

## Single Curved Fiber Sedimentation Under Gravity

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### ABSTRACT

Dynamics of single curved fiber sedimentation under the gravity are simulated by using lattice Boltzmann method. The results of migration and rotation of the curved fiber at different Reynolds numbers are reported. The results show that the rotation and migration processes are sensitive to the curvature of the fiber.

### INTRODUCTION

Understanding of migration and aggregation of curved and flexible fibers are important for the papermaking process. The fiber motions are directly related to flocculation and dispersion. With increasing operation speed, the inertia of fibers and flow in the headbox and forming section cannot be ignored.

Particle suspension with zero or small Reynolds numbers has been studied experimentally and numerically. Segre and Silberger (1961) and Karnis and Mason (1966) observed that the equilibrium position of a neutrally buoyant particle in Poiseuille flow is located between the wall and the tube axis due to inertial effects. Feng, Hu and Joseph (1994) investigated the motion of single circular and elliptic particles settling in a Newtonian fluid. It was found that the center of the channel is the equilibrium position regardless of the initial position of the particle. With inertia, the particle experiences five different regimes of motions: steady motion with and without overshoot, weak, strong and irregular oscillations. An elliptic particle always turns its long axis perpendicular to the fall and drifts to the centerline of the channel during sedimentation.

Huang et al (1997) simulated a circular cylinder in Couette and Poiseuille flows of viscoelastic fluids. The simulation results numerically proved that the equilibrium position of slightly buoyant particles migrating in Newtonian fluids is different to that in viscoelastic fluids. In the Couette flow, the equilibrium position of a slightly buoyant particle is nearer the centerline of the channel in a viscoelastic fluid other than in a Newtonian fluid. In the Poiseuille flow, the equilibrium position of the same particle is farther away from the centerline of the channel in a viscoelastic fluid than in a Newtonian fluid.

Three dimensional rotation and translation of non-spherical particles including ellipsoids and cylinders at small aspect ratio were studied by Qi (1997a, 1997b, 1999). Qi (1999) found that when sedimenting at near zero Reynolds number, the long axis of ellipse particle turns vertically while it turns horizontally when the Reynolds number increases. Migration and orientation behavior of single and multiple ellipsoidal particles in a planar Poiseuille flow were investigated by Qi et al (2002). He showed that at steady state, multi-particles with elliptical shape have higher probability to be oriented along the streamline other than across them. Multiple-particles tend to locate between the wall and the center line of the channel. When a single particle is slightly denser than the fluid, it moves towards the wall. When the particle density is large enough, it moves towards the center.

In pulp and paper processes, fibers may be curved rather than straight. Experiment showed that the rotation of flexible fibers in shear flow at dilute region appears springy, snake-turn and S-turn orbits. The rotation of slightly curved fibers and the viscosity of curved fiber suspensions are different from those of straight fibers (Forgacs and

Mason 1965). Qi (2004) studied flexible fiber in a shear flow at finite Reynolds numbers. A flexible fiber was modeled as a chain of spheres with different stiffness. It was observed that the rotation of a flexible fiber changes from rigid fiber rotation to springy, then the more flexible fiber shows S-turn finally. Considering inertial effects, the rotation is slowed down due to streamline separation.

Some fibers, especially the recycled fibers have some permanent deformations, which make fibers slightly curved. In this study, curved fibers are shaped as an arc. The curved fibers settle under gravity is numerically simulated by using lattice Boltzmann method. Different aspect ratios and curvatures are used to characterize the dynamic property of curved fiber sedimentation.

## NUMERICAL METHODS

Navier-Stokes equations are solved directly by lattice Boltzmann method. This method has been successfully applied to simulate the interaction between fluid and solid particles (Aidun, 1998; Qi, 1999; Ladd 1994). In lattice Boltzmann (LB) method, fluid particles are divided into discrete nodes, every particle resides on the lattice nodes to simulate the nature of flow. The fluid particles move to their nearest neighbors along the links with unit spacing in each unit time step.

To determine the mass density  $\rho$  and momentum density  $\rho\vec{u}$ , following equations are used,

$$\rho = \sum_{\sigma} \sum_i f_{\sigma i} \quad \rho\vec{u} = \sum_{\sigma} \sum_i f_{\sigma i} \vec{e}_{\sigma i} \quad (1)$$

The distribution functions in the single relaxation time scheme is given by

$$f_{\sigma}(\vec{x} + \vec{e}_{\sigma}, t + 1) = f_{\sigma}(\vec{x}_{\sigma}, t) - \frac{1}{\tau} [f_{\sigma}(\vec{x}_{\sigma}, t) - f_{\sigma}^{eq}(\vec{x}_{\sigma}, t)] \quad (2)$$

where  $f_{\sigma}(\vec{x}_{\sigma}, t)$  is the fluid particle distribution function for particles with velocity  $\vec{e}_{\sigma}$  at position  $x$  and time  $t$ ,  $f_{\sigma}^{eq}(\vec{x}_{\sigma}, t)$  is the equilibrium distribution function.

In this study, the simulation is performed using the D3Q15 model. It possesses a rest particle state, six links with nearest neighbors, and eight links with next nearest neighbors.  $f_{\sigma}^{eq}(\vec{x}_{\sigma}, t)$  is taken as

$$f_{\sigma}^{eq}(\vec{x}, t) = A_{\sigma} + B_{\sigma} (\vec{e}_{\sigma} \cdot \vec{u})^2 + C_{\sigma} (\vec{e}_{\sigma} \cdot \vec{u})^2 + Du^2, \quad (3)$$

where  $\sigma = 1$  represents the particles moving to the nearest neighbors,  $\sigma = 2$  represents the particles moving to the second nearest neighbors,  $\sigma = 0$  represents the particles resting at the nodes.

In a widely used class of models (Qian et al., 1992), the kinematic viscosity related to the relaxation time  $\tau$  is given by:

$$\nu = \frac{2\tau - 1}{6} \quad (4)$$

When the particle is suspended in the fluid, the fluid particles and solid particles collide. The detailed method which has been reported by Ladd (1994), Aidun, Lu and Ding (1998) and Qi (1999) is not repeated here.

## RESULTS AND DISCUSSIONS

The curved fiber and its corresponding parameters are shown in Figure 1.

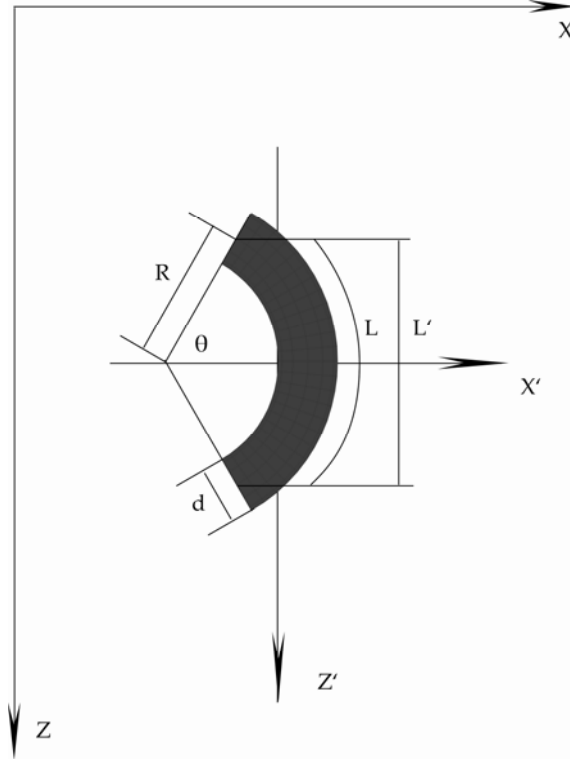


Figure 1: Curved fiber and its parameters

The curl index is defined as  $C = \frac{L}{L'} - 1$  (Page 1985),  $L'$  is the end-to-end length of the curved fiber,  $L$  is the contour length of the curved fiber. The aspect ratio of fiber is given by  $\frac{L}{d}$ ,  $d$  is the diameter of fiber.  $X'$  and  $Z'$  are the primary axes of the curved fiber.  $X$  and  $Z$  are the coordinates of simulation channel.

A single curved fiber is assumed to settle in an infinite long vertical channel filled with the Newtonian fluid along the  $Z$ -direction. The gravity is the only driving force for the motion of the fiber. Once the initial position of the fiber is given, the flow is determined by several parameters:  $\rho_s$ ,  $\rho_f$ ,  $L$ ,  $d$ ,  $C$ , where  $\rho_s$  is the density of solid fiber,

$\rho_f$  is the density of fluid. The density ratio is defined as  $\frac{\rho_s}{\rho_f}$ .

The sedimentations of two curved fibers with different curl indexes and one straight fiber with flat end under the gravity are studied (Figure 2). Fibers studied are at same aspect ratio. The Reynolds number is calculated by

$Re = \frac{VD}{\nu}$ , where,  $V$  is the average terminal velocity of fiber,  $D = 2R \sin \theta$ ,  $\nu$  is the kinematical viscosity of

fluid. For straight fiber,  $D$  is the length of the fiber. When a fiber settles under the gravity, the mean terminal velocities and Reynolds numbers are controlled by the density ratio.

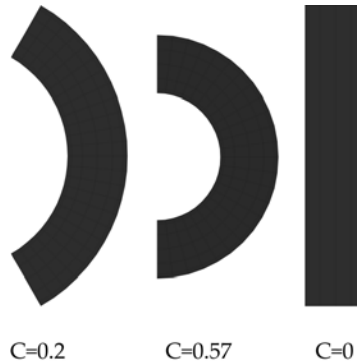


Figure 2: Two curved fibers and one straight fiber are studied

A simulation box of  $140 \times 60 \times 300$  lattice units is used. In this scale, the length of the fiber  $L=40$  and the diameter  $d=8$  give the fiber an aspect ratio of 5. The mass centers of curved and straight fibers are placed at  $0.3W$  at the X-direction. The fibers settle along the Z-direction.  $W=140$  is the width of the channel. Y-direction is periodical boundary. The curl indexes of curved fiber are set as 0.2 and 0.57. Two different density ratios of 1.001 and 1.05 are used.

### 1. Lateral migration

It has been proven in the many studies that the centerline of channel is an equilibrium position of settling. The curved and straight fibers will reach the center of the channel regardless their initial positions. However, the different initial positions may result in different trajectories of migration (Figure 3).

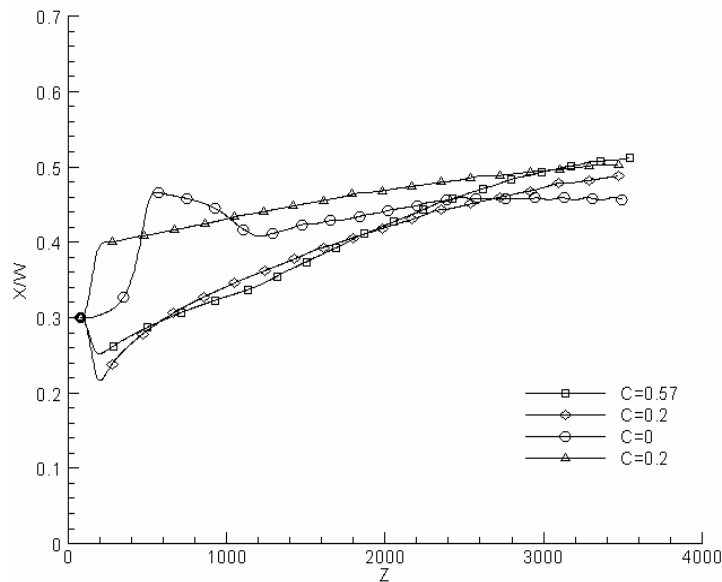


Figure 3: Lateral migration of curved and straight fibers at density ratio of 1.05,  $Re=31.2$  for curved fibers,  $Re=37.7$  for straight fiber. The diamond and square shaped symbols represent the curved fibers with their concave surface towards the left wall. The symbol of triangle represents the curved fiber with its convex surface towards the left wall

In the first case of  $C=0.2$ , the curved fiber with the density ratio of 1.05 is released from the position of (42, 30, 80)

with a zero initial velocity. The concave surface of the fiber is faced to the left wall. The curved fiber shows a lateral migration towards the wall first, and later it is repelled towards the center of the channel. This motion can be clearly shown in the flow streamlines. The curved fiber generates two vortices at the ends of the fiber due to the curvature and rotation. The left vortex is more intense (Figure 4) as the concave surface faced to the left wall, and the vortex suction effect moves the fiber towards the wall. After the curved fiber turns itself perpendicular to the gravity, the intensity of the left vortex, as shown in Figure 5, is reduced and the wall pushes the fiber towards the center of the channel.

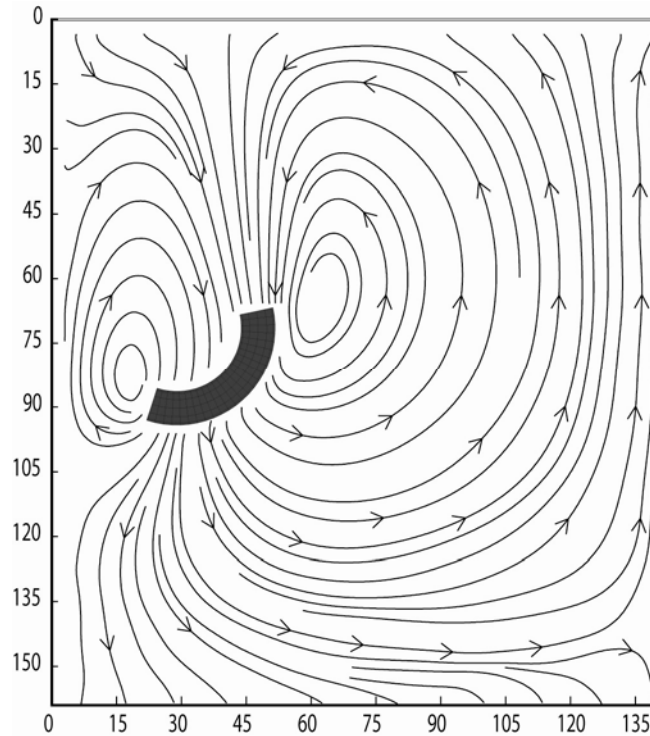


Figure 4: Flow streamlines and the curved fiber with curl index of  $C=0.2$  and the density ratio of 1.05 at time step of 3000

It is necessary to determine whether the initial direction of curved fiber has influence to the migration and rotation. For this purpose, a curved fiber with the same curl index of  $C=0.2$  and density ratio of 1.05 is placed at the same lateral position. The only difference is that the concave surface of the fiber is faced towards the channel center. The results show that this curved fiber has no initial movement towards the wall and the fiber moves to the center of the channel monotonically. It is obvious that the initial direction of concave surface of the curved fiber will influence the trajectory behavior even they have the same initial position.

To compare the influence of curvature to migration and rotation, a curved fiber with curl index of  $C=0.57$  and density ratio of 1.05 is studied. In this case, curved fibers are placed with their concave surface towards the left wall. With higher curl index, the curved fiber shows less movement towards the wall before it starts to move towards the center of channel. In contrary, the straight fiber at same density ratio and aspect ratio moves towards the center of channel without moving towards the wall first.

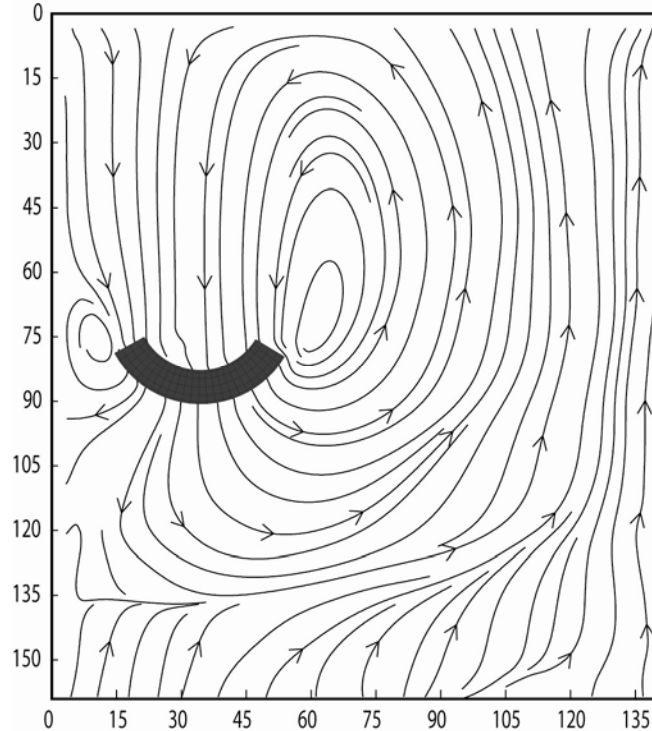


Figure 5: The same as in Figure 4 expect that the time step is 8000

## 2. Rotation

Studies showed that, without the influence of inertia, a non-spherical particle will fall along the stream under gravity with its long axis. While the inertia effect is considered, a non-spherical particle will rotate itself until its long axis becomes horizontal. In this study, the curved and straight fibers sediment under gravity, the inertia effect of flow is considered. The rotation angle is defined as the absolute value of the angle between Z-axis of the fiber and Z-axis of the simulation box.

The results show that the curved fiber always turns itself perpendicular to the gravity at the studied Reynolds numbers. At density ratio of 1.05, the Reynolds number of curved fiber reaches  $Re=31.2$ . The curved fiber turns quickly to be horizontal while the straight fiber takes longer time to turn itself perpendicular to the gravity (Figure 6). The curved fibers with different curl indexes show the same tendency of rotation. With higher curl index, the fiber rotates faster than the one has lower curl index. This can be explained by the different curvatures associated with different moments of inertia. The curved fiber with higher curl index induces a lower moment of inertia of its rotation axis (Figure 7).

The angular velocities shown in Figure 7 also demonstrate that the initial facing direction and curvature result different rotation. When the curved fiber has its concave surface towards the left wall, the fiber turns clockwise to be perpendicular to the gravity. When the curved fiber has its convex surface towards the left wall, the fiber turns anticlockwise to be perpendicular to the gravity. Straight fiber turns anticlockwise as well. The curvature and initial direction of curved fiber have significant influence of rotation.

To study the rotation feature of curved fiber at small Reynolds number, a curved fiber with density ratio of 1.001 is settled. In this case, the Reynolds number is about  $Re=1.3$ . At this small Reynolds number, the curved fiber rotates and finally turns itself perpendicular to the gravity. The straight fiber with same density ratio shows small but non-zero rotation angles (Figure 8).

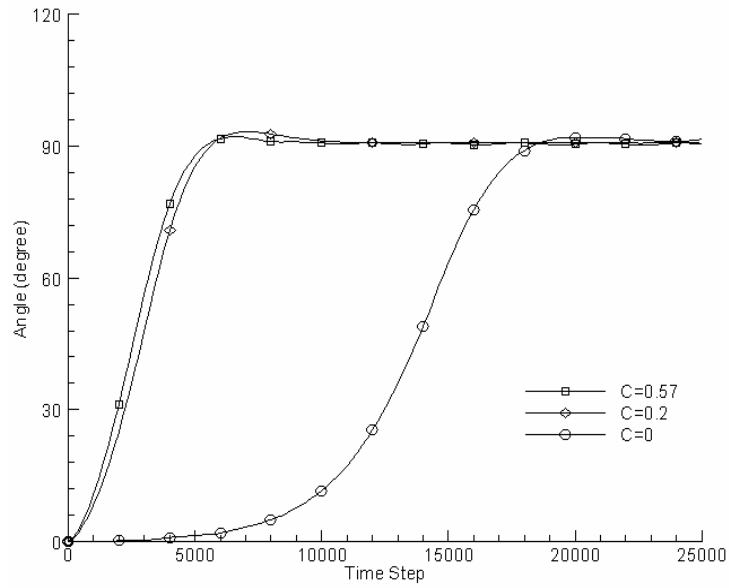


Figure 6: Rotations of curved and straight fibers at density ratio of 1.05. Here, with  $C=0.2$ , the concave surface of the curved fiber is towards the wall

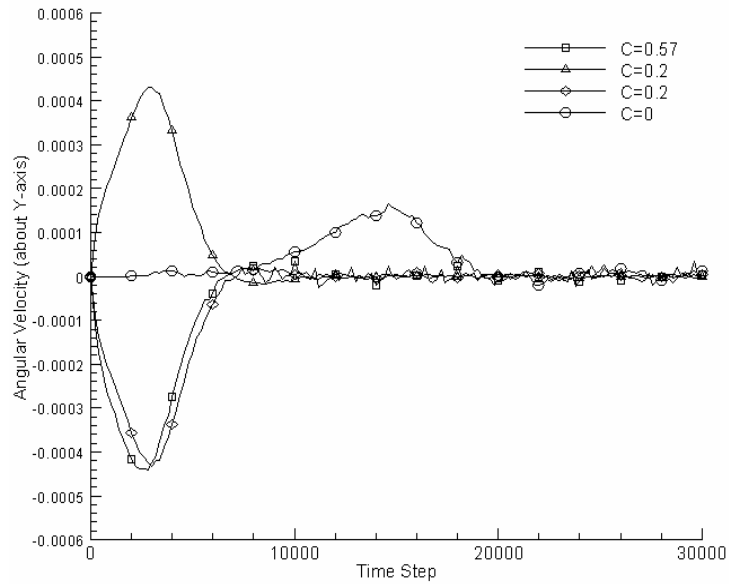


Figure 7: Angular velocities of curved fibers and straight fiber at density ratio of 1.05. Here, symbol of triangle represents the curved fiber with curl index of  $C=0.2$  and with its convex surface towards the left wall, symbol of diamond represents the curved fiber with curl index of  $C=0.2$  but with its concave surface towards the left wall

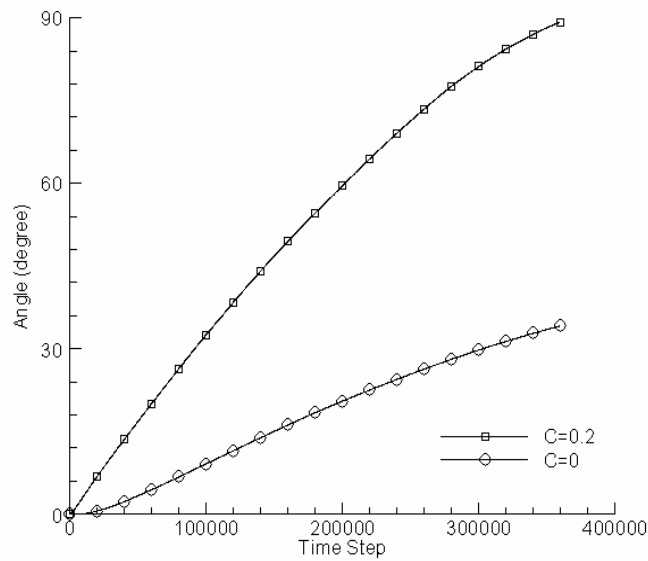


Figure 8: Rotation angles of curved and straight fiber at density ratio of 1.001 and Reynolds number around 1.3

### 3. Settling velocity

The sedimentation of non-spherical particles differs quantitatively from the sedimentation of spherical particles. As shown in this study, the sedimentation of curved fiber differs from that of straight fiber.

At same density ratio, the terminal settling velocities of curved and straight fibers are the same because the mass of these fibers are the same. Figure 8 shows the settling velocities of curved and straight fibers at density ratio of 1.05 when the Reynolds numbers are over 30. The settling velocity increases rapidly at initial times for all the fibers and it is initially oscillated. The settling velocity becomes stable when the fiber is horizontal and close to the center of the channel. The fiber with a larger curl index has smaller overshooting speed and reaches the terminal speed slower than the one with smaller curl index as shown in Figure 9. The straight fiber is the extreme condition which has curl index of  $C=0$ . It has the highest overshooting speed and reaches its terminal speed faster than curved fibers.



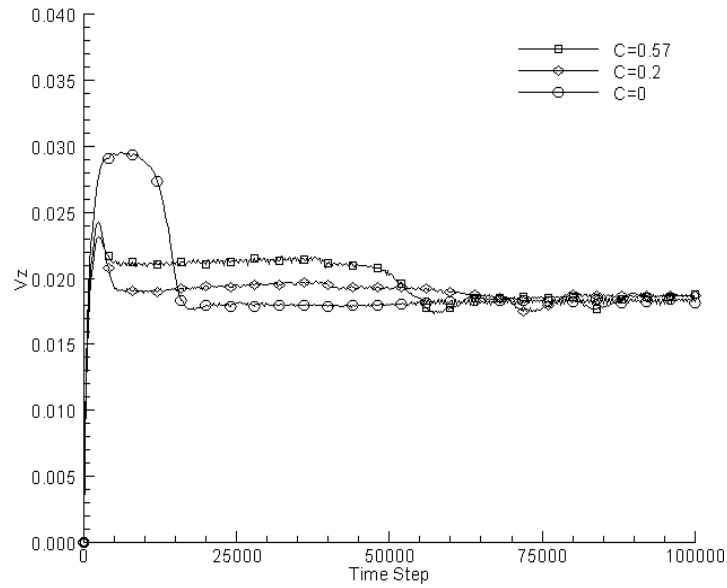


Figure 9: The terminal velocities of curved and straight fibers at the same density ratio of 1.05 are the same. However they have different trajectories. The Reynolds number of straight fiber is  $Re=37.7$ , the Reynolds number for curved fiber is  $Re=31.2$ . The concave surfaces of curved fibers are towards the left wall in this case

## CONCLUSIONS

Rotation and migration of curved and straight fibers settling under gravity are studied by using a lattice Boltzmann method. It is found that the migration and rotation of the fiber are sensitive to the curvature. The curved fiber always turns its long body perpendicular to the direction of gravity at all the Reynolds numbers tested. We find that a fiber with larger curl index reaches the terminal velocity slower than the fiber with smaller curl index. We demonstrate that the initial direction of the concave surface of the fiber will affect the migration trajectories. When the concave surface of the fiber is faced to the wall, the fiber initially moves to the wall due to the suction effect of strong vortex between the wall and the fiber. After the vortex intensity is reduced, the wall effect pushes the fiber to the center of the channel. The curved fiber turns clockwise to be horizontal under this condition. When the concave surface is initially faced to the channel center, the fiber moves to the center monotonically and turns itself anticlockwise to be horizontal. In short, the initial direction of concave fiber will influence the trajectory behavior of rotation and migration even they have the same initial positions.

## ACKNOWLEDGEMENT

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## Contents

- Introduction
- Numerical methods
- Results and discussions
- Conclusions

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## Introduction

- Fiber motions are related to flocculation and dispersion
- Fibers in papermaking process are flexible rather than rigid
- Recycled fibers may have some permanent deformation, become slightly curved
- Inertia effects of fibers and flow need to be considered when increasing operation speed

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## Introduction (Cont.)

- Symmetric particles sedimentation with zero or small Reynolds number have been studied
- Centerline is the equilibrium position of settling
- In Stoke flow, particle settling with its long axis along the gravity
- When Reynolds number increases, the long axis of the particle turns to be perpendicular to the gravity

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## Numerical Method

- Lattice Boltzmann Method (LB)
- Particles are divided by discrete node, move to the nearest neighbor

$$\rho = \sum_{\sigma} \sum_i f_{\sigma} \quad \rho \vec{u} = \sum_{\sigma} \sum_i f_{\sigma} \vec{e}_{\sigma}$$

$$f_{\sigma}(\vec{x} + \vec{e}_{\sigma}, t+1) = f_{\sigma}(\vec{x}_{\sigma}, t) - \frac{1}{\tau} [f_{\sigma}(\vec{x}_{\sigma}, t) - f_{\sigma}^{eq}(\vec{x}_{\sigma}, t)]$$

$$f_{\sigma}^{eq}(\vec{x}, t) = A_{\sigma} + B_{\sigma} (\vec{e}_{\sigma} \cdot \vec{u})^2 + C_{\sigma} (\vec{e}_{\sigma} \cdot \vec{u})^2 + D u^2$$

$$v = \frac{2\tau - 1}{6}$$

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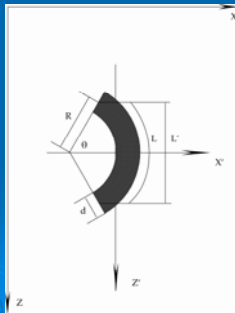
## Parameters

Curl Index (Page)  $C = \frac{L}{L'} - 1$

Aspect Ratio  $\frac{L}{d}$

Density Ratio  $\frac{\rho_s}{\rho_f}$

Reynolds Number  $Re = \frac{VL'}{\nu}$




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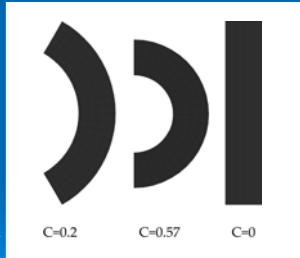
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## Simulation

Simulation Box 140\*60\*300  
Fiber Length L 40  
Fiber Diameter d 8  
Aspect Ratio = 5  
Density Ratio = 1.05 & 1.001  
Settling along Z-direction  
Periodical boundary Y-direction



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## Migration

### ➤ Lateral migration

- Centerline is the equilibrium position
- The direction of the convex surface results different trajectories of migration
- Migration of curved fiber is different from that of straight fiber

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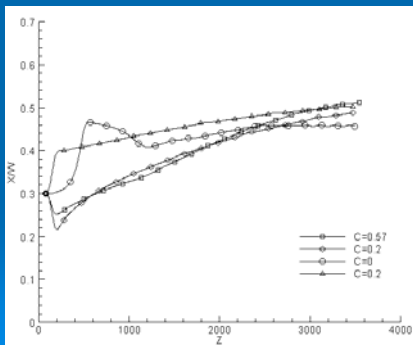
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## Lateral Migration



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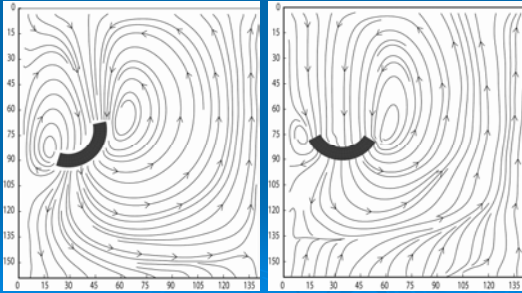
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## Streamlines



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## Rotation

- With considering inertia effects, symmetric particle always turns its long axis to be horizontal
- Curved fiber always turns itself perpendicular to the gravity
- Fiber with higher curl index rotates faster
- Curved fiber turns quickly to be horizontal than straight fiber

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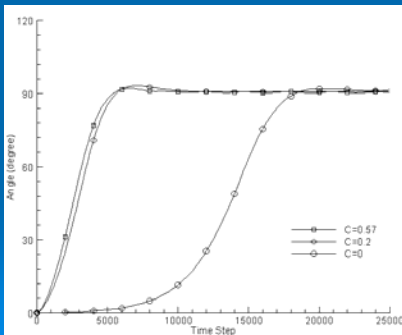
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## Rotation Angles



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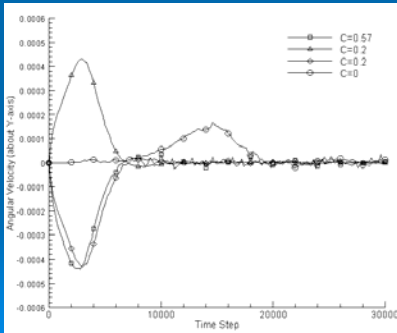
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## Angular Velocity



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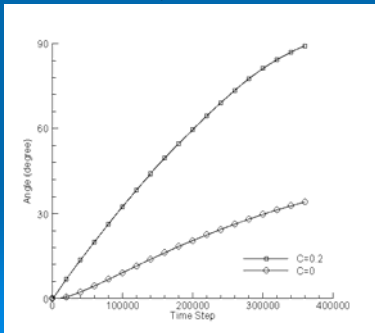
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## Rotation Angle at Small Reynolds Number



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## Settling Velocity

- At same density ratio, terminal velocities are the same
- Settling velocity increases rapidly at initial time and oscillated
- Settling velocity become stable when fiber become horizontal and near the centerline
- Larger curl index results small overshooting speed and reach terminal speed slower

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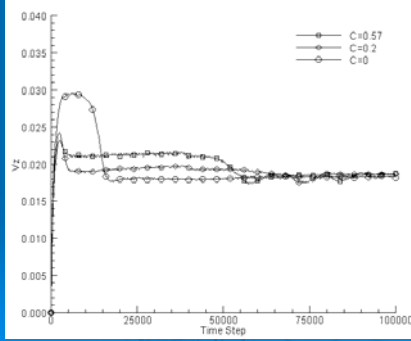
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## Settling Velocity



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## Conclusions

- Migration and rotation is sensitive to the curvature
- Curved fiber always turns itself perpendicular to the gravity
- Curved fiber with larger curl index reaches terminal speed slower
- Initial direction of curved fiber has influence of migration and rotation

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## Acknowledgement

- USDA Forest Products Lab

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Thank You!

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