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T.A. Andreeva* and W.W. Durgin†
Worcester Polytechnic Institute, Worcester, MA, 01609

This paper concerns an experimental study of the effects of the grid-generated turbulence on the propagation of acoustical waves in a wind tunnel. Turbulence effects are investigated using ultrasound time-of-flight method, employing counter-propagating ultrasonic pulses. Turbulence effects are an important source of error in the active probing of the atmosphere using sound sources. The emphasis is on the propagation time fluctuations and their interpretation using theoretical analysis of Kolmogorov.

1. INTRODUCTION

The phenomenon of sound propagation in inhomogeneous, moving media has been investigated intensively and there is vast amount of literature on the subject. The effect of the turbulence on the sound propagation results firstly in the scattering of the sound waves, and secondly, in the fluctuations of the phase and amplitude of the sound waves. In this work we concentrate on the study of the latter effect. The classical analytical approach in which wave propagation equations are simplified and averaged to account for the random environment can be found in Chernov[1], Tatarskii[2]. Along with the classical approach another analytical approximation, so called stochastic Helmholtz equation, as a description for sound propagation through turbulence has been considered by Neubert[3]. Recently Iooss et. al.[4] has studied well-known effect of linear increase of the first-order travel-time variance with propagation distance (Chernov approximation) and introduced a second order term to the travel time variance to account for the nonlinear effect that appears at a certain propagation distance. The experimental results available in the literature, in which measurements of amplitude and phase fluctuations in sound wave are described, are often not in good agreement with analytical predictions. Aside from the limitation of experimental and computational equipment with regard to collection and processing of experimental data, the main reason for the discrepancy between theoretical predictions and experimental data is that the corresponding data were not analyzed in a consistent manner. Extensive experimental material concerning the travel time, phase variance was collected from outdoor experiments[5]. Since atmospheric turbulent flow cannot be controlled, is not well characterized and cannot be considered to be isotropic, the measurements are better taken under laboratory conditions within the turbulent field produced by grid or jet. The difficulty of obtaining laboratory measurements of phase or flight time explains the lack of experimental data in literature. Ho and Kovasznay[6] have performed measurements across an air jet over an extremely short propagation distance. Blanc-Benon[7] used jet-generated turbulence with levels $u_1' = 2.4 m/s$ and $1.1 m/s$, and utilized approximately plane acoustic wave with a pistonlike sound source. In our previous work we investigated experimentally the dependence of the travel time on the propagation distance. The data shows very good correspondence with analytical data for second-order travel time variance obtained by Iooss. More recently, the problem of pulse propagation in inhomogeneous and random media become of interest in applied science (Dacol[8], and references therein). Karweit et. al.[9] in their work described a number of numerical experiments, all of which focus on the travel time variance of a sound propagating through air. The effect of turbulence on the waveform distortion, rise time and peak pressure were investigated numerically by Lipkens[10].

The transit-time ultrasonic method is widely used technique for flow metering [Lynnworth[13], Schmidt[14]]. It allows measuring flow parameters without perturbation very rapidly, providing opportunity to collect significant amount of experimental data. A range of free stream velocities was used that resulted in isotropic turbulence within a specific downstream zone. The ultrasonic transducers were located in the zone.

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* Graduate student, Mechanical Engineering Department, Worcester Polytechnic Institute, 100 Institute Rd., Worcester, MA, 01609; tatiana@wpi.edu

† Professor, Mechanical Engineering Department, Worcester Polytechnic Institute, 100 Institute Rd., Worcester, MA, 01609; wwdurgin@wpi.edu

In the current work, the influence of turbulence on ultrasound waves propagating along and against the direction of the mean flow is considered. Moreover, a number of experiments for collecting data for amplitude fluctuations at the point of the receiving are conducted. These data served to study the amplitude fluctuations as a function distance between transducers and mean flow velocity. The laws of behavior that govern the fluctuations in amplitude and phase of sound are subject to the Kolmogorov's "2/3" law[12]

2. METHODOLOGY

In the experiment we utilize ultrasonic pulses traveling along straight paths from a single source to two receivers, as shown in Fig. 1.

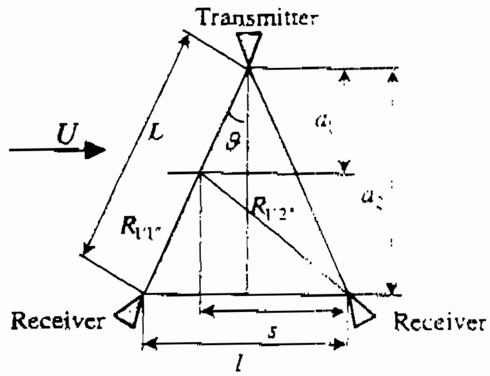

Figure 1 Sketch for the basic relations for the ultrasound measurements.

In order to develop a theoretical model we use the ultrasonic flowmeter equation for sound waves propagating upstream and downstream as following

$$t_{1,2} = \int_0^L \frac{dx}{c \mp u_{1,2}} \approx t_0 \pm \frac{1}{c^2}\int_0^L u_{1,2}\,dx;\; u_{1,2} = U + u_{1,2} \qquad (1),$$

where t_0 is a travel time in the undisturbed media. U is a mean flow velocity, c is a sound speed, u are fluctuations of the mean flow velocity. In order to construct analytical expression for the standard deviation of the travel time we will use theory developed by Krasil'nikov along with Kolmogorov's 2/3 law. We reproduce derivations carried out by Krasil'nikov with changes appropriate for our application. Using (1) we construct expression for the time difference

$$\Delta t = t_1 + t_2 - 2t_0 \approx \frac{1}{c^2}\int_0^L (u_1 - u_2)\,dx = \frac{1}{c^2}\int_0^L \Delta u\,dx \qquad (2).$$

Consequently,

$$\overline{\Delta t^2} = \frac{1}{c^4}\int_0^L dx_1 \int_0^L dx_2\, \overline{\Delta u(x_1)\Delta u(x_2)} \qquad (3),$$

where the overscore indicates time averaging. The value of the mean quantity under this integral can be written as

$$\overline{\Delta u(x_1)\Delta u(x_2)} = \overline{u_1(x_1)u_1(x_2)} + \overline{u_2(x_1)u_2(x_2)} - \overline{u_1(x_1)u_2(x_2)} - \overline{u_1(x_2)u_2(x_1)} \qquad (4).$$

To take into account the correlation of fluctuations at different points of the flow we use the "2/3" law. Following this law, on the basis of the hypotheses of isotropy one may get:

$$\overline{u_i^2} - \overline{u_1 u_2} = \frac{1}{2}C^2 L^{2/3} \qquad (5),$$

where C is a structure parameter having a dimension of $1m^{2/3}\times s^{-1}$ and L is a distance between two transducers. Using (4) it follows from (5)

$$\overline{\Delta u(x_1)\Delta u(x_2)} = -\frac{1}{2}C^2\left(R_{1'1'}^{2/3} + R_{2'2'}^{2/3} - R_{1'2'}^{2/3} - R_{2'1'}^{2/3}\right) \qquad (6).$$

Based on the Figure 1 it is seen, that for small angles

$$R_{1'1'} = \frac{a_2 - a_1}{\cos\vartheta};\; R_{1'1'}^2 = R_{2'2'}^2 = (a_2 - a_1)^2\left(1+\vartheta^2\right) \qquad (7);$$

$$R_{1'1'} = \frac{a_2 - a_1}{\cos\vartheta};\; R_{1'1'}^2 = R_{2'2'}^2 = (a_2 - a_1)^2\left(1+\vartheta^2\right) \qquad (8);$$

$$R_{1'2'}^2 = (a_2 - a_1)^2 + s^2 \qquad (9),$$

$$R_{1'2'}^2 = (a_2 - a_1)^2 + (a_2 + a_1)^2\vartheta^2 \qquad (10).$$

Consequently,

$$\overline{\Delta t^2} = \frac{1}{c^4}C^2\int_0^L da_1\int_0^L da_2\left\{\left[R_{1'2'}^2\right]^{1/3} - \left[R_{1'1'}^2\right]^{1/3}\right\} \qquad (11).$$

The detail evaluation of the integral (9) can be found in Krasil'nikov[11]. The final expression for the standard deviation of the travel time is

$$\sigma = \sqrt{\overline{\Delta t^2}} = const\,C\frac{1}{c^2}L^{1/2}(l)^{5/6},\; l = 2\vartheta L \qquad (12).$$

3. EXPERIMENTAL RESULTS AND DISCUSSION

The experiments were carried out in a wind tunnel of 45.25" length with a 11.75"x 11.62" rectangular test section. Turbulence was produced by a bi-planar grid consisting of a square mesh of aluminum round rods with diameter of 0.25" positioned 1" between centers. The mesh, M, was therefore 1" and the grid solidity was 0.36. In our experiment we have used ultrasonic transducers acted both as a transmitter and as a receiver with a frequency of 100kHz designed for air applications. The experimental arrangement as well as an acquisition system is described in our earlier paper[16]. Two signals shown in Fig. 2 are transmitted signal e_1 and received signal e_2. The travel time t_n shown in Fig. 2 was determined as $\max K_{12}(\tau) = K_{12}(t_n)$ from the cross correlation function $K_{12}(\tau)$. Experiment

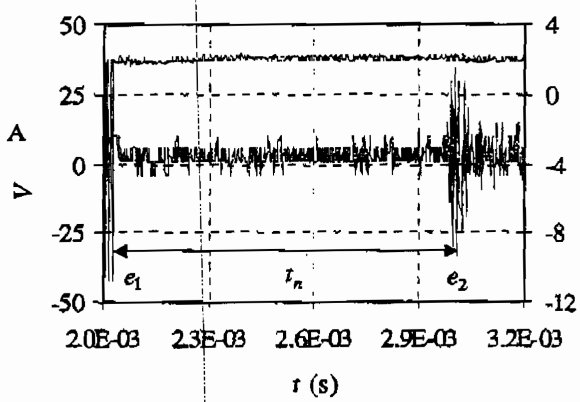

Figure 2 Typical representation of experimental data obtained from the digital data acquisition system CompuScope 82.

was performed for 6 different flow velocities. The Reynolds number based on the mesh size M and other

$\left(\frac{x}{M}\right)$	U (m/s)	Re
0.965	4	5644
0.965	5.5	7761
0.965	7	9877
0.965	8	11288
0.965	9	12700
0.965	10	14111

Table 1 Flow parameters

relevant flow parameters are listed in the Table 4.1.

The signal was sent both in a direction along and opposite to the mean flow. The propagation distance $L = 0.339m$, and the angle $\vartheta = 8^\circ$. The results shown in Fig. 4 are for the structure parameter C that enters Kolmogorov's law as a function of mean flow velocity.

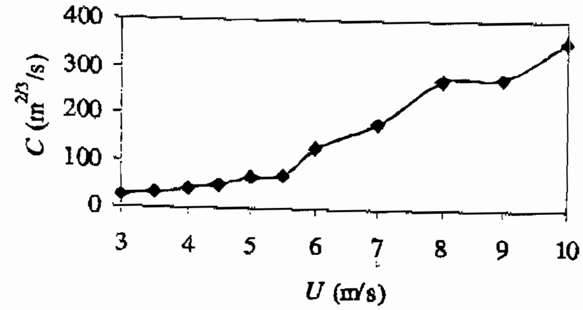

Figure 3 Variation of the structure parameter C versus mean flow velocity

Qualitatively similar results were obtained in the works, published in Tatarskii[1]. There are, however, quantitative differences, that can be attributed to the lack of accuracy of the data obtained from earlier outdoor experiments. It is seen from Fig. 3, that the data points lie on one straight line for mean velocity varying from 2 to 5.5 (m/s) which is a zone of isotropic turbulence according to Mohamed[15] (25-45"downstream from the grid as far as our application is concerned). Although turbulence is not isotropic when mean velocity exceeds 5.5 (m/s), the points are also lie on a line, however, with the different slope. Changing the angle θ we change the path length L.

ϑ (deg)	L (m)
0	0.33
5	0.333448
10	0.339499
15	0.34834
20	0.360278
25	0.375772
35	0.420361

Table 2 Geometrical parameters.

Fig.4 shows dependence of the quantity $\sigma_A = \sqrt{\overline{\left(A - \overline{A}\right)^2}}$, where A is an ultrasound wave amplitude, on the path length. The experimental data was collected for 7 different distances. The dependence of σ_A on L is satisfactory approximated by the formula $\sigma_A = constL^{\alpha}$, where $\alpha \approx 0.34$, as can be seen from the Fig.5. According to the ray theory we ought to have $\alpha = 3/2$. This discrepancy can be explained by the fact, that ray approach is not quite suitable under the conditions of the experiment.

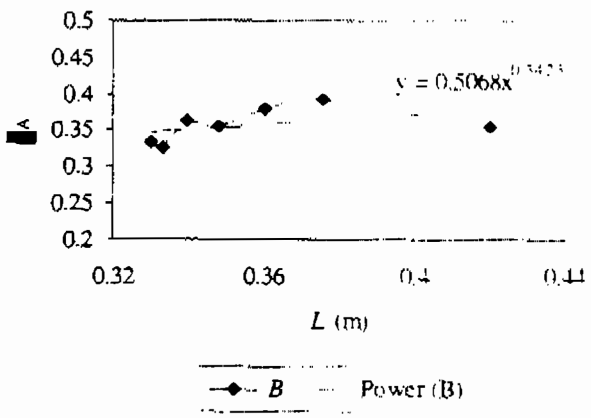

Figure 4 Standard deviation of the amplitude versus the path length L.

On the other hand, the diffraction theory gives the dependence of the amplitude fluctuations on distance such as $\sigma_A \sim L^{1/2}$. Thus, our experimental data agresatisfactory with theoretical predictions. Hamade noticed, that the amplitude of ultrasonic signals received after transmission exhibited fluctuations that increased with increasing velocity. The relation between flow velocity and amplitude fluctuations was nonlinear.

We observed dependence of the standard deviation of the amplitude on the mean flow velocity for two experimental setups. In the first instance, shown in Fig. 5, the ultrasonic waves propagate downstream.

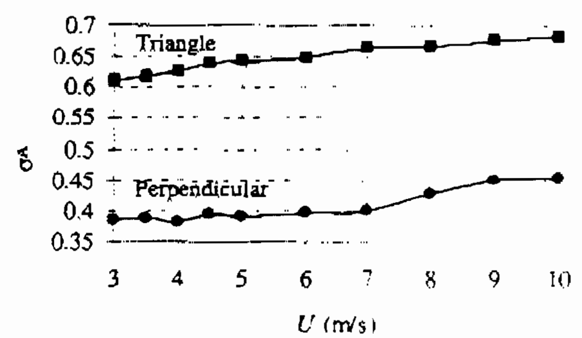

Figure 5 Standard deviation of the amplitude versus the mean flow velocity.

In the second instance the transducers were placed so that the mean flow was directed perpendicular to the ultrasound beam. For both cases the dependence of σ_A on U can be approximated by the formula $\sigma_A = const U^{\beta}$, where $\beta \approx 0.14$ and 0.1 for perpendicular and triangular setups respectively. Greater deviation corresponds to the case of triangular setup.

4. CONCLUSIONS

Methodology based on the ultrasonic technique and supported by the Kolmogorov 2/3 law has been used for investigation of the influence of the grid-generated turbulence on the propagation of the ultrasound waves in the wind tunnel. Experimental data collected in the approximately isotropic flow region, chosen based on the criteria established by Mohamed[15] agreed well with theoretical estimations and explained some discrepancy in data from earlier outdoor experiments. The increase of the structure parameter C with increasing velocity, which is in an agreement with reported earlier results[2]. In the experiment with different path length the results indicated, that standard deviation of the amplitude is proportional to the distance L in the power 0.34 versus 0.5 analytically predicted in diffraction theory, which is a good agreement. There is a nonlinear dependence of the amplitude deviation on the mean velocity, described by the power law with almost identical power (0.14 and 0.1). Overall, obtained results are consistent with theory and experiments performed earlier, which thereby shows, that an ultrasonic method can be utilized for diagnostic of the turbulent flow and obtaining consistent results in the laboratory scale.

It has to be noted, that due to the complexity of the investigated phenomena, in order to achieve sufficient accuracy of the results, more experiments with different setups should be performed and more significant amount of experimental data should be acquired.

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