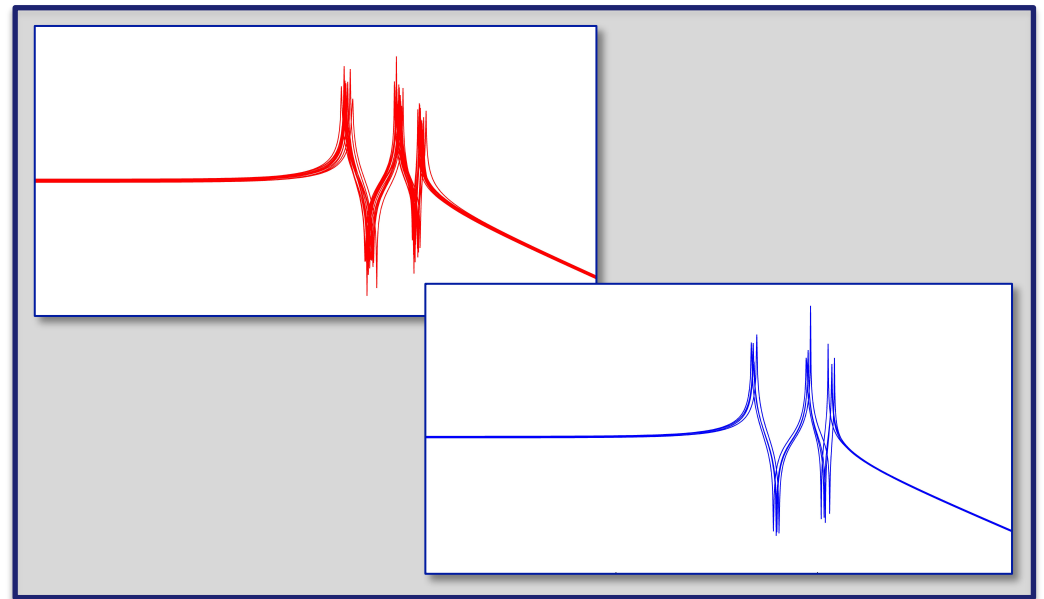


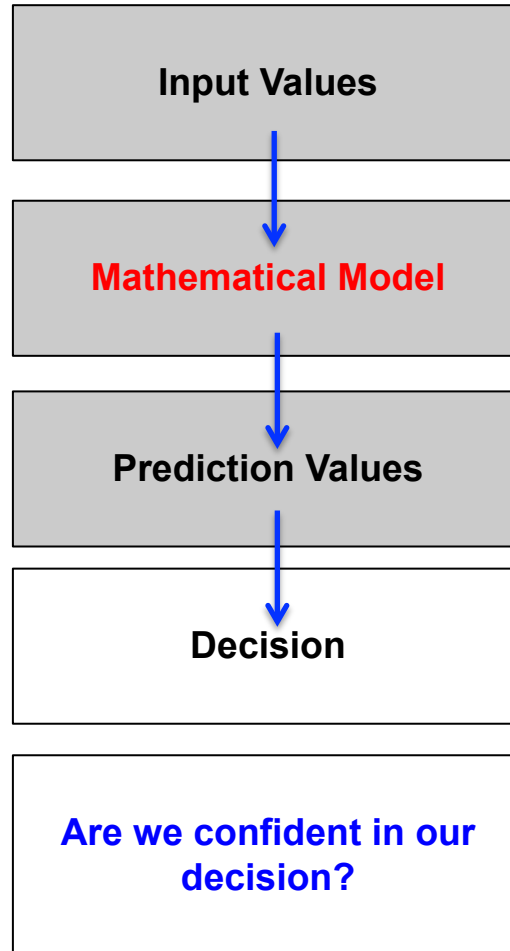
# Investigation of a Statistical Metric for Validation of a Structural Dynamics Model

April Morton  
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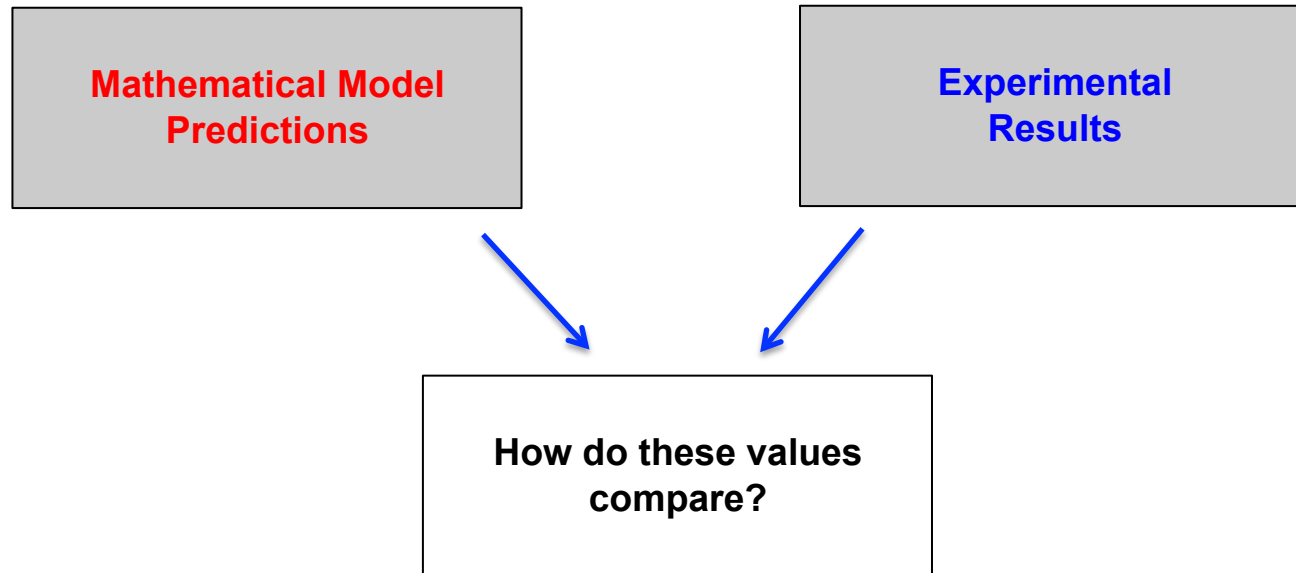
01 August 2011



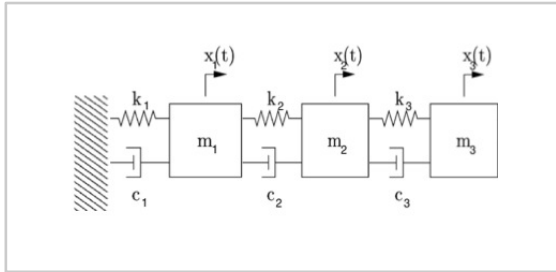
# Mathematical models used to make important predictions must be validated to increase credibility and accuracy



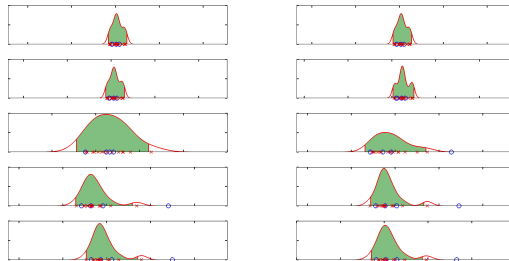
# Model validation is the act of comparing model predictions and experimental data to establish model credibility



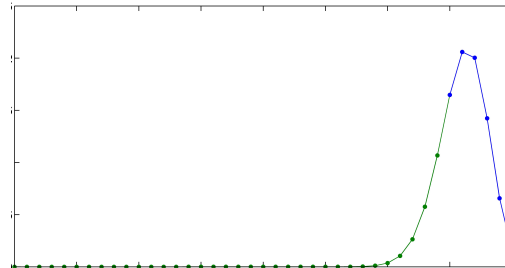
# This presentation outlines a Statistical model validation technique



The Mathematical model

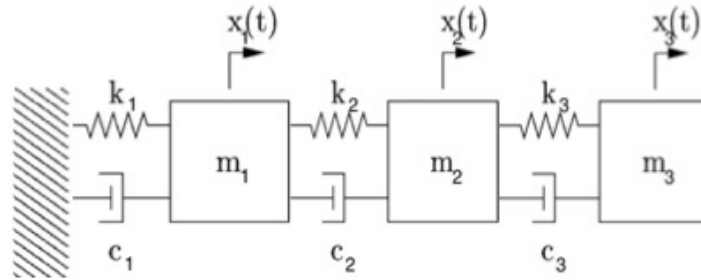


A Statistical Comparison



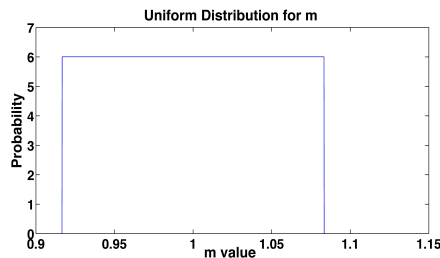
Results

# The spring-mass-damper system has three degrees of freedom and nine randomly varying parameters

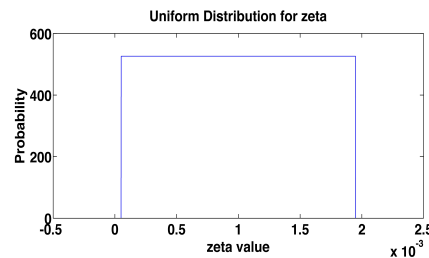


$$M\ddot{x} + C\dot{x} + Kx = F$$

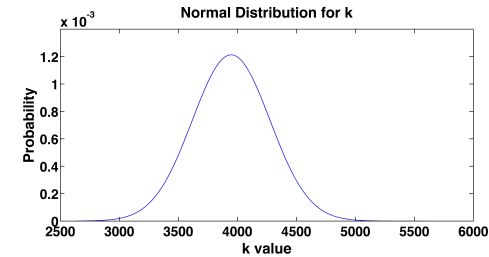
**m**



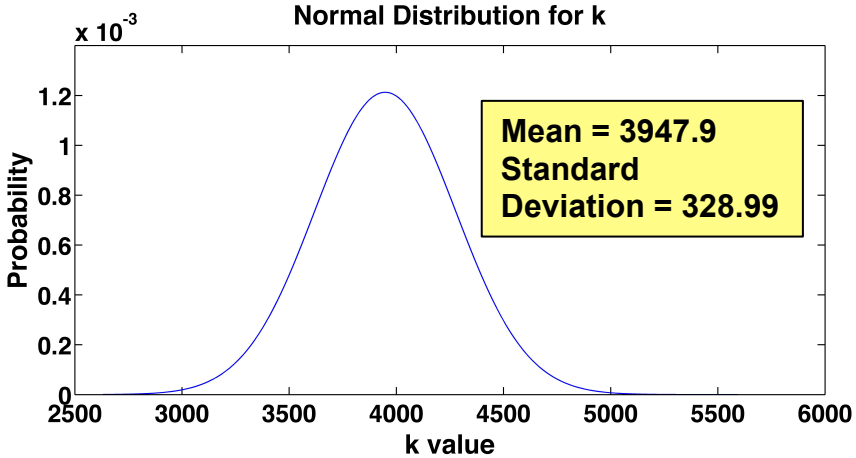
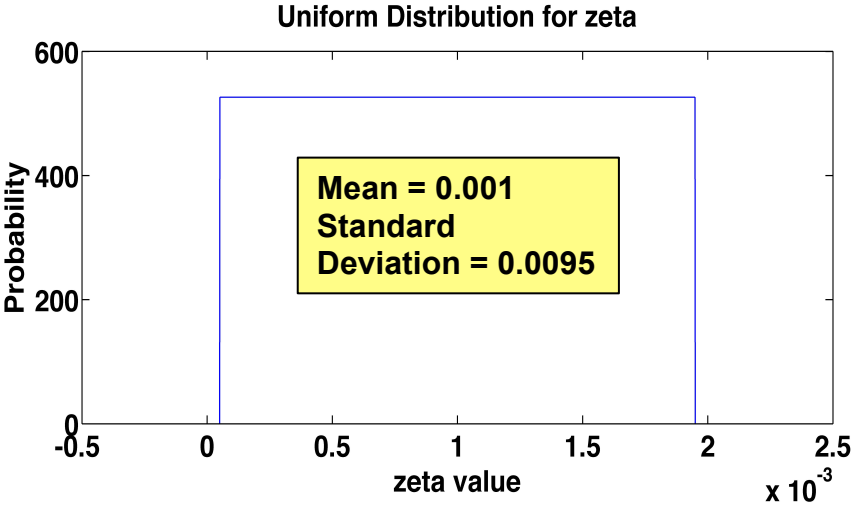
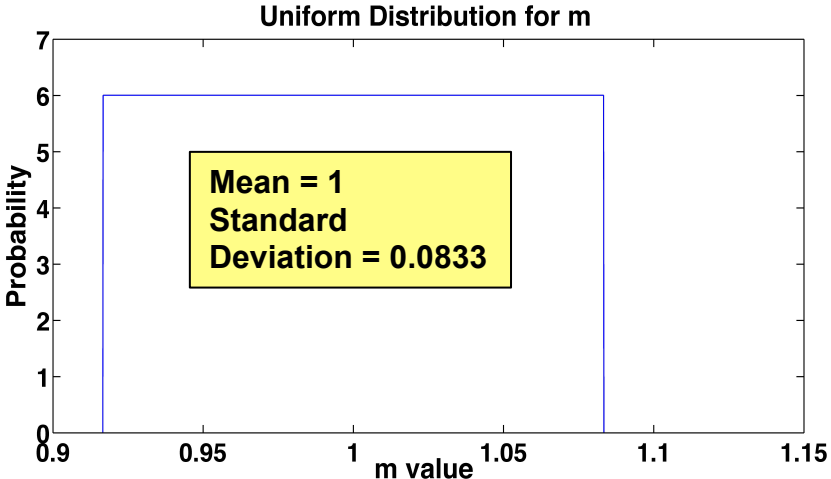
**zeta**



**k**

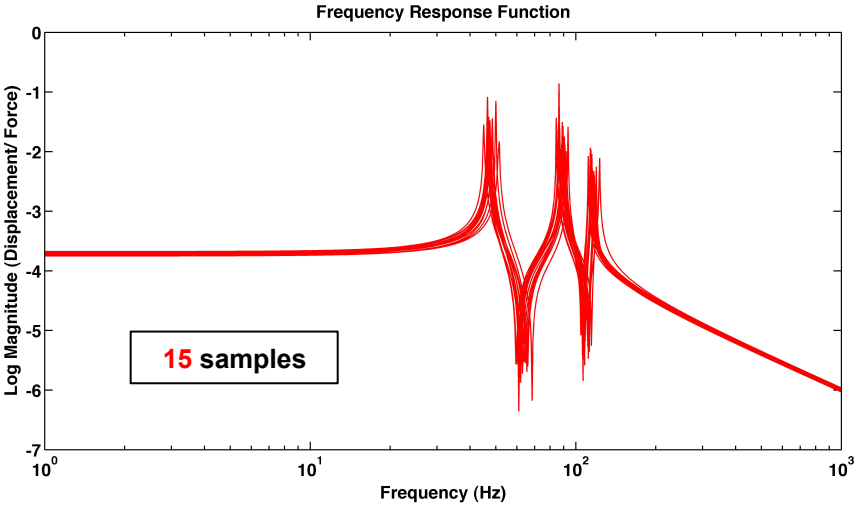


# The means and standard deviations for the distributions affecting $m_n$ , $c_n$ , and $k_n$ are given below

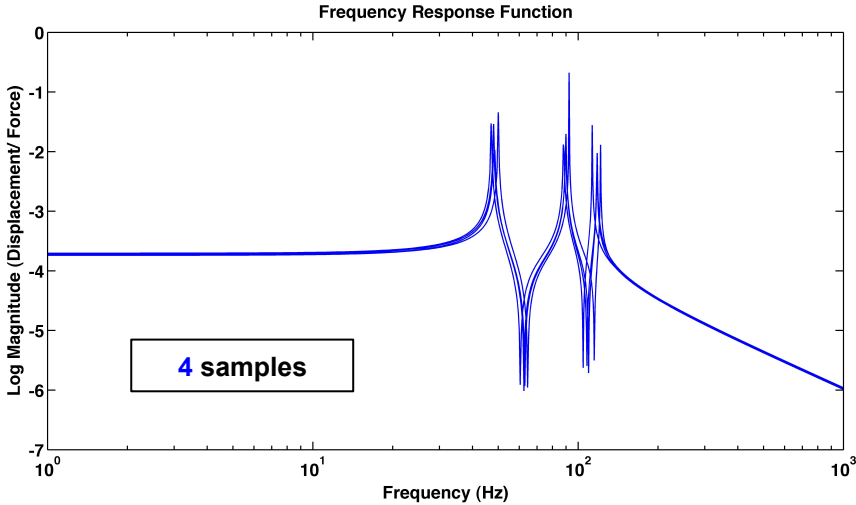


# Generate realizations of the model-predicted and experimental frequency response functions

**Mathematical Model:**  
 $M\ddot{x} + C\dot{x} + Kx = F$



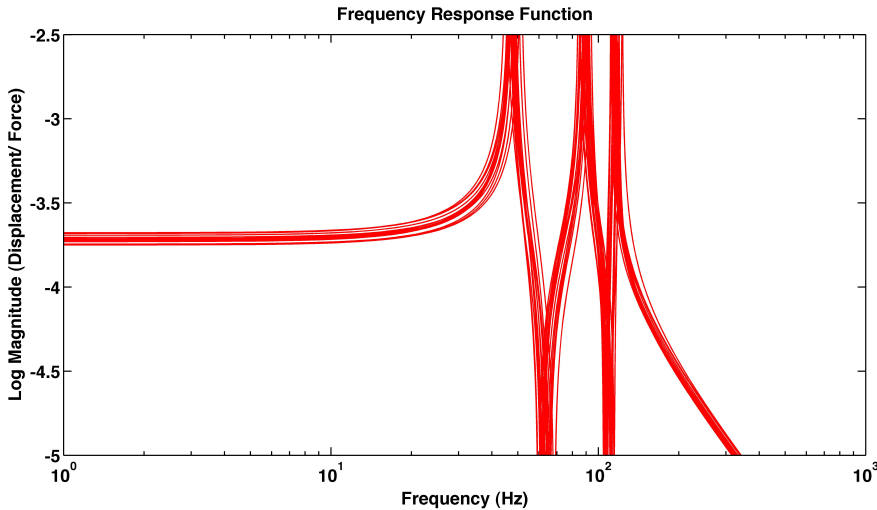
**Physical Experiment:**  
Generated by Model



**Are these measures statistically consistent?**



# Calculate frequency averaged characteristics of the functions to obtain discretized response measures

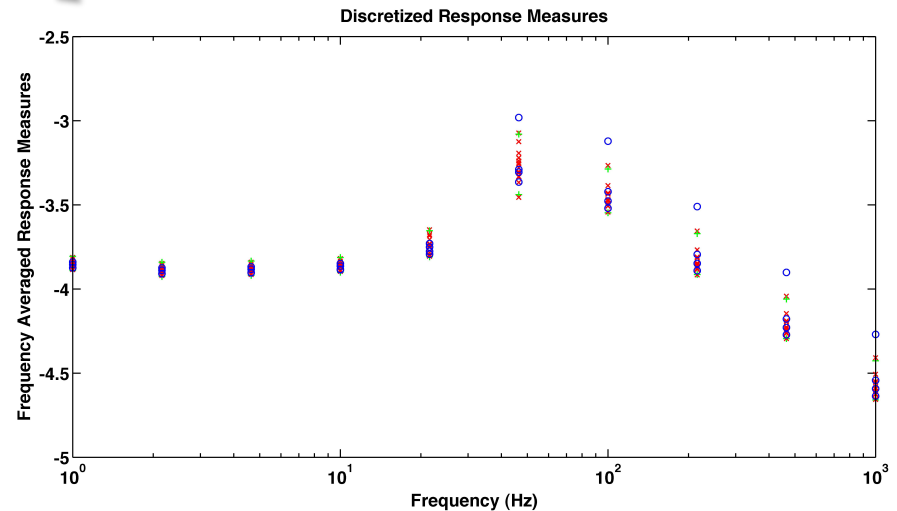


$$response = \int_0^{\infty} (truncated \ gaussian)(magnitude) df$$

$$= \int_0^{\infty} \frac{1}{\sqrt{2\pi\gamma}} e^{-(f-f_j)/2\gamma^2} |H_i^{(mod)}(f)| df$$

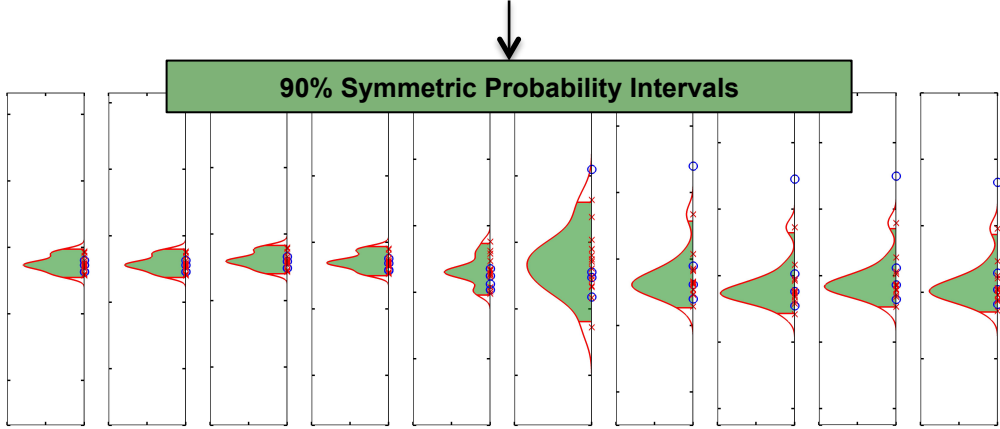
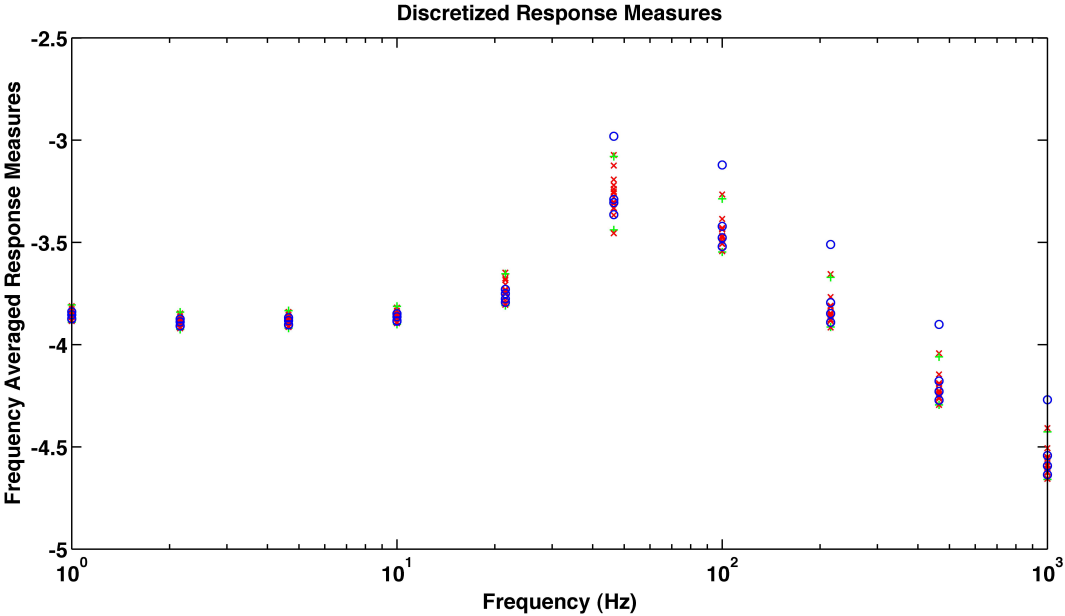
$$\gamma = f_j$$

$$-2\gamma < f_j < 2\gamma$$

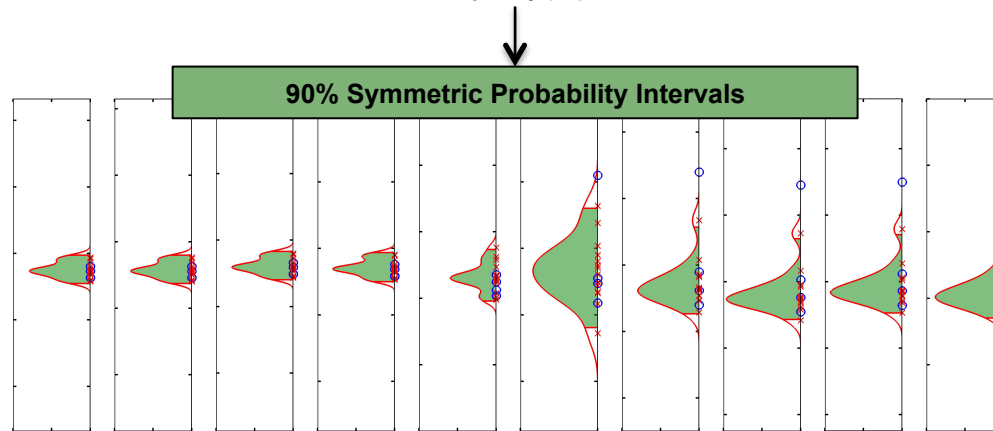
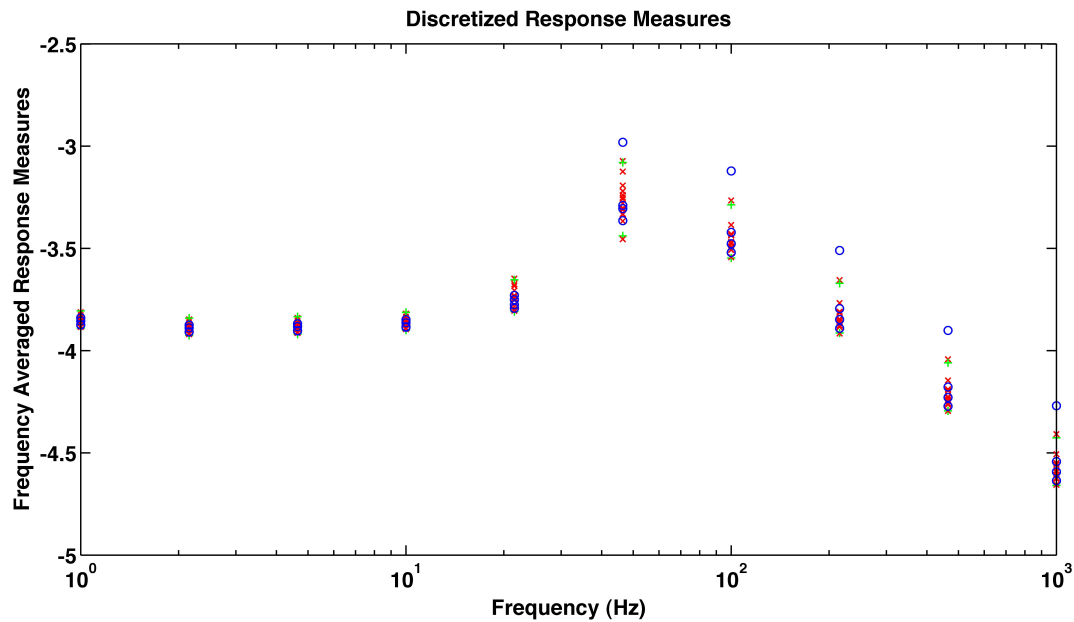




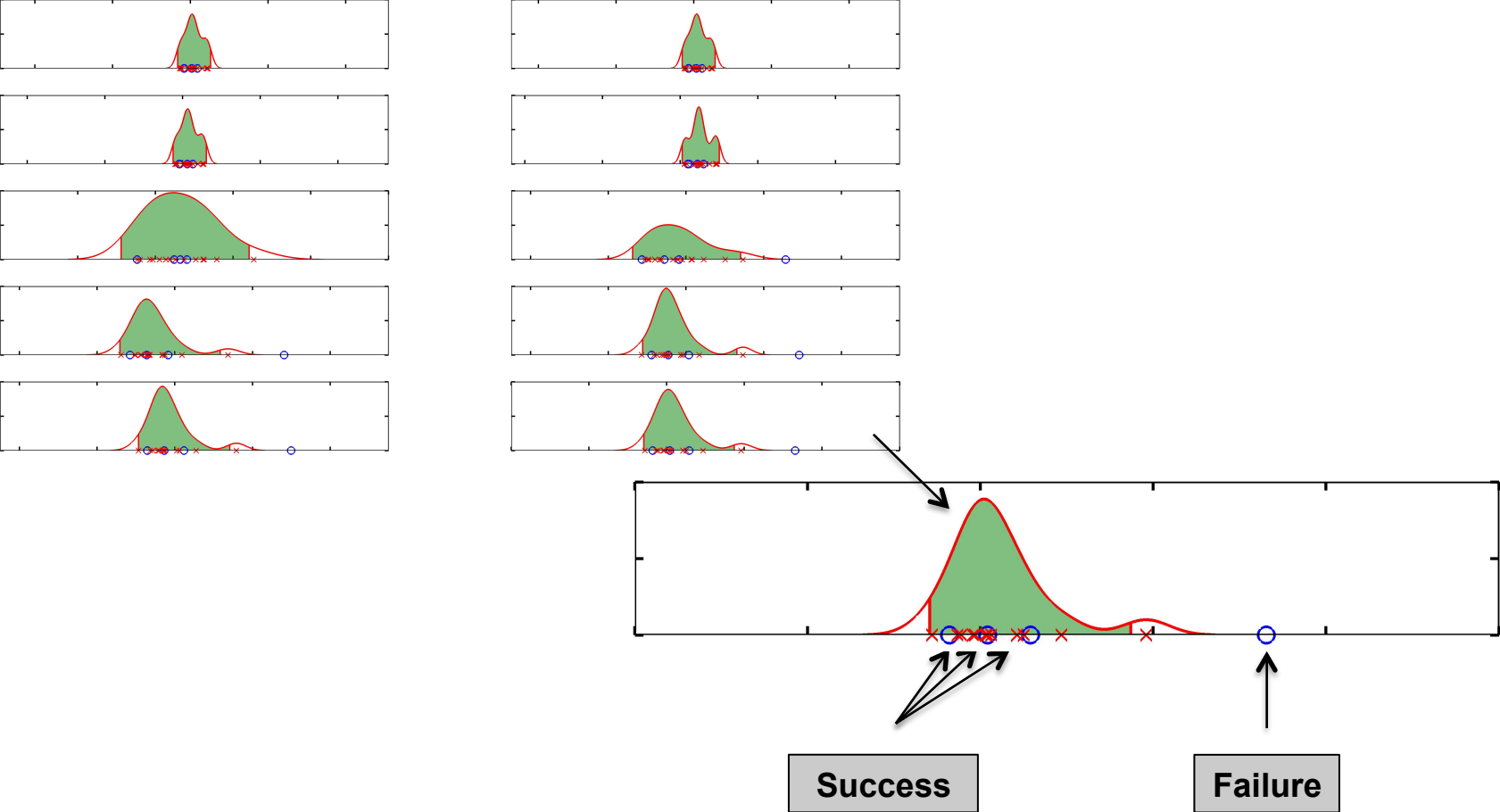
# Form probabilistic descriptions of discrete measures in order to perform validation tests



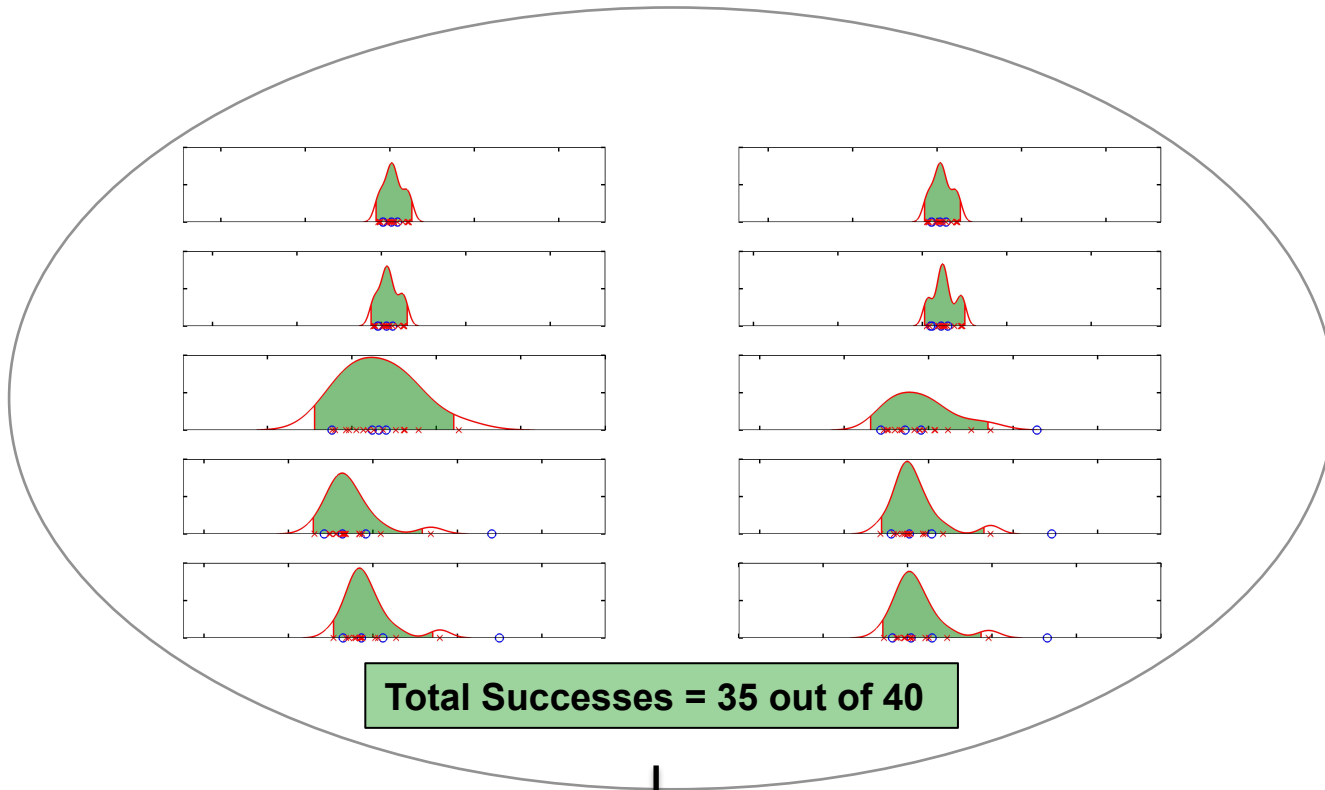
# Test 1: Do experimental points fall within their respective p-valued probability intervals?



# If an experimental response falls within it's 90% symmetric probability interval then it is considered a success

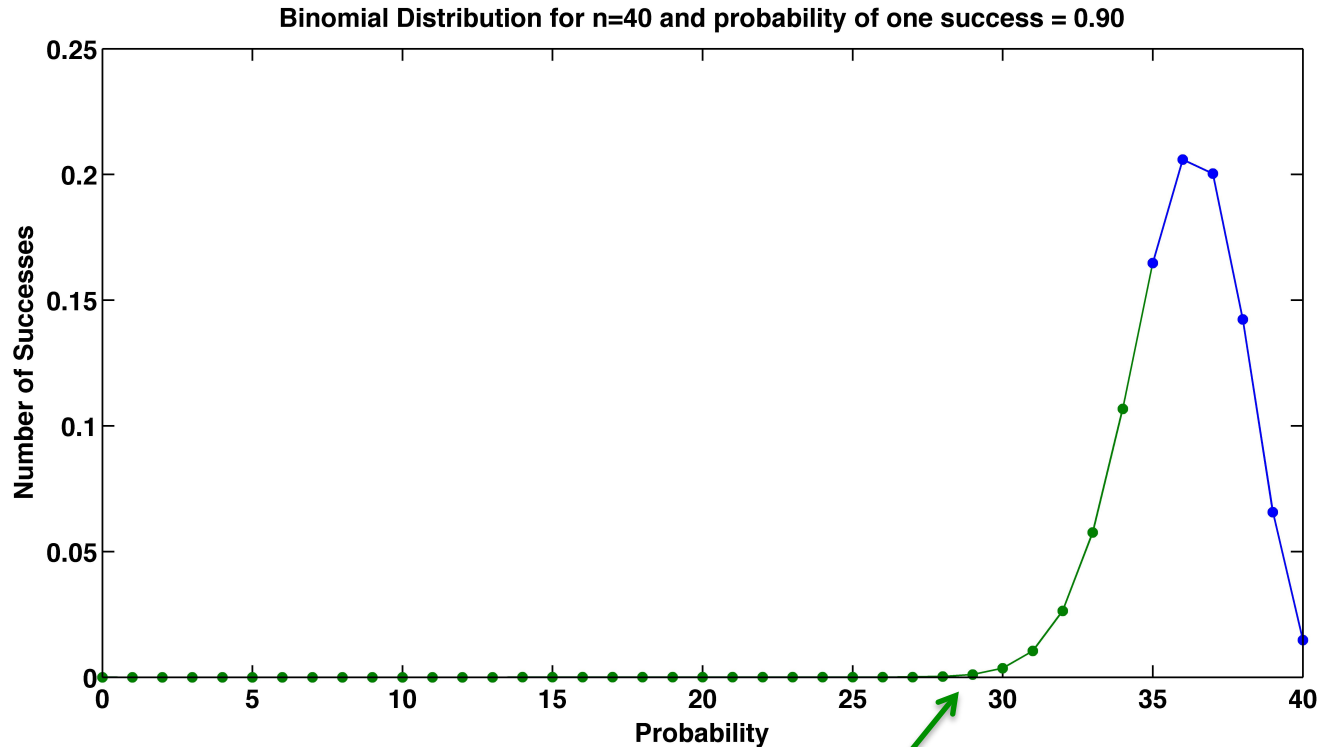


**Test 2:** Is the **collective** number of **successes** “good enough”?



If the model is perfect then what is the probability of seeing 35 (or fewer) successes?

# Generate the appropriate binomial distribution to determine the probability of seeing 35 (or fewer) out of 40 successes



Sum of the green probabilities = 0.3710. Thus there is about a 37% chance that if the model is perfect we will see 35 (or fewer) successes

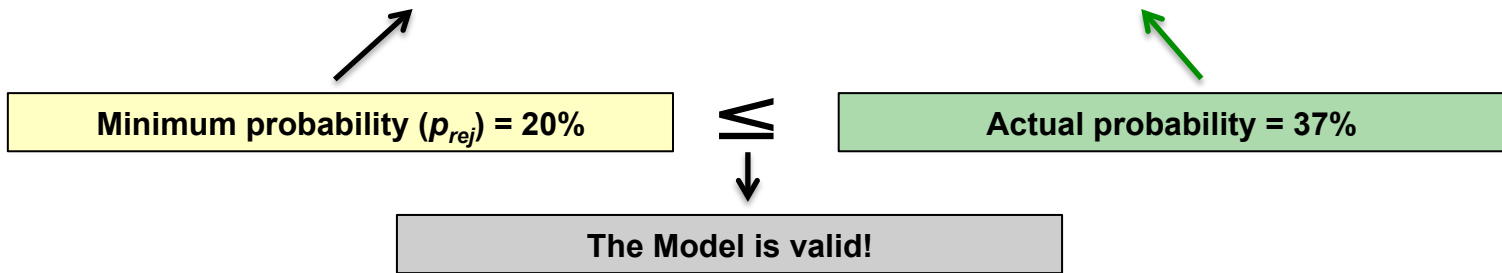
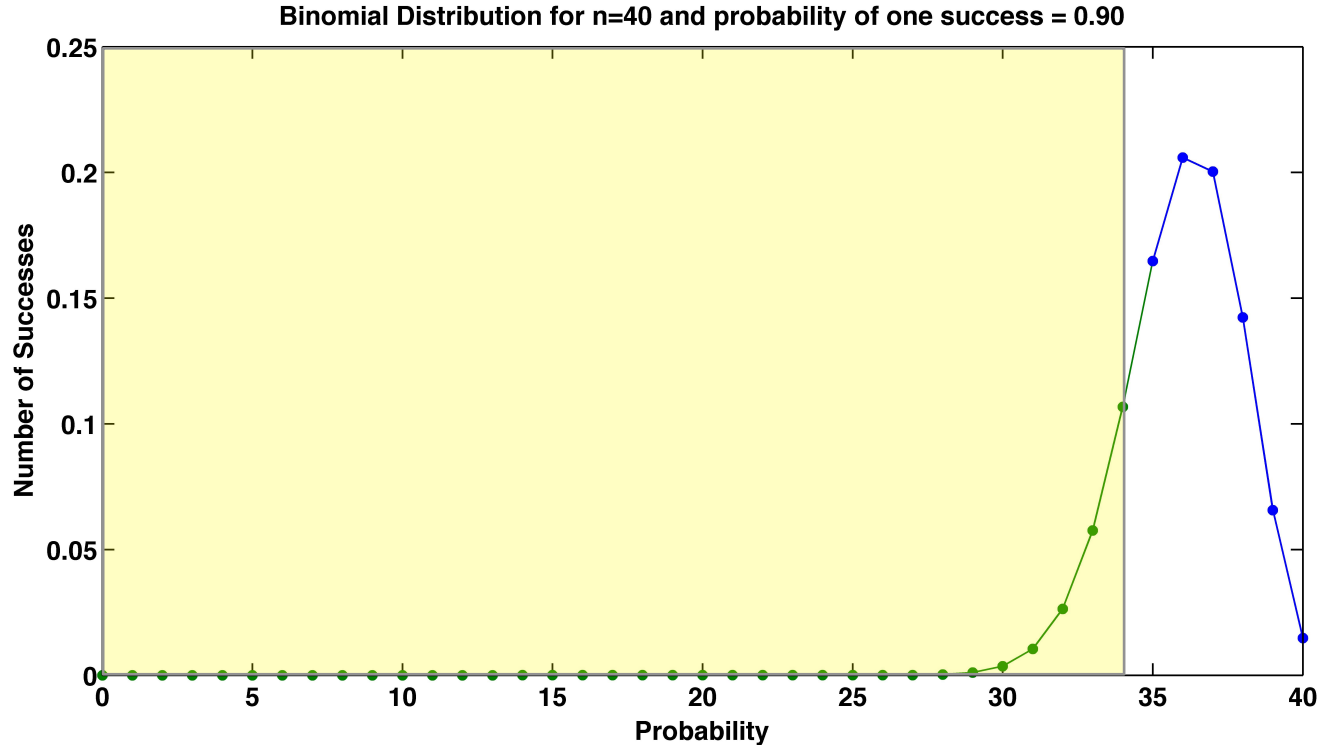
# Choose $p_{rej}$ - the probability of rejecting a perfect model

Does a **37% chance** of seeing **35** successes (or fewer) give us confidence that the model is valid?

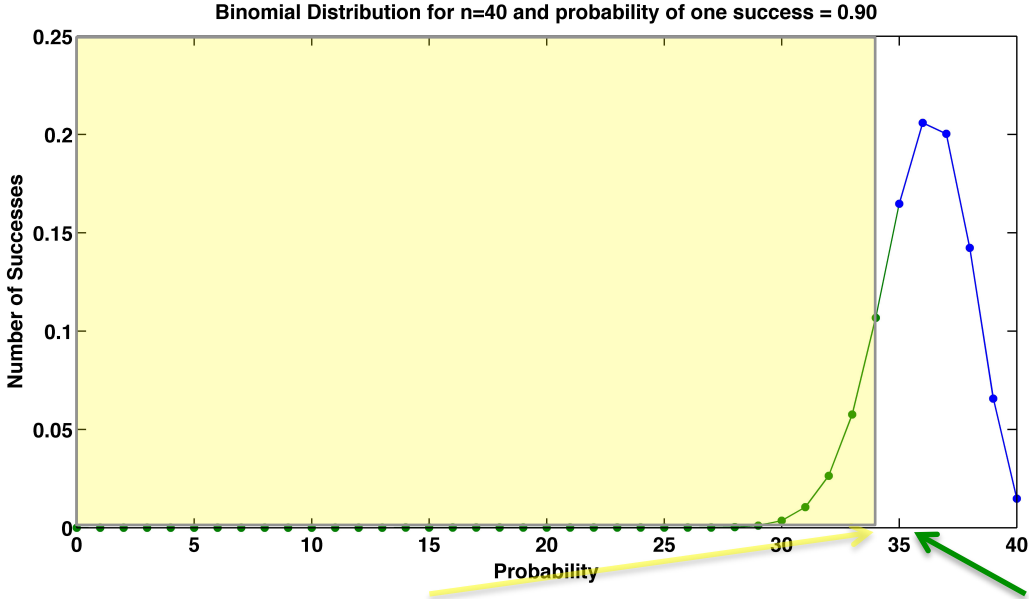


In reality, before beginning this process we must **choose a minimum** probability ( $p_{rej}$ ) we are satisfied with.

# Compare $p_{rej}$ to the computed probability of seeing 35 (or fewer) successes



# Instead of comparing probabilities define $S$ and $S_{val}$ to compare proportions



Minimum number of successes = 34

Actual number of successes = 35

$\neq$



$$S_{Val} = \frac{\text{Minimum Successes}}{\text{Total Comparisons}} = \frac{34}{40} = 0.85$$

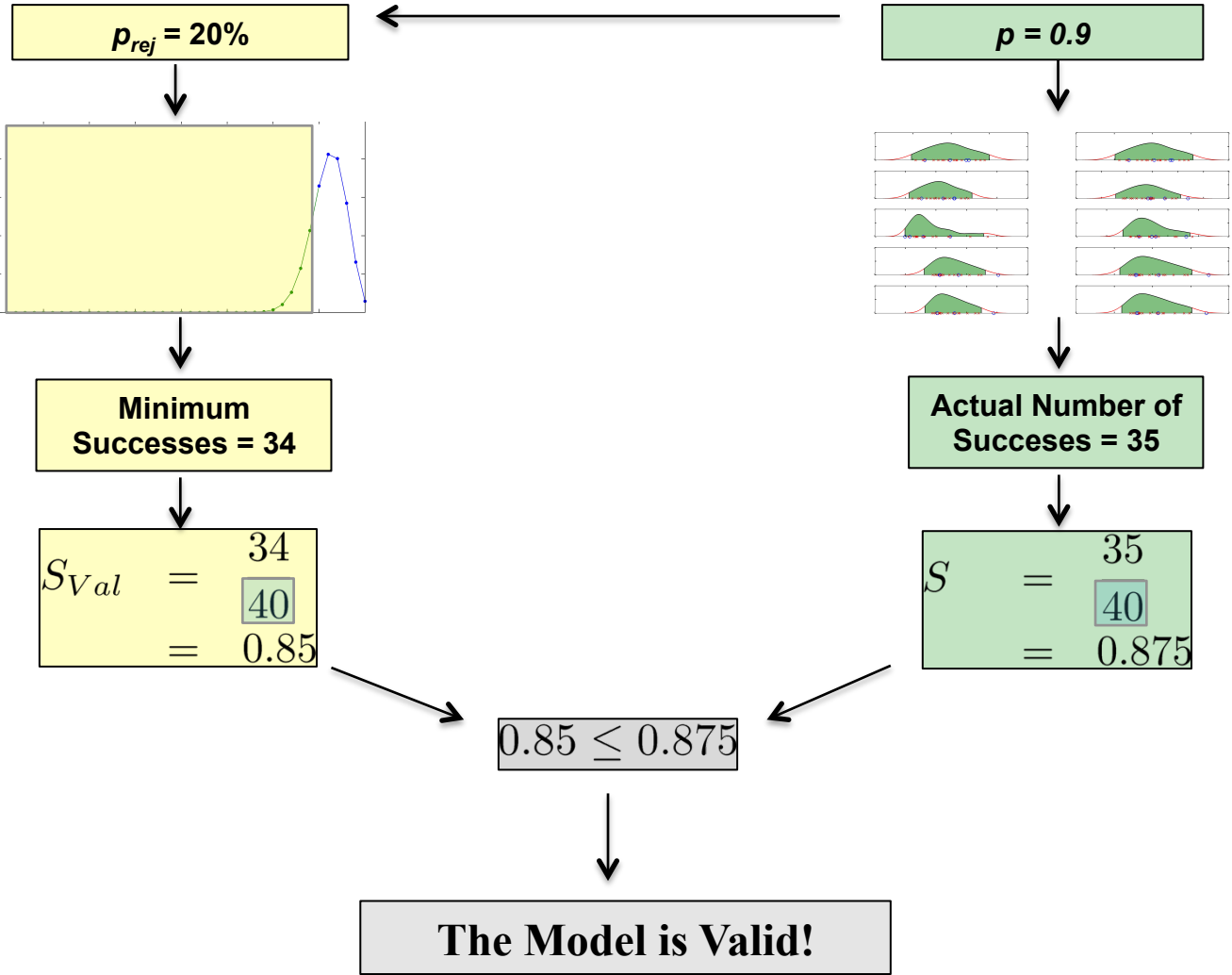
$$S = \frac{\text{Actual Successes}}{\text{Total Comparisons}} = \frac{35}{40} = 0.875$$

$\neq$



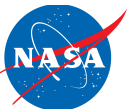


# In summary, our spring-mass-damper system is valid given 90% probability intervals and a probability of rejection of 0.20



# References

- Paez, Thomas L.; Massad, Jordan E.; Hinnerichs, Terry; O’Gorman, Chris; Hunter, Patrick. “Validation of Mathematical Models Using Weighted Response Measures.” *Sandia National Laboratories*. 2007: Web. 13 February 2011.



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# Disclaimer

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