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ORIGINAL ARTICLE

## Fostering German-language learners' constructions of meanings for fractions—design and effects of a language- and mathematics-integrated intervention

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**Abstract** Learning situations that concentrate on conceptual understanding are particularly challenging for learners with limited proficiency in the language of instruction. This article presents an intervention on fractions for Grade 7 in which linguistic challenges and conceptual mathematical challenges were treated in an integrated way. The quantitative evaluation in a pre-test post-test control-group design shows high effect sizes for the growth of conceptual understanding of fractions. The qualitative in-depth analysis of the initiated learning processes contributes to understanding the complex interplay between the construction of meaning and activating linguistic means in school and technical registers.

**Keywords** Fractions · Language learners · Meaning · Conceptual understanding

As in many countries of the world, about 20 % of all students in German schools have to learn mathematics in a language that is not their first language, most of them being students of the second or third immigrant generation (IT NRW 2012). These students with non-German first languages are not only less successful than those with German as first language, but also less successful than second-language learners in comparable countries (OECD 2007, p. 120). However, it is neither immigrant status nor multilingualism that has the most impact on their mathematics achievement, but the language *proficiency* in the language of instruction and assessment (Heinze et al. 2009; Prediger et al. 2013). This also applies to monolingual language learners, mainly with low socio-economic status (Prediger et al. 2013).

As a consequence, programs are needed in all subject areas that enhance all language learners' proficiency in the language of instruction (e.g., Thürmann et al. 2010; MacGregor

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and Moore 1991). This raises the question of how the rich research in mathematics education on the interplay between language and mathematics learning (e.g., Ellerton and Clarkson 1996; Pimm 1987) can be productively used for developing intervention programs. As this aim is especially important for those language learners who are low achievers in mathematics, this article presents the design and effects of an intervention program developed for low-achieving Grade 7 students with language disadvantages.

Fractions were chosen as the specific mathematical topic for the study since fractions are one of the most difficult topics in the middle-school curriculum, especially if the aim is for students to develop conceptual understanding. We have known for a long time that many (monolingual and multilingual) students develop some technical skills, but only limited conceptual understanding and low capacities for applying fractions in varying contexts (Aksu 1997; Hasemann 1981). More than many other areas of mathematics, students seem to have difficulties in constructing adequate conceptual meaning (Aksu 1997; Prediger 2008). This concerns the basic mental models for fractions (Cramer et al. 1997; Streefland 1991; van Galen et al. 2008), as well as operating with fractions, for example for ordering and equivalence of fractions.

We will first present the relevant theoretical background of the study, followed by the principles and design of the intervention. The third major section will outline the research design of the intervention study, which is followed by selected quantitative and qualitative results that show its effects and gives further insight into the complex relationship between the strongly intertwined linguistic and mathematical aspects of the students' learning.

## **Theoretical background: multiple registers for constructing meaning**

### Construction of meanings

Different theoretical perspectives have emphasised the relevance of individual and/or interactive processes of constructing meanings for mathematical concepts and relationships leading to the development of deep conceptual understanding (e.g., Freudenthal 1991), which is here operationalised by mental models (Prediger 2008). Due to the specific ontological nature of mathematical objects as abstract and mostly relational entities, these constructions of meanings as mental models involve the construction of new mental objects and relationships (Steinbring 2005).

This construction must be in line with acquisition of new linguistic, graphical, and symbolic means for expressing them (Schleppegrell 2010). That is why mathematics learning and language learning are deeply interwoven (Pimm 1987).

This article follows a social semiotic perspective by emphasising that language is more than a tool for representation and communication; it is a tool for thinking and constructing knowledge via constructing meanings. The linguistic means and their meanings vary according to the social and cultural contexts and different registers (Schleppegrell 2010).

### Multiple semiotic representations

As the meaning of abstract mathematical concepts and relationships cannot be understood just by referring to real objects, mathematics educators (Duval 2006; Lesh 1979) and psychologists (Bruner 1967) have emphasised the importance of

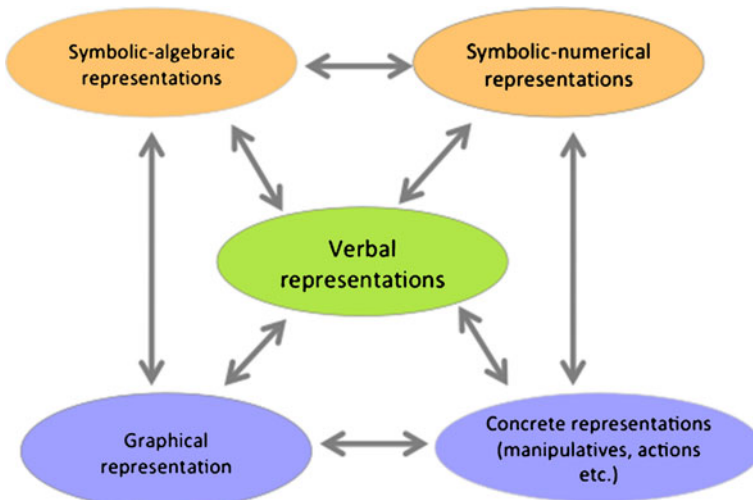
different representations. By transitioning between verbal, symbolic, graphical and concrete representations, students can construct the mental objects and relations to which a mathematical concept refers (Fig. 1). In consequence, connecting representations is established as a fruitful teaching strategy.

From the beginning, language was considered as an integral part of these mathematical representations since, for example, verbalising symbolic expressions is a helpful activity for developing students' understanding. However, this model does not take into account that different linguistic registers are activated in classrooms: that is potentially first- and second-language registers, mathematical technical registers, the school language register, and the students' everyday registers.

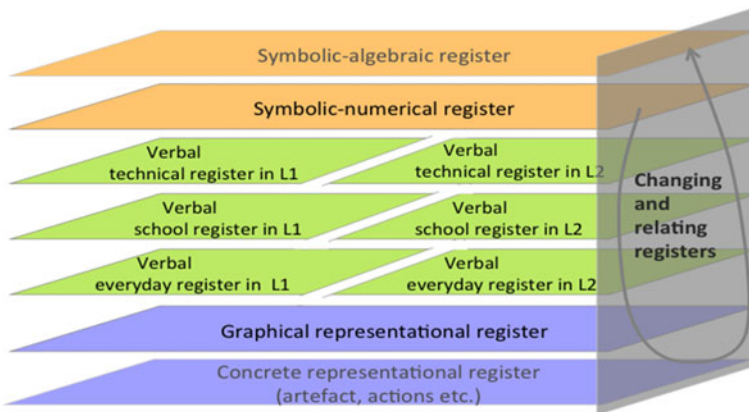
### Multiple registers

For the theoretical underpinning of the construct of registers, it is useful to combine Duval's and Halliday's constructs: The sociolinguist Halliday defines register as "set of meanings, the configuration of semantic patterns, that are typically drawn upon under the specific conditions, along with the words and structures" (Halliday 1978, p. 23). He emphasises its social embeddedness: "A register can be defined as the configuration of semantic resources that a member of a culture typically associates with the situation type... in a given social context" (Halliday 1978, p. 111). Hence, for Halliday, registers are characterised by the types of communication situations, their field of language use, the discourse styles, and modes of discourse. A change of registers involves changes of meanings. The different mathematical representations are not registers in Halliday's sense. However, Duval (2006) does give different mathematical representations a status of different "semiotic registers" and emphasises that the meaning of a mathematical object can change with a shift in representation. In a social semiotic perspective, both can be subsumed as registers.

All these representational and linguistic registers are taken into account in a synthesised model (Fig. 2) which will be explained successively.



**Fig. 1** Transitions between different representations (Bruner 1967; Duval 2006; Lesh 1979 et al.)



**Fig. 2** Relating different registers and representations (Prediger and Wessel 2011)

Pimm (1987), Freudenthal (1991), and others claimed to carefully initiate the transition from the informal everyday register to the formal technical register of mathematics. This idea can be enhanced by the potential of horizontal and diagonal switches between first- and second-language registers as specified by Planas and Setati (2009), Barwell (2009), and others.

Between these two, linguists describe an often hidden, intermediate register, the so-called school register (Prediger et al. 2012), that has been conceptualised as the language of schooling (Schlepppegrell 2004), or as CALP—cognitive academic language proficiency (Cummins 2000). Like the technical register, the school register is characterised by context-reduced, more complex linguistic means than the everyday register and appears conceptually written even if medially oral. But *unlike* the technical register, it is rarely explicitly taught in school. This raises problems for underprivileged language learners who acquire only the everyday register in their families but nevertheless need the school register for higher thinking skills (Clarkson 2009). In contrast, privileged (first- or second-) language learners have access to the school register already in their families (Schlepppegrell 2004; Cummins 2000).

As a consequence, the didactical approaches of initiating transitions from the everyday register to the technical register are extended in several ways (Prediger et al. 2012), two of which are crucial here: 1. Address the school register explicitly. 2. Move dynamically between the three registers throughout a lesson, instead of introducing ideas using the everyday register and moving to the technical register with no return to the other registers.

In principle, the everyday and the school registers also comprise representations other than the verbal representation (see Prediger et al. 2012). However, for practical design purposes, Leisen (2005) suggested to simply consider the different mathematical representations as own registers in Duval's (2006) sense, being one-dimensionally ordered by an increasing degree of abstractness (Fig. 2).

The central design principle for the intervention discussed in this article is consisted of purposefully relating all registers forward and backward. The aim of the intervention study was to seek evidence that this design principle, combined with scaffolding strategies, can substantially enhance the construction of meanings for low-achieving German-language learners.

Many empirical studies have shown that activating the first languages of second-language learners helps to give access to mathematics (e.g. Barwell 2009; Gerber et al. 2005; Planas and Setati 2009; Setati et al. 2011). However, this design strategy has limits for the German-language context with up to seven different immigrant languages in one classroom (mainly Turkish, Russian, Arabian), many monolingual students with limited language proficiency, and mostly monolingual teachers. Thus, this study focuses mainly on other design strategies.

## Didactical design of a language- and mathematics-integrated intervention

### General design strategies

#### *Activities for relating registers*

Concrete activities had to be created and combined to implement the general didactical design strategy of relating registers into an intervention. *Relating* comprises different cognitive activities like *translating*, *(code) switching*, *assigning*, or *contrasting different registers*. In particular the design strategy combines the ideas that the deliberate use of all registers

- can support the development of verbal capacities in the school and technical registers,
- can help to link the first and the second language (the switch was always allowed in the intervention, and not obliged in the intervention), and
- offers opportunities to construct mathematical meanings and relations of crucial concepts (Prediger et al. 2012).

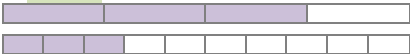
During the iterative development of the intervention, a whole repertory of different types of activities was established (Table 1) and concretised by developing examples for the mathematical topic “ordering fractions”. The choice of suitable activities for each moment in the learning process is a matter of careful orchestration, being guided by the design strategies developed to give the required or “pushed output” (Swain 1985) and macro scaffolding (Hammond and Gibbons 2005). The micro scaffolding needs to spontaneously support the process in the interaction.

#### *Pushed output*

According to “the output hypothesis” (Swain 1985), the act of producing the target language plays a major role for language learning: “[n]egotiating meaning needs ... being pushed toward the delivery of a message that is ... conveyed precisely, coherently and appropriately” (Swain 1985, pp. 248f). Empirical evidence was given for the hypothesis when interactional prompts trigger learners to modify their output (Mackey 2002). Swain (1995) illustrates the different roles that output (i.e., speaking, writing, collaborative dialogue, verbalising) can play besides fostering fluency:

- *Noticing Function*: Producing the target language allows noticing what cannot yet be expressed precisely.

**Table 1** Repertory of activities for relating registers (Prediger and Wessel 2012)

Type of activities	Examples for fractions
A. Translate from one register into another (freely chosen or determined)	<ul style="list-style-type: none"> <li>Here is a given fraction in symbolic form, find a situation for it or draw a picture.</li> <li>Here is a difficult journal text, translate it into your own words.</li> </ul>
B. Find fitting registers, also for consolidating vocabulary	<ul style="list-style-type: none"> <li>On these 15 cards, you find fractions, situations, and drawings. Group those that belong together. Add missing cards.</li> <li>Mathematicians use these words to describe fractions. Connect them with the given example of a symbolic fraction: denominator, numerator, part, whole. What is their meaning in a situation of fair share?</li> </ul>
C. Examine or correct if different registers fit (Swan, 2005)	<ul style="list-style-type: none"> <li>Tim has offered this picture for a sharing situation. Decide whether it is correct.</li> <li>5 kids explained the meaning of <math>3/5</math>. Which explanations are correct? You can use pictures and situations to support your conclusion.</li> </ul>
D. Finding mathematical relations or structures with the help of a certain register	<ul style="list-style-type: none"> <li>Which of the fractions <math>3/4</math> or <math>3/10</math> is bigger? Explain with the help of fractions bars.</li> <li>Find the equivalent fraction to <math>3/4</math> with the help of a picture or situation.</li> </ul> 
E. Systematic variations (Duval, 2006, p. 125)	<ul style="list-style-type: none"> <li>Systematically vary a representation and investigate the effects of the variations in other registers (Examples in Figure 3 and 4)</li> </ul>
F. Collect and reflect different means in a register	<ul style="list-style-type: none"> <li>Collect different questions that ask for <math>3/4</math>.</li> <li>Collect different pictures for <math>3/4</math> and <math>3/10</math>. Which is better to compare the sizes of the fractions? (not used in this intervention)</li> </ul>

- *Hypothesis Testing Function*: Producing language allows testing the meanings and the appropriate use of linguistic means. In reaction to interactional moves (such as clarification requests or confirmation checks), learners modify their output.
- *Meta-linguistic Function*: Placing the focus of attention on language production as a cognitive tool.

### Macro scaffolding

Hammond and Gibbons (2005) located scaffolding at the macro and micro levels: The macro level comprises activities of planning, selecting and sequencing learning arrangements, normally developed before the commencement of the lesson, that take into account the heterogeneity of students' abilities. The micro level concerns providing situational interactional support for the learners during student–teacher interaction beyond pre-planning.

Features of the pre-planned design in the macro scaffolding strategy comprise:

- *Building on students' prior knowledge and experience*: This concerns language abilities relevant for the specific learning content, but also students' prior mathematical experiences and conceptions (Prediger 2008).

- *Selecting and sequencing of tasks*: Sequencing along the three verbal language registers for moving step-by-step towards more in-depth understandings of challenging concepts: for example, students' explanations in everyday language serve as scaffolding for constructing the meaning of a concept and for using a more technical language (Gibbons 2002).
- *Raising meta-linguistic and meta-cognitive awareness*: Initiating explicit reflection by reviewing previous vocabulary and building on it to introduce new content, grounding the introduction of new concepts by working systematically between concrete and more abstract situations, and talking explicitly with students about appropriate language use.

### *Micro scaffolding as leading idea for interactional support*

Interactional support that is guided by the strategy of micro scaffolding includes (Hammond and Gibbons 2005):

- *Linking to prior experience, pointing forward and backward*: Making references to previous lessons, advance organisers for and resuming of content, meta-linguistic and meta-cognitive knowledge.
- *Appropriating and recasting*: Reshaping students' contributions (wordings, ideas, utterances) into a more formal register with the aim of extending students' register use.
- *Cued elicitation and increasing prospectiveness*: Giving verbal or gestural hints to initiate or extend more dialogic and productive student activities such as seeking clarification from the students or asking them for a more detailed explanation.

### **Concrete intervention program: relating registers for fractions with basic models, ordering, and equivalence**

The intervention program following the above design principles was designed for six lessons of 90 min in 2-to-1 sessions (two students with one teacher) to allow for investigating micro scaffolding in detail. The intervention program was conducted by pre-service teachers.

The content and structure of the intervention is summarised in Table 2: It addressed elementary models of fractions (as part of a whole<sup>1</sup>, fraction as quotient, fraction as operator) and basic operations of ordering and finding equivalent fractions in these models. Concrete activities for *relating registers* were designed and orchestrated by covering all types in Table 1.

In line with the strategy of *pushed output*, all activities were accompanied with requests to the students to verbalise their thinking by describing, explaining, and stating reasons. Relating registers activities, which by themselves already initiate pushed output, are especially relevant to Type A (translate from one register into another, freely chosen or determined), Type B (find fitting registers), and Type E

<sup>1</sup> The German word "Anteil" denotes the part-whole relationship, and students are asked to distinguish it from Teil (part) and Ganzes (whole). After discussing with several native speakers, we decided not to translate it since we found no equivalent term in English. "Anteil" is the meaning of a fraction, but the word Bruch (fraction) is reserved for the symbolic expression.

**Table 2** Content of the intervention

Lessons	Main theme	Contents
1	Meaning as part-whole models in different registers	<ul style="list-style-type: none"> <li>• Meaning of fractions as parts of wholes with (verbally given) everyday situations and graphical representations</li> <li>• Fraction bars as graphical representation</li> <li>• Investigating systematic variation of fractions in bars</li> <li>• Linking graphical, symbolic registers, and everyday situations</li> </ul>
2	Partitioning / fair share supported by graphical representations	<ul style="list-style-type: none"> <li>• Fractions as a result of division / fair share</li> <li>• Representing fair share in graphical representations, verbally given everyday situations, and symbolic fractions</li> <li>• Keeping attention to the referent whole</li> <li>• Assigning technical terms and phrases as well as contextual meaning to symbolic fraction</li> </ul>
3	Equivalent fractions	<ul style="list-style-type: none"> <li>• Meaning of equivalence fractions with fraction bars and everyday situations of scoring</li> <li>• Technical terms and phrases for equivalent fractions</li> <li>• Finding equivalent fractions by computation within the symbolic register</li> </ul>
4	Ordering fractions	<ul style="list-style-type: none"> <li>• Order fractions in different register</li> <li>• Assigning same order relations given in different registers</li> <li>• Investigating systematic variation of fractions and reflecting on order relations for fractions with same numerator/ denominator</li> </ul>
5	Fractions as operators (x/y of sets of discrete objects)	<ul style="list-style-type: none"> <li>• Unitising of quantities and specifying fractions as operators in concrete and graphical register</li> <li>• Computing fractions as operators within the symbolic register</li> </ul>
6	Fractions as operators	<ul style="list-style-type: none"> <li>• Determining and representing parts and part-whole relationships for given quantities in graphical representation (arrays) and word problems</li> </ul>

(investigate and describe systematic variations and its effects in other registers). Collaborative dialogue and verbalising were initiated in think–pair–share settings, discussions, individual work with think-aloud protocols, and working in pairs.

The strategy of *macro scaffolding* was realised by offering support (use of different registers, lists of words and sentence structures to use in written productions) and by sequencing the task material, for example, from oral communication (when the mathematical concept was clarified in students' individual everyday language) to written production (when the use of a more academic language with technical terms was aimed at). Every lesson initiated storage activities for technical words and relevant sentence structures.


*Micro scaffolding* was initiated in the interaction between peers or teacher and learner. The teachers were trained in the typical micro scaffolding features.








Figure 3 shows a typical activity from the first lesson of the intervention in which a systematic variation of a fraction is conducted (Task a) and the effects in different registers are investigated (Task b, together Type E) in order to construct mathematical relationships. Guided by the pushed-output strategy, students were encouraged to verbalise their observation of structural relationships.



**Parts of a Chocolate Bar**

a) Kenan and his friends share a chocolate bar. He wants to know which part he gets when he shares with one friend, or with two, three or four friends. He prepared the following table. Complete the table.



Chocolate bar divided by friends:	My Picture	Kenan gets:
1 bar - 2 friends		$\frac{1}{2}$
1 bar - 3 friends		
1 bar - 4 friends		
1 bar - 5 friends		

b) Examine the table carefully and consider the following:

- What happens with Kenan's part of the chocolate bar?
- Why does Kenan's part of the chocolate bar change?

Fig. 3 Typical activity: systematic variation in different registers

For showing a typical sequencing of tasks, Fig. 4 presents the next activities. They are similarly structured according to Type E, but now focus on the construction of non-unit fractions and the quasi-cardinal perspective (Task c): one fifth, two fifths, three fifths... The sequence of these tasks in Figs. 3 and 4 show how the design principle of macro scaffolding was applied to composing activities. The steps from oral description and explanation for the first variation in Fig. 3 to the oral description and explanation in Fig. 4 initiate the construction of mathematical meaning in combination with opportunities to create and develop the respective task-related key linguistic means in a context-embedded, face-to-face situation with the teacher as a mediating support. On the base of these learning opportunities, Task e finally asks students to prepare a written product which demands the more complex linguistic form of less personal and more abstract, context-reduced written language. A further macro scaffolding support is given by the structured list of words.

**Research design and methods for the intervention study**


Research questions and empirical design




Within *Phase 1* of the larger design research study, the sketched intervention program for enhancing conceptual understanding of fractions was developed iteratively in several design experiment cycles. This article reports on the intervention study in *Phase 2* (of Fig. 5) that investigated effects by two triangulating research questions:

1. *Effect size*: To what extent do students who participate in the language- and mathematics-integrated intervention improve their achievement in the fraction test?

**More and more fifth**

c) Now Kenan produces fifths with fraction bars. Complete the table.



Anteil that Kenan wants to draw:	My Picture
$\frac{1}{5}$	
$\frac{2}{5}$	
....	[the original work sheet has lines for $\frac{3}{5}$ and $\frac{4}{5}$ here]
$\frac{5}{5}$	

d) Examine the table precisely and consider the following:

- What happens with the coloured part of the fraction bar?
- Why does the coloured part change?

e) Your research:

- How and why does the Anteil change?
- Write down your findings so that another student can understand what is happening with the Anteil and why the Anteil changes.
- You can use the following words:

What changes?	How does it change?
- the numerator	- more
- the denominator	- less
- the number of friends	- bigger
- the number of chocolate bars	- smaller
	- the same

**Fig. 4** Typical macro scaffolding: first orally, then in written form with a scaffold

2. *Processes*: What is the situational potential of the intervention activities to initiate students' constructing of meanings and activating of linguistic means in the school and technical register?

For investigating *effect sizes* in the pre-intervention-posttest-design, the usual classrooms were defined as base line. That is why the control group was taught by their regular teacher with the usual fraction textbook repetition program. Since it was expected that scores for the intervention group would be greater (due to their more focused support), the quantitative research concentrated on *effect sizes*.

For understanding not only *how much* but also *how* a focus on language issues can enhance the processes of individual and interactive construction of meanings, the learning *processes* were video recorded and then analysed in an explorative in-depth analysis with respect to the intended connections between relating registers, pushed output, and scaffolding and their relationship to constructing meaning. This analysis of processes allowed us to make sure that the growth in understanding could be traced back to the design principles, and not only, for example, to the intensive individual teaching that did occur.

The methods for qualitative and quantitative data gathering, sampling, and data analysis for *Phase 2* are listed in the overview in Fig. 5 and explained in the following sections.

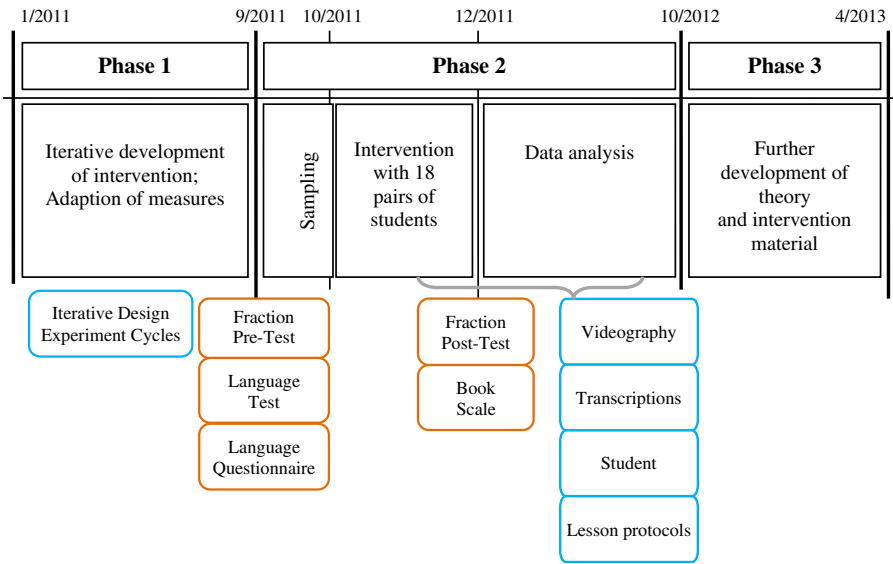


Fig. 5 Mixed-methods design of the study

Methods for data gathering

*Quantitative measures*

*Fraction pre- and post-test.* Students' performance in dealing with fractions was measured by a pre-test at the beginning of Grade 7, and by a post-test 3 months later, after the intervention. For this purpose, a standardised test on fractions (Bruin-Muurling 2010) was adapted to German curricula and piloted with 156 students (first pilot phase) and 212 students (second pilot phase).

The test included 41 items covering: specifying and drawing fractions in part-whole and part-group models, finding locations on the number-line, ordering fractions according to size and explaining order in a contextual situation or graphical representation, finding equivalent fractions with given numerator / denominator and explaining equivalence in a contextual situation or graphical representation, multiplying in a part-of situation, subtracting with proper and improper fractions, and fraction as operators in word problems. Pre- and post-tests had the same content (with 28 equal and 13 items modified in numbers). With Cronbach's Alpha at 0.856 (41 items) in the second pilot study and 0.835 (41 items) in the main study, the test showed a satisfactory internal consistency.

*German Language Test.* Students' language proficiency was assessed by a C-Test, which offers an economical and highly reliable measure of a complex construct of general language proficiency of first- and second-language learners (Grotjahn 1992). The C-Test applied in this study (Kniffka et al. 2007) consisted of five German sub-texts with gaps. With Cronbach's Alpha at 0.96 (N=262 in a pilot study, cf. Linnemann 2010) between the five sub-texts, the test shows a very good internal consistency.

*Questionnaire for language background and socio-economic background.* A students' questionnaire was used to ask for students' and parents' countries of birth, *languages* spoken in the family and with friends, and the number of years students had been living in Germany for new immigrants. As the *socioeconomic background* has often been specified as a further relevant factor that affects the learning success (OECD 2007), it was measured by the book scale with graphical illustrations that is widely used and has shown good retest scores (mean  $r=0.80$ , cf. Paulus 2009).

### *Data gathering for qualitative analysis*

As the intervention study in Phase 2 took place in a larger research context of design research in the learners' perspective (see Prediger 2013 for details of Phase 1), the quantitative analysis of learning effects was complemented by a qualitative analysis for exploratively reconstructing the situative potential of the design strategies and intervention activities in the learning processes.

Eighteen pairs of students took part in the 540 min of the intervention program. These 18×540 min were video-taped and relevant episodes were selected according to research question 2. The data corpus for the qualitative analysis comprised selected videos, all materials from the program and lesson protocols of the teachers, as well as all documents written by the students during the intervention.

### Participants

The participants of the study were 72 seventh-grade second language learners with below average math performance and limited German language proficiency.

A sample of 303 students from 14 classes in a German urban region was tested on the above measures. From the 48 % of multilingual students in the sample (Family language  $x$  or  $G + x$ ), we selected those with limited, but still sufficient, German *language proficiency* to participate in the intervention program; they were identified by having scores between 50 and 90 points on the German Language Test (Kniffka et al. 2007). For identifying *low achievement in mathematics*, a cut-off score for the fraction pre-test was 19 points, the mean value in the sample of the pilot study.

Among all students who satisfied these criteria, 36 pairs of students were randomly assigned to the intervention or the control group. Pairs of students from each group were matched on mathematics achievement, language proficiency, and family languages (see Table 3). The variance test showed no significant differences between the intervention and the control groups in the fraction pre-test ( $F [1, 70]=.173, p>.05$ ) and the Language Test ( $F [1, 70]=.01, p>.05$ ). The groups were also comparable in their distribution of age, family languages, and socio-economic background.

### Methods for the quantitative analysis

To analyse the effects and effect sizes of the intervention, the differences between the test means on the fraction post-test comparing the intervention and control groups were tested for a statistical significance level of 0.05. A repeated measures analysis of variance (ANOVA) was conducted with the fraction test pre-post difference as dependent variable, group and time as main factors, and group by time as an interaction factor.

**Table 3** Sample and sampling

	Complete sample ( <i>N</i> =303)	Intervention group ( <i>N</i> =36)	Control group ( <i>N</i> =36)
Achievement in fraction pre-test	<i>m</i> =13.28 SD=5.98	<i>m</i> =10.83 SD=4.46	<i>m</i> =10.42 SD=4.03
Achievement in German Language Test	<i>m</i> =84.20 SD=15.22	<i>m</i> =76.33 SD=11.40	<i>m</i> =76.08 SD=9.79
Socio-economic background (complete sample only with <i>N</i> =252)	Low=34 % Medium=39 % High=27 %	Low=53 % Medium=39 % High=8 %	Low=40 % Medium=40 % High=20 %
Family languages (G: only German; G + x: German and other language; x: only other language)	G=52 % G + x=35 % x=13 %	G=0 % G + x=64 % x=36 %	G=0 % G + x=72 % x=28 %
Age	<i>m</i> =12.57 SD=0.74	<i>m</i> =12.78 SD=0.13	<i>m</i> =12.97 SD=0.13

Inter-group effect sizes were measured by partial eta squared ( $\eta^2$ ) in the variance analysis that reflects the percentage of variance explained by the independent variables in the sample data. According to Cohen (1988), partial  $\eta^2$  in a variance analysis measures effect sizes between groups:  $\eta^2 < 0.01$  counts as low,  $0.01 \leq \eta^2 < 0.13$  counts as medium, and  $\eta^2 > 0.13$  counts as high effect sizes.

Additionally, the intra-group effect sizes were measured by “*d*” in Glass’ meta-analysis (1976) that reflects the differences of means within each group as a percentage of a standard deviation. In general,  $d < 0.2$  counts as low and  $d > 0.8$  as high (Cohen 1988).

### Methods for the qualitative analysis

The qualitative analysis was conducted for explorative reconstructing of the situational potential of the design strategies and intervention activities for, first, the individual and interactive processes of constructing meanings for mathematical concepts and relations and, second, students’ activating linguistic means in the school register and the technical register. These two dimensions of the research question guided the selection of episodes. The selected episodes were transcribed. The data analysis referred to the videos, the transcripts, and the students’ written documents.

For *Step I* of the data analysis, a *sequential analysis* was carried out on the data by a systematic extensive interpretation (Beck and Maier 1994) for reconstructing the individual and interactional processes of constructing mathematical meanings in terms of mental models for fractions (Prediger 2008).

In *Step II*, several *qualitative coding procedures* for students’ utterances were conducted with respect to different linguistic aspects. In this article, the linguistic aspects are restricted to IIa, the reference context for students’ descriptions of systematic variation activities, and IIb, the identification of moves within the verbal register in students’ utterances.

*Codes for Step IIa.* To carve out the specific potential of the intervention activity Type E of systematic variation as shown in Figs. 3 and 4, the categories <object> (what changes?), <operation> (how does it change?), and <effects> (what are the effects of the change?) are applied (Link 2012). For example, in Fig. 3 the changing <object> is the number of friends, the <operation> is “always one more,” and the <effect> is that Kenan’s part of the chocolate bar becomes smaller, which can be identified and explained either in the symbolic register (the denominator in the fraction becomes bigger which results in a smaller fraction) or in the graphical register (the coloured part of the fraction bar becomes smaller).

*Codes for Step IIb.* For coding the learners’ and teacher’s linguistic utterances and their moves along the continuum of verbal registers from everyday to school and technical register, the linguistic characterisations of Koch and Österreicher (1985), Schleppegrell (2004), and Cummins (1986, 2000) suggested the following categories of continua: from context-embedded (using deictic means, gestures etc.) to context-reduced; from personal to less personal, more abstract; from exemplary to more general; from less complex sentence structures to more complex sentences structures; and from individually invented or everyday terms to more technical terms; and, for each category, moves back from the technical to the everyday register.

In *Step III*, students’ development of mathematical meaning and linguistic means were reconstructed in their interplay with concrete elements of the design, namely relating registers, pushed output, and macro and micro scaffolding. By the method of *contrasting selected moments* in their intended and realised initiation of students’ processes, we reconstructed typical pathways, obstacles, conditions, and means for the teaching-learning process.

## Selected results from the empirical analysis

Effect sizes: significant progress in conceptual understanding of fractions—results and discussion

As the higher post-test means in Table 4 suggest, both groups have a significant increase in their fraction test scores over time. But whereas the control group has only a low-medium effect (intra-group effect size measured by  $d=0.42$ , according to Glass 1976), the intervention group has a very strong effect ( $d=1.22$ ), which reflects more than a standard deviation.

This difference is confirmed by the significant interaction effect in the variance analysis ( $F_{(\text{Group} \times \text{Time})} [1, 70]=10.78, p<0.01$ ). Thus, the groups develop unequally over time with the intervention group having a significantly higher increase than the control group, as expected. The related measure for the inter-group effect size of  $\eta^2=0.13$  can be interpreted as a high effect (Cohen 1988), especially for such a short treatment. If the item scores in the fraction test are restricted to the contents of the intervention, the inter-group effect size increases to  $\eta^2=0.164$ . In contrast, the intervention group has no significant advantage in the items on non-treated contents.

**Table 4** Comparing achievement of intervention and control group in pre- and post-test

	Achievement in fraction pre-test	Achievement in fraction post-test	Intra-group effect-size d
Intervention group ( $N=36$ )	$m=10.83$ $SD=4.46$	$m=16.58$ $SD=4.90$	+1.22
Control group ( $N=36$ )	$m=10.42$ $SD=4.03$	$m=12.33$ $SD=4.89$	+0.42
Inter-group and time effects	$F_{(\text{Group})} [1, 70]=6.59$ $F_{(\text{Time})} [1, 70]=43.12$ $F_{(\text{Group} \times \text{Time})} [1, 70]=10.78$	$p<0.05$ $p<0.001$ $p<0.01$	$\eta^2=0.09$ $\eta^2=0.38$ $\eta^2=0.13$

The high effect sizes are remarkable and support—by triangulation—the relevance of qualitatively investigating details in the processes.

Selected aspects of processes: case studies on the situational potential of the activities

In order to give some insights into the in-depth analysis of the teaching–learning processes, two connected case studies from the first intervention lesson are presented. In this lesson, the part-whole model for fractions and structural relationships were addressed by systematic variations in different registers of increasing degree of abstractness (as a relating register activity of Type E, see Figs. 3 and 4).

We focus on research question 2 by using a small extract from the large video data set that clearly identifies the specific potential of macro and micro scaffolding in combination with the design strategy of pushed output. Hence we asked, with respect to systematic variation as a form of macro scaffolding activities that were sequenced along the language continuum from oral to written language production

- How could these sequenced activities initiate meaning constructions and the production of content-related language?
- How do the strategies of micro scaffolding and pushed output support these learning processes?

*The case of Learta and Ismet: interactive development of linguistic means for expressing the effects of systematic variation*

Learta (L in the following transcript) and Ismet (Is) are in Grade 7. Ismet's first language is Kurdish; he is 13 years old and immigrated from Turkey at the age of 8 years. Learta's first language is Albanian; she is 12 years old and immigrated from Kosovo 2 years ago.

In the selected scene from the first intervention lesson, the students had already worked on Task a) of the variation activity (Fig. 3). The transcript starts with Task b) which explicitly asks for the structural relationship between the lines. This relationship had not yet been considered by the students but is now put on the table by the teacher (abbreviated T).

- 
- 83 L *[reads aloud Task b in Fig. 3]*
- 84 Is Come on, I know something.
- 85 T Wait, give Learta also a short chance to think about it.
- 86 Is Whoa, ehm, ehm.
- 87 L What would—Anteil is the fraction, isn't it?
- 88 T Correct *[pause of 5 s]*
- 89 Is Let me explain//
- 90 L // because always more friends yes, at first 2 and then 3 and then 4 and then 5
- ... *From line 90 to 97 Ismet proposes an answer considering the variation. The teacher comes back to Learta's proposal in line 90 and focuses on the changing number of friends*
- 97 T Okay, so the ratio changes, because always more friends//
- 98 Is //yea
- 99 T come in addition. And can you tell me, ehm, if that, is that good how it improves for Kenan? Does he get more, does he get less?
- 100 L No
- 101 T How can you see that in the fraction?
- 102 L The fewer friends there are, the more he gets.
- 103 Is No, yes, the more he gets, but the main thing is, that everybody get similar big eh piece.
- 

*Step I* (summary): The first sequential analysis shows that the students interactively constructed the intended meaning and structural relationship: By filling in the table, they related the verbally given situation to its graphical representation in a fraction bar and its contextual realisation. They became aware that the situation varied systematically from line to line, with the number of chocolate bars staying always one, and the number of friends increasing from two to five. They realised that this systematic variation resulted in a decrease of Kenan's part of the chocolate bar. During this interaction process, they successively elaborated the linguistic means for describing the structural relationship. The resulting linguistic pattern "the fewer ... the more" (Learta in line 102) can be qualified as a phrase of the school register. The more detailed analysis shows how this expression interactively emerged.

*Step IIa*: While still struggling with the concepts Anteil and fraction (line 87), Learta's first approach (in line 90) for an explanation already concentrated on the changing number of friends as the changing object being affected by the variation ("because always more friends yes"), that is, <object of variation> and the <operation> increase. The next part of her utterance focuses more on the <operation>: "at first 2 and then 3 and then 4 and then 5." However, Learta did not yet explicitly refer to the <effect> that this changing number has, and the concept development for the order of fractions has not yet been finalised.

*Step IIb* for line 90: From a linguistic point of view, she moves to a generalisation by inserting "always" in her description and structures her explanation with "first" and "then." Typically of oral communication, she referred directly to the context situation and exemplified with the numbers of the task, knowing that both listeners understood without further explication.



*Step IIa/b* for lines 97–99: The teacher enforced the so-far missing focus on <effects>, when she summed up and repeated in a complete sentence why the fraction is changing: “Okay, so the Anteil changes, because always more friends come in addition” (line 97 and 99). This reaction can be classified as micro scaffolding in form of focusing and extending by repeating the task’s question of involving the changing fraction (→ Step III). By asking “And can you tell me, ehm, if that, is that good how it improves for Kenan? Does he get more, does he get less?” (line 99), the teacher initiates a shift of attention from <object> and <operation> to the <effect> for Kenan’s part of the chocolate bar. Doing so, she also offered vocabulary of description: more, less (→ Step III).

*Step IIa/b* for lines 100–102: Leara’s answer “no” in line 100—which is probably a “no” to the teacher’s question “if the change is good for Kenan”—became more detailed when the teacher wanted to know how she could see that in the fraction (in line 101): “The fewer friends there are, the more he gets.” (line 102). She seemed to restructure and combine the teacher’s offer of the phrase fragment in line 97 and 99 “so the Anteil changes, because always more friends come in addition” and the question regarding the effect on Kenan’s part (line 99). She adapts “the more” into “the fewer” for expressing <object> and <operation> extremely economically, and explained the <effect> that Kenan gets more in the situations of sharing with fewer friends. The fact that she inverted the operation (decrease instead of increase of friends) gives hints that she grasped the structural relationship that is inherent in the order of fractions and condensed it into the elegant formulation “the fewer ... the more.”

*Step III:* The short excerpt suggests that the interactive construction of meanings for structural relationships can very well be integrated with the initiation of rich content-related language production. The *push of output* worked quantitatively: Leara and Ismet both wanted to formulate an answer to the question and seemed highly motivated (Ismet in line 84: “Come on, I know something”), but also qualitatively: For describing the interplay of changes and effects, the phrase fragment “the more..., the less...” and “the less..., the more...” was crucial here; it was interactively developed by student and teacher in a setting of minimal micro scaffolding. At the same time, they worked on becoming successively aware of the structural connections on ordering fractions as part-whole relationships (Anteile).

However, the case of Ismet’s micro learning process in the succeeding parts suggests that the intended macro scaffolding of working along the language continuum can function only under certain conditions for the process of interaction when the mathematical meaning is intended to be constructed. As the teacher’s attention was focused mostly on Leara and her thinking, Ismet’s utterance in line 103 remained unanswered. In the later process of writing down their investigations (Task e) of Fig. 4, Ismet correctly noted the changing <object> and <operation> (“the numerator is always getting one more, the denominator stays the same”). For describing the <effects> of this variation, he began to write “the fraction shows us”. He orally started four times and repeated this phrase in his search for expressing the part-whole relationship signified by the fraction. As his search was not noticed by the teacher, a micro-scaffolding could not take place. In the end, he gave up and crossed out the phrase. This same scenario was not an isolated case and we reconstructed this for several other cases found in our data set. This gives hints to the serious limits of micro scaffolding in regular classrooms with more than two students.

*The case of Asim and Hadar: transition from everyday to technical register*

Asim (A in transcript below) and Hadar (H) are both 12 years old and in Grade 7. Asim's first language is Persian, and he emigrated from Iran 10 years ago. Hadar's first language is Bosnian. His parents emigrated from Bosnia; he was born in Germany.

In the following scene from the first intervention lesson, both students were working on the activities in Fig. 4 in which non-unit fractions were systematically varied from one fifth to five fifths, fostering the quasi-cardinal perspective on fractions. The excerpt starts at the end of Task d) and then shows the change in Task e) when transitioning from oral to written treatment:

- 
- 124 H Ehm. Why does the Anteil change?  
 125 T Yes, what do you think? Think about it shortly.  
 126 A Because Kenan just gets more, because he gets two, then three, four, five, so he gets more.  
 127 T (approving) Mmh.  
 128 H Because the numerator always gets bigger.  
 129 T Yes.  
 130 H He gets always more.
- ... *Teacher gives new instruction for part d): "Write down your findings so that another student can understand what is happening with the Anteil and why the Anteil changes." Students individually write down answers:*
- Asim's written text* "Because the numerator gets always bigger, that is why Kenan gets always one more. And the denominators stay the same."  
*Hadar's written text* "When the numerator gets bigger, one gets more Anteil."
- 

The analysis focuses on the comparison of the oral descriptions and explanations for the changing fraction from line 124 to 130 with their written texts.

*Step IIb* for lines 126, 128/130: Both oral explanations were rather context-embedded, with Asim (in line 126) being closer to the contextual situation of fair share than Hadar (in line 128/130) who already included the technical term "numerator" to formulate an inner-mathematical reason which he linked to the context situation of Kenan always getting more chocolate. Hadar generalised his explanation by adding "always."

*Step IIa* for lines 126, 128/130: Hadar explicitly referred to <object>, <operation> in technical terms, and to the <effect> in contextual terms. Asim used exemplary language referring to the task ("he gets two, then three, four, five") by which he implicitly expressed <object> and <operation>. In contrast, the <effect> is explicitly described in contextual terms.

*Step IIa/b for written answers:* It is striking that both Asim and Hadar wrote more abstract and context-reduced answers that contained more mathematics-specific technical terms to describe fractions and part-whole relationships. Asim adopted Hadar's notion of the numerator always getting bigger as a reason why Kenan gets more

chocolate pieces. Here, a moment of micro scaffolding between the peers can be identified ( $\rightarrow$  Step III). Asim still linked the technical description to the context, so that his product can be categorised at a point of transition between context-embedded and context-reduced writing. Impressively, Asim also extended his investigation to the fractions' denominator: "stay the same." In contrast, Hadar completely left out the context of chocolate in his description by just arguing within a wholly mathematical context, "When the numerator gets bigger, one gets more Anteil." His explanation contained technical terms to describe all three components of variation: <object>, <operation>, and <effect>. As Hadar left out the context, the explanation became more abstract and less personal because he used an impersonal construction: "one gets more Anteil." Hadar generalised by using the conditional construction beginning with "when" and offered a rather short and concise sentence. Both written answers contained characteristics of written-like language with higher linguistic demands with respect to more lexical resources (technical terms) and more complex grammatical constructions.

*Step III:* Asim's and Hadar's case illustrates the transition from oral context-embedded talk to the technical register when the sequencing of tasks as a feature of macro scaffolding is used as an organisational principle to structure the learning process. This could be observed for all 36 students. The move from oral description to written production was designed in such a way that the oral communication could support the construction of meanings of ordering fractions. For all 36 students, it also served as a supply of linguistic means by a macro scaffolding move and opportunities for situational micro scaffolding support.

## Discussion and conclusion

The article set out to present a language- and mathematics-integrated intervention based on the theoretically grounded design strategies of relating registers, pushed output, and scaffolding for students with limited German language proficiency, a particularly vulnerable group of students in Germany. The quantitative and qualitative research on effects and processes investigated *to what extent and how* the intervention can contribute to fostering students' construction of meaning and as a consequence improve students' achievement in the fraction test.

The quantitative analysis provided empirical evidence for a significant growth of achievement in the fraction test. Of course, a small sample always suffers from limited representativeness, and a follow-up test would have strengthened the result on a sustainable growth of understanding. Despite these methodological limitations, the large effect sizes between pre- and post-test ( $d=0.42$ ) are especially noteworthy, considering the quite short time of intervention.

Whereas these quantitative results alone might have been explained on the basis of the intensity of individual teaching, the case studies gave some first insights into *how* the design elements offered fruitful learning opportunities, since they showed the deep interconnection of linguistic and mathematical learning processes: Hence, it appears that a mathematically substantial activity like systematic variation of related registers can offer opportunities for rich mathematical talk (pushed output) and deliberate moves between everyday, school, and technical registers, even for vulnerable learners. With the help of macro and micro scaffolding, low-achieving students successfully proceeded

from their everyday language to a *language for thinking and talking about structural relationships*. These complex interplays, and especially the crucial role of the students' resources in the everyday register for constructing meaning (before transitioning to more formal registers), although not reported here, were also found for the other 32 students in the further qualitative analysis.

However, the analysed case studies also showed the fragility of teaching, even for the nearly optimal 2-to-1 situation which clearly lacks ecological validity for regular classrooms: Although the orchestration and sequencing of activities in the material is crucial, the learning processes heavily rely on the adaptivity of the situated support with sensitive micro scaffolding by the teacher. Adaptivity here refers to the *moments* of micro scaffolding as well as to the *choice* of concrete linguistic means being relevant for the growth of conceptual understanding. Whereas the specification of relevant linguistic means can be further investigated in laboratory settings like in this study, finding the crucial moments for micro scaffolding in a classroom with 30 students remains a huge challenge even for well-trained teachers. This became evident in the analyses of unsuccessful interactions in the design experiments.

In spite of these limits of representativeness and ecological validity, the results give hope that the process of acquiring the language of instruction as a language for developing conceptual understanding can be enhanced by the combination of the design strategies relating registers and scaffolding. Although the study was conducted with second-language learners, the qualitative insights into the processes justify optimism with respect to transfer: The design strategies also seem to be fruitful for other learners with or without language difficulties.

Further research and development should extend these results to regular classrooms and to other mathematical topics.

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