

# Global Shear Wave Velocity Database for Probabilistic Assessment of the Initiation of Seismic-Soil Liquefaction

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Engineering practitioners commonly use penetration-based methods (SPT & CPT) for assessment of seismic liquefaction triggering hazard. On the horizon, shear wave velocity ( $V_s$ ) may offer engineers a third tool that is lower cost and provides more physically meaningful measurements. Development of the shear wave velocity liquefaction method has been hampered by a paucity of published velocity profiles; particularly in deeper soil deposits (>10m) and deposits subjected to high cyclic stress ratios ( $CSR > 0.3$ ). A review of the literature reveals that most historic liquefaction sites fitting this depth and CSR criteria are located in Asia, though most of these sites remain untested for  $V_s$ .

To remedy this scarcity of data, we set out to assemble a global  $V_s$  dataset by acquiring new data in Japan, Taiwan, China, India, and the United States (US). These data are merged with the exiting catalog of published velocity data. To acquire new field data, we use the recently developed continuous swept-sine wave spectral analysis of surface waves test (CSS-SASW). The CSS-SASW test has proven to be extremely reliable at rapidly gathering high signal-to-noise dispersion data sufficient to invert 20-40 meter  $V_s$  profiles. So far, we have acquired new velocity profiles at nearly 300 liquefaction-evaluation sites throughout Asia and the US, mostly at sites previously tested by conventional penetration methods. This new dataset represents the majority of the worlds documented sites of liquefaction occurrence since instrumental recording.

To correlate the global shear wave velocity data set with likelihood of initiation of seismic-soil liquefaction, we utilize high-order probabilistic tools (Bayesian updating) developed for structural reliability. A multi-parameter limit-state function for liquefaction triggering is modeled and evaluated based on the means, distributions and uncertainties of each model-

variable. Each case history is then sub-divided into 'quality'-ranking categories based on the conjugate-uncertainties of CSR and  $V_{s1}$ . A low-pass cut-off of the coefficient of variation is used filter-out poorly constrained sites. Finally for the probabilistic analysis, the Bayesian updating procedure is used to iteratively compute coefficients for the limit-state function that minimize model error. The intended outcome of this effort is a new evaluation of the  $V_s$ -liquefaction-triggering boundary in light of a global data set and modern limit-state probabilistic tools.

**Keywords:** *liquefaction, surface wave, shear wave velocity, SASW, probability, reliability, Bayes*

## INTRODUCTION

This paper describes elements of an ongoing investigation to comprehensively re-evaluate the probabilistic shear wave velocity-liquefaction resistance correlation. The project elements include 1) re-assessment of field data published in the literature for Bayesian analysis parameterization; 2) expansion of the worldwide catalog of shear wave velocity profiles through new field data collection to close gaps where only penetration-based profiles are reported; 3) parameter estimation of sites for earthquake motion load and soil capacity against seismic-liquefaction resistance; 4) estimation of probabilistic bounds for seismic-soil liquefaction occurrence using structural reliability methods.

Up until now, the most comprehensive study of the application of field-based shear wave velocity ( $V_s$ ) measurements to seismic-liquefaction assessment has been presented by Andrus and Stokoe [1] on 59 spatially independent sites. For each of these sites, one or more profiles were presented. Of the many noteworthy findings of Andrus and Stokoe [1] in their assessment of the available data set, was the paucity of shear wave velocity data in the cyclic stress ratio (CSR) region

above 0.3, especially at high shear wave velocities near the region where the liquefaction resistance boundary likely resides, and the general lack of data compared with what is available for conventional penetration-based methods of field investigation. On the other hand, the modern standard and cone penetration-liquefaction resistance correlation (SPT & CPT) are evaluated from many hundreds of spatially independent SPT and CPT logs [e.g. 2,3,4,5,6], data sets that extend the pioneering efforts by Seed et al. [7], and Xie [8]. These new ‘global’ catalogs of penetration resistance data have extensive suites of field data from China, Japan, Taiwan, Turkey and the USA where relatively few  $V_s$  profiles are reported, and many of these new data are critical for constraining the liquefaction resistance boundary.

### NEW DATA COLLECTION

Our project seeks to elevate the  $V_s$  liquefaction-catalog to a level par with the penetration-based methods by conducting new investigations at sites previously documented using conventional tests. The catalogs developed for SPT and CPT correlations [e.g. 3,4] serve as initial ‘road maps’ for new site investigation, and subsequent on-site access to local knowledge, observations, datasets, and domestic publications allows us to fine-tune and expand our data collection efforts, particularly in the identification of non-liquefaction sites. Stokoe and his colleagues have conducted extensive surveys of some United States liquefaction sites, whereas, the large catalog of SPT & CPT sites in Asia and the northern California have fewer velocity logs and relatively spotty coverage across earthquake events.

At first step, we identified Asia and the United States liquefaction-investigation sites documenting conventional exploration and missing shear wave velocity logs. This list served as a roadmap for our field efforts. Recent well-documented historic events, spanning in time from the disastrous 1948 Fukui City earthquake up to the most recent 2003 Miyagi earthquake, are the principal target of our field investigation. To efficiently re-evaluate these documented sites without drilling apparatus we are using the Spectral Analysis of Surface Waves (SASW) method [9]; multistation-SASW methods [10]; and passive ambient-signal micro-tremor array methods [11,12,13], all relatively new non-invasive surface wave techniques for evaluating the shear wave velocity characteristics of soil. Surface wave

methods are particularly useful for rapid, lightweight, high-resolution surveys of liquefaction sites where characterization of the near surface (typically <15m) are needed. Surface wave techniques also work well for accurately profiling difficult materials such as gravely deposits and stiff soils where conventional truck-based penetration methods are not practical. Using surface wave methods, it is possible to routinely produce detailed shear wave velocity profiles of the upper 30 meters of the soil column.

Starting in 2001, we visited and profiled approximately 300 liquefaction and no-liquefaction evaluation sites Asia and the US using surface wave techniques (Figure 1). Nearly all of well-documented liquefaction sites in East Asia, originally evaluated only by conventional penetration apparatus, have been re-tested in our study using surface wave methods. A listing, by earthquake event, of the new seismic-liquefaction test sites is presented in Table 1. The earthquake events and liquefaction evaluation sites listed here represent the vast majority of the world’s well-documented case histories of liquefaction occurrence and non-occurrence in modern times.

<b>EARTHQUAKE INVESTIGATED</b>	<b>NEW SITES</b>
<b>1948 Fukui, Japan</b>	<b>11</b>
<b>1964 Alaska, USA</b>	<b>22</b>
<b>1964 Niigata, Japan</b>	<b>6</b>
<b>1968 Tokachi-Oki, Japan</b>	<b>12</b>
<b>1973 Miyagi-Oki (a) , Japan</b>	<b>11</b>
<b>1975 Haicheng, China</b>	<b>Oct. 2003</b>
<b>1976 Tangshan China</b>	<b>Oct. 2003</b>
<b>1978 Miyagi-Oki (b, c) , Japan</b>	<b>11</b>
<b>1983 Nihonkai-Chubu, Japan</b>	<b>8</b>
<b>1989 Loma Prieta, USA</b>	<b>36</b>
<b>1993 Hokkaido-Nansei, Japan</b>	<b>24</b>
<b>1993 Kushiro-Oki, Japan</b>	<b>10</b>
<b>1994 Kushiro, Japan</b>	<b>10</b>
<b>1995 Hyogo-Nambu, Japan</b>	<b>83</b>
<b>1998 Sanriku, Japan</b>	<b>2</b>
<b>1999 Chi Chi, Taiwan</b>	<b>14 + Nov. 2003</b>
<b>2000 Tottori, Japan</b>	<b>3</b>
<b>2001 Geiyo (Hiroshima), Japan</b>	<b>6</b>
<b>2002 Denali Fault Alaska, USA</b>	<b>26</b>
<b>2003 Sendai, Japan</b>	<b>11</b>

Table 1. New surface wave liquefaction test sites.

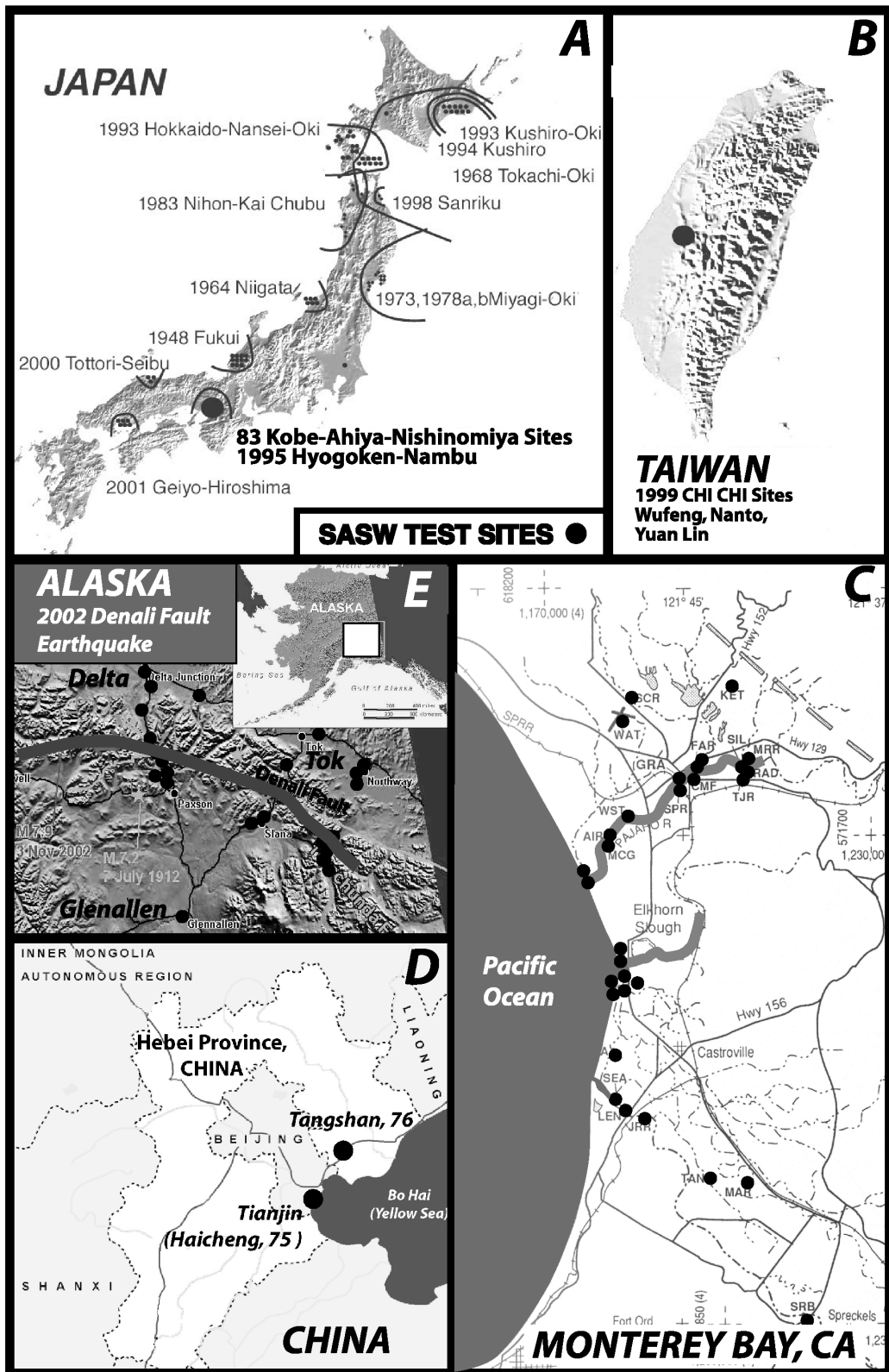


Figure 1. New shear wave velocity profiles at liquefaction evaluation sites (clockwise from top-left): (A) 168 - Japanese test sites (83 in Kobe area) collected in 2001-2; (B) 14 - Chi Chi, Taiwan, earthquake sites collected in 2001, and new sites collected in Nov. 2003; (C) 36 - Moss Landing, Pajaro, Salinas River sites and inner Bay Area sites (not shown) collected 2001-2003; (D) Tangshan and Haicheng, China, sites collected in October 2003; (E) 26 - Denali Fault earthquake sites, central Alaska and 22 - 1964 Alaska earthquake (not shown).

Merging this new data set with the 59 sites cataloged by Andrus and Stokoe [1], and data from the Kocaeli, Turkey event of 1998 allows us to finally evaluate the shear wave velocity data set in light of a truly global catalog. An important focus of our field-study is to target and test sites that straddle the liquefaction resistance boundary at  $CSR > 0.3$  where prior data were sparse (e.g. Hyogoken-Nambu 1995; Hokkaido Nansei-Oki 1993; Chi Chi, 1999; Fukui 1948; Kushiro-Oki 1993).

## SURFACE WAVE METHODS for PROFILING SHEAR WAVE VELOCITY

The Spectral Analysis of Surface Waves (SASW) method is a relatively new class of seismic tools useful for evaluating the stiffness characteristics of soil deposits. For our field-testing, we use a variety of passive-signal and active-source tests appropriate for the wide range of environments and site accessibilities at liquefaction sites. Surface wave methods are especially useful for profiling gravelly sand deposits where sampling is difficult and penetration tests fail to accurately quantify soil properties [9]. Gravelly sands are typical of soils found in the high CSR boundary zone region. Surface wave methods are perhaps the most efficient field tests for profiling the near-surface, principally because all measurements are made at ground level with highly portable equipment, the apparatus is non-destructive to the soil, and computed profiles typically yield a highly-detailed shear wave velocity model of the near surface. Unlike drilling and probing methods, SASW apparatus is lightweight, portable, easily deployable by one or two people, and usually requires no permitting. Surface wave apparatus can be deployed at locations impossible to reach by vehicle or truck. To reach many of the most remote locations in Figure 1, we have transported our surface wave test systems by canoe, raft, helicopter, and backpack.

Passive arrays are useful for capturing long-wavelength, low frequency ambient surface waves that radiate into the study area from afar, and are particularly useful for obtaining deep ( $>50\text{m}$ )  $V_s$  profiles. In coastal areas where historic liquefaction observations have most commonly been made, ocean waves serve as an excellent source of low-frequency surface waves, although distant cultural vibrations and even footsteps works well to generate low-frequency signals. Two passive-signal methods we use are the spatial autocorrelation (SPAC) method [10,11,12] and the frequency-wavenumber ( $f$ - $k$ )

method [10,13,14]. We use circular arrays for both SPAC and  $f$ - $k$  testing, though these 2-d arrays are often impractical for testing in highly vegetated and crowded urban areas. Passive signal methods can be of limited use in shallow-layer liquefaction surveys if higher frequency ( $>10$  Hz) surface waves are deficient.

Active source tests we use utilize surface waves generated in alignment with a 1-D array of seismometers. The surface-wave source can be either transient (impact-random wave) or continuous (random-wave or frequency-controlled wave source). Obviously, whatever the source, 1-D arrays are spatially easier and more practical to set up in the field than 2-D arrays. We set up our seismometer arrays in a traditional 2-receiver SASW-mode [15] or a 10-to-15-receiver-multichannel MASW mode [13,16]. For extremely remote site investigation, we have backpacked-in, or carried by helicopter, 2-receivers, cables, a spectrum analyzer, hammer, and battery into the field and tested using the traditional transient-SASW method [15]. When a field vehicle can be driven to the site, far cleaner phase-velocity data can be obtained using a continuous frequency-controlled source.

Our field configuration for over 95% of our liquefaction evaluation test sites is the continuous sine wave surface wave test (CSS-SASW and CSS-MASW). To generate harmonic frequency-controlled surface waves we use a signal generator and electromechanical vertical-excitation shaker. The signal generator produces a sweep of  $n$ -cycle single-frequency harmonic waves that are amplified and sent to the shaker. Surface waves generated by the source radiate past either four 1-Hz seismometers arrayed in pairs of two (SASW, Figure 2), or ten-to-fifteen 1-Hz sensors (MASW configuration of [13]).

From the active and passive array data, Raleigh-wave phase velocities can be computed directly from the cross-power spectrum (SASW method), averaged-coherence (SPAC method), or the wave number associated with the peak of the power spectrum at each frequency step ( $f$ - $k$  method). The dispersion curve used to invert the shear wave velocity profile is a plot of either phase-velocity-versus-frequency, or -wavelength. The ability to perform near-real time frequency domain calculations and monitor the progress and quality of the test allows us to adjust various aspects of the data-collection to optimize the capture of these dispersion measurements. These aspects include amplitude of the source-wave generation, frequency-

range, frequency step between wave bursts, number of cycles-per-frequency, and the receiver spacing.

Inversion of the experimental dispersion measurements is done using a numerical approach that employs a constrained least squares fit [similar to 13]. This inverse problem solves for a model shear wave velocity profile whose theoretical dispersion curve is a least squares “best-fit” with the experimental field data.

### BAYESIAN UPDATING for PROBABILISTIC ASSESSMENT of LIQUEFACTION

Bayesian updating is a rigorous probabilistic framework within which we experiment to identify a model that best describes the bounding frontier between regions of high- and low-likelihood of liquefaction occurrence. The curves in this bounding region express our degree-of-belief that initial triggering of liquefaction has, or will, occur. The model for seismic soil liquefaction is formulated in a traditional manner that limit-state models are formulated for single-component structural reliability problems: That is, capacity minus load. An example of one of the limit-state models we are testing is presented in equation 1.

$$g_{V_{s1}} = V_{s1}(1 + \Theta_1 FC) + \Theta_2 FC - \Theta_3 \ln(CSR) - \Theta_4 \ln(M_w) - \Theta_5 \ln(\sigma'v) + \Theta_6 + \epsilon \quad (1)$$

where  $V_{s1}$  is effective stress normalized shear wave velocity; FC is fines content (est. from other tests); CSR is earthquake-induced cyclic stress ratio;  $M_w$  is moment magnitude;  $\sigma'v$  is effective stress; and  $\Theta_6$  is the standard normal variate for an unbiased model, and  $\epsilon$  is the model error term. We formulate a limit state model with positive-capacity and negative-load terms and solve, through iterative Bayesian updating, for the best-fit model parameter,  $\Theta$ 's, that minimize the model error term,  $\epsilon$ . The generalized model is a limit-state expression of strengths minus stresses. When the model tips into negative terrain, we have some degree-of belief that failure has, or will, occur. The Bayesian updating process used to solve for the model parameter,  $\Theta$ 's, and model error term,  $\epsilon$ , involves selecting the critical liquefaction evaluation layer from the shear wave velocity logs and then computing the mean and variances for each model parameter [4,17]. For our data set, critical layers are selected from adjacent SPT or CPT logs using the NCEER-workshop guidelines [18]. To do this, we have tried to collect our new data set at sites already tested by penetration testing and where fines content information is available.

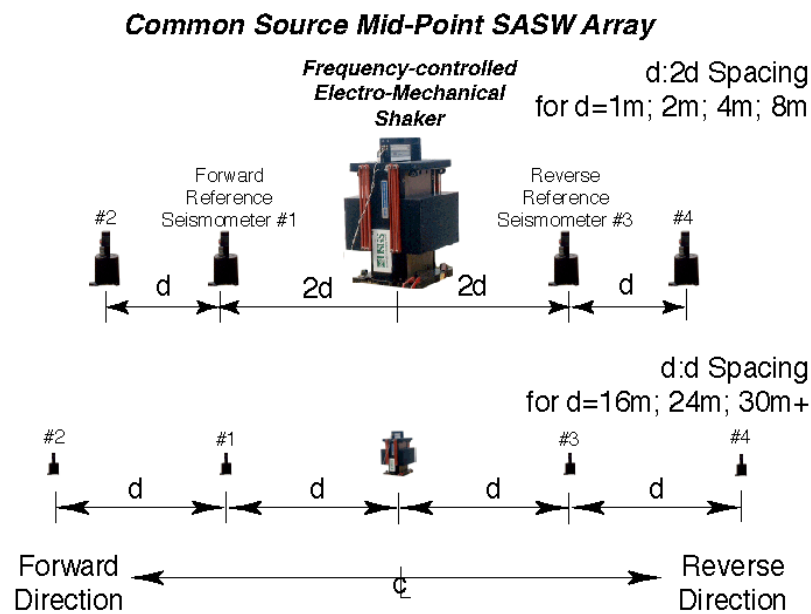


Figure 2. One of the USGS-surface wave systems, composed of 1-Hz sensors and a 100 kg electro-mechanical shaker used to build a 4-sensor SASW array. The shaker apparatus allows for frequency-controlled swept-sine CSS-SASW analysis. Array separation changes from d:2d to d:d as forward and reverse sensors are configured for large array separations.



Each variable in the limit-state function (CSR,  $V_{s1}$ , etc.) is assessed for their distribution statistics. At each site, the mean and coefficient of variation (c.o.v.) are determined for the variables needed to compute cyclic stress ratio (CSR) and effective stress normalized shear wave velocity  $V_{s1}$ . The CSR variable is computed from a composite suite of independent variable each having independent distributions. The composite c.o.v. for CSR is estimated using a first-order Taylor series expansion about the mean. Fig. 3 presents the product of the CSR- $V_{s1}$ -statistics, with diamonds marking the mean-value points for each site and 1-sigma bars marking the standard deviation.

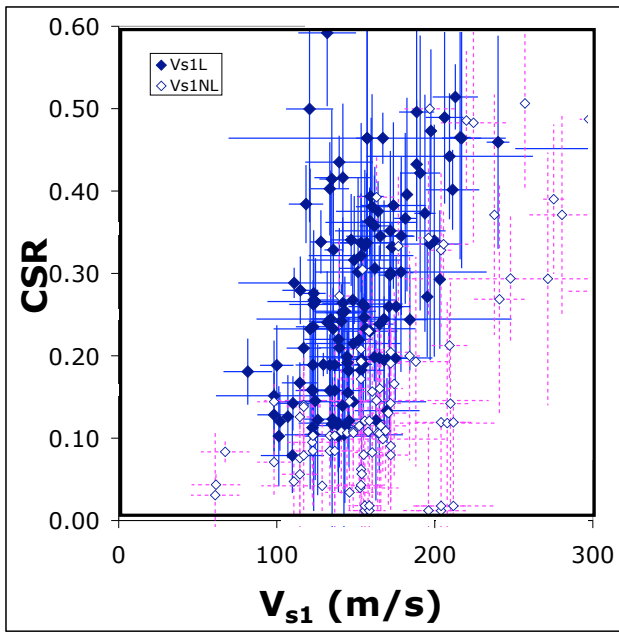


Figure 3. CSR- $V_{s1}$  plot of approximately 60% of our data set, with mean and standard deviation bars

Bayesian updating involves forming an experimental likelihood function, selecting a non-informative prior distribution, calculating a normalizing constant, and then processing through the posterior statistics [4,17] (equation 2). Starting with a non-informative prior distribution allows for the computation of an unbiased posterior distribution [19]. The experimental process of Bayesian updating is a search for the optimal likelihood function that minimizes model error.

$$f(\Theta) = c \cdot L(\Theta) \cdot p(\Theta) \quad (2)$$

Equation 2 presents Bayes rule, where  $f(\Theta)$  is the posterior distribution;  $c$  is the normalizing constant;  $L(\Theta)$  is the likelihood function; and  $p(\Theta)$  is the non-informative prior distribution.

The likelihood function for liquefaction triggering is the product of the probabilities of observing  $k$  liquefied sites and  $n-k$  non-liquefied sites (equation 3), where  $\Theta_v$  are the model variables and  $\Theta_m$  are the model parameters.

$$L(\Theta_v, \Theta_m, \varepsilon) \propto P \left[ \prod_{i=1}^k \{ \hat{g}(\Theta_{vi}, \Theta_{mi}) + \varepsilon_i \leq 0 \} \prod_{i=k+1}^n \{ \hat{g}(\Theta_{vi}, \Theta_{mi}) + \varepsilon_i > 0 \} \right] \quad (3)$$

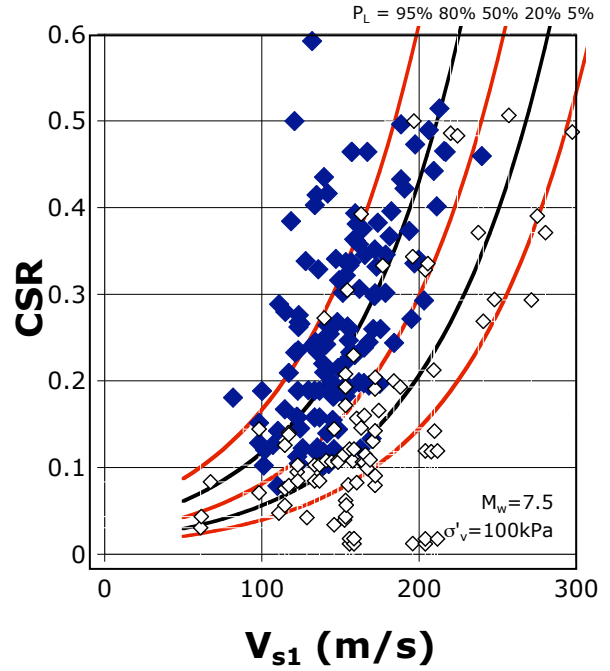


Figure 4. Preliminary probabilistic liquefaction triggering contours for ~60% of the global  $V_{s1}$  data set processed to date.  $P_L$  is the probability (degree-of-belief) that liquefaction has occurred.

Combining the uncertainties from the variables and model error term into a cumulative error term,  $\sigma_\Sigma$ , the likelihood function can be written in the form of equation 4, where  $\Phi$  is the standard normal cumulative distribution function.

$$L(\Theta_v, \Theta_m, \sigma_\Sigma) \propto \prod_{i=1}^k \Phi \left[ - \frac{\hat{g}(\Theta_{vi}, \Theta_{mi})}{\sigma_\Sigma} \right] \prod_{i=k+1}^n \Phi \left[ \frac{\hat{g}(\Theta_{vi}, \Theta_{mi})}{\sigma_\Sigma} \right] \quad (4)$$

The equal probability contours for  $V_{s1}$  are presented in Figure 4 for approximately 60% of the dataset processed to date. These preliminary contours are generated by a mean value-first order-second moment (MVFOSM) estimation of the failure surface, and will be assessed for quality using higher order first- and second-order reliability methods (FORM and SORM) and Monte Carlo simulations.

## CONCLUSIONS

This paper presents our procedures for expanding the worldwide data set of shear wave velocity at liquefaction evaluation sites, and processing these data within a Bayesian framework for probabilistic assessment of seismic-triggering of liquefaction. Since 2001, we have investigated approximately 300 Asia and US liquefaction evaluation sites using SASW, MASW and microtremor methods. The elements of our investigation are (1) cataloging locations of all documented liquefaction and non-liquefaction sites, (2) identifying critical layers and their textural characteristics; (3) re-evaluating sites by active-source and passive-signal surface wave methods; and, (4), applying Bayesian and structural reliability methods to assess the likelihood of liquefaction occurrence. The goal of this project is a formal re-evaluation of the liquefaction resistance assessment methodology by shear wave velocity in light of a new global data set and Bayesian probabilistic data processing.

## ACKNOWLEDGEMENTS

Components of this work are supported through funds or cost sharing by the United States Geological Survey (USGS); the Pacific Earthquake Engineering Research Center (PEER); Pacific Gas and Electric Company (PG&E); National Chung Hsing University, Taiwan; Kobe University; the PG&E/Alyeska Corp./USGS cooperative research and development agreement; and the China Seismological Bureau.

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