Article

Chinese Science Bulletin

SPECIAL TOPIC
Control Problems in Quantum Systems

June 2012 Vol.57 No.18: 2247–2251
doi: 10.1007/s11434-012-5163-6

# Quantum state transformation and general design scheme on teleportation protocols

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Received October 21, 2011; accepted December 29, 2011; published online April 24, 2012

We present a scheme for probabilistic transformation of special quantum states assisted by auxiliary qubits. In our scheme, if quantum states can be rewritten in a particular form, it is possible to transform such states into other states using lower-dimensional unitary operations that can be more easily realized in physical experiments. Furthermore, as an important application, we propose a generalized scheme that helps construct faithful quantum channels via various probabilistic channels when considering the existence of nonmaximally-entangled states.

**probabilistic teleportation, auxiliary particle, bell-state measurement, state transformation**

**Citation:** Jiang M, Huang X, Zhou L L, et al. Quantum state transformation and general design scheme on teleportation protocols. Chin Sci Bull, 2012, 57: 2247–2251, doi: 10.1007/s11434-012-5163-6

Over the past decade or so, there have been dramatic developments in quantum information processing from both theoretical and experimental aspects [1–3]. Quantum control plays a key role in quantum information technology as quantum gates are not hardwired as in traditional chips, but instead involve sophisticated manipulations of quantum systems [1–11]. In particular, if information is represented in quantum mechanical states, quantum information processing is essentially the manipulation of these quantum states. However, controlling a quantum system at a desired level is entirely different to control classical counterparts and raises an entirely new set of issues in control theory. Note that a teleportation-based strategy can accomplish some tasks such as distant feedback of a coherent quantum state that usually cannot be completed by coherent controls with classical feedback in a similar manner as Markovian quantum feedback [4]. In detail, quantum teleportation can be used to construct feedback controllers with the output from the quantum system to be controlled fed back into an actuator via teleportation to alter the dynamics of the system [5]. In that way, quantum teleportation performs distant transmissions that can be viewed as part of a quantum feedback control loop. Alternatively, we can consider the implementation of an arbitrary unitary gate upon a distant quantum system. This teleportation of a quantum unitary gate can be viewed as a quantum remote control [9,10]. Therefore, a detailed study of quantum teleportation would greatly contribute to a better understanding of quantum control. Due to its potential application, quantum teleportation has drawn the attention of many researchers [12–28]. Moving onwards from teleportation of a one-particle or two-particle unknown state, some researchers have focused on the teleportation of multi-particle states [29–33], controlled teleportation [34,35], and probabilistic teleportation [36–40]. Dong et al. [40] considered probabilistic and controlled quantum teleportation given that the quantum channel was not in the form of an exact GHZ state. By far, much of the many probabilistic or deterministic teleportation protocols is based on the nonmaximally-entangled states. In our previous work [32,33], we developed a scheme for establishing a faithful quantum channel for both indirect and direct teleportation that could be applied in a teleportation

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network where intermediate relay agents exist between a sender and a receiver.

In most probabilistic teleportation protocols, various unitary gates are performed on some qubits and one auxiliary qubit held by a sender or a receiver involved in the implementation of teleportation circuit. However, no unified scheme has been reported to explain the cause and effect of the high-dimensional unitary operations. In other words, the general problem remains still open to construct teleportation protocols via various quantum states obtained in the physical realization.

In this paper, we present a general construction of teleportation protocol via various quantum states assisted by auxiliary qubits. First, we consider the generalized form of a special class of quantum state and present a scheme to implement probabilistic transformation with minimal resources. Next, we apply our method and theoretical analysis to further propose a general scheme for teleportation protocols based on various nonmaximally-entangled channels. Finally, we give a brief discussion and summary.

## 1 State probabilistic transformation assisted by ancillary particles

Let us start with one multi-particle state, assumed normalized, that can be expanded in the following manner:

$$\left|\varphi\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m} = \sum_i a_i \left|\{j_l\}\right\rangle_{A_1A_2\cdots A_n}\left|\{t_f\}\right\rangle_{B_1B_2\cdots B_m}, \quad (1)$$

where $\left|\{j_l\}\right\rangle = \left|j_1j_2\cdots j_n\right\rangle (j_l \in \{0,1\}(l=1,2,\cdots n))$ denote the basis computational states of $n$ particles belonging to the particle set $\{A\}$ whereas $\left|\{t_f\}\right\rangle = \left|t_1t_2\cdots t_m\right\rangle (t_f \in \{0,1\}$ $(l=1,2,\cdots m))$ denote the basis computational states of $m$ particles belonging to the particle set $\{B\}$. It should be noted that in stating (1) each basis computational state $\left|\{t_f\}\right\rangle_{B_1B_2\cdots B_m} = \left|t_1t_2\cdots t_m\right\rangle_{B_1B_2\cdots B_m} (t_f \in \{0,1\}(l=1,2,\cdots m))$ can only be combined with at most one basis computational state $\left|\{j_l\}\right\rangle_{A_1A_2\cdots A_n} = \left|j_1j_2\cdots j_n\right\rangle_{A_1A_2\cdots A_n} (j_l \in \{0,1\}(l=1,2,\cdots n))$.

With use of one auxiliary qubit and an $n$+1 dimensional unitary gate, we can obtain the following state with a certain probability:

$$\left|\varphi'\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m} = \sum_i b_i \left|\{j_l\}\right\rangle_{A_1A_2\cdots A_n}\left|\{t_f\}\right\rangle_{B_1B_2\cdots B_m}. \quad (1)$$

Here, as for $\left|\varphi\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m}$ in eq. (1) above, $\left|\varphi'\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m}$ is normalized.

Proof. Assume $k_i=a_i/b_i$ and $k_m = \min\{k_i\}(i=0,\ 2,\ \cdots 2^{n-1})$. The specific circuit implementation of the whole process is illustrated in Figure 1.

First, we introduce an ancillary particle $\left|0\right\rangle_r$ and perform the following unitary operation $U_{B_1B_2\cdots B_n r}$ on particles $B_i$ ($i$=1, 2,…$n$) and the ancillary particle $r$:

$$U_{B_1B_2\cdots B_n r} = \begin{bmatrix} U_0 & & & \\ & U_2 & & \\ & & \ddots & \\ & & & U_{2^{n-1}} \end{bmatrix}. \quad (2)$$

Here

$$U_i = \begin{bmatrix} \dfrac{k_m}{k_i} & \sqrt{1-\dfrac{k_m^2}{k_i^2}} \\ \sqrt{1-\dfrac{k_m^2}{k_i^2}} & -\dfrac{k_m}{k_i} \end{bmatrix}. \quad (3)$$

Next, after performing this unitary operation, the state of particles $A_1, A_2, \cdots A_n, B_1, B_2, \cdots B_m$ and $r$ reads

$$\begin{aligned} &\left|\varphi\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m} \\ &= k_m \sum_i b_i \left|\{j_l\}\right\rangle_{A_1A_2\cdots A_n}\left|t_i\right\rangle_{B_1B_2\cdots B_m}\left|0\right\rangle_r \\ &\quad + \sum_i b_i \sqrt{k_i^2-k_m^2}\left|\{j_l\}\right\rangle_{A_1A_2\cdots A_n}\left|t_i\right\rangle_{B_1B_2\cdots B_m}\left|1\right\rangle_r. \end{aligned} \quad (5)$$

Finally, we perform a measurement on the state of the auxiliary particle $r$ spanned by the basis $\{\left|0\right\rangle, \left|1\right\rangle\}$. If the result is $\left|0\right\rangle_r$, then the state of particles $A_1, A_2, \cdots A_n, B_1, B_2, \cdots B_m$ will have been transformed to $\left|\varphi'\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m}$; i.e. the transformation is successfully realized. However, if the result is $\left|1\right\rangle_r$, then the transformation will have failed. The probability of successful transformation can be calculated as $\left|k_m\right|^2$.

Note that if the multi-particle state can be written in the form (1), the particle set $\{A_i\}$($i$=1, 2,…$n$) will be named as our particular particle set. Thus, it is essential to our scheme to find the particular particle set $\{A_i\}$($i$=1, 2,…$n$). Furthermore, the transformation between $\left|\varphi\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m}$ and $\left|\varphi'\right\rangle_{A_1A_2\cdots A_n B_1B_2\cdots B_m}$ can be regarded as the transformation between the following two states:

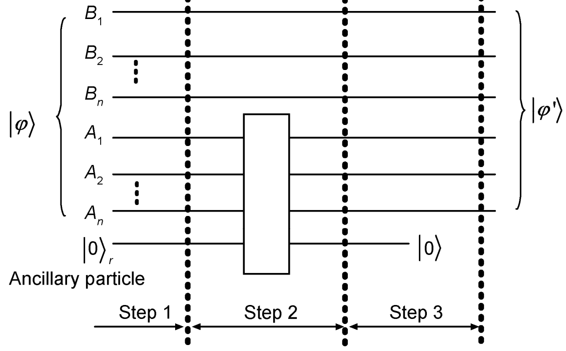


**Figure 1** Circuit representation for the probabilistic transformation of quantum states assisted by an auxiliary qubit.

$$|\varphi\rangle^*_{A_1A_2\cdots A_n} = \sum_i a_i |\{j_l\}\rangle_{A_1A_2\cdots A_n}, \tag{6}$$

and

$$|\varphi'\rangle^*_{A_1A_2\cdots A_n} = \sum_i b_i |\{j_l\}\rangle_{A_1A_2\cdots A_n}. \tag{7}$$

Consider the extreme case when, in the beginning, the particle set $\{A_i\}(i=1, 2,\ldots n)$ represent all the particles comprising the joint system. In the following, we show that it is possible to transform the initial state to any target state composed of all or less than $n$ particles with a certain probability.

Based on the above formalism, we will give a brief description of the protocol in regard to the probabilistic transformation of quantum states as shown in Figure 1.

(1) Depending upon the relationship between initial and target states in the entangled joint system, pick the particular particle set $\{A_i\}(i=1, 2,\ldots n)$;

(2) Next, according to the transformation relationship between the original and target states, import an auxiliary qubit, design a unitary gate operation and perform it on $\{A_i\}(i=1, 2,\ldots n)$ and the auxiliary qubit;

(3) Finally, perform a von Neumann measurement on the auxiliary qubit. If the measurement outcome fulfills the requirement, the transformation has succeeded; otherwise, it has failed.

## 2 Construction of the faithful channel via various quantum states

Based on the analysis in the above section, we will further explore the scheme for construction of the faithful channel via various quantum states.

As a starting point, let us take a more concrete example [27] where the channel is in the following state:

$$\begin{aligned}|\varphi\rangle_{34567} &= \alpha|00000\rangle_{34567} + \beta|01100\rangle_{34567}\\ &\quad + r|10011\rangle_{34567} + k|11111\rangle_{34567}.\end{aligned} \tag{8}$$

It can be written either as

$$\begin{aligned}|\varphi\rangle_{34567} &= \alpha|00\rangle_{34}|000\rangle_{567} + \beta|01\rangle_{34}|100\rangle_{567}\\ &\quad + r|10\rangle_{34}|011\rangle_{567} + k|11\rangle_{34}|111\rangle_{567}\end{aligned} \tag{9}$$

or as

$$\begin{aligned}|\varphi\rangle_{34567} &= \alpha|00\rangle_{56}|000\rangle_{347} + \beta|10\rangle_{56}|010\rangle_{347}\\ &\quad + r|01\rangle_{56}|101\rangle_{347} + k|11\rangle_{56}|111\rangle_{347}.\end{aligned} \tag{10}$$

That is to say, particle set {3, 4} or {5, 6} can be regarded as the particular particle set. Therefore, to obtain a faithful channel, either sender Alice holding the particle set {3, 4} or receiver Bob holding the particle set {3, 4} can take appropriate measurements to play the role of channel modulator. Finally, if we make use of the scheme proposed in Section 1, then the initial system can with a certain probability be transformed into the following un-normalized form:

$$\begin{aligned}|\varphi\rangle_{34567} &= |00000\rangle_{34567} + |01100\rangle_{34567}\\ &\quad + |10011\rangle_{34567} + |11111\rangle_{34567}\\ &= (|00\rangle_{36} + |11\rangle_{36})(|00\rangle_{45} + |11\rangle_{45})(|0\rangle_7 + |1\rangle_7)\\ &\quad + (|00\rangle_{36} - |11\rangle_{36})(|00\rangle_{45} + |11\rangle_{45})(|0\rangle_7 - |1\rangle_7)\end{aligned} \tag{11}$$

signifying that a faithful channel has been established to implement either deterministic or controlled teleportation. Note that the above analysis gives a theoretical interpretation and is totally consistent with the probabilistic teleportation protocol in [34].

Similar theoretical analysis can also be applied to the inexact GHZ state form [22,35,36]:

$$\begin{aligned}|\mathrm{GHZ}\rangle_{123} &= \alpha|000\rangle_{123} + \beta|111\rangle_{123}\\ &= \alpha|0\rangle_1|00\rangle_{23} + \beta|1\rangle_1|11\rangle_{23},\end{aligned} \tag{12}$$

where the sender, receiver and controller possess particles 1, 2 and 3, respectively. It is obvious that each participant can perform modulation operations to obtain a deterministic channel for controlled teleportation. Next, let us further analyze one universal example.

Assume that Alice and Bob share the following quantum state as a channel [33,34]:

$$\begin{aligned}|\varphi\rangle_{X_1Y_1X_2Y_2\cdots X_nY_n} &= \sum_{p_1=0}^{p_1=1}\sum_{p_2=0}^{p_2=1}\cdots\\ &\quad \sum_{p_n=0}^{p_n=1} a_{p_1,p_2,\cdots p_n}|p_1p_2\cdots p_n\rangle_{X_1X_2\cdots X_n}\\ &\quad \otimes|p_1p_2\cdots p_n\rangle_{Y_1Y_2\cdots Y_n}\end{aligned} \tag{13}$$

where Alice and Bob hold particles $\{X_i\}(i=1, 2,\ldots n)$ and $\{Y_i\}$ $(i=1, 2,\ldots n)$, respectively. Here, according to the analysis of Section 1, it can be shown that either $\{X_i\}(i=1, 2,\ldots n)$ or $\{Y_i\}$ $(i=1, 2,\ldots n)$ can play the role of the particular particle set $\{A_i\}(i=1, 2,\ldots n)$.

Next, either Alice or Bob imports an auxiliary qubit, performs the appropriate operation and makes a measurement to adjust the channel.

Finally, if the measurement succeeds, the joint system will be transformed into the following form:

$$|\varphi\rangle_{X_1Y_1X_2Y_2\cdots X_nY_n} = \prod_{i=1}^{n}(|00\rangle_{X_iY_i} + |11\rangle_{X_iY_i}). \tag{14}$$

This demonstrates that a faithful channel composed of $n$ EPR pairs has been formed between sender and receiver that can be used to teleport the unknown state of a $n$-qubit system.

In the event that the measurement fails, the state of the joint system will be

$$\begin{aligned}|\varphi\rangle_{X_1Y_1X_2Y_2\cdots X_nY_n} &= \sum_{p_1,p_2,\cdots p_n=0}^{1}\Big(\sqrt{a^2{}_{p_1p_2\cdots p_n} - a^2{}_m}\\ &\quad |p_1p_2\cdots p_n\rangle_{X_1X_2\cdots X_n}|p_1p_2\cdots p_n\rangle_{Y_1Y_2\cdots Y_n}\Big).\end{aligned} \tag{15}$$

Nevertheless, it can be found that the remaining particles $\{X_i\}(i=1, 2,\ldots n)$ and $\{Y_i\}$ $(i=1, 2,\ldots n)$ still have the possibility of forming a deterministic channel if appropriate operations are applied to the joint system. In this way, the joint system comprising particles $\{X_i\}(i=1, 2,\ldots n)$ and $\{Y_i\}$ $(i=1, 2,\ldots n)$ can be reused as an important communication resource rather than be abandoned directly after the initial failed attempt. Therefore, a subsequent attempt can be initiated to establish a faithful channel.

One can see that since either $\{X_i\}(i=1, 2,\ldots n)$ or $\{Y_i\}$ $(i=1, 2,\ldots n)$ dominates the same position of the particular particle set $\{A\}$ mentioned in Section 2, either the sender or the receiver can perform iterative channel modulations until either one succeeds. Interestingly, an arbitrary choice of high-dimensional gate operator can greatly reduce the implementation complexity of terminal node in a network.

Therefore, when any quantum state obtained from a physical realization is considered as a candidate for a quantum channel, it should first be written in the form of state (1). Next, choose a particular particle set; this particular particle set $\{A_i\}(i=1, 2,\ldots n)$ must be kept by one participant to be able to implement local high-dimensional quantum gates during execution of a faithful teleportation protocol. The precious entangled qubit resource can subsequently be reused.

## 3 General discussion

With the proliferation of quantum information results, it is possible to adopt more information technologies to quantum control theory. Since quantum teleportation could be viewed as a feed-forward process and can accomplish some distant quantum feedback control tasks, it can be used to obtain and transmit feedback information. In realizing this generalized teleportation via various channels, we initially presented a scheme for quantum state probabilistic transformations assisted by ancillary particles that furnished the following advantages: (1) it provided a way to identify and group a new particular particle set; and (2) with use of this new particle set, it enabled applications of lower-dimensional local operations that implement the state transformation with a certain probability. Next, based on this scheme, we analyzed the existing probabilistic teleportation scheme and constructed in an elegant and simple manner a general teleportation protocol via various quantum states obtained from the potential physical realization. Finally, we provided some concrete examples to explore the feasibility of our scheme. These results have helped to highlight the role of our scheme and provided new insights into the mechanisms underlying quantum state transformations and quantum remote control.

*This work was supported by the National Natural Science Foundation of China* (*61104002, 60904034 and 61071214*)*, the Natural Science Foundation of Jiangsu Province* (*BK2011283 and BK2011377*) *and the Natural Science Foundation of the Jiangsu Higher Education Institutions of China* (*10KJB510024*).


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