Time and Determinism

Thomas Müller

Abstract This paper gives an overview of logico-philosophical issues of time and determinism. After a brief review of historical roots and 20th century developments, three current research areas are discussed: the definition of determinism, space-time indeterminism, and the temporality of individual things and their possibilities.

1 Introduction

Perennial philosophical questions of time and determinism, often in combination, have triggered important developments in logic. In return, such logical developments have provided tools for philosophical inquiry, and their use has led to progress on the philosophical side. The logico-philosophical discussion of time and determinism thus stands out as a field in which the interaction between formal methods and philosophical subject matter has succeeded in an exemplary fashion. In this paper we will trace relevant developments in philosophical logic and mention three active research topics: the definition of determinism, indeterminism and space-time, and temporal/modal first-order logic. Our focus will be on philosophical problems for which formal methods have proven to be helpful, rather than on the formal results

T. Müller (✉)
Fachbereich Philosophie, University of Konstanz,
Fach 17, 78457 Konstanz, Germany
e-mail: t.mueller@uu.nl; Thomas.Mueller@uni-konstanz.de

T. Müller
Department of Philosophy, Utrecht University,
Janskerkhof 13, 3512 BL Utrecht, Netherlands

themselves.[1] We will stress the usefulness of branching histories frameworks for tackling these problems. For reasons of space, we will have to leave by the wayside the impact of discussions of time and determinism on computer science and artificial intelligence.[2]

2 Historical Roots

In the Western logical tradition, philosophical puzzles of time and determinism have been discussed explicitly from Aristotle onwards.[3] A central issue is, and remains to be, the open future and the so-called problem of future contingents. The idea that the future is indeterministic or open—that there are different alternative possibilities for the future—has several sources. We commonly acknowledge indeterminism with respect to non-human natural affairs such as the weather, but the most important source of our idea of indeterminism arguably lies in our own free actions affecting the future. Aristotle's vivid example from *De Interpretatione* is still used in present-day discussions:[4]

> [I]t is necessary for there to be or not to be a sea-battle tomorrow; but it is not necessary for a sea-battle to take place tomorrow, nor for one not to take place—though it is necessary for one to take place or not to take place. (19^a30 ff.)

So what about the assertion "There will be a sea-battle tomorrow"? Two conflicting intuitions come to the fore, apparently creating a problem for the core logical notion of truth. One the one hand, it seems clear that the future is open in the relevant respects: As of now, it is not yet determined whether there will be a sea-battle tomorrow or not. On the other hand, we understand and often make assertions about the future, even in indeterminate cases. And don't we, in assertion, intend to speak the truth—even though we can, of course, fail, and then speak falsely? Aristotle appears to deny this:

> Clearly, then, it is not necessary that of every affirmation and opposite negation one should be true and the other false. (19^a39 ff.)

Does this force us to accept a different status, perhaps a third truth value, for assertions about future contingents, and reject the logical principle of bivalence? Or can we give a different account of what is going on?

---

[1]For a comprehensive overview of technical developments in some areas of temporal logic, see Gabbay et al. [15, 16].

[2]Øhrstrøm and Hasle [38] give an informative overview of the philosophical roots. Apart from the literature referred to in note 1, see also Van Eijck and Stokhof [48] for an overview of dynamic logics, and Xu's article in this volume for the logic of agency.

[3]This author cannot competently relate to other important logical traditions, and makes no claim to scholarly competence. See Gabbay and Woods [17] for recent overview articles on the Greek, Indian and Arabic traditions, and see notes 4 and 6 for some scholarly references.

[4] For scholarly commentary and translations, see Ackrill [1] and Weidemann [49]. We quote from the former work, p. 53.

One enticing idea is to re-interpret the original assumption that the world is not (yet) determined as regards some future happening, in an epistemic way. Surely if it is not yet determined what will happen tomorrow as regards the sea-battle, we *cannot know* what will happen. But perhaps not knowing is all that is at issue, really: Couldn't it be that it will be one way or the other, determinately, but there is (perhaps in principle) just no way for us to find out, except waiting and seeing? This move would short-circuit the logical problems of truth: Our assertions would be plainly true or false. The move would also provide an account of (epistemic) future indeterminacy despite the (metaphysical) truth of determinism, i.e., despite there being just one single possible future course of events.[5]

The epistemic story could be further strengthened if determinism—the metaphysical claim that there is just one real possibility for the future—could be shown to be unavoidable anyway. And indeed, future contingents have often been denounced as a *logical* impossibility, which would amount to establishing determinism on logical grounds. An important early argument to that effect was provided by Diodorus Cronus: the so-called "master argument", which claims that only that is possible which either is the case or will be the case.[6] Irrespective of the merits of that particular argument, the general idea of lifting a notion of possibility from purely temporal notions (in this case, "Diodorean possibility") has been important in the development of temporal logic in the 20th century (see Section 3.2 below).

The ancient and medieval commentary tradition treats the problem of future contingents at length. In the Christian era it takes on new importance in that even the epistemic reading becomes problematic once an omniscient god enters the picture. Many subtle issues arise due to a presupposition of human freedom (otherwise, God's moral judgment about humans seems unfair) and God's omniscience implying that for every ϕ, even if ϕ's topic is a free human act, God knows whether ϕ.[7]

3 Time and Determinism: Logical Developments in the 20th Century

The rise of modern symbolic logic in the late 19th century took place in the context of mathematics, for which issues of time and determinism apparently play no role. This subject-related aspect was strengthened by the influence of the Kantian tradition (certainly in the case of Frege), in which matters of modality were not seen as pertaining to the *content* of a judgment, making a symbolic representation of modality superfluous. If time is mentioned, it is in the form of a set of times over which we can quantify (a time coordinate, familiar from physics). Thus Frege claims that the appearance that one and the same sentence, "The tree outside is green", is

---

[5]Note that whether one favours a metaphysical or an epistemic account of indeterminism, we have to acknowledge that we apparently sometimes make assertions about that which we *cannot* know. So on both accounts, some pragmatic story is needed to explain that fact.—For a recent contribution to the question whether determinism and indeterminism are epistemically distinguishable, see Werndl [50].

[6]See, e.g., Gaskin [18] for details. See also Knuuttila [22] for pointers to more recent literature.

[7]God's relation to time is another difficult issue, and there are many more. For a useful introduction to the medieval discussion, see Adamson [2].

true at one time and false at another, is misleading. It is rather that the sentence is elliptical, leaving out the respective date; the full sentence is, e.g., "The tree outside is green on 8 July 2012", which, if true, is true at all times (a so-called "eternal sentence").

Corresponding to this syntactical idea about the role of time for logic, there is the semantic point that the proper notion of content for logical purposes should also be eternal: The subject matter of logic should be free from modality and temporal indexicality. Quine in *Word and object* [44] gives a forceful and influential exposition of this view.

Worries about the adequacy of this approach, together with an interest in the formalization of arguments from ancient and medieval sources, which required a different treatment of time and modality, triggered some important logical developments. Here we only mention three. Łukasiewicz's interest in Aristotle's logic and the question of future contingents led to the development of formal many-valued logic in the 1920s (Section 3.1). In the middle of the 20th century, Prior worked out a formal logic of tense (Section 3.2). Relatedly, a formal semantics for indexical notions was established (Section 3.3).

3.1 Many-Valued Logic and Future Contingents

Jan Łukasiewicz, who studied Aristotle extensively, built a system with three in place of two possible semantic values (extensions) for sentences, thereby rejecting the principle of bivalence. He introduced a middle value, $1/2$, as the semantic value of future contingents, corresponding to possibility in the sense of an open future. Later he extended his system to four values [26]. Many-valued logic is now a major research area in logic, with many real-world applications.

A recalcitrant problem with Łukasiewicz's approach to the issue of time and modality, already with respect to Aristotle's sea-battle example, is that he originally rules a disjunction with two middle-value disjuncts to have the middle value, while it seems clear that "either tomorrow there will be a sea-battle, or tomorrow there will not be a sea-battle" should be true and not indeterminate. Alternative many-valued truth tables face similar problems. Indeed it seems that an extensional treatment of future contingents is bound to be unsatisfactory, and that an intensional logic is needed.

3.2 Tense Logic, Relational Semantics and Branching Histories

Arthur Prior created formal tense logic in the 1950s, in part as an explicit reaction and alternative to Łukasiewicz's extensionalism [42]. Prior started from the observation that there are certain stable structural dependencies between differently tensed sentences. For example, if it is true that ϕ, then it will always be true that ϕ has been the case. In tense logic, unmodified (atomic) sentences are present-tensed, and temporal modifications are represented as one-place operators, often written P (past, "it was the case that") and F (future, "it will be the case that"). Their semantics is relational, naturally building on a linearly or partially ordered set of moments at which formulae are to be evaluated. Thus, e.g., $P\phi$ is true at a moment m iff there is an earlier moment m' s.t. ϕ is true at m'.

At the time Prior was establishing tense logic, other authors were conducting semantical work in the foundations of modal logic that also led to relational semantics—a true story of multiple discovery.[8] The intuitiveness of thinking of time as an ordering made tense logic a good testbed for developing ideas in relational semantics, including correspondence theory, incompleteness, the standard translation, and hybrid logic. The inexpressibility of one quite natural demand for a temporal ordering, irreflexivity, made expressive limitations of modal languages vivid and highlighted the topic of admissible rules. Temporal logic has found its way into many applications in computer science and artificial intelligence (see note 1), forming the backbone of, e.g., model checking and models for concurrent processes.

Philosophically, future contingents have remained among the pressing issues in temporal logic. A crucial question is what the proper semantics for the future operator F should look like. Clearly, if the semantics is based on a linear ordering of moments, the clause for F should just be the mirror image of that for P given above: $F\phi$ is true at a moment m iff there is a later moment m' s.t. ϕ is true at m'. A linear ordering implies that with respect to any moment, there is just one single possible future course of events, which amounts to determinism. Thus, temporal possibility can only be spelled out in the Diodorean fashion mentioned in Section 2 above: ϕ is (Diodorean-)possibly true at m if it is either true at m, or will be true at some later moment. It is however quite clear that this analysis does not capture an interesting notion of the open future—as the sea-battle example shows, we are interested in different, incompatible possibilities *for the same time*. One way to spell this out, suggested to Prior by Kripke,[9] was to think of moments as forming not a linear total ordering, but a tree-like partial ordering. This ordering is a relational (Kripke-) frame in the sense of modal logic, and together with a valuation constitutes a model. Maximal linear chains in the frame, called *histories*, represent complete possible courses of events. A Y-shaped branching tree with two histories can represent the sea-battle scenario quite faithfully: There are two different moments at the same time, tomorrow (two points on the different branches), and one of them represents the occurrence, the other, the non-occurrence of a sea-battle. The question remains, however, how we are to understand the semantics of "There will be a sea-battle" on such an approach. Several solutions have been proposed, the most prominent one being the so-called Ockhamist solution. See Thomason [46, 47] for two important early contributions to the discussion. Ockhamism forms the background for the important *Stit* modal logic of agency.[10] Still, other ways of using branching histories are being explored. See Øhrstrøm [37] and Malpass and Wawer [29] for recent defenses of a so-called "Thin Red Line" reading of branching histories, according to which one of the histories can be singled out as the real one without thereby falling prey to determinism. An important clarification in the debate has been reached by MacFarlane

---

[8]See Copeland [13] and Goldblatt [19] for historical details.
[9]See Ploug and Øhrstrøm [41] for the history.
[10]See Belnap et al. [8] as well as the separate paper by Xu in this volume.

[27], who differentiates terminologically the task of giving a formal account of the recursive semantics of *F* for use in embedded sentences, and of giving an account of stand-alone sentences in terms of a non-recursive so-called "postsemantics".[11]

3.3 Logical Treatment of Indexicals

Within the context of tense logic, the semantics of temporal indexicals such as "now" or "tomorrow" also became a focus of interest. It seems obvious that "now" refers to the time of utterance, so that, e.g., "now" is redundant in "It is now raining". Subtle issues however arise in embedded contexts, e.g., "Yesterday I believed that it would now be raining". Kamp established a number of important formal results about "now" in the 1960s [20]; a general semantic framework for indexicals was given by Kaplan [21]. This "two-dimensional semantics" has found many other philosophical applications as well [12]. For a comprehensive scheme for classifying various parameters of truth, expanding Kaplan's considerations, see Belnap et al. ([8], Ch. 6B). There is a direct line from these considerations to the new semantic relativism of, e.g., MacFarlane [27, 28].

4 Current Topics

We single out three of the many interesting and worthwhile topics of current philosophico-logical investigation, thereby indicating areas in which logic and philosophy continue to interact to improve our understanding of time and determinism: the definition of determinism (Section 4.1), determinism and space-time (Section 4.2), and first-order logic (Section 4.3). As in the rest of this paper, the selection is explicitly this author's, who cannot claim to provide an unbiased overview.

4.1 Defining Determinism

What is a proper formal definition of determinism? There appears to be a major disconnect between the rather simple logical handling of determinism and indeterminism in tense logic, the finely differentiated discussions of the status of determinism with respect to scientific theories in philosophy of science, and the importance of determinism for other philosophical debates, e.g., about free will.

Logical frameworks that allow one to differentiate deterministic from indeterministic models, like the branching histories framework mentioned in Section 3.2 above, often come with an extremely simple definition of determinism: In fact, for any framework employing branching histories, a frame is deterministic iff it has just one

---

[11]See also Belnap [5] as well as the pragmatic account of word-giving in Belnap et al. ([8], Ch. 5C).— For an extension of Ockhamism using local future possibilities ("transitions") rather than histories, see Rumberg [45].

history, and indeterministic otherwise, and this property of the frame is also used to classify models.[12]

In a possible worlds framework that rejects branching histories, such as the "$T \times W$" approach to combining time and modality (see Thomason [47]) or Lewis's modal realism [25], a purely frame-based definition is no longer appropriate. Once the set of possible worlds W has more than one member, the resulting structure has more than one maximal chain, i.e., more than one history (all of which are isomorphic to the set of times, T)—but still, a model based on such a frame could be deterministic.[13] The basic idea for defining determinism in the $T \times W$ framework is that determinism means that the present state fixes future states uniquely; this idea can be traced back to Laplace [24]. Formally this can be spelled out as follows: A model $\langle W, T, <, V \rangle$ is (future-)deterministic iff for any two $w, w' \in W$, if there is some $t \in T$ for which $V(w, t) = V(w', t)$, then for any $t^* > t$, we also have $V(w, t^*) = V(w', t^*)$.[14] A similar definition (with some added subtleties) was given by Montague [30], but it is only appropriate for rather simple applications. Briefly, there are two issues. One is that time translation shouldn't matter, so that one can't insist on fixing one and the same time in both w and w'—one has to allow for different times t and t' at which to evaluate V. Secondly and more crucially, in physics one has to take into account the widespread phenomenon of gauge freedom, which means that several different mathematical representations can correspond to the same physical state at a time. So one has to connect $V(w, t)$ and $V(w', t')$ not via identity, but via some less stringent mapping, and similarly for the identification of the worlds after the respective times.[15] The resulting definition of determinism, directed at the classification of scientific theories, goes as follows: "a theory is deterministic if, and only if: for any two of its models, if they have instantaneous slices that are isomorphic, then the corresponding final segments are also isomorphic" [11].[16] This sounds like a logically stringent definition that invites widespread application, but as a matter of fact it seems that after the time of Montague, who discussed two simple classical theories, this definition has hardly ever been used to assess real physical theories. Rather, what practitioners interested in the determinism or indeterminism of a theory do is to study that theory's defining equations and assess whether these equations admit unique solutions given initial values. This practice, however, seems more congenial to a branching histories view of indeterminism than to a

---

[12]In an extreme case, this could mean that a model with two isomorphic histories on which the valuation is exactly the same, would count as indeterministic. In order to exclude such cases, one can require that a *faithful* model must reflect any branching in the underlying frame by a corresponding difference in valuations. See, e.g., Placek and Belnap [39].

[13]A note on terminology: We here continue to use "model" in the logical sense, i.e., as a frame with a valuation, incorporating different incompatible possible courses of events (histories) in one structure. In philosophy of science, models are often required to represent just one temporal course of events, i.e., to be what we here call a history plus valuation.

[14]A mirror-image, past-determinism, is defined by replacing "$t^* > t$" with "$t^* < t$".

[15]For a comprehensive and authoritative discussion of the relevant issues, see Earman [14].

[16]In this quotation, "model" is used in the way common in philosophy of science, designating a single history admitted by the scientific theory in question; see note 13 above.—One has to talk about slices rather than states at times to account for space-time theories, see Section 4.2.

"possible worlds plus mappings between them" approach. E.g., in the widely discussed example of simple indeterminism in Newtonian mechanics, Norton's dome [36], the defining ordinary differential equation has an infinite set of solutions for certain initial data, but (allowing for temporal translations) only two non-isomorphic temporal developments. A branching representation can capture the infinite size of the solution space faithfully by picturing that solution space as one unified structure with infinitely many histories branching off at specific moments.[17] It seems beneficial, therefore, to define indeterminism of scientific theories formally in terms of branching structures, rather than holding on to the current situation in which there is an official formal definition that is not actually used and which does not reflect the work of the practitioners. The logical resources of branching histories make possible a simpler and more congenial definition of determinism and indeterminism that reflects the concrete temporal structure of possibilities [35].

Such a change in the definition of determinism could also reflect back on the role that determinism plays in various philosophical debates, e.g., on free will. The most important issue seems to be the following: In the $T \times W$ framework, objective uncertainty about what the future will bring is affirmed, but it is ultimately explained epistemically, as uncertainty about which world $w \in W$ one is located in. Each of these worlds has a linear, deterministic development. This fixes a deterministic picture of the temporal development of individuals that is far removed from our ordinary self-conception as agents affecting the future course of things. In this author's view, the present state of the free will debate is still too much permeated by the idea that ultimately, only determinism gives a truly scientific picture of the world. Incorporating an adequate model of indeterminism into the scientific discussion about determinism of theories could help to remove this misconception.

4.2 Determinism and Space-Time

A related area in which the interaction between logic and philosophy of science should be strengthened, concerns determinism and space-time. The definition of determinism in philosophy of science has been heavily influenced by considerations of space-time theories; Einstein's "hole argument" played a decisive role (see Butterfield [10]). The field shows the tension mentioned above: officially, determinism in space-time theories is defined via mappings of "models" (single space-times), but in fact, the behaviour of differential equations is studied. There is however available a branching histories framework for discussing space-time (in)determinism: Belnap's "branching space-times" [4].[18] As this framework represents different possibilities in one unified formal structure, it provides a formal handle for studying EPR-like modal correlations such as arise, e.g., in quantum mechanics; see Müller et al. [34]. Further practical relevance of the framework may come from its closeness to formal structures used to study concurrent processes in computer science (see Wroński [52]).

---

[17]For an explicit model, see Müller and Placek [35].

[18]For a discussion of topological issues, see Placek et al. [40] and Müller [33].

4.3 Individuals and Their Possibilities

Perhaps the biggest lacuna in the development of logical tools for understanding time and determinism, in the view of this author, is the absence of individual things from the scene. It is quite understandable that in computer science, which is driving most technical developments in modal and temporal logic, the interest in predicate logic is limited—applications require tractable systems. For philosophical purposes, however, an adequate modeling of things, their temporal development and their possibilities is certainly required.

Time and determinism interact with the metaphysical discussion about things in two ways, which are mostly treated separately. First, there is the question of the temporal development of things, of identity through successive changes—the question of *persistence*. Second, there is the question of what is possible for a thing—the issue of *de re* modality—that is variously discussed under the headings of laws, dispositions, or powers. Logically speaking, what seems to be required is a combination of temporal-modal (intensional) logic with quantification theory: quantified modal logic. Several systems have been proposed since the 1940s, starting with the syntactic investigations of Barcan [3]. An important semantical investigation, employing relational frames, is Kripke [23]. Most of these systems combine modality and quantification without explicit formal attention to time; an exception is Wölfl [51].

Almost all such systems (including Wölfl) work with a formal representation of things that is only adequate for a metaphysical picture of "bare particulars": Things are represented as mere labels, as pegs to hang properties on, but their representation includes no internal variation that would reflect change. A thing is simply present at a world (or at a world/time index) or not. Change is rather represented on the side of the extension of properties at worlds (or world/times). Thus, typically, one has a domain of things per world, the union of which determines the total domain of possibilia. Variables are then interpreted along the lines of the idea of "rigid designation", i.e., as picking out the same thing in each world (or nothing, if the thing doesn't exist in that world).

This way of looking at things in time may suffer from the same extensionalist idea mentioned above in connection with Łukasiewicz—that everything in logic has to boil down to extensions. Established temporal and modal logics deviate from this idea and introduce intensional operators at the sentential level: For example, the semantic value (extension) of $P\phi$ at m isn't a (truth-)function of the semantic value of ϕ at m, but depends on the course of values of ϕ at different moments m'. I.e., the extension of $P\phi$ at m is a function of the *intension* of ϕ.

A similar move at the level of terms is also possible, but has not been researched much. Bressan [9] proposed a framework of quantified modal logic in which variables do not function as rigid designators, but as terms with a (possibly non-constant) intension and an extension in each case;[19] predication is generally intensional. In

---

[19]"Case" is Bressan's general word for temporal/modal alternatives.—Bressan's original modal system ML^ν is higher order and quite complex. Its expressive resources are comparable to Montague's more well-known IL [31], but ML^ν does not require explicit type conversions as in IL.

case-intensional first oder logic [7], which derives from Bressan's work, a predication $\Theta(\alpha)$ is true or false at a case (e.g., at a moment in time) depending on the intension (course of extensions at moments) of α. Thereby, modal properties *de re* (such as, e.g., dispositions) are represented as properties of a thing directly, classifying the intension of a term α. The persistence conditions of a thing can also be expressed via intensional predication, employing Bressan's ground-breaking idea of absolute properties. In this way, the two aspects relating things with the issues of time and determinism, persistence and modality *de re*, come together in one formal system.[20] The formal aspects of the respective limitations of indeterminism due to things around us [43] still need to be spelled out more clearly. Embarking on this project would hopefully allow formal logic to make good on another insight due to Aristotle, viz., that an argument for indeterminism can be based on very simple considerations of things around us:

> [I]n things that are not always actual there is the possibility of being and of not being [...]. Many things are obviously like this. For example, it is possible for this cloak to be cut up, and yet it will not be cut up but wear out first. But equally, its not being cut up is also possible [...]. Clearly, therefore, not everything is or happens of necessity [...]. ($19^a 9$ ff.)

We know many things about the world around us, just by being part of it. It remains a task of philosophical logic to reflect that knowledge adequately. Time and determinism remain core topics of philosophical reflection in the 21st century, and new formal methods can continue to help clarify our thinking about these topics.

References

1. Ackrill, J.L. (1966). *Aristotle's Categories and De Interpretatione,* translated with notes by J. L. Ackrill, Clarendon Aristotle Series. Oxford: Oxford University Press.
2. Adamson, P. (2010). Freedom and determinism. In R. Pasnau (Ed.), *The Cambridge History of Medieval Philosophy* (Vol. 1, pp. 399–413), Cambridge: Cambridge University Press.
3. Barcan, R. (1946). A functional calculus of first order based on strict implication. *Journal of Symbolic Logic*, *11*, 1–16.
4. Belnap, N. (1992). Branching space-time. *Synthese*, *92*(3), 385–434. See also the postprint 2003, available on philsci-archive.

---

[20]See Belnap and Müller [6] for a system explicitly based on branching histories, which puts the general CIFOL machinery to work for a discussion of indeterminism.

5. Belnap, N. (2002). Double time references: speech-act reports as modalities in an indeterminist setting. In F. Wolter, H. Wansing, M. De Rijke & M. Zakharyaschev (Eds.), *Advances in Modal Logic* (Vol. 3, pp. 37–58). Singapore: World Scientific.

6. Belnap, N., & Müller, T. (2014). BH-CIFOL: Case-intensional first order logic. (II) Branching histories. *Journal of Philosophical Logic*, *43*, 835–866.

7. Belnap, N., & Müller, T. (2014). CIFOL: Case-intensional first order logic. (I) Toward a logic of sorts. *Journal of Philosophical Logic*, *43*, 393–437.

8. Belnap, N., Perloff, M., & Xu, M. (2001). *Facing the future: agents and choices in our indeterminist world*. Oxford: Oxford University Press.

9. Bressan, A. (1972). *A general interpreted modal calculus*. New Haven, CT: Yale University Press.

10. Butterfield, J. (1989). The hole truth. *The British Journal for the Philosophy of Science*, *40*(1), 1–28.

11. Butterfield, J. (2005). Determinism and indeterminism. In *Routledge Encyclopedia of Philosophy*, Vol. 3. London: Routledge.

12. Chalmers, D. (2006). Two-dimensional semantics. In E. Lepore & B. Smith (Eds.), *Oxford Handbook of Philosophy of Language* (pp. 575–606). Oxford: Oxford University Press.

13. Copeland, B. (2002). The genesis of possible worlds semantics. *Journal of Philosophical Logic*, *31*(2), 99–137.

14. Earman, J. (2006). Aspects of determinism in modern physics. In J. Butterfield & J. Earman (Eds.), *Handbook of the Philosophy of Physics* (pp. 1369–1434). Amsterdam: Elsevier.

15. Gabbay, D.M., Hodkinson, I., & Reynolds, M. (1994). *Temporal logic. Mathematical foundations and computational aspects* (Vol. 1). Oxford: Oxford University Press.

16. Gabbay, D.M., Reynolds, M., & Finger, M. (2000). *Temporal logic. Mathematical foundations and computational aspects,* Vol. 2. Oxford: Oxford University Press.

17. Gabbay, D.M., & Woods, J. (2004). *Handbook of the history of logic. Volume 1: Greek, Indian and Arabic logic*. Amsterdam: Elsevier.

18. Gaskin, R. (1995). *The sea battle and the master argument: Aristotle and Diodorus on the metaphysics of the future*. Berlin: de Gruyter.

19. Goldblatt, R. (2006). Mathematical modal logic: a view of its evolution. In D.M. Gabbay & J. Woods (Eds.), *Handbook of the history of logic. Volume 7: Logic and the modalities in the 20th century* (pp. 1–98). Amsterdam: Elsevier.

20. Kamp, H. (1971). Formal properties of 'now'. *Theoria*, *37*, 227–273.

21. Kaplan, D. (1989). Demonstratives. An essay on the semantics, logic, metaphysics, and epistemology of demonstratives and other indexicals. In J. Almog, J. Perry & H.K. Wettstein (Eds.), *Themes from Kaplan* (pp. 481–564). Oxford: Oxford University Press.

22. Knuuttila, S. (2012). A history of modal traditions. In D.M. Gabbay, F.J. Pelletier & J. Woods (Eds.), *Handbook of the history of logic. Volume 11: Logic: A history of its central concepts* (pp. 309–339). Amsterdam: Elsevier.

23. Kripke, S. (1963). Semantical considerations in modal logic. *Acta philosophica Fennica*, *16*, 83–94.

24. Laplace, P. (1820). *Théorie Analytique des Probabilités.* Paris: V. Courcier. English translation: A philosophical essay on probabilities. Dover: New York, NY, 1951.

25. Lewis, D.K. (1986). *On the plurality of worlds*. Oxford: Blackwell.

26. Łukasiewicz, J. (1957). *Aristotle's syllogistic*. Oxford: Oxford University Press, second, enlarged edition.

27. MacFarlane, J. (2003). Future contingents and relative truth. *The Philosophical Quarterly*, *53*(212), 321–336.

28. MacFarlane, J. (2014). *Assessment sensitivity: Relative truth and its applications*. Oxford: Oxford University Press.

29. Malpass, A., & Wawer, J. (2012). A future for the thin red line. *Synthese*, *188*, 117–142.

30. Montague, R. (1962). Deterministic theories. In D. Willner (Ed.), *Decisions, Values and Groups* (pp. 325–370). Oxford: Pergamon Press Reprinted as Chap. 11 of [32].

31. Montague, R. (1973). The proper treatment of quantification in ordinary English. In J. Hintikka, J. Moravcsik & P. Suppes (Eds.), *Approaches to natural language: Proceedings of the 1970 Stanford workshop on grammar and semantics* (pp. 221–242). Dordrecht: D. Reidel. Reprinted as Chap. 8 of [32].

32. Montague, R. (1974). *Formal philosophy: selected papers of Richard Montague*. In R.H. Thomason (Ed.), New Haven, CT: Yale University Press.

33. Müller, T. (2013). A generalized manifold topology for branching space-times. *Philosophy of Science*, *80*, 1089–1100.

34. Müller, T., Belnap, N., & Kishida, K. (2008). Funny business in branching space-times: infinite modal correlations. *Synthese*, *164*, 141–159.

35. Müller, T., & Placek, T. (2015). Defining determinism. *British Journal for the Philosophy of Science*. forthcoming.

36. Norton, J.D. (2008). The dome: an unexpectedly simple failure of determinism. *Philosophy of Science*, *75*, 786–799.

37. Øhrstrøm, P. (2009). In defence of the thin red line: a case for Ockhamism. *Humana.mente*, *8*, 17–32.

38. Øhrstrøm, P., & Hasle, P. (1995). *Temporal logic. From ancient ideas to artificial intelligence*. Dordrecht: Kluwer.

39. Placek, T., & Belnap, N. (2012). Indeterminism is a modal notion: branching spacetimes and Earman's pruning. *Synthese*, *187*(2), 441–469.

40. Placek, T., Belnap, N., & Kishida, K. (2014). On topological issues of indeterminism. *Erkenntnis*, *79*, 403–436.

41. Ploug, T., & Øhrstrøm, P. (2012). Branching time, indeterminism and tense logic. Unveiling the Prior-Kripke letters. *Synthese*, *188*(3), 367–379.

42. Prior, A.N. (1957). *Time and modality*. Oxford: Oxford University Press.

43. Prior, A.N. (1962). Limited indeterminism. *The Review of Metaphysics*, *16*(1), 55–61.

44. Quine, W. (1960). *Word and object*. Cambridge, MA: MIT press.

45. Rumberg, A. (2015). Transition Semantics for Branching Time. Forthcoming.

46. Thomason, R.H. (1970). Indeterminist time and truth-value gaps. *Theoria*, *36*, 264–281.

47. Thomason, R.H. (1984). Combinations of tense and modality. In D. Gabbay & G. Guenthner (Eds.), *Handbook of philosophical logic, vol. II: extensions of classical logic, Vol. 165 of Synthese Library, Studies in epistemology* (pp. 135–165). Dordrecht: D. Reidel Publishing Company.

48. van Eijck, J., & Stokhof, M. (2006). The gamut of dynamic logics. In D.M. Gabbay & J. Woods (Eds.), *Handbook of the history of logic. Volume 7: Logic and the modalities in the 20th century* (pp. 499–600). Amsterdam: Elsevier.

49. Weidemann, H. (1994). *Aristoteles, Peri hermeneias,* übersetzt und erläutert von Hermann Weidemann, No. 1. II in *Aristoteles. Werke in deutscher Übersetzung*. Berlin: Akademie Verlag.

50. Werndl, C. (2011). On the observational equivalence of continuous-time deterministic and indeterministic descriptions. *European Journal for the Philosophy of Science*, *1*(2), 193–225.

51. Wölfl, S. (1999). Combinations of tense and modality for predicate logic. *Journal of Philosophical Logic*, *28*, 371–398.

52. Wroński, L. (2013). Branching space-times and parallel processing. In H. Andersen, D. Dieks, W.J. Gonzalez, T. Uebel & G. Wheeler (Eds.), *New challenges to philosophy of science* (pp. 135–148). Berlin: Springer.