the canonically induced morphism $m(g)$ from $m(U)$ to $m(V)$. If an observer P in $\hat{S}$ observes m over V, represented by a morphism $t_V : m(V) \longrightarrow P(V)$, then the composition of $t_V : m(V) \longrightarrow P(V)$ with the canonical morphism $m(g)$ gives the morphism: $m(U) \xrightarrow{m(g)} m(V) \xrightarrow{t_v} P(V)$. Then the image $\mathrm{Im}(t_V \circ m(g))$ of the composition of those morphisms can be physically interpreted as the information of m over U to P over V. An expression as "An electron moves from point A to point B taking all available paths simultaneously" is inadequate. This is because expressions like "path" and "simultaneously" are the concepts assuming the following: such an electron were observed besides the two states at A and B. Our theory focuses on all the possible factorizations $\{W\}$ by linearly t-ordered morphisms from V to U via W where V and U are the corresponding objects in t-site S to A and B, respectively (see [14]). Note that if a morphism from V to U is a micromorphism, such a proper factorization W does not exist except for the trivial ones, i.e., a morphism V to U can be factored only via either V' or U' where V' and U' are isomorphic to V and U, respectively. When a morphism from V to U is a micromorphism, such an electron cannot be observed after the state corresponding to A and before the state corresponding to B.

For the projection morphism p_j from m to each component of the decomposition

$$m = \prod_{j \in J} m_j \xrightarrow{p_j} m_j, \tag{1.1}$$

suppose that m_j is observed by P over a generalized time period W. When P and W are compatible (Definition 0.1), we have the induced morphism $s_W : m_j(W) \longrightarrow P(W)$. Then the composition morphism

$$s_W \circ p_j(W) : m(W) = \left(\prod_{j \in J} m_j \right)(W) \xrightarrow{p_j(W)} m_j \xrightarrow{s_W} (W) P(W) \tag{1.2}$$

is regarded as the information (measurement) of the (macro) object m via the observation of the (micro) object m_j by P over the generalized time period W. However, the converse: an observation morphism of the (macro) object m over P over a generalized time period cannot be composed with the projection morphism p_j. Namely, a measurement of a macro object (presheaf) m does not give any information of the micro objects (subsheaves) of which m consists.

For a given state $m(V)$ of m over V, assume that there exists an object V' in such a way $\tau_m(V')$ precedes $\tau_m(V)$. That is, there exists a linear t-order sequence of objects in t-site S as

$$\cdots \longrightarrow V'' \longrightarrow V' \longrightarrow V. \tag{1.3}$$

In what will follow, such an inverse (projective) limit of (1.1) is to play an important role for u-singularities and Planck scale objects for the given state of m over V.

A definition of a t-topos theoretic light cone is given in [11]. We will give another definition of a light cone using the presheaf γ associated with a photon.

Definition 1.1 Let γ be a photon presheaf which is observed over a generalized time period V. Then consider all the *cone-sequences* going through V:

$$\{\cdots \longleftarrow V^2 \longleftarrow V^1 \longleftarrow V \longleftarrow V^{-1} \longleftarrow \cdots\} \tag{1.4}$$

where all the morphisms involved in (1.4) are linearly t-ordered. Then we define the light cone with respect to the state $\gamma(V)$ as follows. The light cone for the state $\gamma(V)$ is the collection of all the objects and the induced morphisms from (1.4):

$$\{\kappa_\gamma(V^l), \tau_\gamma(V^l)\} \quad \text{for } l = \pm 1, \pm 2, \ldots, \tag{1.5}$$

where κ_γ and τ_γ are the space and time presheaves associated with a photon presheaf γ. That is, the *light cone with respect to* V can be interpreted as the collection of all the possible sequences:

$$\{\cdots \longleftarrow \kappa_\gamma(V^2) \longleftarrow \kappa_\gamma(V^1)\kappa_\gamma(V) \longleftarrow \kappa_\gamma(V^{-1}) \longleftarrow \cdots\} \tag{1.6}$$

and

$$\{\cdots \longleftarrow \tau_\gamma(V^2) \longleftarrow \tau_\gamma(V^1) \longleftarrow \tau_\gamma(V)\tau_\gamma \longleftarrow (V^{-1}) \longleftarrow \cdots\} \tag{1.7}$$

for all the cone-sequences of objects going through V as (1.4) in the t-site S. In general, in terms of t-site S, we can also define the notion of a light cone as follows: V and V' are said to be in a light cone if there exists a *cone-sequence* between V and V'.

2 Entropy and Limits

We will define the notions of entropies for a decomposition as in (0.1) of m and for a covering as in (0.2) of V of objects in the t-topos $\hat{S}$ and t-site S, respectively.

Definition 2.1 The t-entropy of the state $m(V)$ for a micro decomposition $m = \prod_{j\in J} m_j$ and a micro covering $\{V \overset{\varphi_k}{\longleftarrow} V_k\}_{k\in K}$ is defined as the number of compatible pairs $\{m_j(V_k)\}$.

Definition 2.2 The formal entropy of $m(V)$ for the decomposition and the covering is defined by the product of cardinalities of J and K.

Note 2.3 For compatible pairs $m_j(V_k)$ and $m_{j*}(V_{k*})$, there need not be a linear t-order between $\tau_{m_j}(V_k)$ and $\tau_{m_{j*}}(V_{k*})$. The rest of the non-compatible pairs between the decomposition and the covering are the collection of non-observable (non-measurable) particle associated subsheaves. Hence, then there is no associated space and time with such non-compatible pairs.

Definition 2.4 The absolute entropy of $m(V)$ is the maximum number of compatible pairs for all decompositions and coverings of m and V, respectively.

Next, we will consider limits of such sequences as in (0.1) and (0.2) and a sequence as in (1.3). Considering a further decomposition of each subsheaf m_j in (0.1), we get a sequence of morphisms

$$m = \prod_{j\in J} m_j \longrightarrow \prod_{i\in I} m_{j_i} \longrightarrow \cdots, \tag{2.1}$$

which is a sequence of the projection morphisms as in (1.1). Also, for such a covering as in (0.2), since a covering of a covering is a covering of V (e.g., [11]), we have a sequence as

$$\{V \xleftarrow{\varphi_k} V_k\}_{k\in K} \longleftarrow \{V \xleftarrow{\varphi_{ki}} V_{k_i}\}_{k\in K, i\in I} \longleftarrow \cdots. \tag{2.2}$$

We will discuss the inner relations among

(i) u-singularities,
(ii) an inverse limit appearing in (1.3), and
(iii) a direct limit of (2.1) and an inverse limit of (2.2).

Our study of "singularities" is categorical, which is neither in Morse-Thom topological catastrophe theory nor the differential geometric theories of general relativity. That is one of the main reasons for us to choose the notion of a topos by replacing "points" with either "regions" or objects of a category as mentioned in Prologue. The terminology "u-"singularities is meant to be a characterization in terms of "*universal*" mapping property of direct and inverse limits. That is, our *u-singularities* are defined (and hopefully captured) in terms of categorical notions of limits. For the general notions of direct (inductive) and inverse (projective) limits, see treatises [9, 12, 13]. First, the direct limit of the sequence of morphisms in (2.1) may be called an *ur-subplanck decomposition*. (Note that in [11], this notion of a ur-subplanck decomposition is introduced. However, the direct sum used in [11] should be replaced by the direct product as in (2.1). Hence, the direct limit is appropriate, not the inverse limit used in [11].) The inverse limit of (2.2) is called an *ur-subplanck covering* of V. By the definition of the t-topos theoretic entropy, the mass of the particle ur-state $m(V)$ is in general greater than the totality of the measured total mass of all the compatible pairs for any decomposition as in (0.1) of m and any covering as in (0.2) of V.

We define the u-singularities for the state $m(V)$ as the states of stationary conditions in the following sense. After a finite number of the processes of refinements of the coverings in (2.2), there appear stationary objects V_α so that for the inverse limit of the covering sequence of (2.2), the coverings are consisting of isomorphic objects to V_α's. Namely, we get a covering $\{V \xleftarrow{\varphi_a} V_\alpha\}$ as the inverse limit of (2.2). See Definition 2.6 below. The t-entropy for such a limit pair of the ur-subplanck decomposition of m and the ur-subplanck covering of V, can be computed in the following sense. By identifying the isomorphic objects, the number of compatible pairs, i.e., the t-entropy of the state $m(V)$, is determined by the number of m_{j_ω} which are compatible with those V_α, where m_{j_ω} are subobjects of m appearing in the direct limit of (2.1). See Definition 2.5 below. For each refinement (or as the inverse limit) of a covering as in (2.2), the corresponding time periods (not generalized time periods as objects of S) $\tau(V_{k_i})$ are shorter than $\tau(V_k)$ as noted earlier. Then for the corresponding decomposition (or as the direct limit) as in (2.1), some of the compatibly paired objects can be physically interpreted as corresponding to short-lived particles causing more severe curvatures in spacetime in the classical sense. Such a condition can be interpreted as the non-smoothness of spacetime in microcosm. Note that as noted before an assignment of scaling for an object like $\tau(V_k)$ can be given by *FUNC* (the fundamental composition principle in [2, 3]. See the following Epilogue).

Definition 2.5 A presheaf m in $\hat{S}$ is said to be *fundamental* when m cannot be decomposed into a product of proper subsheaves.

Definition 2.6 An object V in S is said to be *fundamental* when an isomorphism $V \longleftarrow V'$ is the only covering of V.

Note 2.7 Such fundamental presheaves in Definition 2.5 can be considered to correspond to presheaves associated with elementary particles. In the definition of an ur-subplanck decomposition defined as a direct limit of (2.1), each decomposed subsheaf m_{j_ω} in the above is fundamental. Note also that such an object V_α defined in the above as an inverse limit, i.e., an ur-subplanck covering, is a fundamental object in S.

For m in $\hat{S}$ and V in S, if $m(V)$ is defined, we have a linearly t-ordered sequence for such a V as in (1.3).

Next we will consider such a sequence as (1.3) for a fundamental presheaf denoted as m_ω. Then we have the following sequence from (1.3):

$$\cdots \longleftarrow m_\omega(V'') \longleftarrow m_\omega(V') \longleftarrow m_\omega(V). \tag{2.3}$$

The direct limit of (2.3) is associated with the u-singularity induced by the inverse limit $\varprojlim(V) \underset{\text{def}}{=} \varprojlim(\cdots \longrightarrow V'' \longrightarrow V' \longrightarrow V)$ of the linear t-order sequence (1.3). Then $\tau_{m_\omega}(\varprojlim(V))$ must not be preceded in linear t-order, hence by any usual classical time, provided that such a (particle associated) presheaf m_ω has survived from the earliest universe. As such a fundamental presheaf m_ω, we may consider cosmic background radiation. If the classical notion of the big bang indicates correctly an earliest universe state, it is reasonable to assume the following. For any such presheaf m_ω and such specified object V in the above, they have a common (isomorphic) inverse limit corresponding to a big bang state. However, there is no reason for such a preasheaf m_ω to be compatible with such an inverse limit object of linear t-order sequence. Notice that this common (isomorphic) object in the t-site is related to the stationary object V_α appearing in the inverse limit of the covering (2.2) whose u-singularities represent gravitational fluctuations. Note that both cases of a big bang and a black hole satisfy the following principle called "Ancestor's Rule." That is, each of us has 2 to the n-th ancestors in our n generations back. For the increasing population, however, the number of the ancestors is small for a large n. Note that the entropies in Definitions 2.1, 2.2, 2.4 of decrease as time recedes.

Epilogue

Our basic approach toward quantum behavior of a particle (elementary or not) is to capture an ur-particle state as a reified pair of the associated presheaf m and an object V of t-site. When presheaf m does not have an object to be reified, m is said to be in an ur-wave state. This ur-wave state includes the case of the double slit experiment because of the indetermination of such a choice of an object of the t-site. Let V and V' be two objects determining the corresponding ur-particle states of presheaf m. Moreover, suppose $m(V)$ is observed first and $m(V')$ is observed later. Then there is a morphism f from V to V' in the t-site, inducing a linear time order in the usual sense. Let us focus on factorizations of such a morphism from V to V'. Suppose there is no intermediate linearly ordered time state between the states $m(V)$ and $m(V')$ via any proper factorization. That is, if there do not exist morphisms g and h and an object W in t-site satisfying $f = h \circ g$ for $V \xrightarrow{g} W \xrightarrow{h} V'$, where neither g nor h is isomorphic, then such a morphism f is said to be a micromorphism. When applied to the notion of a light cone of a particle in a microcosm, such a t-topos theoretic light cone is a light cone with holes, i.e., missing states where the associated particle presheaf does not have objects from t-site to be reified as we have mentioned earlier. One of the missing elements in our approach of t-topos is the aspect of dynamics. In t-topos theory there is a notion for such a relativistic dynamics in terms of the space and time presheaves depending upon a particle (locally defined) and the notion of the product of those presheaves. However, further study is needed to develop the t-topos theory to treat further applications. The development of t-topos methods is still at the earlier stage. The t-topos aspect of the time delay effect, for example near a black hole, is yet to be formulated. In the near future, our plan is to investigate the t-topos theoretic interpretations of Hawking radiation and quantum tunneling. See our forthcoming papers, e.g., [17]. Categorically speaking (not in the mathematical sense), our theory may belong to a hidden variable approach (with direct experimental applications) as indicated in [18]. A similarity between back hole type singularity and a big bang type singularity is the concept of u-singularity, i.e., the categorical notion of a limit (inverse or direct). Namely, for a compatible pair of a presheaf and an object (generalized time period) of the t-site, a black hole type singularity is described as limits of micro decompositions of the given presheaf and of micro coverings of the object of the t-site. Meanwhile, a micro big bang type singularity is given as a limit of a linearly t-ordered sequence for such a compatible object of the t-site with an arbitrary fundamental presheaf. We may even consider the totally incompatible (non-reified) state of fundamental presheaves and fundamental objects of S. We may call such a state as a *ur-bang*, which should not be called a "pre-big bang" state. It is the "unmatched melting pot" of t-topos and t-site objects without any compatible pairs. Some results from particle physics may tell us how many fundamental objects are in t-topos and t-site at the big bang. In order to make the t-topos theory into a quantitative theory, we may be able to use the so called the fundamental composition principle as in [2] and [3] for V defined for an operator in a Hilbert space H corresponding to a physical quantity. Namely, the following diagram consisting of the vertical morphism of Hilbert space H induced by a function from real number R to R:

$$\begin{array}{ccc} H & \xrightarrow{v} & R \\ \downarrow & & \downarrow \\ H & \xrightarrow{v} & R \end{array}$$

is commutative. See [10] for details. For the mathematical foundations for t-topos theory, see the forthcoming [16]. In this paper, sheaf cohomology per se does not appear. However, sheaf cohomology via coverings is crucial for Penrose's work as mentioned in Prologue and also for Mallios and Raptis, De Rham cohomology, i.e., the hypercohomology with coefficient in the cochain complex of differential forms, plays an important role in [4] and [5]. Volovich's p-adic string theory as in [19] requires the computation of the 1^{st} p-adic cohomology group associated with Fermat curve over a finite field to obtain the Veneziano amplitude (see the references in [19]). More general treatments of cohomologies can be found in [9] and [12].

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