

Adaptive control of rotary inverted pendulum system with time-varying uncertainties

Yung-Feng Chen · An-Chyau Huang

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Abstract In this paper, an adaptive controller is proposed to balance a rotary inverted pendulum with time-varying uncertainties. The goal of the control is to bring the pendulum close to the upright position regardless of the various uncertainties and disturbances. Its underactuated dynamics is first decoupled by Olfati's transformation into a cascade form, and then an adaptive controller is designed to deal with the uncertainties in the new space. Based on the Lyapunov-like theory, the closed loop stability and boundedness of all internal signals can be proved. The simulation results show that the proposed scheme is capable of giving good performance, as desired.

Keywords Rotary inverted pendulum · Adaptive control · Time-varying uncertainties

1 Introduction

The rotary inverted pendulum (Fig. 1) was first introduced by Furuta et al. [13]. Since it contains the well-known underactuated dynamics, many reports can be found in the literature for its stabilization [1, 3, 5, 25]. Most of the controls for the rotary inverted pendulum

fall into one of several categories [6]. For example, some considered the problem of stabilizing the pendulum around the unstable vertical position [1, 6, 25, 26, 32]. Some swung the pendulum from its hanging position to its upright vertical position [3, 9, 14, 15, 31]. Some others tried to create oscillations around its unstable vertical position [2, 12, 30]. In this paper, we would like to consider the control problem of stabilizing the pendulum around the unstable vertical position when subjected to time-varying uncertainties.

Several robust controllers were proposed for dealing with uncertainties and disturbances in the Furuta system. Yu et al. [34] proposed a robust controller to stabilize the Furuta pendulum under bounded perturbation. Khanesar et al. [22] used a fuzzy sliding controller to drive a rotary inverted pendulum to the vertical position subject to bounded uncertainties and disturbances. Park et al. [29] presented a swing-up and stabilization control with coupled sliding mode control. Some other robust designs can be found in the recent papers by Iraj et al. [21], Muske et al. [24], Ashrafioun and Whitman [4], and Uchiyama et al. [33]. A common assumption to these robust designs is that the variation bounds of the uncertainties and disturbances have to be available; otherwise, the design is not feasible.

The other approach to deal with system uncertainties is the adaptive control. However, few reports can be found. Matsuda et al. [23] proposed a variable structure system type adaptive controller to stabilize the pendulum at the upright position. Hirata et al. [16]

Y.-F. Chen (✉) · A.-C. Huang
Department of Mechanical Engineering, National Taiwan University of Science and Technology, 43, Keelung Road, Section 4, Taipei, Taiwan, ROC
e-mail: d9503602@mail.ntust.edu.tw

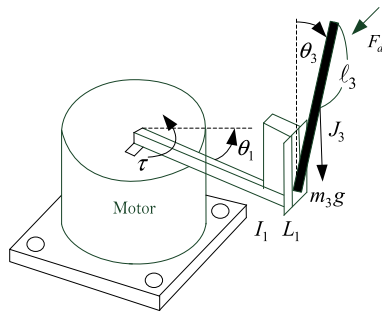


Fig. 1 The Furuta pendulum system

presented a robust adaptive control for the stabilization problem of the Furuta pendulum. Similar to the traditional adaptive design, the uncertainties in these approaches are assumed to be linearly parameterized as a multiplication of a known regressor matrix and an unknown constant vector. If this regressor form cannot be achieved, the adaptive design fails. In this paper, we consider the case when the system dynamics contains time-varying uncertainties that cannot be represented in the regressor form. Therefore, we may not apply the traditional adaptive design. In addition, let us assume that the variation bounds of these time-varying uncertainties are not available; the conventional robust designs are not feasible either. Here, we would like to design a function approximation technique (FAT) based adaptive controller to cover the time-varying uncertainties [7, 8, 17–20].

The rotary inverted pendulum is well-known to be an underactuated system [7, 11] where fewer actuators are used to drive the system than its degree of freedom. To decouple the underactuated dynamics, Olfati’s transformation [27, 28] is used to transform the

system into a special cascade form [7]. Together with the FAT-based adaptive controller, the closed loop stability can be justified with the Lyapunov-like method. Simulation cases are designed to justify the effectiveness of the proposed method.

This paper is organized as follows. Section 2 derives the system dynamics and introduces Olfati’s transformation. The FAT-based adaptive controller is proposed in Sect. 3. Section 4 presents the simulation results. The last section concludes the paper.

2 System dynamics and Olfati’s transformation

Consider the rotary inverted pendulum shown in Fig. 1 where I_1 is the moment of inertia of the arm, L_1 is the arm length, m_3 is the mass of the pendulum, l_3 is the distance to the center of gravity of the pendulum, J_3 is the inertia of the pendulum around its center of gravity, θ_1 is the angular displacement of the arm, θ_3 is the angular displacement of the pendulum, F_d is the external force disturbance, and τ is the torque applied to the arm. The system dynamics can be represented by the set of differential equations

$$\begin{aligned} \dot{\theta}_1 &= \theta_2, \\ \dot{\theta}_2 &= f_1(\boldsymbol{\theta}) + b_1\tau, \\ \dot{\theta}_3 &= \theta_4, \\ \dot{\theta}_4 &= f_2(\boldsymbol{\theta}) + b_2\tau + F_d, \end{aligned} \tag{1}$$

where $\boldsymbol{\theta} = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4]^T$ is a vector of states, and functions $f_1, f_2, b_1,$ and b_2 are defined as

$$\begin{aligned} f_1(\boldsymbol{\theta}) &= \frac{(J_3 + m_3\ell_3^2)m_3\ell_3\dot{\theta}_3(L_1\dot{\theta}_3 \sin \theta_3 - \ell_3\dot{\theta}_1 \sin(2\theta_3))}{\det(D(q))} - \frac{m_3^2\ell_3^2L_1 \cos \theta_3(g \sin \theta_3 + 0.5l_3\dot{\theta}_1^2 \sin(2\theta_3))}{\det(D(q))}, \\ f_2(\boldsymbol{\theta}) &= \frac{m_3\ell_3 \sin \theta_3((I_1 + m_3L_1^2 + \ell_3^2 \sin^2 \theta_3)g - m_3\ell_3L_1^2\dot{\theta}_3^2 \cos \theta_3)}{\det(D(q))} \\ &\quad + \frac{m_3\ell_3^2\dot{\theta}_1 \sin(2\theta_3)[m_3\ell_3L_1\dot{\theta}_3 \cos \theta_3 + 0.5\dot{\theta}_1(I_1 + m_3L_1^2 + m_3\ell_3^2 \sin^2 \theta_3)]}{\det(D(q))}, \\ b_1(\theta_3) &= \frac{(J_3 + m_3\ell_3^2)}{\det(D(q))}, \quad b_2(\theta_3) = \frac{-m_3\ell_3L_1 \cos \theta_3}{\det(D(q))}, \\ \det(D(q)) &= (J_3 + m_3\ell_3^2)(I_1 + m_3\ell_3^2 \sin^2 \theta_3) + J_3m_3L_1^2 + m_3^2\ell_3^2L_1^2 \sin^2 \theta_3. \end{aligned}$$

Suppose f_1 and f_2 are unknown functions without knowing their variation bounds, while b_1 is known. The uncertain function b_2 is assumed to be bounded as $b_{\min} \leq b_2 \leq b_{\max}$ with known bounds. The nominal value of b_2 is defined as $b_m = \sqrt{b_{\min} b_{\max}}$ so that we may represent b_2 as $b_2 = b_m \Delta b$ where Δb is the uncertainty satisfying $0 < \delta_{\min} \equiv \frac{b_{\min}}{b_m} \leq \Delta b \leq \frac{b_{\max}}{b_m} \equiv \delta_{\max}$.

Since the system is well-known to be underactuated, we would like to apply Olfati's transformation [27, 28] to represent the system in a special cascade form so that the underactuated dynamics can be eliminated. By using the coordinate transformation

$$\begin{aligned} z_1 &= \theta_1 + \frac{J_3 + m_3 \ell_3^2}{m_3 \ell_3 L_1} \ln |\sec \theta_3 + \tan \theta_3|, \\ z_2 &= \theta_2 + \frac{J_3 + m_3 \ell_3^2}{m_3 \ell_3 L_1 \cos \theta_3} \theta_4, \end{aligned} \quad (2)$$

$$z_3 = \theta_3,$$

$$z_4 = \theta_4,$$

we have the system dynamics in the new space given by

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= z_3 + d(\mathbf{z}), \\ \dot{z}_3 &= z_4, \\ \dot{z}_4 &= f_2(\mathbf{z}) + b_2 u + F_d, \end{aligned} \quad (3)$$

where $\mathbf{z} = [z_1 \ z_2 \ z_3 \ z_4]^T$ is the new state vector and

$$d(\mathbf{z}) = -z_3 + f_1(\mathbf{z}) + \frac{J_3 + m_3 \ell_3^2}{m_3 \ell_3 L_1 \cos \theta_3} f_2(\mathbf{z}).$$

It is seen that system (3) is in a special cascade form and d is a mismatched time-varying uncertainty without knowing its variation bound. To deal with the mismatched uncertainty, a backstepping-like design called multiple-surface sliding control is employed with the function approximation technique to give appropriate compensation. To unify the derivation, we would like to represent (3) as

$$\begin{aligned} \dot{z}_1 &= z_2 + d_1, \\ \dot{z}_2 &= z_3 + d_2, \\ \dot{z}_3 &= z_4, \\ \dot{z}_4 &= f_2(\mathbf{z}) + b_2 u + F_d, \end{aligned} \quad (4)$$

where $d_1 = 0$ and $d_2 = d(\mathbf{z})$.

3 Controller design

Define $y = z_1$ as the output signal of system (4) and $y_d(t)$ is its desired trajectory. Let us consider the error signals

$$s_i = z_i - z_{id}, \quad i = 1, \dots, 4, \quad (5)$$

where z_{id} is the desired value for z_i . These error signals can be regarded as a set of surfaces in some error space. Let $z_{1d} = y_d(t)$. Then the time derivative of s_1 can be found as

$$\dot{s}_1 = s_2 + z_{2d} + d_1 - \dot{y}_d. \quad (6)$$

To stabilize (6), we may regard z_{2d} as a virtual control, and it is selected to be $z_{2d} = \dot{y}_d - c_1 \frac{s_1}{\phi_1}$ where c_1 is a positive constant and $\phi_1 > 0$ is the thickness of the boundary layer for the surface $s_1 = 0$. With this selection of z_{2d} , (6) becomes

$$\dot{s}_1 = s_2 - c_1 \frac{s_1}{\phi_1}. \quad (7)$$

It can be seen that if s_2 is small, then s_1 will also be small. Taking the time derivative of s_2 , we have

$$\dot{s}_2 = s_3 + z_{3d} + d_2 - \dot{z}_{2d}, \quad (8)$$

where $\dot{z}_{2d} = \ddot{y}_d - \frac{c_1}{\phi_1} (z_2 - \dot{y}_d)$. To stabilize (8), we may regard z_{3d} as a virtual control which can be designed as

$$z_{3d} = \dot{z}_{2d} - \hat{d}_2 - c_2 \frac{s_2}{\phi_2}, \quad (9)$$

where c_2 and ϕ_2 are positive constants, and \hat{d}_2 is an estimate of the mismatched uncertainty d . Hence, (8) can be derived as

$$\dot{s}_2 = s_3 + (d_2 - \hat{d}_2) - c_2 \frac{s_2}{\phi_2}. \quad (10)$$

If s_3 is small and some update law can be designed so that $d_2 - \hat{d}_2$ is small, then (10) implies that the magnitude of s_2 is small. To evaluate the dynamics of s_3 , let us take its time derivative as

$$\dot{s}_3 = s_4 + z_{4d} - \dot{z}_{3d}, \quad (11)$$

where \dot{z}_{3d} is a function of the uncertainty d_2 , so it is also unknown. To simplify the derivation, let us define

$d_3 = -\dot{z}_{3d}$ and we may select $z_{4d} = -\hat{d}_3 - c_3 \frac{s_3}{\phi_3}$ to have

$$\dot{s}_3 = s_4 + (d_3 - \hat{d}_3) - c_3 \frac{s_3}{\phi_3}. \tag{12}$$

Finally, the dynamics of s_4 is found as

$$\dot{s}_4 = f_2 + b_2 u - \dot{z}_{4d}.$$

Define $f = f_2 - \dot{z}_{4d}$, and we have

$$\dot{s}_4 = f + b_m \Delta b u. \tag{13}$$

We may thus design the control law as

$$u = \frac{1}{b_m} \left[-\hat{f} - c_4 \frac{s_4}{\phi_4} - u_r \right], \tag{14}$$

where \hat{f} is an estimate of f and $c_4 > 0$ and $\phi_4 > 0$ are constants and u_r is a robust term to be designed. Substituting the control law u into (13), we have

$$\begin{aligned} \dot{s}_4 &= f - \Delta b \hat{f} - c_4 \Delta b \frac{s_4}{\phi_4} - \Delta b u_r \pm \hat{f} \pm c_4 \frac{s_4}{\phi_4} \\ &= (f - \hat{f}) - c_4 \frac{s_4}{\phi_4} + (1 - \Delta b) \left(\hat{f} + c_4 \frac{s_4}{\phi_4} \right) \\ &\quad - \Delta b u_r. \end{aligned} \tag{15}$$

At this stage, we have already obtained the dynamics of error surfaces in (7), (10), (12), and (15). Let us collect them to form the error dynamics for the whole system as

$$\dot{s}_1 = s_2 - c_1 \frac{s_1}{\phi_1}, \tag{16a}$$

$$\dot{s}_2 = s_3 - c_2 \frac{s_2}{\phi_2} + (d_2 - \hat{d}_2), \tag{16b}$$

$$\dot{s}_3 = s_4 - c_3 \frac{s_3}{\phi_3} + (d_3 - \hat{d}_3), \tag{16c}$$

$$\begin{aligned} \dot{s}_4 &= -c_4 \frac{s_4}{\phi_4} + (f - \hat{f}) + (1 - \Delta b) \left(\hat{f} + c_4 \frac{s_4}{\phi_4} \right) \\ &\quad - \Delta b u_r. \end{aligned} \tag{16d}$$

If a robust term u_r can be designed so that the term $(1 - \Delta b)(\hat{f} + c_4 \frac{s_4}{\phi_4})$ in (16d) can be properly covered, and an update law for \hat{f} can be found such that $\hat{f} \rightarrow f$, then we may have the convergence of s_4 . Likewise, if an update law for \hat{d}_3 can be derived to satisfy $\hat{d}_3 \rightarrow d_3$, then (16c) implies the convergence of

s_3 . Again, if we may design an update law for \hat{d}_2 such that $\hat{d}_2 \rightarrow d_2$, then (16b) gives $s_2 \rightarrow 0$. Finally, with the convergence of s_2 , we may conclude the convergence of s_1 from (16a). To design these update laws, we would like to apply the function approximation technique [8, 17–20]. Define $d_4 = f$ and $\hat{d}_4 = \hat{f}$, then we have the representations

$$\begin{aligned} d_i &= \mathbf{w}_i^T \mathbf{z}_i + \varepsilon_i, \\ \hat{d}_i &= \hat{\mathbf{w}}_i^T \mathbf{z}_i, \end{aligned} \quad i = 1, \dots, 4 \tag{17}$$

where \mathbf{z}_i is a known vector of basis functions, \mathbf{w}_i and $\hat{\mathbf{w}}_i$ are respectively the coefficient vector and its estimate, and ε_i is the approximation error. Note that $d_1 = 0$, and the representation in (17) for d_1 and \hat{d}_1 is only for convenience in the following derivation. Hence, we may rewrite (16a)–(16d) as

$$\begin{aligned} \dot{s}_1 &= -\eta_1 s_1 + s_2, \\ \dot{s}_2 &= -\eta_2 s_2 + s_3 + \tilde{\mathbf{w}}_2^T \mathbf{z}_2 + \varepsilon_2, \\ \dot{s}_3 &= -\eta_3 s_3 + s_4 + \tilde{\mathbf{w}}_3^T \mathbf{z}_3 + \varepsilon_3, \\ \dot{s}_4 &= -\eta_4 s_4 + \tilde{\mathbf{w}}_4^T \mathbf{z}_4 + \varepsilon_4 \\ &\quad + (1 - \Delta b) \left(\hat{f} + c_4 \frac{s_4}{\phi_4} \right) - \Delta b u_r, \end{aligned} \tag{18}$$

where $\tilde{\mathbf{w}}_i = \mathbf{w}_i - \hat{\mathbf{w}}_i$ are approximation errors for the coefficient vectors, and $\eta_i = \frac{c_i}{\phi_i}$, $i = 1, \dots, 4$ are positive constants. Define a Lyapunov-like function

$$V_4 = \frac{1}{2} s_4^2 + \frac{1}{2} \tilde{\mathbf{w}}_4^T \mathbf{\Gamma}_4 \tilde{\mathbf{w}}_4, \tag{19}$$

where $\mathbf{\Gamma}_4$ is a positive definite matrix. Taking the time derivative of (19) along the trajectory of (18), we have

$$\begin{aligned} \dot{V}_4 &\leq -\eta_4 s_4^2 + s_4 \varepsilon_4 + (1 + \delta_{\max}) |\hat{f} + \eta_4 s_4| |s_4| \\ &\quad - \delta_{\min} s_4 u_r + \tilde{\mathbf{w}}_4^T (s_4 \mathbf{z}_4 - \mathbf{\Gamma}_4 \dot{\hat{\mathbf{w}}}_4). \end{aligned} \tag{20}$$

Select the update law for $\hat{\mathbf{w}}_4$ as

$$\dot{\hat{\mathbf{w}}}_4 = \mathbf{\Gamma}_4^{-1} (s_4 \mathbf{z}_4 - \sigma_4 \hat{\mathbf{w}}_4), \tag{21a}$$

$$u_r = \frac{1 + \delta_{\max}}{\delta_{\min}} |\hat{f} + \eta_4 s_4| \text{sgn}(s_4), \tag{21b}$$

where σ_4 is a positive number. Hence, (20) can be written as

$$\dot{V}_4 \leq -\eta_4 s_4^2 + |s_4| |\varepsilon_4| + \sigma_4 (\tilde{\mathbf{w}}_4^T \mathbf{w}_4 - \|\tilde{\mathbf{w}}_4\|^2). \tag{22}$$

By using the inequalities

$$-\eta_4 s_4^2 + |s_4| \varepsilon_4 \leq -\frac{1}{2} \left(\eta_4 s_4^2 - \frac{\varepsilon_4^2}{\eta_4} \right),$$

$$\tilde{\mathbf{w}}_4^T \mathbf{w}_4 - \|\tilde{\mathbf{w}}_4\|^2 \leq -\frac{1}{2} (\|\tilde{\mathbf{w}}_4\|^2 - \|\mathbf{w}_4\|^2),$$

Eq. (22) can be derived as

$$\begin{aligned} \dot{V}_4 &\leq -\frac{1}{2} \left(\eta_4 s_4^2 - \frac{\varepsilon_4^2}{\eta_4} \right) - \frac{1}{2} \sigma_4 (\|\tilde{\mathbf{w}}_4\|^2 - \|\mathbf{w}_4\|^2) \\ &\leq -\alpha_4 V_4 + \left[\frac{\alpha_4}{2} - \frac{\eta_4}{2} \right] s_4^2 + \left[\frac{\alpha_4}{2} \lambda_{\max}(\mathbf{\Gamma}_4) - \frac{\sigma_4}{2} \right] \\ &\quad \times \|\tilde{\mathbf{w}}_4\|^2 + \frac{\sigma_4}{2} \|\mathbf{w}_4\|^2 + \frac{1}{2\eta_4} \varepsilon_4^2, \end{aligned}$$

where α_4 is selected as $\alpha_4 \leq \min\{\eta_4, \frac{\sigma_4}{\lambda_{\max}(\mathbf{\Gamma}_4)}\}$ to give

$$\dot{V}_4 \leq -\alpha_4 V_4 + \frac{\sigma_4}{2} \|\mathbf{w}_4\|^2 + \frac{1}{2\eta_4} \varepsilon_4^2. \tag{23}$$

Hence, $\dot{V}_4 < 0$ whenever

$$(s_4, \tilde{\mathbf{w}}_4) \in E_4 \equiv \left\{ (s_4, \tilde{\mathbf{w}}_4) \mid V_4 > \varphi_4 \equiv \frac{\sigma_4}{2\alpha_4} \|\mathbf{w}_4\|^2 + \frac{1}{2\alpha_4\eta_4} \sup_{\tau \geq t_0} \varepsilon_4^2(\tau) \right\}.$$

Note that φ_4 is a positive constant, and hence we may conclude that $(s_4, \tilde{\mathbf{w}}_4)$ is uniformly ultimately bounded. In addition, given any $\mu_4 > 0$, there exist $T_4 \geq t_0 \geq 0$ such that

$$V_4(t) \leq \varphi_4 + \mu_4 \quad \text{for } t \geq T_4.$$

Along the same line, we can choose the Lyapunov-like functions

$$V_i = \frac{1}{2} s_i^2 + \frac{1}{2} \tilde{\mathbf{w}}_i^T \mathbf{\Gamma}_i \tilde{\mathbf{w}}_i, \quad i = 1, 2, 3. \tag{24}$$

By taking the time derivatives of these functions along the trajectory of (18), we may have the update laws to be selected as

$$\dot{\hat{\mathbf{w}}}_i = \mathbf{\Gamma}_i^{-1} (s_i \mathbf{z}_i - \sigma_i \hat{\mathbf{w}}_i), \quad \sigma_i > 0, \quad i = 1, 2, 3. \tag{25}$$

After some rearrangements, the time derivative of V_i becomes

$$\dot{V}_i \leq -\alpha_i V_i + \frac{\sigma_i}{2} \|\mathbf{w}_i\|^2 + \frac{1}{2\eta_i} (\varepsilon_i + s_{i+1})^2 \tag{26}$$

where $\alpha_i \leq \min\{\eta_i, \frac{\sigma_i}{\lambda_{\max}(\mathbf{\Gamma}_i)}\}$, $i = 1, 2, 3$. For $t_0 \leq t < T_{i+1}$, we may have $\dot{V}_i < 0$ whenever

$$(s_i, \tilde{\mathbf{w}}_i) \in E_i \equiv \{(s_i, \tilde{\mathbf{w}}_i) \mid V_i > \varphi_i\}, \tag{27}$$

$\varphi_i \equiv \frac{\sigma_i}{2\alpha_i} \|\mathbf{w}_i\|^2 + \frac{1}{2\alpha_i\eta_i} [\sup_{\tau \geq t_0} |\varepsilon_i(\tau)| + \sqrt{2 \max\{V_{i+1}(t_0), \phi_{i+1} + \mu_{i+1}\}}]^2$. This implies that V_i is bounded for all $t \in [t_0, T_{i+1}]$, i.e., before convergence of V_{i+1} , V_i is bounded. Define $\varphi'_i \equiv \frac{\sigma_i}{2\alpha_i} \|\mathbf{w}_i\|^2 + \frac{1}{2\alpha_i\eta_i} [\sup_{\tau \geq t_0} |\varepsilon_i(\tau)| + \sqrt{2(\phi_{i+1} + \mu_{i+1})}]^2$, then after the convergence of V_{i+1} (i.e., when $t \geq T_{i+1}$) to $\varphi'_{i+1} + \mu_{i+1}$, V_i is bounded. We have thus proved that $(s_i, \tilde{\mathbf{w}}_i)$ is uniformly ultimately bounded. Therefore, we have established the order of convergence from s_4 to s_1 . During the convergence of s_i , the boundedness of s_j , $j = i - 1, \dots, 1$ is ensured. Specifically, when $t \geq T_1$, we have

$$|s_1(t)| = |z_1 - z_{1d}| = \sqrt{2V_1} \leq \sqrt{2(\phi'_1 + \mu_1)}. \tag{28}$$

Hence the proposed controller ensures that the output error of the rotary inverse pendulum system will be bounded by some constants adjustable by controller parameters. In this work, the performance is achieved subject to mismatched time-varying uncertainties.

4 Computer simulations

To verify the effectiveness of the proposed design, let us consider the system in Fig. 1 again. The initial condition is assumed to be $\boldsymbol{\theta}(0) = [1 \ 0 \ -1 \ 0]^T$, and we would like to bring the desired state to the origin. The actual values of system parameters [10] are $m_3 = 5.38 \times 10^{-2}$ kg, $J_3 = 1.98 \times 10^{-4}$ kg m², $\ell_3 = 0.113$ m, $g = 9.8$ m/s², $L_1 = 0.215$ m, $I_1 = 1.75 \times 10^{-2}$ kg m². The parameters for modeling b_2 can be computed to be $b_{\min} = -81.79$, $b_{\max} = -40.08$, $b_m = 57.26$, $\delta_{\max} = -0.69997$, $\delta_{\min} = -1.4284$. The weighting matrices are chosen as $\mathbf{\Gamma}_2 = 32\mathbf{I}$, $\mathbf{\Gamma}_3 = 35\mathbf{I}$, $\mathbf{\Gamma}_4 = 40\mathbf{I}$, and the constant σ is set to zero. The external disturbance is applied to the system during 4 to 6 seconds with $F_d = 20 \sin(20t)$ as shown in Fig. 2. The initial weighting matrices for the function approximation are set to $\hat{w}_i(0) = [0 \ 0 \ \dots \ 0]^T$, $i = 2, 3, 4$. The 21-term Fourier series is used as the basis in the function approximation. Two cases are considered in the simulation study.

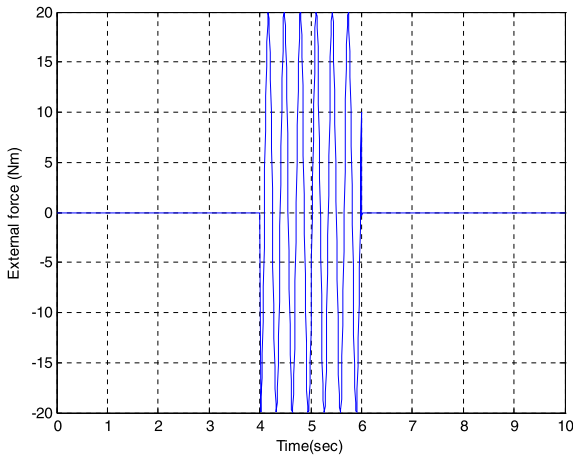


Fig. 2 External force

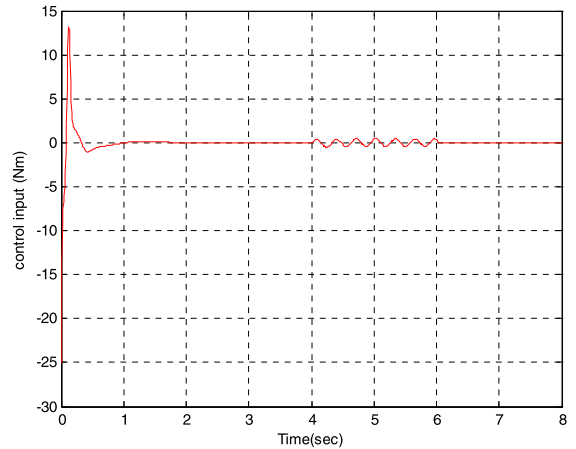


Fig. 4 Control input

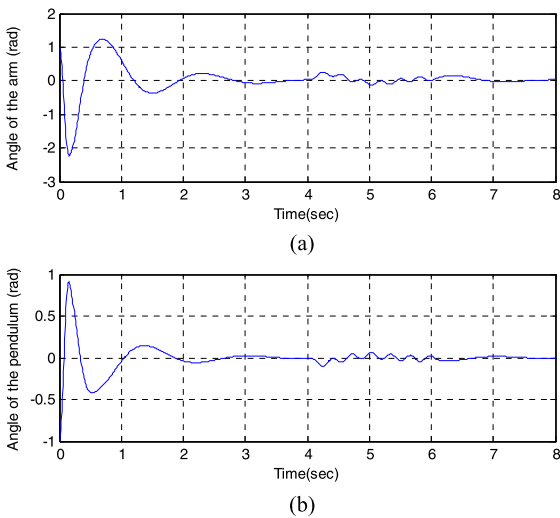


Fig. 3 System response (a) arm angle (b) pendulum angle

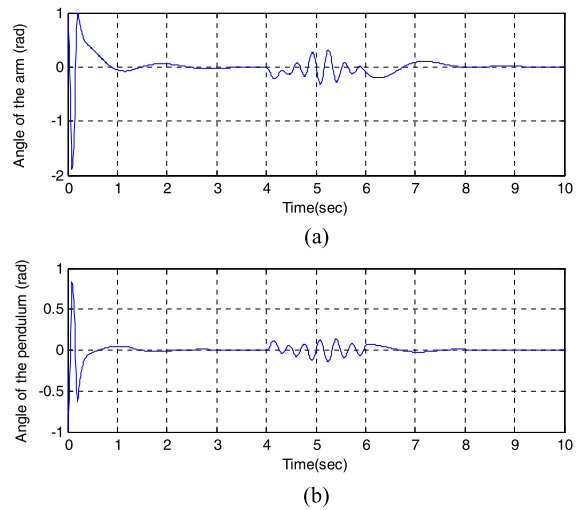


Fig. 5 System response (a) arm angle (b) pendulum angle

4.1 CASE 1. Constant parameters

In this case, system parameters m_3 and ℓ_3 are assumed to be unknown constants. The controller parameters are selected as $c_1 = 50, c_2 = 20, c_3 = 0.06, c_4 = 1 \times 10^{-9}$ and $\phi_i = 1, i = 1, \dots, 4$. The simulation results are shown in Figs. 3 and 4. Figure 3 shows that both the arm and pendulum positions converge to their desired values in 3 seconds regardless of the system uncertainties. With the presence of the external disturbances during 4 to 6 seconds, the pendulum exhibits some deviations in the trajectories. However, the proposed controller is robust enough to keep the

states in the neighborhoods of the equilibrium points and bring them back to the desired values right after the absence of the disturbance. Figure 4 shows that the control law is realizable.

4.2 CASE 2. Time-varying parameters

In this case, we consider the case when both values of the pendulum mass and center of gravity are time-varying as

$$m_3 = 0.0538 + 0.04 \cos 50t \text{ kg,}$$

$$\ell_3 = 0.113 + 0.1 \sin 4t \text{ m.}$$

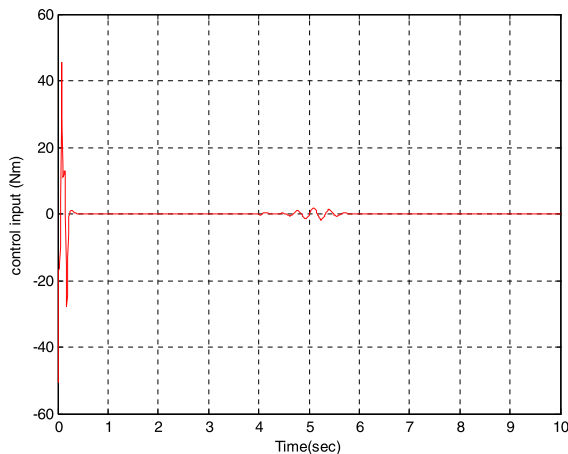


Fig. 6 Control input

All other system parameters remain the same as those in the previous case. The controller parameters are selected as $c_1 = 60$, $c_2 = 30$, $c_3 = 0.06$, $c_4 = 1 \times 10^{-8}$ and $\phi_i = 1, i = 1, \dots, 4$. The simulation results are presented in Figs. 5 and 6. Figure 5 shows that both the arm and pendulum converge to their desired values in 3 seconds even with the time-varying parameters. The robustness of the controller is still sufficient to regulate the system in a bounded region near the equilibrium point when the external force appears during 4 to 6 seconds. Figure 6 depicts the control effort.

5 Conclusions

We have proposed an adaptive controller for a rotary inverted pendulum system subject to time-varying uncertainties and external disturbances. We first apply Olfati's transformation to the system dynamics, and then the function approximation technique is utilized to estimate the uncertainties. Based on the Lyapunov-like stability theory, the closed-loop stability is proved to be ultimately uniformly bounded. Experimental results show that the proposed design is able to give proper performance regardless of various uncertainties and disturbances.

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References

1. Acosta, J.A.: Furuta's pendulum: a conservative nonlinear model for theory and practice. *Math. Probl. Eng.* (2010). Article ID 742894
2. Aguilar, L.T., Boiko, I., Fridman, L., Iriarte, R.: Generating self-excited oscillations via two-relay controller. *IEEE Trans. Autom. Control* **54**(2), 416–420 (2009)
3. Astrom, K.J., Aracil, J., Gordillo, F.: A family of smooth controller for swinging up a pendulum. *Automatica* **36**(2), 1841–1848 (2008)
4. Ashrafioun, H., Whitman, A.M.: Closed-loop dynamic analysis of a rotary inverted pendulum for control design. *J. Dyn. Syst. Meas. Control* **134**, 024503 (2012)
5. Bartolini, G., Pisano, A., Usai, E.: Second-order sliding-mode control of container cranes. *Automatica* **38**(10), 1783–1790 (2002)
6. Aguilar-Ibañez, C., Suárez-Castañón, M.S., Gutiérrez-Frias, O.O.: The direct Lyapunov method for the stabilisation of the Furuta pendulum. *Int. J. Control* **83**(11), 2285–2293 (2010)
7. Chen, Y.F., Huang, A.C.: Controller design for a class of underactuated mechanical systems. *IET Control Theory Appl.* **6**(1), 103–110 (2012)
8. Chien, M.C., Huang, A.C.: Adaptive control of flexible-joint electrically-driven robot with time-varying uncertainties. *IEEE Trans. Ind. Electron.* **54**(2), 1032–1038 (2007)
9. Fantoni, I., Lozano, R.: Stabilization of the Furuta pendulum around its homoclinic orbit. *Int. J. Control* **75**(6), 390–398 (2002)
10. Fantoni, I., Lozano, R.: *Nonlinear Control for Underactuated Mechanical Systems*. Springer, Berlin (2001)
11. Fang, Y., Dixon, W.E., Dawson, D.M., Zergeroglu, E.: Nonlinear coupling control laws for an underactuated overhead crane system. *IEEE/ASME Trans. Mechatron.* **8**(3), 418–423 (2003)
12. Freidovich, L., Shiriaev, A., Gordillo, F., Gomez-Estern, F., Aracil, J.: Partial-energy-shaping control for orbital stabilization of high-frequency oscillations of the Furuta pendulum. *IEEE Trans. Control Syst. Technol.* **17**(4), 853–858 (2009)
13. Furuta, K., Yamakita, M., Kobayashi, S.: Swing-up control of inverted pendulum. In: *Proceedings of the International Conference on Industrial Electronics, Control and Instrumentation (IECON'91)*, pp. 2193–2198 (1991)
14. Gordillo, F., Acosta, J.A., Aracil, J.: A new swing-up law for the Furuta pendulum. *Int. J. Control* **76**(8), 836–844 (2003)
15. Hera, P.X.L., Freidovich, L.B., Shiriaev, A.S., Mettin, U.: New approach for swing up the Furuta pendulum: theory and experiments. *Mechatronics* **19**, 1240–1250 (2009)
16. Hirata, H., Takabe, T., Anabuki, M., Ouchi, S.: Vss robust adaptive control including a self-tuning controller for a rotary inverted pendulum. *Electron. Commun. Jpn., Part 2, Electron.* **94**(6), 48–58 (2011)
17. Huang, A.C., Chen, Y.C.: Adaptive sliding control for single-link flexible-joint robot with mismatched uncertainties. *IEEE Trans. Control Syst. Technol.* **12**(5), 770–775 (2004)
18. Huang, A.C., Chen, Y.C.: Adaptive multiple-surface sliding control for non-autonomous systems with mismatched uncertainties. *Automatica* **40**(11), 1939–1945 (2004)

19. Huang, A.C., Chien, M.C.: Adaptive Control of Robot Manipulators—A Unified Regressor-Free Approach. World Scientific, Singapore (2010)
20. Huang, A.C., Kuo, Y.S.: Sliding control of nonlinear systems containing time-varying uncertainties with unknown bounds. *Int. J. Control* **74**(3), 252–264 (2001)
21. Iraj, H., Saleh, M.: Controller design for rotary inverted pendulum system using evolutionary algorithms. *Math. Probl. Eng.* (2011)
22. Khanesar, M.A., Teshnehlab, M., Shoorehdeli, M.A.: Fuzzy sliding mode control of rotary inverted pendulum. In: IEEE International Conference on Computational Cybernetics, pp. 57–62 (2007)
23. Matsuda, N., Izutsu, M., Ishikawa, J., Furuta, K., Artrom, K.J.: Swing-up and stabilization control based on natural frequency for pendulum systems. In: American Control Conference, June 2009, pp. 5291–5296 (2009)
24. Muske, K.R., Ashrafiuon, H., Nersesov, S., Nikkhah, M.: Optimal sliding mode cascade control for stabilization of underactuated nonlinear systems. *J. Dyn. Syst. Meas. Control* **134**, 021020 (2012)
25. Nair, S.: A normal form for energy shaping: application to the Furuta pendulum. In: Proc. of IEEE Conference on Decision and Control, Dec. 2002, vol. 1, pp. 516–521 (2002)
26. Olfati-Saber, R.: Fixed point controllers and stabilization of the cart-pole and rotating pendulum. In: Proceedings of the 38th Conference on Decision and Control, Dec. 1999, vol. 1, pp. 1174–1181 (1999)
27. Olfati-Saber, R.: Normal forms for underactuated mechanical systems with symmetry. *IEEE Trans. Autom. Control* **47**(2), 305–308 (2002)
28. Olfati-Saber, R., Megretasi, A.: Controller design for a class of underactuated nonlinear systems. In: Proc. of IEEE Conf. on Decision and Control, Dec. 1998, vol. 4, pp. 4182–4187 (1998)
29. Park, M.S., Chwa, D.: Swing-up and stabilization control of inverted-pendulum systems via coupled sliding-mode control method. *IEEE Trans. Ind. Electron.* **56**(9), 3541–3555 (2009)
30. Shiriaev, A.S., Freidovich, L.B., Robertsson, A., Johansson, R., Sandberg, A.: Virtual-holonomic-constraints-based design of stable oscillations of Furuta pendulum: theory and experiments. *IEEE Trans. Robot.* **23**(4), 827–832 (2007)
31. Sukontanakarn, V., Parnichkun, M.: Real-time optimal control for rotary inverted pendulum. *Am. J. Appl. Sci.* **6**, 1106–1115 (2009)
32. Turker, T., Gorgun, H., Cansever, G.: Lyapunov’s direct method for stabilization of the Furuta pendulum. *Turk. J. Electr. Eng. Comput. Sci.* **120**(1), 99–110 (2012)
33. Uchiyama, N.: Robust control of rotary crane by partial-state feedback with integrator. *Mechatronics* **19**(8), 1294–1302 (2009)
34. Yu, C.H., Wang, F.C., Lu, Y.J.: Robust control of a Furuta pendulum. In: SICE Annual Conference, Aug. 2010, pp. 2559–2563 (2010)