Asymmetric Grading Error and Adverse Selection: Lemons in the California Prune Industry

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Grading systems are often introduced to address the classic adverse selection problem associated with asymmetric information about product quality. However, grades are rarely measured perfectly, and adverse selection outcomes may persist due to grading error. We study the effects of errors in grading, focusing on asymmetric grading errors—namely when low-quality product can erroneously be classified as high quality, but not vice versa. In a conceptual model, we show the effects of asymmetric grading errors on returns to producers. Application to the California prune industry shows that grading errors reduce incentives to produce more valuable, larger prunes.

Key words: adverse selection, grading errors, product quality, prunes

Introduction

Adverse selection is a concern in many agricultural product markets due to imperfect information about product quality. Grading is one way to mitigate this problem. The price premiums and discounts associated with commodity grades provide incentives for market participants to alter the distribution of quality characteristics. Various researchers (e.g., Matsumoto and French; Lichtenberg) have studied farmers' incentives to alter cultural practices in response to grade-based prices, while others have investigated handlers' incentives to alter the distribution of quality through blending and cleaning products such as grain (Hennessy 1996a; Hennessy and Wahl; Giannakas, Gray, and Lavoie). Other authors have utilized hedonic models to investigate the failure of grading systems to provide appropriate signals (Naik; Bierlen and Grunewald).

Grading almost always involves error, but this aspect has received comparatively little attention and represents the focus of this study. We present a conceptual and empirical analysis of the economics of size-based grading for agricultural commodities in the presence of grading error. We develop a formal model to show that most sizing methods have an inherent adverse selection bias due to grading error, which acts to discourage the production of high-quality product. In our model, all agents have perfect information regarding product quality and the measurement error. It is assumed that

each producer provides product of mixed quality which must be subjected to grading. These latter characteristics are particularly descriptive of agricultural markets where grading is mandated by the government or through marketing orders. We show that the imperfect measurement associated with the assignment of size grades can result in an adverse selection outcome: low-quality goods driving out high-quality goods. Notably, the adverse selection outcome in this context emerges without resorting to asymmetric information and heterogeneous producers.

The model and our main results pertain to the case of asymmetric or one-way grading errors—product can masquerade as having higher than its actual quality, but not lower than actual quality. However, our main results do not require this assumption, which we make both for purposes of exposition and for compatibility with our application to California prunes. Prunes are graded by size into one of five categories, and they are subject to asymmetric errors in grading. We examine the effect of this measurement error on prices and on the incentives for producers to adopt cultural practices to grow prunes of larger sizes.

A Review of Past Research on
Grading Errors

Grading errors can emerge both as a consequence of sampling errors and from imperfect testing. Most commodity grading is done on a sampling basis, because grading is costly and often involves destruction of the tested product. Starbird studied sampling error in grading processing tomatoes for worm damage and concluded that a primary factor motivating pesticide applications in the industry was to reduce the risk that shipments would be rejected due to erroneous test results. Lichtenberg similarly observed that sampling error may play a role in growers' application of pesticides to meet cosmetic standards for fruits and vegetables.

The effects of imperfect testing have been investigated conceptually by Heinkel; De and Nabar; Mason and Sterbenz; and Hennessy (1996b). Heinkel showed that ex post testing and imposition of penalties for low quality could attenuate Akerlof's classic lemons problem in the used automobile market. However, Heinkel also found that, as the accuracy of the test diminishes, dealers' incentives to perform maintenance on low-quality automobiles similarly diminishes, because the inaccuracies in testing reduce the chance to avoid penalties for selling low-quality cars.

While testing or grading is mandatory in the situations studied by several researchers (Starbird; Lichtenberg; and Heinkel), studies by De and Nabar and by Mason and Sterbenz investigate the effects of errors in testing on sellers' incentives to undertake voluntary product certification. De and Nabar show that, whereas low-quality sellers have no incentive to certify their product under perfect testing, they may undertake certification when grading errors are present in hopes of obtaining an erroneous high-quality certification. Mason and Sterbenz's model is similar to that of De and Nabar, but they allow producers to conceal test outcomes if they wish. In this case, imperfect testing is even more likely to produce inefficient, pooling outcomes wherein both low- and high-quality producers engage in testing. Inaccurate tests lead to the same incentive for low-quality producers to undertake testing (De and Nabar). In addition, because highly accurate tests increase the price of certified units and unfavorable outcomes need not

be revealed, increased accuracy may paradoxically cause low-quality producers to undertake the test in hopes of obtaining a high-quality certification.

The antecedent work most closely related to this study is by Hennessy (1996b), who investigates the effect of imperfect testing on producers' incentives to invest in quality-improving capital. With imperfect testing, high-quality product can erroneously be classified as low quality and vice versa. Thus, the market prices for high-quality product must reflect the fact that the measured grade contains some low-quality product and vice versa. The market accordingly fails to reward properly the technological investment, and this results in underinvestment and market failure.[1]

Our work is differentiated from the prior literature in two dimensions. First, we emphasize the economics of asymmetric grading error, wherein low-quality product may receive a high-quality rating, but the converse cannot occur. We argue that this type of error is the norm for size-based grading methods. For example, in systems used to grade or sort fruit, vegetables, nuts, or grain by size, the product is conveyed across screens or cylinders with holes of increasing size or diverging belts or rollers. Small product may not fall into its designated category and may, instead, travel on to categories reserved for larger product, but large product cannot physically fall into the categories designated for smaller product. Thus, a portion of lower-quality goods receives a higher-quality ranking, but the converse cannot occur.[2]

Second, we develop expressions that capture the effects of grading error on the prices paid to growers by grade. Our approach is then implemented empirically for the California prune industry. The unique analytics of asymmetric grading error enable us to provide an intuitive explanation of how prices are affected. To our knowledge, the study is the first to quantify empirically the effects of imperfect testing on prices and producer behavior.

A Theoretical Model of Errors in Grading

Consider a farm product which is sorted and graded based on a single quality characteristic, i.e., size, and a grading system characterized by the one-way measurement errors described above. One outcome of this type of error is that the measured quantity of products in each grade is not the actual quantity of the product meeting the grade standard. For a product sorted into n grades, with 1 being the highest-quality grade and n being the lowest-quality grade, let the actual or true percentage of product in grade i be w_i. The measured share of product classified as grade i (m_i) is equal to the portion of the product that is correctly sorted and graded as grade i, plus the portion of product of a lower grade ($j > i$) that is incorrectly assigned grade i.

For simplicity, the subsequent analysis is set forth in the context of four grades. However, the theory generalizes seamlessly to n grades ($n = 5$ in our subsequent application), and can be presented compactly, if not intuitively, in matrix form. We provide this development in the appendix, along with a discussion of symmetric measurement errors.

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[1] Hennessy argues that this market failure provides an incentive for vertical integration between the producing and processing sectors.

[2] See Henderson and Perry for a detailed discussion of the engineering processes used in grading food products by size.

The following expressions relate measured to actual shares in each of the four grades, where s_j^i is the share of actual grade j product misclassified as grade i:

(1)
$$m_1 = w_1 + s_2^1 w_2 + s_3^1 w_3 + s_4^1 w_4,$$

(2)
$$m_2 = \left(1 - s_2^1\right) w_2 + s_3^2 w_3 + s_4^2 w_4,$$

(3)
$$m_3 = \left(1 - s_3^1 - s_3^2\right) w_3 + s_4^3 w_4,$$

and

(4)
$$m_4 = \left(1 - s_4^1 - s_4^2 - s_4^3\right) w_4,$$

where

$$\sum_{i=1}^{j-1} s_j^i \leq 1, \quad \text{for } j = 2, 3, 4.$$

For instance, the measured grade 1 share (m_1) will consist of actual grade 1 product plus the actual shares in other grades multiplied by the probability of those grades masquerading as grade 1. Note that $s_j^i > 0$ only for $i < j$, reflecting the asymmetry in grading errors—the product can move up to a higher grade, but it cannot move down. The condition that the s_j^i values do not sum to more than one is a physical constraint, and a strict inequality means that at least some product is graded correctly.

We assume that the distribution of the quality characteristic across all producers is known, and that there exists perfect information concerning the probabilities of grading errors. We define V_i as the farm price that would emerge for product of grade i in the absence of any grading error. We need make no assumptions about the manner in which farm prices are set, or the competitive relationships involved in that price-setting process. For example, the market for the farm product could be perfectly competitive, in which case V_i would represent the per unit retail value of product of grade i less all per unit marketing and processing costs. Alternatively, farm prices could be determined under any of the various forms of imperfect competition. For example, in our subsequent application to prunes, farm prices are determined prior to harvest through negotiations between a grower bargaining association and handlers, in which case the hypothetical V_i would represent the outcome of the negotiation process in the absence of grading error. For purposes of exposition, we often refer to V_i as the "value" of product correctly classified into grade i.

Under these assumptions, it is straightforward to show that the producer price (P_i) paid for all grades $i < n$ will be discounted relative to the actual value (V_i), because product measured as grade i is "contaminated" by product from the lower grades. This is true for all grades except the lowest grade, which can contain only product of the lowest grade by construction of the grading process. Hence, the producer price for the lowest grade is equal to its true market value.

In our model with four grades, this means that

(5)
$$P_4 = V_4.$$

However, measured grade 3 consists of product from both grade 4 and grade 3. Therefore, under perfect information, the producer price for grade 3 must represent a weighted average of the true market value of grade 3 and grade 4 products, with the weights corresponding to the relative quantities of grade 3 and grade 4 products that are classified as grade 3:

$$(6) \qquad P_3 = \frac{V_3\left(1 - s_3^1 - s_3^2\right)w_3 + V_4 s_4^3 w_4}{m_3}.$$

The numerator in (6) is the expected total value of product measured as grade 3, so dividing by m_3 yields a willingness to pay for a unit of the product measured as grade 3. Because grade 3 products can also be classified as grades 1 or 2, the weight on the true market value of actual grade 3 is the proportion of grade 3 products that was correctly graded.

Similarly, measured grade 2 will consist of products from grades 2, 3, and 4. The grower price for grade 2 is thus a weighted average of the true market values of these grades. Grade 2 products can be classified erroneously as grade 1, so the weight on the true market value of grade 2 in equation (7) is the proportion of grade 2 products that remain in grade 2:

$$(7) \qquad P_2 = \frac{V_2\left(1 - s_2^1\right)w_2 + V_3 s_3^2 w_3 + V_4 s_4^2 w_4}{m_2}.$$

An analogous result holds for P_1:

$$(8) \qquad P_1 = \frac{V_1 w_1 + V_2 s_2^1 w_2 + V_3 s_3^1 w_3 + V_4 s_4^1 w_4}{m_1}.$$

We wish to ascertain the error in valuation of the product due to mistakes in grading, or the difference between the actual value (V_i) and the grower price (P_i) for each grade. If buyers and sellers all know the various parameters determining the size distribution of the product and also the probabilities of grading errors, then it is a matter of indifference whether P's or V's are negotiated prior to production. There is a simple linear relationship between them. However, in our application, only the P's are observable, and from them we can infer the V's.

Given the above relationships, we can determine the V_i's by proceeding recursively from the lowest grade. Because there is no error in the lowest grade, the producer price for grade 4 then equals the actual market value of that grade, as in (5).

Next, consider grade 3. Given (6), we can substitute for m_3 using (3), substitute (5), and solve to obtain

$$(9) \qquad P_3 - V_3 = -\left(P_3 - P_4\right)\frac{s_4^3 w_4}{\left(1 - s_3^1 - s_3^2\right)w_3} \le 0.$$

Notice that only V_3 is unknown in this expression. This relationship has the intuitive interpretation that product in grade 3 sells for less than the true value of grade 3

product, because grade 3 is contaminated by product from grade 4; $V_3 = P_3$ only if s_4^3, the probability of such pollution, is zero. The magnitude of the value-price difference is determined by the difference in grower prices for grades 3 and 4 and the amount of grade 4 product measured in grade 3, relative to the amount of actual grade 3 product that remains in the grade.

Turning now to grade 2, we perform a similar set of operations to obtain

$$(10) \qquad P_2 - V_2 = -(P_2 - P_3)\frac{s_3^2 w_3}{\left(1 - s_2^1\right)w_2} - (P_2 - P_4)\frac{s_4^2 w_4}{\left(1 - s_2^1\right)w_2}$$

$$+ (V_3 - P_3)\frac{s_3^2 w_3}{\left(1 - s_2^1\right)w_2} \leq 0.$$

Only V_2 is unknown because V_3 is defined in (9). Equation (10) has a similar common-sense interpretation. The market price of grade 2 is discounted based on the relative amounts of grade 3 and grade 4 product that receive a grade of 2, and the differences in grower prices between grade 2 and the lower grades. The final term adjusts for the fact that grade 3 product is actually more valuable than the price paid for grade 3 to growers, based on (9).

Finally, again relying on the recursive nature of the approach, we can form a similar expression for grade 1:

$$(11) \qquad P_1 - V_1 = -(P_1 - P_2)\frac{s_2^1 w_2}{w_1} - (P_1 - P_3)\frac{s_3^1 w_3}{w_1} - (P_1 - P_4)\frac{s_4^1 w_4}{w_1}$$

$$+ (V_2 - P_2)\frac{s_2^1 w_2}{w_1} + (V_3 - P_3)\frac{s_3^1 w_3}{w_1} \leq 0,$$

which is interpreted identically to its predecessors. Alternatively, the error in the valuation can be expressed in terms of the V_i's as follows:

$$(12) \qquad P_3 - V_3 = -(V_3 - V_4)\frac{s_4^3 w_4}{m_3},$$

$$(13) \qquad P_2 - V_2 = -(V_2 - V_3)\frac{s_3^2 w_3}{m_2} - (V_2 - V_4)\frac{s_4^2 w_4}{m_2},$$

and

$$(14) \qquad P_1 - V_1 = -(V_1 - V_2)\frac{s_2^1 w_2}{m_1} - (V_1 - V_3)\frac{s_3^1 w_3}{m_1} - (V_1 - V_4)\frac{s_4^1 w_4}{m_1}.$$

These expressions indicate that the reduction in the market price for grade i from its true market value depends on the difference in value between grade i and lower grades and the share of lower-quality commodity classified as grade i.

Because the producer price for grade i is a weighted average of the true market values of grade i product and lower-grade products which are incorrectly classified as grade i,

the producer is paid less than market value for the portion of the farm product that is correctly sorted and graded for all grades except the lowest. However, the producer is paid more for product that truly meets a lower grade standard but ends up in a higher grade.[3]

The net effects on revenues from each grade can be seen in equations (15)–(18). Let Q denote the total number of units of the product produced, so that $w_i Q$ represents the number of units of product of grade i. The farm revenue (R_i) obtained for the product truly belonging to each grade i standard can then be expressed as

(15) $$R_1 = P_1 w_1 Q,$$

(16) $$R_2 = \left[P_1 s_2^1 + P_2 \left(1 - s_2^1 \right) \right] w_2 Q,$$

(17) $$R_3 = \left[P_1 s_3^1 + P_2 s_3^2 + P_3 \left(1 - s_3^1 - s_3^2 \right) \right] w_3 Q,$$

(18) $$R_4 = \left[P_1 s_4^1 + P_2 s_4^2 + P_3 s_4^3 + P_4 \left(1 - s_4^1 - s_4^2 - s_4^3 \right) \right] w_4 Q.$$

The per unit farm value (v_i) of grade i product is R_i divided by the actual quantity $w_i Q$ in grade i:

(19) $$v_1 = P_1,$$

(20) $$v_2 = P_1 s_2^1 + P_2 \left(1 - s_2^1 \right),$$

(21) $$v_3 = P_1 s_3^1 + P_2 s_3^2 + P_3 \left(1 - s_3^1 - s_3^2 \right),$$

(22) $$v_4 = P_1 s_4^1 + P_2 s_4^2 + P_3 s_4^3 + P_4 \left(1 - s_4^1 - s_4^2 - s_4^3 \right).$$

In order to interpret equations (19)–(22), recall that the producer price for each grade is lower than the true market value of the grade, i.e., $P_i < V_i$, except for the lowest grade, where $P_4 = V_4$. Because $P_1 < V_1$ [from equation (11)] and $v_1 = P_1$ [as shown in equation (19)], $v_1 < V_1$, and the farm unit value of the highest grade is lower than the true market value of the grade. Similarly, since $P_4 = V_4$ [from equation (5)] and $v_4 > P_4$ [from equation (22)], the farm unit value of the lowest grade is higher than the true market value of the grade, i.e., $v_4 > V_4$, due to some product migrating into higher grades. For intermediate grades, the relationship between the unit farm value and the true market value is ambiguous. However, the comparison can be made using the relationships derived thus far. Using (20), for instance, we can show that

(23) $$v_2 - V_2 = \left(P_1 - P_2 \right) s_2^1 - \left(V_2 - P_2 \right).$$

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[3] Hennessy (1996b) refers to this problem as an externality, because producers of high-quality product (large prunes in our application) do not receive the full value of their production; instead, part of the value spills over into the lower grades, to the benefit of producers whose product is concentrated in those grades. This positive externality accordingly results in underproduction of high-quality product, relative to low-quality product. Similarly, the presence of low-quality product masquerading as higher-quality product is a negative externality, lowering the price paid for all product of the highest grade.

Similarly, for grade 3,

$$(24) \qquad v_3 - V_3 = \left(P_1 - P_3\right)s_3^1 + \left(P_2 - P_3\right)s_3^2 - \left(V_3 - P_3\right).$$

Whether product of intermediate grade i is under- or overvalued depends on which of two opposite effects is larger: the gain in farm value obtained by a portion of grade i product migrating into higher grades [the first term on the right-hand side of (23) or the first two terms on the right-hand side of (24)], or the loss in farm value because $P_i < V_i$ due to the measurement error [the second term on the right-hand side of (23) or the third term on the right-hand side of (24)]. Undervaluation is more likely to occur for higher quality intermediate grades, because there are relatively few higher grades for the product to migrate into and relatively more lower grades from which product can migrate into that grade. A somewhat distinctive feature of this model is that we can derive the classic adverse selection outcome, wherein low-quality product is overvalued and overproduced, relative to high-quality product, even though all agents have symmetric information and, indeed, perfect information as it pertains to the probabilities of grading error.[4]

The appendix contains a general, n-grade version of the theoretical model above in matrix form. We also discuss in the appendix a generalization to the case of symmetric (or two-way) measurement errors. The only real consequence of this generalization is that the comparisons of prices and values are less definitive than for the case of one-way errors. The result that the top grade is undervalued (i.e., $P_1 < V_1$) is unaffected by these changes, while the condition that the lowest grade is correctly valued is replaced by the proposition that it will be overvalued (i.e., $P_4 > V_4$) because of the presence of higher-grade product. Results for the intermediate grades become indeterminate, depending on particular magnitudes of w_i's, V_i's, and s_j^{i}'s. P_2, for instance, may exceed V_2 when errors are symmetric, if enough grade 1 product moved down to grade 2 relative to the amounts of grades 3 and 4 moving up into grade 2.

Application to California Prunes

California produces nearly all U.S. prunes and about 70% of the world's supply. The harvesting of prunes occurs in mid-August to mid-September, using a mechanical shaker which is attached to the tree trunk. Once harvested, prunes are dried. The dried prunes are then cured and aerated for a period of about 30 days. After drying and curing, the fruit is delivered to a packer's warehouse. Packers process the dried prunes by rehydrating, grading, sizing, packaging, and reinspecting to meet the particular specification of the trade. Size is the main quality criterion for dried prunes and is the crucial characteristic in determining prune value. The largest prunes can be sold in

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[4] The problem can be viewed within the classic asymmetric information context of adverse selection problems, if one wishes to think of the graded product itself as a "player" in the underlying game. The prunes in our application can be thought to know their type (size), but they may choose to masquerade as a different type and, accordingly, receive a higher payoff by not falling through their designated screen. Actions to improve the accuracy of the grading process make masquerading more difficult for the prunes, and thus have the same effect as do tests, warranties, and licenses in traditional adverse selection problems. That is, improved grading helps to generate a separating outcome wherein types are distinguished, versus a pooling outcome wherein types are not identified.

gourmet retail packs at a premium price. Moderately large prunes can be pitted and sold as pitted prunes, while the smallest prunes are useful only for juice, paste, and other industrial products.

Prunes in California are marketed under both a federal and a state marketing order. The federal marketing order authorizes the industry to regulate and set standards for the prune grading system. The Dried Fruit Association (DFA) of California is the inspection agent. Packers maintain their own screen graders and may set screen lengths and sizes to suit their own needs. In particular, they may sort and sell prunes into various size categories with various prices. However, official grading for the purpose of determining payments to growers is done using a five-screen grader, and is based on a 40-pound sample collected at the time the prunes are graded by the processor. Prunes that are smaller than the diameter of the screen openings may fall through the holes in the screen and be classified accordingly. The first screen is designed to eliminate trash, while the next four screens are for prune sizing. Prior to 1998, the Undersize screen had 23/32-inch diameter holes, the D screen had 24/32-inch diameter holes, the C screen had 26/32-inch diameter holes, and the B screen had 30/32-inch diameter holes. Prunes in the A category, or "overs," do not fall through any screen and therefore go over the end of the grader. Results from the grading process for each sample are summarized on a grade sheet prepared by the DFA. Under current practices, payments to growers depend on the percentage of prunes within each grade, but these payments are unaffected by variations in the size or weight of prunes within each grade.

For several years, industry participants have complained of an "oversupply" of small prunes. We interpret this concern to refer to the modest value of small prunes in the market place, relative to larger prunes, and not literally to a market disequilibrium condition. Prune size may be enhanced through cultural practices, such as pruning, shaker thinning, and delaying harvest. Field sizing may also be used to eliminate the smallest prunes and to avoid incurring the cost of handling them. Various industry publications have encouraged growers to adopt these practices, although with limited success to date. The adoption in 1996 of payments based upon the grading system described here was an attempt to provide incentives to growers to increase prune size. Rather than receiving one price for their entire crop, based on the average prune size in the sample, growers were paid a separate price for the percentage of crop in each screen grade, based on the sample.

Despite this change, the problem of oversupply of small prunes has persisted. In early 1998, the U.S. Department of Agriculture (USDA) approved an increase to 24/32-inch holes for the Undersize screen as a way to remove more small prunes from the saleable market. In addition, the diameters of the holes in the D and C screens were raised to 26/32-inch and 28/32-inch, respectively. These actions may help to address the imbalance in production, but they do not address the incentive and adverse selection problems caused by asymmetric grading error.

The higher prices that are offered for larger prunes should induce practices that increase prune size, but, as our theoretical model shows, the presence of asymmetric grading error both reduces the premium for large prunes and increases the farm value of smaller prunes, thereby attenuating the payoff from increasing prune size. In our empirical work, we seek to measure how much grading error reduced price incentives to produce larger prunes for the 1996 crop year. We used the formulas from the conceptual model to estimate the differences between the grower price and the actual value

$(P_i - V_i)$, and between the per unit farm value and actual value $(v_i - V_i)$ for each grade of prunes for this crop year. These estimates were based on detailed information for two 40-pound samples of prunes provided by the Prune Bargaining Association (PBA), and on the grade sheets completed for every sample graded by the DFA in 1996. The latter represent actual shipments and payments to growers, whereas the former were used to estimate probability distributions for prune weights within each screen grade.

Data

The PBA sample data consist of the measured grade and the actual weight for each of over 7,000 prunes contained in the two 40-pound samples. After each sample was run through the DFA grader, the weight of each individual prune was recorded. Thus, for each prune in the sample, we knew which screen it fell through and its actual weight. In other words, the measured (based on screen size) and actual (based on weight) size distributions are known for these two samples. We also know the distribution of weights within each screen.

Figure 1 shows these data for the first PBA sample. Each panel shows the sample distribution of weights of prunes falling through a given screen. For instance, the first panel represents all prunes measured as grade A. The vertical dividing lines indicate break points between actual grades. These break points are not based on the marketing order, but, rather, are based on industry rules-of-thumb concerning the weights that separate the two classes. Of the measured A-screen prunes in the first PBA sample, approximately 80% were actually of grade-A weight. Among the rest, approximately 19% were grade-B weight prunes. The remaining prunes were actually either of grade C or grade D weight, the latter too few to show up on our histogram.

The second panel represents the actual size distribution for prunes receiving a grade of B. In this case, 21% of the prunes measured in the B screen were actually grade C-weight prunes, and 3% were grade D-weight prunes. However, 11% of the B-screen prunes weighed enough to be considered grade A. This apparent moving down of A-grade prunes into the B screen is not inconsistent with our theoretical model. It arises because we are estimating true grades based on weight rather than size. Prunes that fit through the B screen are, by definition, B-grade prunes or smaller based upon their size. It does not make sense to say that a prune should have fallen through a higher grade's screen when, in fact, it fit through the smaller hole in the screen where it was found.

The appearance of undergrading is due to the dichotomy of actual grades determined by weight (consistent with industry practices), and measured grades (for purposes of paying producers) that are based on screens that discriminate according to size. There is not a one-to-one relationship between prune weight and prune size. For example, a prune that is long and narrow may fall through the B screen but weigh enough to be considered grade A. Similarly, a B-screen prune with particularly high moisture content may weigh enough to be considered grade A.

In the subsequent empirical work, we treat nominally undergraded prunes as if they actually belonged to the lower grade where they were measured. In addition to the reasons just provided, an economic justification for this choice is that a prune with the physical characteristics that cause it to be undergraded, relative to its weight, on the
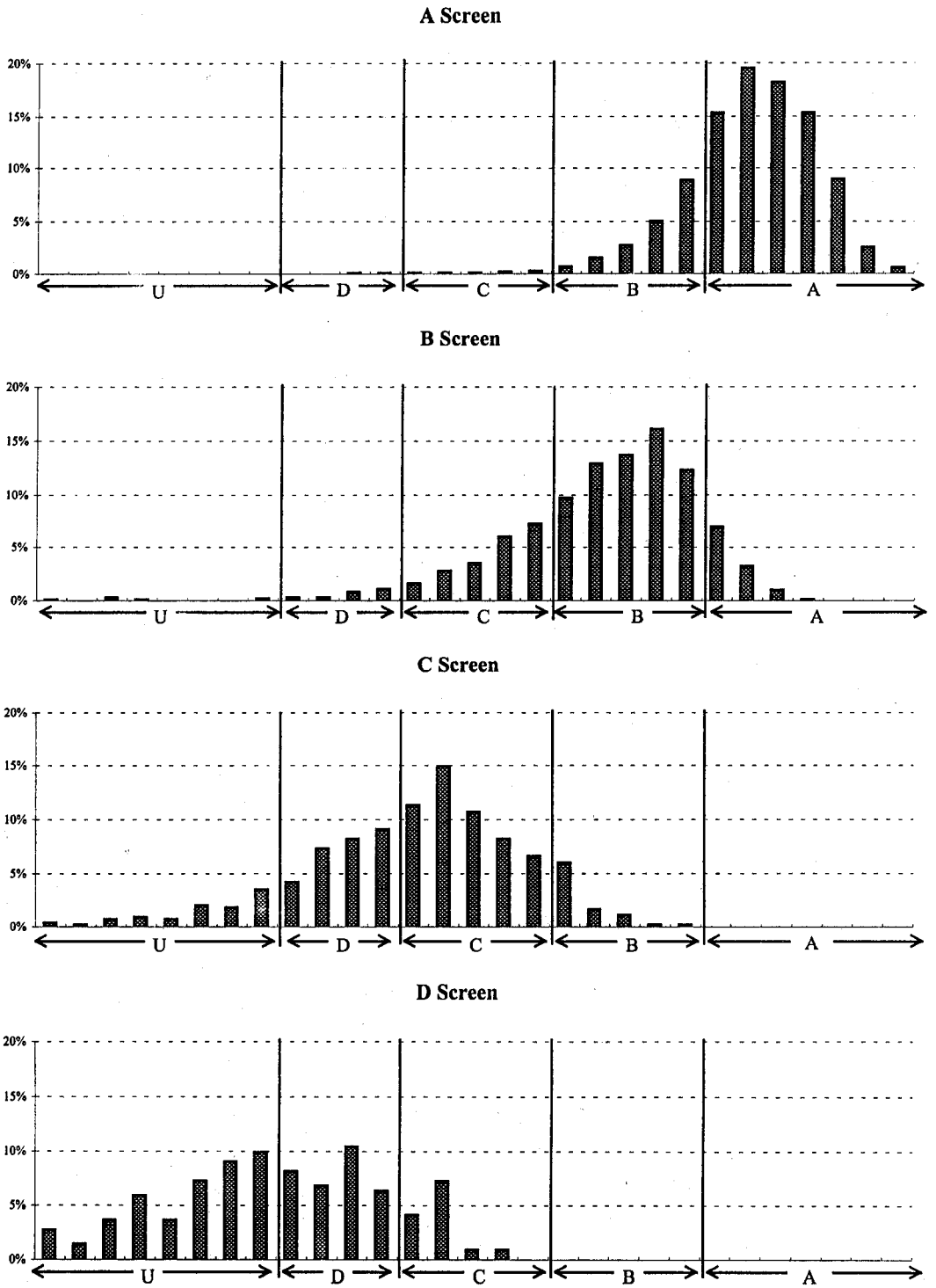
Figure 1. Relative frequencies for each screen in sample 1 (by measured prune size)

DFA grader would also tend to reduce its grade on a processor's grading system, because both are based on size rather than weight. Because end use and wholesale value of a prune are determined by the processors' grades, such prunes would be used and valued as if they were in fact of the lower grade. Allowing for undergrading—by defining grades using weight and allowing for symmetric grading errors, rather than reclassifying the prunes in question—did not materially affect the results of the empirical analysis. Modified versions of tables 1–5 that allow prunes to be undergraded are available from the authors upon request.[5]

The shipments data represent the grading sheets for 1,487 actual shipments from the 1996 crop year. Each sheet reports the total weight and the average size in each of the measured grades A, B, C, D, and U, based on the 40-pound sample taken from each shipment after drying. With only the total weight of prunes in each screen grade, we cannot determine the extent to which grading error is present in any particular shipment. We used the PBA samples to estimate theoretical size distributions for each grade, which we then applied to the shipments data, as discussed below.

Estimation of Prune Size Distributions

Because the actual and measured distributions of the two 40-pound samples are known, this information was used to infer the actual size distributions for each of the 1,487 actual shipments. Based on analysis of the sample data, it appeared reasonable to model the size distributions for the prunes, within each measured grade, using the Gamma probability distribution, which allows for an asymmetry in the distribution of prune sizes.[6] A unique Gamma distribution for the weights of individual prunes was estimated for each measured grade in each shipment. Because we had only summary information on the weight within each grade, it was necessary to use prior information to identify the two parameters of the distribution. We chose to assume that the coefficient of variation within each measured grade was the same as for our two detailed samples combined.

This coefficient of variation was used in conjunction with the reported average prune size to estimate the two parameters for the Gamma distribution for each grade of each 1996 shipment. The Gamma probability density function is written as

$$(25) \qquad\qquad f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} e^{-(x/\beta)} x^{\alpha-1}, \quad x \geq 0,$$

where $\Gamma(\alpha)$ denotes the usual Gamma function, and the mean and variance are $\alpha\beta$ and $\alpha\beta^2$, respectively (e.g., Mittelhammer, p. 187). We can infer the value of the α parameter based on the coefficient of variation (CV) observed in each grade:

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[5] An alternative empirical approach would be to define true grade based on size rather than weight. For example, the individual prunes in the PBA sample could, in principle, have been measured for size rather than for weight. "True" size would then be the smallest screen size the prune could physically fit through. Deviations of measured screen from true screen would necessarily be one-way, or asymmetric, in this analysis. We must use weight as a proxy for true screen size because we have no such data concerning the physical dimensions of individual prunes.

[6] Goodness-of-fit tests supported the Gamma, relative to the log-normal or Beta distributions, which also have the desired asymmetry.

(26)
$$CV^2 = \left(\frac{\sigma}{\mu}\right)^2 = \frac{\alpha\beta^2}{(\alpha\beta)^2} = \frac{1}{\alpha}.$$

Hence, we set

(27)
$$\alpha = \frac{1}{(CV)^2}.$$

We then estimated the β parameter by setting the average size in each grade-shipment combination equal to its theoretical expected value, $\alpha\beta$, and solving for β using the α that was based on the CV.

Thus, for each shipment, we have a set of estimated probability distributions from the Gamma(α,β) family describing the probability distribution of weights of individual prunes in each measured grade. By evaluating the estimated cumulative distribution function at the break points between actual grades, we were able to estimate the proportions of prunes of an actual grade that were measured in each of the five grades. The averages of these proportions over all 1996 shipments are reported in tables 1 and 2.

Table 1 contains m_i and w_i, the measured and actual proportions of prunes in a grade, for each grade, averaged over all shipments of the 1996 crop. Differences between the actual and measured proportions are readily apparent, but the degree of measurement error is further clarified in table 2. Each row of table 2 refers to the actual prune grade and each column refers to the measured prune grade. Cells contain s_j^i, the proportion of the prunes actually belonging to row j's grade that received column i's grade, so that the diagonal elements represent proportions of correctly graded prunes. The numbers below the diagonal represent the percentage of prunes of each actual grade migrating to higher grades.

Table 2 shows that the probability of grading errors is greatest in the lower grades. This result is not surprising, because products in these grades have the greatest opportunity to migrate into higher grades, as shown in the conceptual model. All A-quality prunes are graded correctly by construction of the grading process, and 85% of B-quality prunes are graded correctly, with the remaining 15% masquerading as A-quality prunes. However, only 56% of C-quality prunes are graded correctly, with 42% masquerading as B prunes and 2% masquerading as A prunes. Only 38% of true D-quality prunes were graded as D, with 50% and 12% migrating into the C and B screens, respectively.

Tables 1 and 2 contain the grading information necessary to specify equations (1)–(4). The grower prices for each grade, based on the outcome of negotiations between the handlers and the PBA, are presented in table 3. This information, along with the grading information in tables 1 and 2, is sufficient for specifying equations (5)–(8) and equations (19)–(22), and solving for the actual value of each grade (V_i) and average per unit farm values (v_i). These values are included in table 3, as are the $P_i - V_i$ differentials and the average $v_i - V_i$ differentials for each grade. All grades are undervalued, except for the lowest grade U (undersized), i.e., the price spread is negative, meaning that the grower price is lower than the actual value of prunes of grades A through D. The price of grade A prunes is lower than its true value by 2.3¢/pound (or by 4%), while B-grade prunes are undervalued by 3.4¢/pound (or 8%). By construction, the grower price for the U grade exactly equals its true value, zero.

Table 1. Proportions of Shipments (by Weight) Measured as and Actually Belonging to Each Grade in the 1996 Crop

Prune Grade	Proportions of Prune Shipments	
	Measured (m_i)	Actual (w_i)
A	0.36	0.29
B	0.44	0.42
C	0.13	0.18
D	0.04	0.06
U	0.03	0.05

Table 2. Proportions of Actual Grade Products Classified into Each Measured Grade (by Weight) for the 1996 Crop

Actual Prune Grade	Measured Prune Grade				
	A	B	C	D	U
A	1.00				
B	0.15	0.85			
C	0.02	0.42	0.56		
D	0.00	0.12	0.50	0.38	
U	0.00	0.02	0.17	0.25	0.56

Table 3. Grower Price, Actual Value, and Per Unit Farm Value for Each Grade of the 1996 Crop

Prune Grade (i)	Price (P_i)	Actual Value (V_i)	Farm Value (v_i)	Price minus Actual Value ($P_i - V_i$)	Farm Value minus Actual Value ($v_i - V_i$)
	<————————————————— (cents per pound) —————————————————>				
A	54.25	56.53	54.25	-2.28	-2.28
B	41.00	44.43	42.96	-3.43	-1.47
C	21.75	26.09	30.45	-4.34	4.36
D	7.00	10.70	18.54	-3.70	7.84
U	0.00	0.00	6.21	0.00	6.21

The average spread between the per unit farm value and the actual value of prunes in each grade is shown in the last column of table 3. Since A-grade prunes cannot masquerade as other grades, their per unit farm value equals their grower price, and thus differs from their true value by the same amount as the price, 2.3¢/pound. For B-grade prunes, the average per unit farm value is 3% lower than the actual value. This negative spread indicates that the downward adjustment in grower prices due to measurement error decreases the average revenue earned on B-grade prunes more than the masquerading of B-grade prunes as A-grade prunes increases it. This relationship is reversed for grades C, D, and U, for which per unit grower revenue is larger than the actual value. Here, the increase in revenue from each grade masquerading as higher grades outweighs the decrease in revenue resulting from lower grower prices. On average, grade C and D prunes earn per unit farm values of 17% and 73%, respectively, in excess of their true values. Furthermore, undersized prunes, which have no value, earn over 6¢/pound solely due to measurement error.

Distribution Effects of Grading Error

The preceding calculations describe the effects of grading error on different grades of prunes. A different set of calculations concerns the distribution effects of grading error across producers. The errors in valuation of the form $P_i - V_i$ will be the same for every grower, since the V_i's are calculated based on industry averages for the w_i's, m_i's, and s_j^i's. However, the per unit farm values will vary between shipments, to the extent that any of these parameters depart from industry averages.

We calculated a measure of the value-price "spread" for every shipment, defined as the difference between the total value per unit of the shipment across all grades and the total revenue per unit received by the grower. A positive value for the spread represents the rent captured by the processor for that shipment effectively or, alternatively, value not captured by the grower. Conversely, a negative spread would indicate revenue earned by the grower in excess of the true value of the shipment. Our analysis indicates that a shipment with relatively large prunes should have a positive spread, while one with predominately small prunes will have a negative spread due, for example, to the relatively larger effect on payments of C-quality prunes receiving the price for B's.

Figure 2 shows the empirical distribution of the spreads calculated for all 1,487 shipments in our sample. This spread varies between –6.6¢ and +2.5¢/pound. The distribution is slightly skewed, in that 753 shipments have negative spreads and 734 have positive ones. To the extent that over- or undervaluation can be related to particular characteristics of the shipments, it is reasonable to draw conclusions regarding the effects of grading errors on grower incentives. For instance, to test our expectations concerning the effects of prune size on the spread, we sorted shipment records into three sets, corresponding to average prune sizes for the entire shipment falling in the range of grades A, B, and lower, respectively. The three panels in figure 3 show the empirical distributions of the spread variable for these three groups. For shipments with an overall average prune size corresponding to grade A, nearly all of the calculated spreads were positive, indicating that growers were not paid the full value of these shipments. For the shipments with overall average prune size corresponding to grade B, the spread variable is nearly symmetric around zero, with a few large negative values contributing
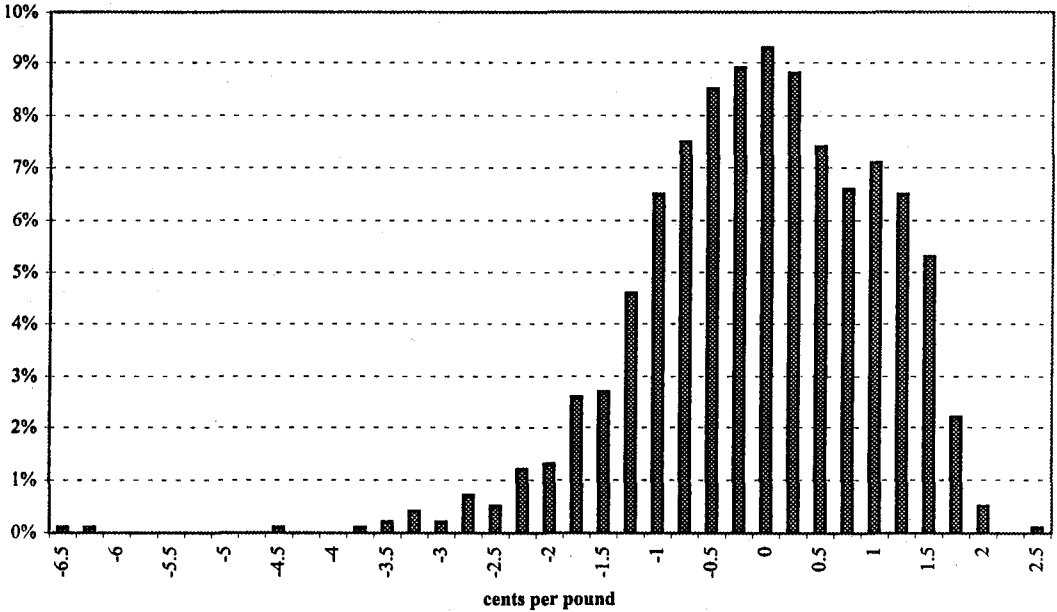
Figure 2. Frequency distribution of price spreads for all (1,487) observations

to an average spread of −0.18¢/ pound. Finally, for the shipments with the smallest average size, the spread is nearly always negative, averaging −1.34¢/pound. For these shipments, growers are paid, on average, more than the true value of the product.

A more formal analysis of the empirical importance of these effects is based on the simple correlations between the spread and various shipment characteristics. By regressing the spread for each shipment on various characteristics, we were able to measure the importance of these effects. Confirming our observations from figure 3, we found a statistically significant negative relationship between the average prune size in the shipment (expressed as the number of prunes needed to make a pound) and the price spread. This indicates that shipments with relatively smaller prunes gain (i.e., have a lower spread) at the expense of growers of larger prunes. The average prune size variable alone explains half of the variation in spreads. Similarly, the spread had a statistically significant positive relationship with the percentage of prunes measured as A-quality ($R^2 = 0.28$), and statistically significant negative relationships with the average count (the number of prunes to make a pound) in each screen ($R^2 = 0.76$). A final regression showed that as the share of *actual* A- and B-quality prunes increases, so does the spread ($R^2 = 0.33$).

Table 4 reports the results of a regression that combines this share variable with the variables measuring average size in each screen; the signs of the individual coefficients reveal the same patterns as the separate regressions we have described, and this regression explains nearly 79% of the variation in spread. These various regressions all serve to reaffirm the fundamental observation that grading error causes growers who produce relatively large prunes to cross-subsidize those who produce relatively small ones—the adverse selection outcome.
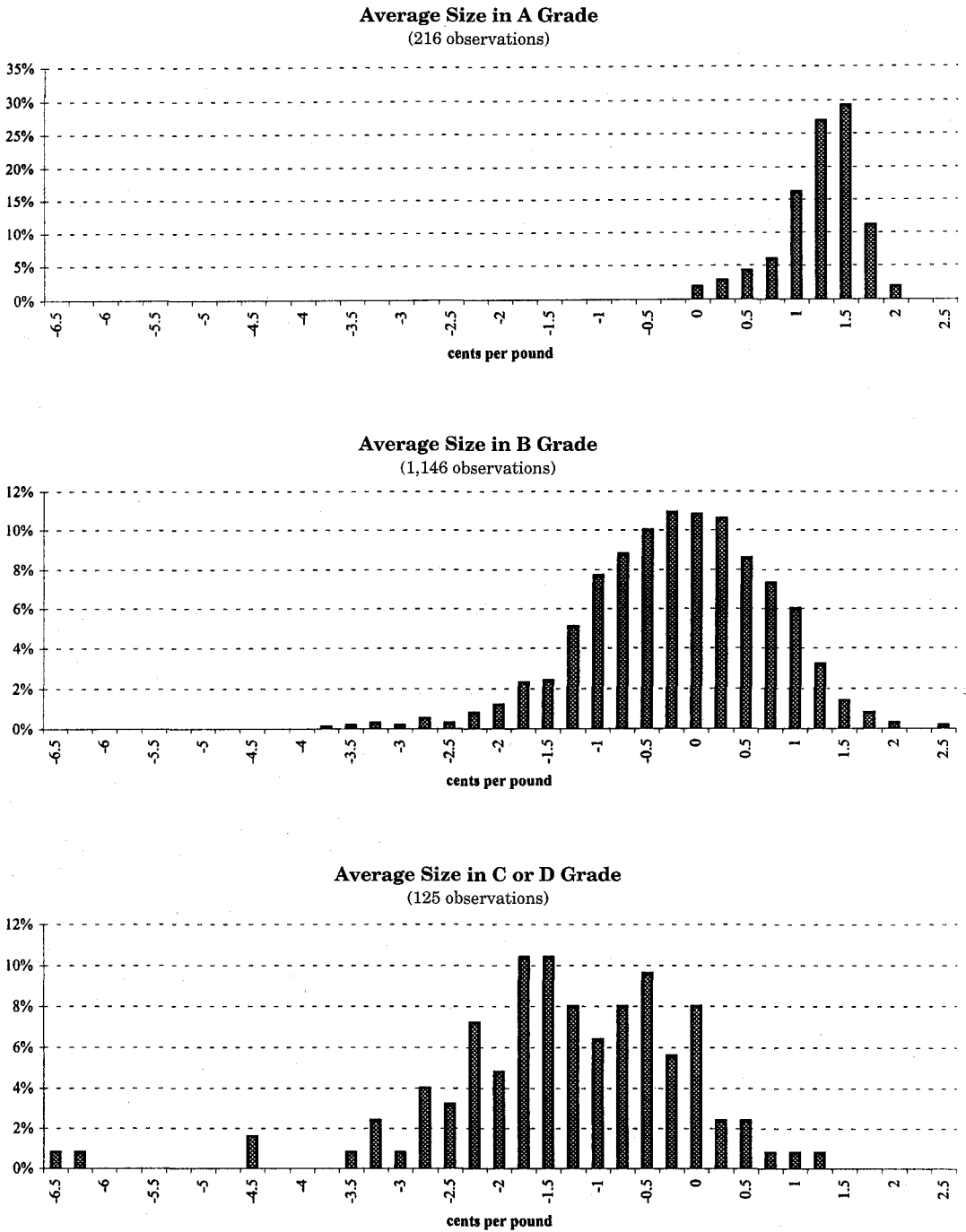
Average Size in A Grade
(216 observations)

Average Size in B Grade
(1,146 observations)

Average Size in C or D Grade
(125 observations)

Figure 3. Frequency distribution of price spreads by average size

Table 4. Regression Results for Computed Spread Variable

Explanatory Variable	Estimated Coefficient	Standard Deviation	t-Statistic
Intercept	14.0078	0.3053	45.876
Average Size:			
A Screen	-0.0437	0.0040	-11.029
B Screen	-0.0684	0.0040	-16.464
C Screen	-0.0749	0.0023	-33.169
D Screen	-0.0021	0.0003	-6.138
$w_A + w_B$	1.4089	0.0980	14.379

*The Effects of Grading Error on the Returns
to Shaker Thinning*

How important are these errors in influencing farmers' production practices, such as shaker thinning, to increase prune size? To explore this question, we evaluated the return to shaker thinning based on 1996 prices under current grading practices versus a hypothetical regime of no grading error. Data on shaker thinning were from a trial conducted by the PBA. A representative orchard was chosen; one row was mechanically thinned, and a sample from the eventual harvest was graded on the DFA grader. An adjacent row was treated as a control, and a sample from its (unthinned) harvest was also submitted to the DFA grader. The measured proportions of the sample in each grade (the m_i's) from the thinned and unthinned rows are reported in table 5, as are the actual proportions (the w_i's) for each row, which we derived using the estimated Gamma distributions as discussed previously. The relevant prices to evaluate the return to thinning under current practices are the actual PBA prices, P_i. However, under the hypothetical regime of perfect grading, the prices would be the actual values, V_i.

Thinning increases the proportion of large prunes harvested, but it also reduces the yield—from 4.3 dry tons/acre on the control to 3.2 dry tons/acre in the PBA trial. Shaker thinning costs about $60/acre. However, the smaller yield also reduced the grower's harvesting, hauling, and drying charges from $1,166/acre to $783/acre based on the PBA's estimates. Considering both the cost and revenue effect of shaker thinning, the thinned crop yielded $365 more net profit per acre than the unthinned crop, given current grading practices. The return to shaker thinning under no grading error was estimated to be $499/acre, or an increase of 37% over the return with grading error.

Thus, thinning was profitable even in the presence of grading error, but would have been considerably more profitable in its absence. Returns from thinning will depend upon the particular characteristics of an orchard. For example, the return from thinning a tree with a very heavy fruit set and correspondingly small average prune size will exceed the return from thinning a tree with only a moderate fruit set. Thus, it seems certain that grading error is an important factor limiting the number of orchards that are thinned.

Table 5. The Impact of Shaker Thinning on Prune Size Distribution

Description	CURRENT GRADING SYSTEM		NO GRADING ERROR	
	Unthinned	Thinned	Unthinned	Thinned
Grades:	Share of Crop in Each Measured Grade		Share of Crop in Each Actual Grade	
A	0.11	0.39	0.07	0.34
B	0.45	0.46	0.38	0.48
C	0.28	0.10	0.32	0.13
D	0.09	0.02	0.12	0.03
U	0.07	0.03	0.10	0.03
Yield (tons/acre)	4.3	3.2	4.3	3.2
Total Revenue/Acre ($)	2,676	2,717	2,633	2,809
Total Costs/Acre ($)	1,166	843	1,166	843
Net Profit/Acre ($)	1,510	1,874	1,467	1,966
Returns to Thinning	$365/acre (24%)		$499/acre (34%)	

Conclusions

This study examines an adverse selection problem caused by an asymmetric measurement error in grading, a characteristic of most grading systems where size is the main quality criterion. The problem occurs when sizing systems such as screens are used where products of small size, and therefore lower quality, can fall into larger size categories, but the converse cannot occur.

We developed a theoretical model which shows that when this error in sorting is present, the prices of all grades will be lower than their true values, except for the lowest grade, whose price equals its true value, since no lower quality product exists. Moreover, the revenue that prune growers receive for each grade depends on the trade-off between the gain obtained from product of that grade moving into higher grades and the loss in revenue due to the grade's price being less than its value. The theoretical model showed that the per unit farm value is lower than the true value of the highest grade, but higher than actual value for the lowest grade. The net effect is indeterminate for intermediate grades.

The empirical application to the California prune industry illustrates the potential importance of these effects. The price of grade A prunes is lower than its true value by 4%, and the price of grade B prunes is lower by 8%. We translated these effects into measures of average revenue for each actual grade of prunes, and found that farm value fell below actual value for the highest two grades, while farm values of the lower three grades exceeded their true values. The implication is that the incentives to produce large prunes are reduced. These findings are consistent with the pattern of "oversupply" of small prunes in recent years and illustrate that continuing to produce relatively

greater numbers of small prunes, rather than undertaking shaker thinning or other cultural practices to produce larger prunes, may well be a rational response to current incentives. The industry can partially address the problem of oversupply of small prunes by improving the accuracy of the grading process. Examples would include increasing screen length or adding additional screens on the DFA grader. Alternatively, the industry might consider a graduated payment system that offers premiums and discounts based on average prune size in each measured grade, rather than a single price per grade, as is the current practice.

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Appendix:
Generalization of the Theoretical Model
in Matrix Form

Let $\mathbf{w}$ denote the vector of shares of product in each grade in the absence of measurement error. Let $\mathbf{m}$ denote the vector of shares of product measured in each grade, under a grading process with errors. Define the matrix $\mathbf{A}$ as

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ s_2^1 & (1-s_2^1) & 0 & 0 \\ s_3^1 & s_3^2 & (1-s_3^1-s_3^2) & 0 \\ s_4^1 & s_4^2 & s_4^3 & (1-s_4^1-s_4^2-s_4^3) \end{bmatrix}.$$

Generalization to n grades is straightforward:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 \\ s_2^1 & (1-s_2^1) & 0 & \cdot & \cdot & \cdot \\ s_3^1 & s_3^2 & (1-s_3^1-s_3^2) & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ s_n^1 & s_n^2 & \cdot & \cdot & s_n^{n-1} & (1-s_n^1-s_n^2-...-s_n^{n-1}) \end{bmatrix}.$$

The matrix $\mathbf{A}$ can be thought of as the grading-error transformation matrix. It is a triangular matrix, reflecting the one-way nature of grading errors. The rows must sum to one since they are probabilities for events that exhaust the possibilities for product of any given *actual* size. For a prune of actual grade i, the probabilities in row i thus define a multinomial process that characterizes the random variable actual screen grade. Generalization to the case of two-way grading errors would simply involve moving some of the probability to columns above the diagonal.

Define the total crop size as Q. We know from text equations (1)–(4) that

$$\mathbf{m} = \mathbf{A'w} \Rightarrow \mathbf{m}Q = \mathbf{A'w}Q.$$

These expressions give the mapping from actual to measured grades. Multiplying each vector by the total number of units of product Q converts the probability distribution to a distribution of counts of the product in each grade. Alternatively,

$$\mathbf{w} = (\mathbf{A'})^{-1}\mathbf{m}.$$

By premultiplying by a diagonal matrix of prices, the vector of payments π, based on *measured* grades of the product, is given by

(A1) $$\pi = Q \cdot \text{diag}(\mathbf{p})\mathbf{m} = Q \cdot \text{diag}(\mathbf{p})\mathbf{A'w}.$$

We noted in the text that payments could be expressed in terms of either the observed prices for measured grades or the underlying values of correctly measured product. Reflecting this observation, the expressions in text equations (5)–(8) are

$$\text{diag}(\mathbf{m})\mathbf{p} = \mathbf{A}'\text{diag}(\mathbf{w})\mathbf{V}$$

or

(A2)
$$\mathbf{p} = [\text{diag}(\mathbf{m})]^{-1}\mathbf{A}'\text{diag}(\mathbf{w})\mathbf{V}.$$

Adding-Up

It is straightforward to show that payments for the crop can be expressed in terms of market prices or the underlying values for each correctly measured grade. Working backwards from text equations (19)–(22),

$$\mathbf{v} = \mathbf{Ap} \quad \text{or} \quad \mathbf{p} = \mathbf{A}^{-1}\mathbf{v}.$$

This lets us show the adding-up relationship. Take (A1) and premultiply by a unit vector ι to obtain total revenue:

(A3)
$$\iota'\pi = \iota'Q \cdot \text{diag}(\mathbf{p})\mathbf{m} = \iota'Q \cdot \text{diag}(\mathbf{p})\mathbf{A}'\mathbf{w}.$$

Since the product of ι' and $\text{diag}(\mathbf{p})$ is just $\mathbf{p}'$, and since $\mathbf{v} = \mathbf{Ap}$, this implies that total revenue is

(A4)
$$\iota'\pi = Q \cdot \mathbf{p}'\mathbf{m} = Q \cdot \mathbf{p}'\mathbf{A}'\mathbf{w} = Q \cdot \mathbf{v}'\mathbf{w}.$$

Since v_i was defined in the text as $R_i/(w_i Q)$, we can also write

$$\mathbf{R} = \text{diag}(\mathbf{w})\mathbf{v} \quad \text{or} \quad \mathbf{v} = [\text{diag}(\mathbf{w})]^{-1}\mathbf{R}.$$

The adding-up result is that

$$\iota'\mathbf{R} = \iota'\text{diag}(\mathbf{w})\mathbf{v} = \iota'\pi.$$

From text equations (15)–(18),

$$\mathbf{R} = \mathbf{Bp} = \mathbf{BA}^{-1}\mathbf{v} = \text{diag}(\mathbf{w})\mathbf{v},$$

where $\mathbf{B} = \text{diag}(\mathbf{w})\mathbf{A}$. Our other adding-up result is from (A2):

$$\begin{aligned}
\mathbf{m}'\mathbf{p} &= \mathbf{m}'[\text{diag}(\mathbf{m})]^{-1}\mathbf{A}'\text{diag}(\mathbf{w})\mathbf{V} \\
&= \iota'\mathbf{A}'\text{diag}(\mathbf{w})\mathbf{V} \\
&= \iota'\text{diag}(\mathbf{w})\mathbf{V} \\
&= \mathbf{w}'\mathbf{V}.
\end{aligned}$$

Comparing $\mathbf{p}$ *and* $\mathbf{V}$

Finally, alternative expressions can be derived for comparing $\mathbf{p}$ and $\mathbf{V}$:

(A5)
$$\mathbf{p} - \mathbf{V} = \left([\text{diag}(\mathbf{m})]^{-1}\mathbf{A}'\text{diag}(\mathbf{w}) - \mathbf{I}\right)\mathbf{V},$$

generalizing equations (5) and (9)–(11) in the text, or

$$\mathbf{V} - \mathbf{p} = \left(\mathbf{I} - [\text{diag}(\mathbf{m})]^{-1}\mathbf{A}'\text{diag}(\mathbf{w})\right)\mathbf{p}.$$

Symmetric Errors

We used the assumption that the **A** matrix is triangular, reflecting asymmetric grading errors, in developing the results in the text in a recursive manner. However, the expressions in this appendix do not require that particular structure for **A**. Each result holds for the case of the two-way or symmetric grading errors. The key in either case is to think of the mapping from **m** to **w** as a simple linear redefinition of the number of units of the product in various grades. As long as this is understood, it is a matter of indifference how prices are expressed for a given crop.

Our interest is in the effect of grading errors on the share of revenues accruing to each measured grade, and how grading error affects the differences between actual values and market prices for measured grades. In equation (A5) above, since the **A** matrix has no zero elements in the first column, the first row of its transpose has no zero elements, and the equation for $P_1 - V_1$ is unaffected by making errors symmetric (except that the 1 on the diagonal will be reduced, to the extent that grade 1 product moves down in grade). For each comparison between **P** and **V** for other grades, additional terms will appear in the generalizations of equations (5) and (12)–(14) from the text. For instance, for grade 2, there now will be a term involving V_1, since the 0 appearing in the first row and second column of **A** is replaced by a positive fraction. This reflects the possible appearance of grade 1 product in measured grade 2, which raises the average value of that grade—making the comparison between P_2 and V_2 ambiguous, whereas it was clear that $P_2 < V_2$ for the asymmetric case. Similar results apply for the other intermediate grades. For the lowest grade, which can now include higher-grade product, equation (5) in the text is replaced with the result that $P_4 > V_4$.