

Fuzzy Model for Selection of Underground Mine Development System in a Bauxite Deposit

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Abstract In this paper, a fuzzy programming model, incorporating fuzzy measures of costs and ore reserves, is developed to evaluate different design alternatives in the context of the selection of the underground mine development system. The bauxite deposit is usually mined using the sublevel mining method. This method extracts the ore via sublevels, which are developed in the ore body at regular vertical spacing. In such an environment, we consider the development system as a weighted network interconnecting all sublevels with surface breakout point using the minimum cost of development and haulages. Selection of the optimal development system is based on the application of Convex Index and composite rank. The uncertainties related to the future states of transportation costs are modeled with a special stochastic process, the Geometric Brownian Motion. The results indicate that this model can be applied for solving underground mine development problems.

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الخلاصة

تم في هذه الورقة العلمية تطوير نموذج برمجة غامض يتضمن تدابير غامضة للتكاليف واحتياطات الخام لتقييم بدائل التصميم المختلفة في سياق اختيار نظام تطوير منجم تحت الأرض. وعادة ما يتم استخراج ترسبات البوكسايت باستخدام طريقة تعدين المستوى الثانوي. وهذه الطريقة تستخرج الخام عبر مستويات ثانوية تطورت في جسم الخام عند تباعد عمودي منتظم. وفي مثل هذه البيئة أخذنا في الاعتبار تطوير النظام كشبكة مرجحة تربط جميع المستويات الثانوية مع نقطة اختراق السطح باستخدام الحد الأدنى لتكاليف التطوير والشحن. ويستند اختيار نظام التنمية الأمثل إلى تطبيق مؤشر محدب ورتبة مركب. وقد تمت نمذجة الشكوك المتعلقة بالحالات المستقبلية لتكاليف النقل باستخدام عملية عشوائية خاصة والحركة البراونية الهندسية. وتشير النتائج إلى أن هذا النموذج يمكن تطبيقه لحل مشاكل تطوير منجم تحت الأرض.

Keywords Mining · Development · Networks and graphs · Fuzzy sets · Stochastic processes · Decision support system

1 Introduction

The investment environment associated with the mining industry is unique when compared with the environment encountered by typical manufacturing industries. Some of the characteristics of mining which are often proclaimed as unique are as follows: capital intensity, long preproduction periods, high risk and nonrenewable resource [1]. Mines' development system investments provide a good example of irreversible investment. Such investment requires careful analysis because, once the investment takes place, it cannot be recouped without significant loss a value. Obviously, the selection of an underground mine development system belongs to strategic planning. Tahernejad et al. [2] emphasize that a lack of scientific planning, poor management and a lack of clear strategies are the most important problems of Iran's dimensional stone mines.

The problem discussed in this paper is ‘Kostari’ mine, a small-scale open-pit bauxite mine located in Bosnia–Herzegovina. Production of bauxite from the open-pit mine is approaching the end. The management of the company has estimated that the remaining reserves of bauxite can only be mined by an underground method. The question of which underground mine development system is suitable for accessing and exploiting a deposit is one that mine engineers and planners are faced with, when investigating the most efficient production system. Basically, there are three main development systems to gain access to an ore body: vertical shaft, decline (ramp) and adit. These three systems can be mutually combined, and in that, the number of potential alternatives is increased. Generally speaking, the process of selection of an underground mine development system encompasses the identification, evaluation and selection among alternatives.

To solve the problem of selecting a suitable development system, we consider an underground mine development system as a network interconnecting all access points with surface breakout point, using the minimum cost of development and haulage. The major task is to design the lowest cost-feasible development system, respecting all operational constraints.

Many researches considered an underground mine development system as a network optimization problem, [3–7].

In our case, conditions prevailing on the surface and deposit are not suitable to apply adit as the development system. Shaft, Decline and Shaft–Decline development systems are identified as potential alternatives for the evaluation process. From each sublevel access point, we span the ore transportation network to the surface in three alternative directions. Each direction corresponds to one development system. Every section of the network is weighted by an adequate fuzzy cost function, which combines the cost needed to build up the section and the cost of ore transportation along it. The values of this function are changed over the project time, using a stochastic process, Geometric Brownian Motion, to simulate them. A set of potential alternatives is preference-ranked, according to the ascending order of Convex Index. By evaluating the networks spanned from each sublevel, it can be seen how depth, sublevel ore reserves and fixed production rate affect the efficiency of the development system.

2 Fuzzy Sets Theory

In order to deal with vagueness of human thought, Zadeh [8] first introduced the fuzzy set theory. This theory was oriented to the rationality of uncertainty, owing to imprecision or vagueness. A fuzzy set is a class of objects with a continuum of grades of membership. The role of fuzzy sets is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal

is to find a good approximate solution [9]. Modeling using fuzzy sets has proved to be an effective way of formulating decision problems, where the information available is subjective and imprecise [10].

2.1 Linguistic Variable

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language [11]. As an illustration, age is a linguistic variable if its values are assumed to be fuzzy variables, labeled young, not young, very young, not very young, etc., rather than the numbers 0, 1, 2, 3... [12].

2.2 Fuzzy Numbers

A fuzzy number \tilde{M} is a convex normalized fuzzy set \tilde{M} of the real line R [12]:

– it exists such that one $x_0 \in R$ with $\mu_{\tilde{M}}(x_0) = 1$ (x_0 is called mean value of \tilde{M})

– $\mu_{\tilde{M}}(x)$ is piecewise continuous.

Triangular fuzzy number can be defined as a triplet (a, b, c) . The membership function is defined as [13]:

$$\mu_{\tilde{M}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c \end{cases} \quad (1)$$

3 Methodology

3.1 Problem Formulation

Considering a directed network (in the context of the ore-haulage direction) that is composed of a finite set of nodes and a set of directed arcs, we denote each arc by an order pair (i, j) , where i and j are different nodes, respectively. The arc length is the distance needed to traverse (i, j) from node i to j . It is denoted by $l(i, j)$. Formally, the problem is to find the fuzzy least cost path from the origin node (access point) to the destination node (surface breakout point). Figure 1 shows an example of the development network with several possible development paths, and Fig. 2 presents an adequate directed graph of the possible ore-haulage paths for the production area (sublevel) 1–2.

The fuzzy least cost path problem can be formulated as the following form. The objective function

$$\begin{aligned} \tilde{F}(X, t) &= \sum_{i=1}^n \sum_{j=1}^n (\tilde{f}_i + \tilde{f}_{ij}) \cdot x_{ij} \\ &= \sum_{i=1}^n \sum_{j=1}^n (\tilde{C}_i^{DA} + \tilde{C}_{ij}^{DH} + \tilde{C}_{ij}^H(t)) \cdot x_{ij} \end{aligned} \quad (2)$$

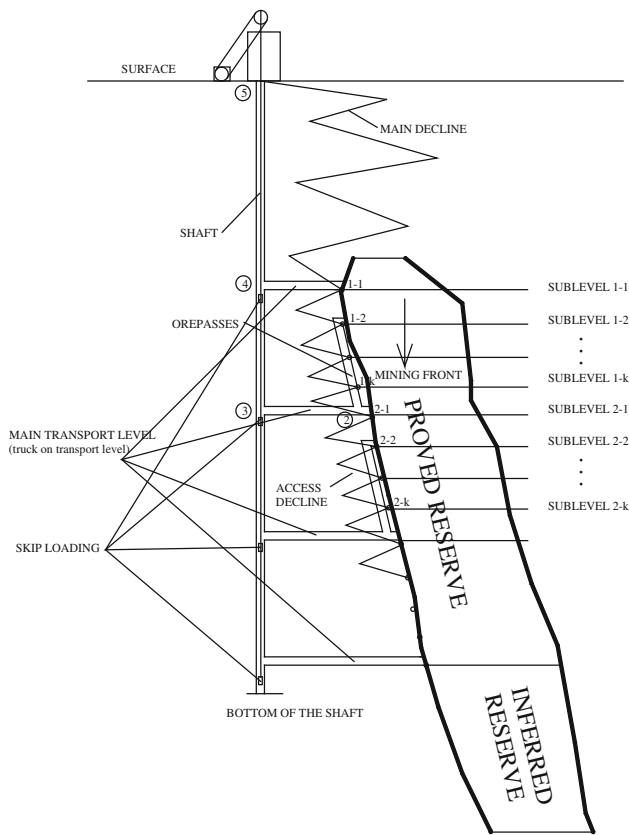


Fig. 1 Development network with possible paths

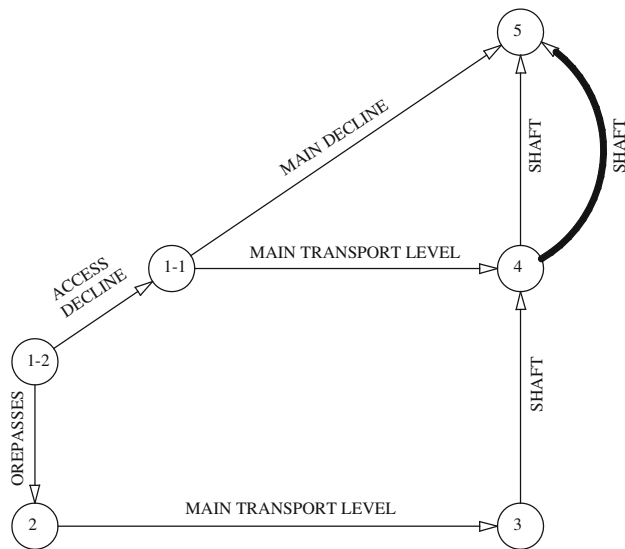


Fig. 2 The graph of the ore-haulage system for the production area 1-2

has to be minimized, subject to:

$$\sum_{j=1}^n x_{ij} - \sum_{j=1}^n x_{ji} = \begin{cases} 1, & i = 1 \\ -1, & i = n \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i,j) \in A \quad (4)$$

$$\tilde{C}_i^{DA} \begin{cases} = 0 & \text{if } \tilde{C}_{ij}^H > 0 \\ > 0 & \text{if } \tilde{C}_{ij}^H = 0 \end{cases} \quad (5)$$

where \tilde{f}_i —the triangular fuzzy cost function in node i is added to the outgoing edge from node i , but not to an incoming edge; \tilde{f}_{ij} —the triangular fuzzy cost function between node i and node j (for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, n$); x_{ij} —the decision variable defines binary variables x_{ij} , where $x_{ij} = 1$ if the form $(i$ to $j)$ is on the path and $x_{ij} = 0$ otherwise; \tilde{C}_i^{DA} —total cost of building (driving) of development forms that are used only for the purpose of accessing production area; ore haulage will not be done along them (\$); \tilde{C}_i^{DH} —total cost of building (driving) of development form that is used for the purpose of accessing production area, and to haul the ore along it (\$); $\tilde{C}_{ij}^H(t)$ —the cost of ore haulage along the development form, which changes over project time (\$).

The objective function, defined by Eq. (2), refers only to one sublevel. Let $G(P)$ be a graph of P , where $P = \{p_1, p_2, \dots, p_y\}$ is a set of fuzzy possible paths from sublevel to surface (see Fig. 2). The solution of Eq. (2) is defined by the least cost path $p = \min(p_1, p_2, \dots, p_y)$ having the minimum Convex Index. The path p is the most suitable path, and we assign rank 1 to it. The rest of the paths are ranked further, in ascending order of their Convex Index values, and we assign values 2, 3, ..., y to them, respectively. It means there is one rank order of given paths p_1, p_2, \dots, p_y (for example, p_3, p_1, p_2 ; with assigned values $p_3 \rightarrow 1, p_1 \rightarrow 2, p_2 \rightarrow 3$) for one sublevel. Suppose there are k sublevels that should be mined during the planned period. If we take into consideration the previous assumption, then there are k rank orders of given paths, one for each sublevel.

According to above discussion, our problem can be represented as an alternatives, attributes, evaluations (A, X, E) model. We consider: a finite set of alternatives, i.e., development systems $A(p_1, p_2, \dots, p_y)$; a finite set of attributes, i.e., assigned values $X(x_1, x_2, \dots, x_k)$, according to solution of Eq. (2); and a set of evaluations of alternatives with respect to attributes.

$$E = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1k} \\ x_{21} & x_{22} & \dots & x_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{y1} & x_{y2} & \dots & x_{yk} \end{bmatrix} \quad (6)$$

The main goal of our model formulation was to determine the development system that should be used to access all k sublevels, and to haul ore from them to the surface. The solution is based on the composite ranking of given alternatives. The optimal development system for a given ore deposit is

selected by the following equation:

$$DS = \min \left[\sum_k x_{1k}, \sum_k x_{2k}, \dots, \sum_k x_{yk} \right] \quad (7)$$

The complete procedure will be described in Sect. 3.3.

3.2 The Cost Functions

The general fuzzy cost function for development form that is used for access and ore haulage is:

$$\tilde{f}_{ij}(t) = \tilde{C}_{ij}^{DH} + \tilde{C}_{ij}^H(t) = \tilde{\delta} \cdot l \cdot \lambda^{-1} + \tilde{c}(t) \cdot \tilde{Q} \cdot l \quad (8)$$

where l = the total length of the development form (m); \tilde{Q} = the estimated quantity of ore to be transported along the development form (t), $\tilde{Q} = \tilde{P}_r \cdot \tilde{R} \cdot \tilde{P}_r$ = estimated reserves (t); \tilde{R} = reserve recovery ratio (%); $\tilde{\delta}$ = the unit cost of building of a development form (\$/m); $\tilde{c}(t)$ = the unit cost of ore haulage (\$/tm); λ = number of sublevels that will be accessed from the specified development form, or number of sublevels that will use it as a haulage path or both ($\lambda = 1, 2, \dots, k$).

We first analyze the cost of vertical development form, called ‘shaft’. The length of the vertical shaft is the difference in height between the top and the base of the shaft. In the development network, we treat the shaft as a vertical line segment with variable cost of the form:

$$\tilde{f}_s(t) = \tilde{\delta} \cdot |z - z_0| \cdot \lambda^{-1} + \left[\frac{\tilde{a}_s(t)}{n_s} + \tilde{b}_s(t) \cdot |z - z_0| \right] \cdot \tilde{Q} \quad (9)$$

where \tilde{a}_s and \tilde{b}_s are operational parameters associated with the hoist costs, n_s is the number of parts of the shaft between loading point and surface. For example, for the (1–2), (2), (3), (4), (5) ore-haulage path (see Fig. 2), n_s takes a value of 2, and for the (1–2), (1–1), (4), (5) ore-haulage path, n_s takes a value of 1. The shaft cost function for the second ore-haulage path is different from the shaft cost function for the first ore-haulage path, and its value is represented by a ‘bold curve’ between nodes (4) and (5).

The cost function of the horizontal development form, called the ‘drive’, is:

$$\tilde{f}_D(t) = (\tilde{\delta}_D \cdot \lambda^{-1} + \tilde{c}_D(t) \cdot \tilde{Q}) \cdot \sqrt{(x - x_0)^2 + (y - y_0)^2} \quad (10)$$

When we define the cost function of the declined development form called the ‘ramp’, the physical constraint related to transportation path gradient has to be involved. This operational constraint shows that each ramp must have a maximum allowable absolute gradient of r , where r depends on the type of mining transportation equipment, such as haulage trucks. Let x_o, y_o, z_o denote coordinates of point A , and x, y, z denote coordinates of point B in 3-D space. The gradient of the line

Table 1 The transformation of linguistic terms to positive triangular fuzzy numbers

Description	Fuzzy number
VL	(30,40,50) or (0.3,0.4,0.5)
L	(40,50,60) or (0.4,0.5,0.6)
M	(50,60,70) or (0.5,0.6,0.7)
H	(60,70,80) or (0.6,0.7,0.8)
VH	(70,80,90) or (0.7,0.8,0.9)

Table 2 Expert estimation of reserve recovery ratio

Expert				Aggregated value
E_1	E_2	...	E_p	$1/p \otimes (\tilde{R}_1 \oplus \tilde{R}_2 \oplus \dots \oplus \tilde{R}_p)$
\tilde{R}_1	\tilde{R}_2	...	\tilde{R}_p	

(β), connecting these two points, is equal to absolute value of the slope from A to B . The cost function of the ramp is:

$$\tilde{f}_R(t) = \begin{cases} (\tilde{\delta}_R \cdot \lambda^{-1} + \tilde{c}_R(t) \cdot \tilde{Q}) \cdot \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} & \text{if } \beta \leq r \\ (\tilde{\delta}_R \cdot \lambda^{-1} + \tilde{c}_R(t) \cdot \tilde{Q}) \cdot |z - z_0| \cdot \sqrt{1 + r^{-2}} & \text{if } \beta > r \end{cases} \quad (11)$$

The cost function of the ore-pass is a little bit different from the general cost function because there is no second term related to haulage cost. Gravitation is used as a way to transport ore. The cost function of the ore-pass is:

$$\tilde{f}_p = \begin{cases} (\tilde{\delta}_p \cdot \lambda^{-1} \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}) & \text{if } \alpha \leq 90^0 \\ \tilde{\delta}_p \cdot \lambda^{-1} \cdot |z - z_0| & \text{if } \alpha > 90^0 \end{cases} \quad (12)$$

where α is the gradient of the ore-pass.

The ore reserve quantities to be excavated and transported depend directly on the way of stoping, called the ‘mining method’. Efficiency of the mining method can be expressed by the reserve recovery ratio. Fuzzy linguistic variables used to describe recovery ratio are as follows: very low (VL), low (L), medium (M), high (H) and very high (VH). The next step transforms the fuzzy linguistic variables to triangular fuzzy numbers, as shown in Table 1.

To estimate an adequate value for the reserve recovery ratio, it is necessary to get opinions of experts dealing with underground mining methods. Suppose we have p experts and each of them has given their opinion. The final value is expressed by an aggregated fuzzy number obtained by averaging the fuzzy opinions of the experts. Table 2 presents the expert estimation process.

The concept of the triangular fuzzy numbers is also applied to the estimation of the unit cost of construction of development form. For example, the cost of a shaft-sinking operation depends directly on rock mass properties. Generally, it is very

hard to predict with certainty what may happen when sinking at depth. Accordingly, we cannot define the cost of sinking as a crisp value; we use triangular fuzzy number $\tilde{\delta}(\delta_1, \delta_2, \delta_3)$ to define it.

The uncertainties related to the future states of costs of transportation are modeled using a special stochastic process, Geometric Brownian Motion. Certain stochastic processes are functions of a Brownian motion process and have many applications in finance, engineering and the sciences. Some special processes are solutions of Itô-Doob-type stochastic differential equations [14]. We applied a continuous time process using the Itô-Doob-type stochastic differential equation to describe movement of unit costs of transportation in this study:

$$dc_t = \mu c_t dt + \sigma c_t dW_t \tag{13}$$

where μ is the drift and σ is the volatility, W_t is a normalized Brownian motion. In order to estimate the parameters of the Brownian motion process (μ, σ) , we run the following regression:

$$dx_{t+1} = \beta_0 + \beta_1 x_t + \varepsilon.$$

The main objective of using simulation in the selection of the development system is to determine the distribution of the unit costs of transportation for every year of the project. In this way, we obtain the sequence of probability density functions of unit costs, $c_i \sim (pdf_i, \mu_i, \sigma_i)$, $i = 1, 2, \dots, T$, where T is a total project time. Sequence of obtained pdf_i of unit costs can be transformed into a sequence of triangular fuzzy numbers of unit costs, $c_i \sim TFN_i$, $i = 1, 2, \dots, T$, i.e., $c_1 \sim pdf_1 \rightarrow c_1 \sim TFN_1$; $c_2 \sim pdf_2 \rightarrow c_2 \sim TFN_2$; ...; $c_i \sim pdf_i \rightarrow c_i \sim TFN_i$. The way of transformation is based on the following fact;

the support of the membership function and the pdf are the same, and the point with higher probability (likelihood) has the higher possibility. For more details, see [15]. The uncertainty in the parameter is modeled by triangular fuzzy number with the membership function, which has the support of $\eta - 2\sigma < X < \eta + 2\sigma$, set up for around 95 % confidence interval of normal distribution function. If we take into consideration that the triangular fuzzy number is defined as a triplet (a, b, c) , then a and c are lower bound and upper bound obtained from lower and upper bound of 5 % of the distribution, and the most promising value b is equal to mean value of the distribution [16].

The production plan can be defined approximately, as follows:

$$\tilde{t}_\phi \in [\tilde{t}_{\phi-1}, \tilde{t}_{\phi-1} + \tilde{t}_s] \quad \phi = 1, 2, \dots, k \tag{14}$$

where

$$\tilde{t}_{\phi-1} = \tilde{t}_c + \frac{\sum_{\phi=1}^k \tilde{Q}_{\phi-1}}{Y_p} \tag{15}$$

$$\tilde{t}_s = \frac{\tilde{Q}_\phi}{Y_p} \tag{16}$$

$\tilde{t}_{\phi-1}$ —point when the previous sublevel is mined and ϕ -th sublevel is started to be mined (year);

$\tilde{t}_{\phi-1} + \tilde{t}_s$ —point when the ϕ -th sublevel is mined and $\phi + 1$ sublevel is started to be mined (year);

\tilde{t}_c —construction period (year); Y_p —yearly production rate (t/year); \tilde{t}_s —time of mining of ϕ -th sublevel (year); k —total number of sublevels.

According to the defined production plan, unit costs of transportation related to the ϕ -th sublevel are expressed as follows:

$$\tilde{c} = f(E(\tilde{t}_\phi)) \tag{17}$$

where $E(\tilde{t}_\phi)$ —de-fuzzified value of fuzzy triangular number $\tilde{t}_\phi = (a, b, c)$ It is obtained according to the following equation: $(a + b + c)/3$.

In this way, the interval of time within each sublevel to be mined is transformed into a crisp interval time; i.e.:

$$t_\phi \in [E(\tilde{t}_{\phi-1}), E(\tilde{t}_{\phi-1} + \tilde{t}_s)] \quad \phi = 1, 2, \dots, k \tag{18}$$

Unit costs of transportation related to mining of the ϕ -th sublevel are calculated as follows:

$$\tilde{c}_\varphi(t) = \frac{\tilde{c}[E(\tilde{t}_{\phi-1})] + \tilde{c}[E(\tilde{t}_{\phi-1} + \tilde{t}_s)]}{2} \quad \varphi = 1, 2, \dots, k \tag{19}$$

where $\tilde{c}[E(\tilde{t}_{\phi-1})]$ —unit costs of transportation in the year when mining of the ϕ -th sublevel is started; $\tilde{c}[E(\tilde{t}_{\phi-1} + \tilde{t}_s)]$ —unit costs of transportation in the year when mining of the ϕ -th sublevel is over.

3.3 Model of Selection of Development System

The selection of the underground mine development system in a fuzzy environment is the task of finding the shortest path in a fuzzy weighted network. In this paper, we apply an algorithm for the fuzzy shortest path problem based on the Convex Index [17]. To apply this algorithm, it is necessary to introduce the following definitions.

Definition 1 The α -cut interval is obtained as follows for all $\alpha \in [0, 1]$:

$$M_\alpha^L = \alpha \cdot (b - a) + a; \quad M_\alpha^U = c - \alpha \cdot (c - b)$$

Definition 2 Convex Index (CoI); Let \tilde{M} be a triangular fuzzy number, then $CoI(\tilde{M}) = \lambda \cdot (M_\alpha^L) + (1 - \lambda) \cdot (M_\alpha^U)$ where $[M_\alpha^L, M_\alpha^U]$ is the α -cut interval of $\tilde{M} = (a, b, c)$, for all $\alpha, \lambda \in [0, 1]$, where λ is the index of optimism. If \tilde{A} and \tilde{B} are two triangular numbers, then in the Convex Index, we have $\tilde{A} < \tilde{B}$ if $CoI(\tilde{A}) < CoI(\tilde{B})$

According to the above derivation, we can pose our problem for one access point (sublevel) as follows: **Input**: for each access point (sublevel), a mine designer creates a directed haulage network of n nodes with fuzzy edge cost functions. **Output**: the fuzzy least cost function and corresponding least cost development haulage path.

Algorithm for solving the problem of selection of underground mine development system is composed of the following steps:

1. **Step1**. For every year of the project time, simulate unit costs $c_i(t)$, $i = 1, 2, \dots, T$ of transportation, with respect to the form of development (shaft, decline, . . .), using Eq. (13).
2. **Step2**. Transform $c_i \sim (pdf_i, \mu_i, \sigma_i)$, $i = 1, 2, \dots, T$ into adequate fuzzy triangular number $c_i \sim \text{TFN}_i$.
3. **Step3**. Construct a haulage network from first sublevel to the surface.
4. **Step4**. Form the possible haulage paths from source vertex (access point) to destination node (surface breakout point), and compute the corresponding cost functions $F_i(X, t) = (a_i(t), b_i(t), c_i(t))$, $i = 1, 2, \dots, y$, for possible y paths.
5. **Step5**. Calculate α -cut interval for triangular fuzzy number for all possible path cost functions $F_i(X, t) = (a_i(t), b_i(t), c_i(t))$, $i = 1, 2, \dots, y$, using Definition 1. Set $F_{i(\alpha)} = [F_{i(\alpha)}^L, F_{i(\alpha)}^U]$, $i = 1, 2, \dots, y$.
6. **Step6**. Calculate Convex Index $CoI(\tilde{F}_i) = \lambda \cdot (F_{i(\alpha)}^L) + (1 - \lambda) \cdot (F_{i(\alpha)}^U)$, for all possible path cost functions $F_i(X, t) = (a_i(t), b_i(t), c_i(t))$, $i = 1, 2, \dots, y$, using Definition 2.
7. **Step7**. Determine the actual least cost development haulage path with the minimum CoI_i and assign a value of 1 to it.
8. **Step8**. Make a rank order of the rest of paths according to ascending order of Convex Index, and assign value to each path $(2, 3, \dots, y)$.
9. **Step9**. As the ore body is dipping, repeat *Step 3* to *Step 8* for all defined access points (for k sublevels).
10. **Step10**. Create (S, CoI_i) diagram where x -axis denotes sublevel where the access point is located (S), and y -axis denotes the Convex Index (CoI_i).
11. **Step11**. According to *Step 10*, create (S, A_v) diagram, where x -axis denotes the sublevel where the access point is located (S), and y -axis denotes the assigned value (A_v).
12. **Step12**. Form the set of evaluations of alternatives E [see Eq. (6)].
13. **Step13**. For each row of E matrix, compute the sum of all the terms in the row. The alternative that corresponds to the row with the minimum sum of the terms is the optimal development system for given ore deposit [see Eq. (7)].

4 Numerical Example

4.1 A Numerical Example Statement

To illustrate the proposed procedure, we applied it to a study considering the selection of an underground mine development system needed for the exploitation of a bauxite deposit. Since the stripping ratio is approaching the planned value, the management of the company is faced with the problem of increasing production costs at an active open-pit mine. The management has decided to start a project of underground mining of the remaining bauxite reserves. Part of the deposit that should be mined is located between level 710 and level 494. Deposit is inclined at an angle of about 70° . The underground mine should be designed for the capacity of production of 200,000 t/year. According to geological and mining conditions, the sublevel mining method is selected. The height of sublevel is 8 m. Two geological and two mining experts ($E_p = 4$) were consulted to estimate the adequate values of reserve recovery ratio for each sublevel separately. The relevant operational data related to bauxite reserves are shown in Table 3. Data related to costs of construction and transportation are given in Table 4.

4.2 Numerical Example Solution

Three development systems have been evaluated: Decline development system; Shaft development system; and Shaft-Decline development system.

Decline development system: main decline starts from level 768 and ends on level 718. The length of the main decline is 452 m and the gradient is 1:9. First sublevel access decline starts from level 718 and ends on the first mining level 710. The ore is removed from stope using Load Haul Dump vehicles. The ore is then dumped into a mine truck to be hauled to the surface via main decline. Construction period is $\tilde{t}_c = (1 \ 1, 2 \ 1, 5)$ year. There is no need to interrupt the production when lower levels are developed for mining.

Shaft development system: the shaft is sunk from level 768 to level 662. On level 718, the horizontal drive is constructed from the shaft to the deposit. Access decline is constructed from the horizontal drive to sublevel 710, and further via sublevels 702, 694, 686, 678 and 670, to the main transport level 662. Ore-pass connects sublevels 710, 702, 694, 686, 678 and 670 with the main transport level 662. Main transport level 662 connects ore-pass with the loading point located near the shaft. The ore is removed from the stope using Load Haul Dump vehicles and dumped down an ore-pass, where it falls to the main transport level 662. The ore is then dumped into a mine truck and transported to the shaft, to be hoisted up to the surface in skips and emptied into bins at the surface. Construction period is $\tilde{t}_c = (1, 5 \ 1, 7 \ 2)$ year.

Table 3 Bauxite reserves

Sublevel	Estimated reserves (t)			Reserve recovery ratio (%)			Recoverable reserves (t)		
	ai	bi	ci	ai	bi	ci	ai	bi	ci
710	115,067	121,123	127,179	55	65	75	63,287	78,730	95,384
702	123,160	129,642	136,124	47.5	57.5	67.5	58,501	74,544	91,884
694	87,392	91,992	96,592	52.5	62.5	72.5	45,881	57,495	70,029
686	107,449	113,105	118,760	55	65	75	59,097	73,518	89,070
678	150,927	158,870	166,814	52.5	62.5	72.5	79,237	99,294	120,940
670	188,269	198,178	208,086	52.5	62.5	72.5	98,841	123,861	150,863
662	217,348	228,787	240,227	52.5	62.5	72.5	114,108	142,992	174,164
654	194,815	205,069	215,322	52.5	62.5	72.5	102,278	128,168	156,109
646	158,617	166,965	175,313	52.5	62.5	72.5	83,274	104,353	127,102
638	103,038	108,461	113,884	52.5	62.5	72.5	54,095	67,788	82,566
630	63,956	67,322	70,688	47.5	57.5	67.5	30,379	38,710	47,714
622	44,488	46,829	49,171	55	65	75	24,468	30,439	36,878
614	42,542	44,782	47,021	55	65	75	23,398	29,108	35,265
606	54,808	57,692	60,577	55	65	75	30,144	37,500	45,433
598	76,494	80,520	84,546	47.5	57.5	67.5	36,335	46,299	57,069
590	80,859	85,115	89,371	47.5	57.5	67.5	38,408	48,941	60,325
582	78,660	82,800	86,940	47.5	57.5	67.5	37,364	47,610	58,685
574	53,375	56,184	58,994	60	70	80	32,025	39,329	47,195
566	67,426	70,974	74,523	60	70	80	40,455	49,682	59,618
558	130,343	137,203	144,063	60	70	80	78,206	96,042	115,250
550	221,008	232,640	244,272	52.5	62.5	72.5	116,029	145,400	177,097
542	296,061	311,643	327,225	52.5	62.5	72.5	155,432	194,777	237,238
534	305,941	322,043	338,145	60	70	80	183,564	225,430	270,516
526	424,721	447,075	469,429	47.5	57.5	67.5	201,742	257,068	316,864
518	558,222	587,602	616,982	47.5	57.5	67.5	265,155	337,871	416,463
510	599,442	630,992	662,542	52.5	62.5	72.5	314,707	394,370	480,343
502	547,060	575,853	604,645	52.5	62.5	72.5	287,207	359,908	438,368
494	371,424	390,973	410,521	52.5	62.5	72.5	194,998	244,358	297,628

When lower levels are developed, it is necessary to interrupt the production. Construction period is $\tilde{t}_c = (0, 8 \ 1 \ 1, 2)$ year.

Shaft-Divide development system: this development system is the combination of the two previously described systems. The ore is removed from the stope using Load Haul Dump vehicles. The ore is then dumped into a mine truck to be hauled up to the main transport level via decline, and transported to the shaft to be hoisted up to the surface in skips and emptied into bins at the surface. Construction period is $\tilde{t}_c = (1, 4 \ 1, 6 \ 1, 9)$ year. When lower levels are developed, it is necessary to interrupt the production. Construction period is $\tilde{t}_c = (0, 6 \ 0, 8 \ 1, 1)$ year.

The procedure of finding the optimal development system for the given example is executed as follows:

Step1 and Step2. Simulation of unit costs $c_i(t), i = 1, 2, \dots, T$ of transportation with respect to the form of development and transformation of $c_i \sim (pdf_i, \mu_i, \sigma_i), i = 1, 2, \dots, T$ into adequate

triangular fuzzy number $c_i \sim TFN_i$ is presented in Tables 5, 6, 7 and 8.

Step3. Construction of haulage network from the first sublevel S-710 to the surface is presented in Fig. 3.

Step4. Possible haulage paths from sublevel S-710 to the surface and corresponding cost functions. Path p_1 : Ore-pass 710–662; main transport level 662–662; Shaft 662–718–768; corresponding fuzzy cost function: $F_1(278023 \ 313664 \ 352924)$ \$; path p_2 : access decline 710–718; main decline 718–768; corresponding fuzzy cost function: $F_2(72520 \ 93826 \ 119664)$ \$; path p_3 : access decline 710–718; main transport level 718–718; Shaft 718–768; corresponding fuzzy cost function: $F_3(95217 \ 112967 \ 130693)$ \$. **Step 5.** α -cut interval of triangular fuzzy cost function for $\alpha = 0.5$; see Table 9.

Step6. Convex Index of triangular fuzzy cost function for $\lambda = 1$; see Table 10.

Table 4 Costs

Cost	
Haulage and Hoist	
Drive (mine truck). Simulation of Geometric Brownian Motion, yearly time resolution Eq. (13)	Spot value 0.8 \$/tkm Cost volatility 0.04 Drift 0.02
Decline (mine truck). Simulation of Geometric Brownian Motion, yearly time resolution, Eq. (13)	Spot value 1.2 \$/tkm Drift 0.03 Cost volatility 0.06
Shaft (skip system). Simulation of Geometric Brownian Motion, yearly time resolution, Eq. (13)	Parameter a_s Spot value 0.4 \$/t Drift 0.006 Cost volatility 0.014 Parameter b_s Spot value 0.6 \$/tkm Drift 0.0012 Cost volatility 0.024
Construction	
Drive	(1,400 1,600 1,900) \$/m
Decline	(2,000 2,200 2,400) \$/m
Shaft	(20,000 23,000 26,000) \$/m
Ore-pass	(200 250 300) \$/m

Step7 and Step8. Rank order of the haulage paths according to ascending order of Convex Index; see Table 11.

Step9. Rank order of the haulage paths for all sublevels; see Table 12.

Step10. S, Col_i diagram (see Fig. 4).

Step11. S, A_v diagram according to data from Table 12 (see Fig. 5).

Step12. Set of evaluations of underground mine development systems composed of assigned values, i.e., ranks.

$$E = \begin{vmatrix} 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 2 & 2 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 \end{vmatrix}$$

Step13. For the raw data of matrix E, we obtain the sums presented in Table 13.

Haulage paths, from sublevel S-710 to the surface, are weighted by corresponding fuzzy cost functions. Since they are represented in fuzzy format, a ranking procedure based on fuzzy number comparison needs to be applied. Comparison procedure was carried out in Step 5 and 6, where α -cut interval and Convex Index of fuzzy number were used as a basis for comparison. We assigned value of 1 to the haulage path p_2 , value of 2 to p_3 , and value of 3 to p_1 , according to ascending order of Convex Index; Step 7 and 8. The procedures of comparison and assignment were repeated for the rest sublevels (S-702, ..., S-494), and S, Col_i and S, A_v diagrams were created on the basis of obtained outcomes; Steps 9–11. The number of columns of matrix E is equal to the total number of sublevels, while the number of rows corresponds to the number of underground mine development systems which we evaluated. Elements of matrix E are equal to numerical outcomes (ranks) of the assignment procedure; Step 12.

Obtained composite rank order indicates the following rank of underground mine development systems: Shaft-Decline development system; Decline development system; and Shaft development system.

For comparison purposes, Fig. 6 shows the changeover points between proposed underground mine development systems, with respect to mine depth and sublevel ore reserves.

Table 5 Simulation and transformation of costs related to Drive development form

Year	0	1	2	...	25
η (\$/tkm)	0.80000	0.81494	0.83128	...	1.31495
σ	0.00000	0.03260	0.04799	...	0.25327
TFN _i					
$a_i = \eta - 2 \times \sigma$	0.80000	0.74974	0.73531	...	0.80840
$b_i = \eta$	0.80000	0.81494	0.83128	...	1.31495
$c_i = \eta + 2 \times \sigma$	0.80000	0.88013	0.92725	...	1.82150

Table 6 Simulation and transformation of costs related to Decline development form

Year	0	1	2	...	25
η (\$/tkm)	1.20000	1.23811	1.27721	...	2.46461
σ	0.00000	0.07611	0.11310	...	0.73440
TFN _i					
$a_i = \eta - 2 \times \sigma$	1.20000	1.08588	1.05101	...	0.99581
$b_i = \eta$	1.20000	1.23811	1.27721	...	2.46461
$c_i = \eta + 2 \times \sigma$	1.20000	1.39034	1.50341	...	3.93340

Table 7 Simulation and transformation of costs related to Shaft development form-a_s parameter

Year	0	1	2	...	25
η (\$/t)	0.40000	0.40272	0.40538	...	0.46522
σ	0.00000	0.00550	0.00803	...	0.03229
TFN _i					
$a_i = \eta - 2 \times \sigma$	0.40000	0.39172	0.38932	...	0.40062
$b_i = \eta$	0.40000	0.40272	0.40538	...	0.46522
$c_i = \eta + 2 \times \sigma$	0.40000	0.41373	0.42144	...	0.52981

Table 8 Simulation and transformation of costs related to Shaft development form-b_s parameter

Year	0	1	2	...	25
η (\$/tkm)	0.60000	0.60818	0.61562	...	0.80731
σ	0.00000	0.01483	0.02141	...	0.09427
TFN _i					
$a_i = \eta - 2 \times \sigma$	0.60000	0.57853	0.57280	...	0.61876
$b_i = \eta$	0.60000	0.60818	0.61562	...	0.80731
$c_i = \eta + 2 \times \sigma$	0.60000	0.63783	0.65845	...	0.99585

Figures 4, 5, 6 indicate the various changeover points, for various mine depths, sublevel ore reserves and fixed production rates, among the Decline, Shaft and Shaft-Decline development systems. As seen from the mentioned figures, at 200,000 t/year production rate, the first changeover point is at 82 m mine depth. This indicates that Decline is better option up to 82 m than Shaft-Decline, while Shaft option is completely unfavorable. If we neglect for a moment the changeover point at 162 m mine depth, where Decline is a better option than Shaft-Decline, it can be seen that Shaft-Decline has an advantage over Decline from 82 to 266 m mine depth. This transition is caused by significant decrease in ore reserves from sublevel 638 (122 m mine depth) to 598 (162 m mine depth). Although the decrease in ore reserves from sublevel 638 to sublevel 630 was about 50 %, it was not enough for instantaneous transition. Five sublevels, with the same decreased reserves, were needed for the realization of the transition. It indicates the mine depth has a greater influence on the transition than the decrease in ore reserves at deeper sublevels. A very important changeover point is at 210 m mine depth, where Shaft becomes a better option than Decline. It indicates that Decline is a cheaper option up to 210 m, while the Shaft option is economically viable beyond 210 m. With increasing mine depth and sublevel ore reserves, the Shaft development system becomes the more economical system. Supremacy of the Shaft over Decline option, beyond 210 m, is also confirmed by the changeover point at 258 m mine depth, where Shaft is even better than the Shaft-Decline option.

The dynamic and fuzzyfied nature of the model makes the decision-making environment more realistic and the obtained

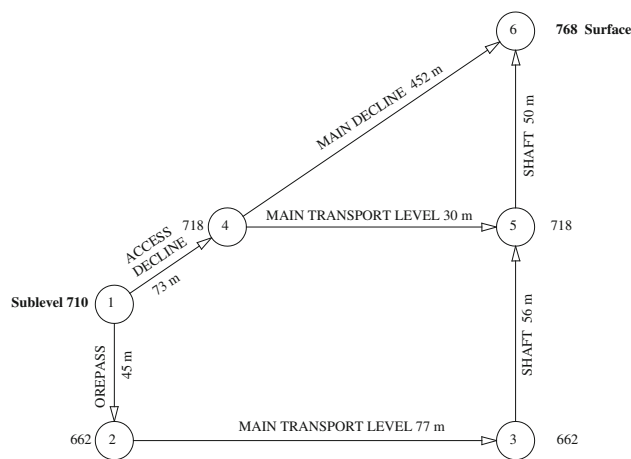


Fig. 3 Haulage network from sublevel S-710 to surface

Table 9 α -cut interval

Triangular fuzzy cost function	Value	$F_{i(0,5)} = [F_{i(0,5)}^L, F_{i(0,5)}^U]$
F_1	(278,023 313,664 352,924)	[295,843, 333,294]
F_2	(72,520 93,826 119,664)	[83,172, 106,745]
F_3	(95,217 112,967 130,693)	[104,091, 121,830]

Table 10 Convex Index

Triangular fuzzy cost function	$CoI(\tilde{F}_i)$
F_1	295,844
F_2	83,173
F_3	104,092

Table 11 Rank order of the haulage paths for sublevel S-710

Triangular fuzzy cost function	$CoI(\tilde{F}_i)$	Rank order
F_1	295844	3
F_2	83173	1
F_3	104092	2

Table 12 Rank order of the haulage paths

Sublevel	System	Rank	Sublevel	System	Rank
710	S;D;SD	3;1;2	598	S;D;SD	3;1;2
702	S;D;SD	3;1;2	590	S;D;SD	3;2;1
694	S;D;SD	3;1;2	582	S;D;SD	3;2;1
686	S;D;SD	3;1;2	574	S;D;SD	3;2;1
678	S;D;SD	3;2;1	566	S;D;SD	3;2;1
670	S;D;SD	3;2;1	558	S;D;SD	3;2;1
662	S;D;SD	3;2;1	550	S;D;SD	2;3;1
654	S;D;SD	3;2;1	542	S;D;SD	2;3;1
646	S;D;SD	3;2;1	534	S;D;SD	2;3;1
638	S;D;SD	3;2;1	526	S;D;SD	2;3;1
630	S;D;SD	3;2;1	518	S;D;SD	2;3;1
622	S;D;SD	3;2;1	510	S;D;SD	2;3;1
614	S;D;SD	3;2;1	502	S;D;SD	1;3;2
606	S;D;SD	3;2;1	494	S;D;SD	2;3;1

Legend: S-shaft; D-decline; SD-shaft-decline

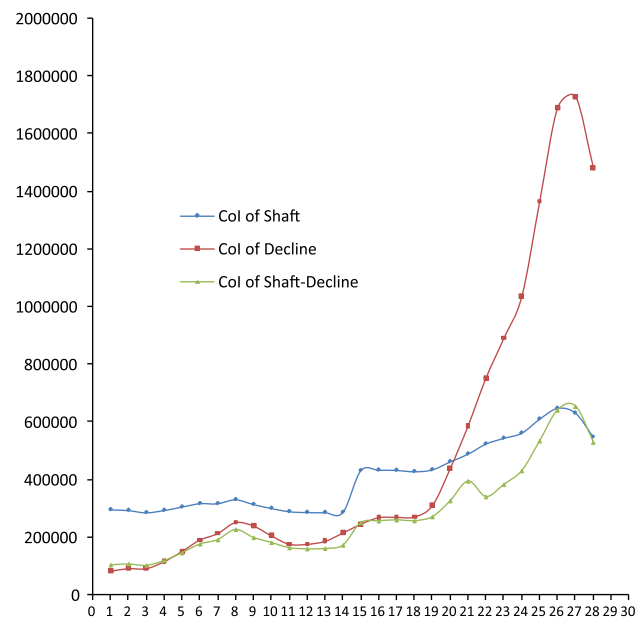


Fig. 4 Convex Index diagram

results more reliable. Network modeling gives the opportunity for the underground mine to be presented almost as a real physical model.

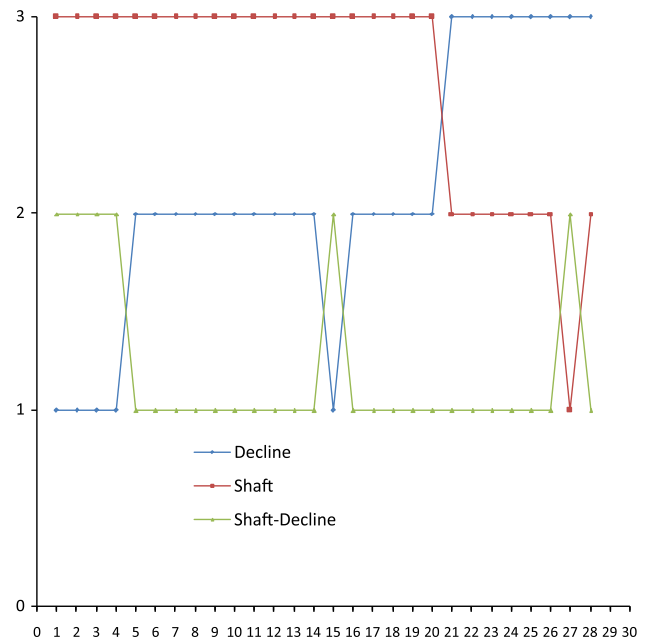


Fig. 5 Rank order diagram

Table 13 Sums of all terms in the rows

	Shaft	Decline	Shaft-Decline
Sum of assigned ranks	75	59	34
Composite rank	3	2	1

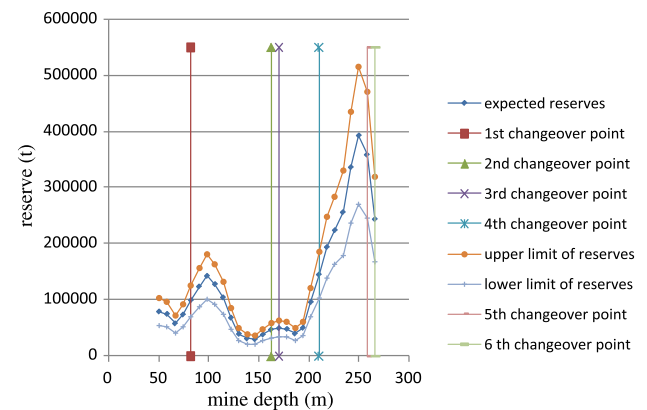


Fig. 6 Changeover points between development systems

5 Conclusion

The selection of an underground mine development system is classified as a strategic decision-making process, which has the most influence on the future of a mine. A major design task is to determine the mine development system needed to provide access to each sublevel and provide haulage paths to transport excavated ore to the surface. We consider the development system as a network interconnecting all sublevels with surface breakout point, using the minimum costs

of development and haulage. The selection model takes into account costs related to the construction of the development system, quantities of ore to be transported, lengths of transportation paths and unit costs of ore transportation. In essence, the selection of the underground mine development system corresponds to the selection of any other investment proposal. If we take into consideration this fact, then the network model enables a decision maker, who is not familiar with underground mining, to view this problem in a completely understandable way. The relevance of the proposed model is supported by the fact that the results were obtained under uncertainty. One of the main advantages of the proposed methodology is the quantification of uncertainties by fuzzification of the input data. In this way, we included risks into the process of strategic decision making, such as the selection of underground mine development system. Future scope of the work may consider to the extension of the model in the sense of a multi-criteria decision-making process, where composite rank is retained as one criterion and new criteria are added. New criteria can include indicators such as equipment reliability, complexity of construction and complexity of transportation route.

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