Hindawi Publishing Corporation
Journal of Control Science and Engineering
Volume 2014, Article ID 936856, 6 pages
http://dx.doi.org/10.1155/2014/936856

Research Article

Improved Robust Stability Criterion of Networked Control Systems with Transmission Delays and Packet Loss

Shenping Xiao, Liyan Wang, Hongbing Zeng, and Wubin Cheng

School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou 412008, China

Correspondence should be addressed to Shenping Xiao; xsph_519@163.com

Received 10 August 2013; Revised 4 December 2013; Accepted 24 December 2013; Published 12 February 2014

Academic Editor: Derong Liu

The problem of stability analysis for a class of networked control systems (NCSs) with network-induced delay and packet dropout is investigated in this paper. Based on the working mechanism of zero-order holder, the closed-loop NCS is modeled as a continuous-time linear system with input delay. By introducing a novel Lyapunov-Krasovskii functional which splits both the lower and upper bounds of the delay into two subintervals, respectively, and utilizes reciprocally convex combination technique, a new stability criterion is derived in terms of linear matrix inequalities. Compared with previous results in the literature, the obtained stability criterion is less conservative. Numerical examples demonstrate the validity and feasibility of the proposed method.

1. Introduction

Network control systems (NCSs) are the feedback control systems in which the control loops are closed via real-time networks [1]. NCSs have great advantages compared to the traditional point-to-point control systems, including high reliability, ease of installation and maintenance, and low cost, and they have been applied in many areas, such as computer integrated manufacturing systems, intelligent traffic systems, aircraft control, and teleoperation. However, due to the insertion of communication network in feedback control loops, time delay caused by data transmission and packet dropout in NCSs is always inevitable, which may degrade system performance or even lead to the potential system instability. Thus it is of significance to cope with the adverse influences of induced delay and packet dropout. Recently, the problem of robust stability analysis for NCSs has attained considerable attention, and a great number of research results have been reported [1–9]. In order to reduce the conservatism, Yu et al. [5] obtained the sufficient condition on the stabilization of NCSs; the admissible upper bounds of induced delay and packet dropout can be computed by using Lyapunov-Razumikhi function techniques and the quasiconvex optimization algorithm. The free weighting matrices method was proposed in [3] to solve the problem of network-based control. However, using too many free weighting matrices makes the system analysis complex; what is more, some terms were neglected directly, which brings the conservative. Peng et al. [9] indicated that if more information of induced delays in NCSs were utilized, conservatism in system analysis could be reduced. Moreover, it can build a bridge to connect quality of control (QoC). In [4], a stability criterion based on maximum allowable network-induced delay rate is proposed for choosing a reasonable sampling period.

Inspired by the above research results, in the present paper, the robust stabilization problem for a class of NCSs with network-induced delay and packet dropout is addressed. The NCSs are modeled as continuous-time linear systems on the basis of the input delay approach. By exploiting the information of lower and upper bounds of the delay, a new Lyapunov-Krasovskii functional is constructed. Moreover, reciprocally convex combination technique is introduced such that less conservative results are obtained. Two numerical examples are given to illustrate the validity and feasibility of the proposed results.

2. System Description

In this paper, the sensor module is assumed to act in a clock-driven fashion with transmission period h; the controller and actuator modules are assumed to act in an event-driven fashion. A new packet will be used by the controller immediately after its arrival. Single packet transmission is considered, where all the data sent or received over the network are sampled at the same sampling instant and assembled together into one network packet.

The plant is described by the following linear plant model proposed in [7]:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ are the plant's state and input vectors, respectively. A and B are known to be real constant matrices with proper dimensions. This control signal is based on the plant's state at the instant $i_k h$. So, the control law can be described as

$$u(t^+) = Kx(i_k h), \quad t^+ \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}), \quad k = 0, 1, 2, 3, \ldots, \tag{2}$$

where K is the state feedback gain matrix, τ_k denotes the network-induced delay, and h is the sampling period, $\{i_1, i_2, i_3, \ldots\} \subset \{1, 2, 3, \ldots\}$; due to the introduction of logical zero-order holder, the actuator will use the latest available control input, so $i_{k+1} > i_k$. If $\{i_1, i_2, i_3, \ldots\} = \{0, 1, 2, 3, \ldots\}$, then no packet dropout occurred in the transmission. And the numbers of consecutive packet dropouts during the time interval $(i_k h, i_{k+1} h)$ can be described as follows:

$$\begin{aligned} i_{k+1} - i_k &= 1, \quad 0 \text{ packet is lost}, \\ i_{k+1} - i_k &= 2, \quad 1 \text{ packet is lost}, \\ &\vdots \\ i_{k+1} - i_k &= p, \quad p - 1 \text{ packets are lost}. \end{aligned} \tag{3}$$

Assume the existence of constants $\tau_m > 0$, $\tau_M > 0$. One has

$$\begin{gathered} \tau_m \le \tau_k, \\ (i_{k+1} - i_k) h + \tau_{k+1} \le \tau_M, \quad k = 0, 1, 2, 3, \ldots, \end{gathered} \tag{4}$$

where τ_m and τ_M indicate the lower and the upper bounds of the total delay involving both transmission delays and packet dropouts, respectively.

From a straightforward combination of (1)–(4), the system can be rewritten as follows:

$$\begin{gathered} \dot{x}(t) = Ax(t) + A_d x(t - d(t)), \\ x(t) = \varphi(t), \quad t \in (-\tau_M, 0), \end{gathered} \tag{5}$$

where $A_d = BK$, the function $d(t) = t - i_k h$ which satisfies $\tau_m \le d(t) \le \tau_M$ denotes the time-varying delay in the control signal, and $\varphi(t)$ is the state's initial function.

The following technical lemmas are introduced, which are indispensable for the proof of the main result.

Lemma 1 (see [13]). *For any positive matrix $M > 0$, scalar $r > 0$, and a vector function $w : [0, r] \to R^n$ such that the integration $\int_0^r w(s)^T M w(s) ds$ is well defined,*

$$r\left(\int_0^r w(s)^T M w(s)\, ds\right) \ge \left(\int_0^r w(s)\, ds\right)^T M \left(\int_0^r w(s)\, ds\right). \tag{6}$$

Lemma 2 (see [14]). *For any positive matrix R, scalars a and b satisfying $a > b$, and a vector function x, the following inequality holds:*

$$\begin{aligned} &\frac{(a-b)^2}{2} \int_b^a \int_s^a x^T(u) R x(u)\, du\, ds \\ &\quad \ge \left(\int_b^a \int_s^a x(u)\, du\, ds\right)^T R \left(\int_b^a \int_s^a x(u)\, du\, ds\right). \end{aligned} \tag{7}$$

Lemma 3 (see [15]). *Let $F_1, F_2, F_3, \ldots F_N : R^m \mapsto R$ have positive values for arbitrary value of independent variable in an open subset W of R^m. The reciprocally convex combination of F_i $(i = 1, 2, \ldots, N)$ in W satisfies*

$$\min \sum_{i=1}^{N} \frac{1}{\eta_i} F_i(t) = \sum_{i=1}^{N} F_i(t) + \max \sum_{i=1}^{N} \sum_{j=1, j \ne i}^{N} W_{i,j}(t) \tag{8}$$

subject to

$$\left\{ \eta_i > 0, \ \sum_{i=1}^{N} \eta_i = 1, \ W_{i,j}(t) : R^m \longmapsto R, \quad W_{j,i}(t) = W_{i,j}(t), \begin{bmatrix} F_i(t) & W_{i,j}(t) \\ W_{i,j}(t) & F_j(t) \end{bmatrix} \ge 0 \right\}. \tag{9}$$

3. Main Results

Theorem 4. *For a given scalar $\tau_M > \tau_m \ge 0$, NCS (5) is asymptotically stable if there exist symmetric matrices $P = [P_{ij}]_{4\times4} > 0$, $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$, $R = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0$, Z_i $(i = 1, 2, 3) > 0$, R_i $(i = 1, 2, 3) > 0$, and proper dimensions matrice S_{12} such that*

$$\begin{gathered} \begin{bmatrix} Z_2 & S_{12} \\ * & Z_2 \end{bmatrix} > 0, \\ \Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0, \end{gathered} \tag{10}$$

where

$$\Theta_{12} = \begin{bmatrix} \frac{\tau_m^2}{8}\Psi R_1 \\ \frac{(\tau_M - \tau_m)^2}{2}\Psi R_2 \\ \frac{\tau_M^2}{8}\Psi R_3 \\ \frac{\tau_m}{2}\Psi Z_1 \\ (\tau_M - \tau_m)\Psi Z_2 \\ \frac{\tau_M}{2}\Psi Z_3 \end{bmatrix},$$

$$\Theta_{11} = \operatorname{diag}\{-R_1 \;\; -R_2 \;\; -R_3 \;\; -Z_1 \;\; -Z_2 \;\; -Z_3\},$$

$$\Theta_{22} = \begin{bmatrix} \Omega_{11} & P_{11}A_d & P_{12} & -P_{12} & \Omega_{15} & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 & 0 & A_d^T P_{12} & A_d^T P_{13} & A_d^T P_{14} \\ * & * & \Omega_{33} & S_{12} & -Q_{12}^T & 0 & P_{22} & P_{23} & P_{24} \\ * & * & * & \Omega_{44} & 0 & -R_{12}^T & -P_{22} & -P_{23} & -P_{24} \\ * & * & * & * & \Omega_{55} & 0 & -P_{23}^T & -P_{33} & -P_{34} \\ * & * & * & * & * & \Omega_{66} & -P_{24}^T & -P_{34}^T & -P_{44} \\ * & * & * & * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & * & * & * & -R_3 \end{bmatrix},$$

$$\Psi = [A \;\; A_d \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0] \tag{11}$$

with

$$\Omega_{11} = A^T P_{11} + P_{11}A + P_{13} + P_{13}^T + P_{14} + P_{14}^T + R_{11} + Q_{11} - Z_1 - Z_3 - \frac{\tau_m^2}{4}R_1 - (\tau_M - \tau_m)^2 R_2 - \frac{\tau_M^2}{4}R_3,$$

$$\Omega_{15} = -P_{13} + Z_1 + Q_{12},$$

$$\Omega_{16} = R_{12} + Z_3 - P_{14},$$

$$\Omega_{17} = A^T P_{12} + P_{23}^T + P_{24}^T + (\tau_M - \tau_m) R_2,$$

$$\Omega_{18} = A^T P_{13} + P_{33} + P_{34}^T + \frac{\tau_m}{2}R_1,$$

$$\Omega_{19} = A^T P_{14} + P_{34} + P_{44} + \frac{\tau_M}{2}R_3,$$

$$\Omega_{22} = -2Z_2 + S_{12} + S_{12}^T,$$

$$\Omega_{23} = Z_2 - S_{12},$$

$$\Omega_{24} = Z_2 - S_{12}^T,$$

$$\Omega_{33} = -Q_{22} - Z_2,$$

$$\Omega_{44} = -R_{22} - Z_2,$$

$$\Omega_{55} = -Z_1 - Q_{11} + Q_{22},$$

$$\Omega_{66} = -Z_3 - R_{11} + R_{22}. \tag{12}$$

Proof. Let us choose a Lyapunov-Krasovskii functional candidate as

$$V(x_t) = \sum_{i=1}^{4} V_i(x_t), \tag{13}$$

where

$$V_1(x_t) = \begin{bmatrix} x(t) \\ \int_{t-\tau_M}^{t-\tau_m} x(s)\,ds \\ \int_{t-(\tau_m/2)}^{t} x(s)\,ds \\ \int_{t-(\tau_M/2)}^{t} x(s)\,ds \end{bmatrix}^T P \begin{bmatrix} x(t) \\ \int_{t-\tau_M}^{t-\tau_m} x(s)\,ds \\ \int_{t-(\tau_m/2)}^{t} x(s)\,ds \\ \int_{t-(\tau_M/2)}^{t} x(s)\,ds \end{bmatrix},$$

$$V_2(x_t) = \int_{t-(\tau_m/2)}^{t} \begin{bmatrix} x(s) \\ x\left(s - \frac{\tau_m}{2}\right) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x\left(s - \frac{\tau_m}{2}\right) \end{bmatrix} ds + \int_{t-(\tau_M/2)}^{t} \begin{bmatrix} x(s) \\ x\left(s - \frac{\tau_M}{2}\right) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x\left(s - \frac{\tau_M}{2}\right) \end{bmatrix} ds,$$

$$V_3(x_t) = \frac{\tau_m}{2}\int_{-\tau_m/2}^{0}\int_{t+s}^{t} \dot{x}^T(u) Z_1 \dot{x}(u)\,du\,ds + (\tau_M - \tau_m)\int_{-\tau_M}^{-\tau_m}\int_{t+s}^{t} \dot{x}^T(u) Z_2 \dot{x}(u)\,du\,ds + \frac{\tau_M}{2}\int_{-\tau_M/2}^{0}\int_{t+s}^{t} \dot{x}^T(u) Z_3 \dot{x}(u)\,du\,ds,$$

$$V_4(x_t) = \frac{\tau_m^2}{8}\int_{t-(\tau_m/2)}^{t}\int_{s}^{t}\int_{u}^{t} \dot{x}^T(v) R_1 \dot{x}(v)\,dv\,du\,ds + \frac{(\tau_M - \tau_m)^2}{2}\int_{t-\tau_M}^{t-\tau_m}\int_{s}^{t}\int_{u}^{t} \dot{x}^T(v) R_2 \dot{x}(v)\,dv\,du\,ds + \frac{\tau_M^2}{8}\int_{t-(\tau_M/2)}^{t}\int_{s}^{t}\int_{u}^{t} \dot{x}^T(v) R_3 \dot{x}(v)\,dv\,du\,ds. \tag{14}$$

Define an extended-state vector as

$$\xi^T(x_t) \triangleq \left[x^T(t) \;\; x^T(t-d(t)) \;\; x^T(t-\tau_m) \;\; x^T(t-\tau_M) \;\; x^T\left(t-\frac{\tau_m}{2}\right) \;\; x^T\left(t-\frac{\tau_M}{2}\right) \;\; \int_{t-\tau_M}^{t-\tau_m} x^T(s)\,ds \;\; \int_{t-(\tau_m/2)}^{t} x^T(s)\,ds \;\; \int_{t-(\tau_M/2)}^{t} x^T(s)\,ds \right], \tag{15}$$

and then NCS (5) is simplified as

$$\dot{x}(x_t) = \psi\xi(x_t). \tag{16}$$

Calculating the derivative of $V_1(x_t)$ along the trajectory of NCS (5) yields

$$\dot{V}_1(x_t) = 2\begin{bmatrix} x(t) \\ \int_{t-\tau_M}^{t-\tau_m} x(s)\,ds \\ \int_{t-(\tau_m/2)}^{t} x(s)\,ds \\ \int_{t-(\tau_M/2)}^{t} x(s)\,ds \end{bmatrix}^T P \begin{bmatrix} \psi\xi(t) \\ x(t-\tau_m) - x(t-\tau_M) \\ x(t) - x\left(t-\frac{\tau_m}{2}\right) \\ x(t) - x\left(t-\frac{\tau_M}{2}\right) \end{bmatrix}. \tag{17}$$

It is easy to get

$$\begin{aligned}\dot{V}_2(x_t) = &\begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_m}{2}\right)\end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22}\end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_m}{2}\right)\end{bmatrix} \\ &- \begin{bmatrix} x\left(t-\frac{\tau_m}{2}\right) \\ x(t-\tau_m)\end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22}\end{bmatrix} \begin{bmatrix} x\left(t-\frac{\tau_m}{2}\right) \\ x(t-\tau_m)\end{bmatrix} \\ &+ \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_M}{2}\right)\end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22}\end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_M}{2}\right)\end{bmatrix} \\ &- \begin{bmatrix} x\left(t-\frac{\tau_M}{2}\right) \\ x(t-\tau_M)\end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22}\end{bmatrix} \begin{bmatrix} x\left(t-\frac{\tau_M}{2}\right) \\ x(t-\tau_M)\end{bmatrix}.\end{aligned} \tag{18}$$

The time derivative of $V_3(x_t)$ can be represented as

$$\begin{aligned}\dot{V}_3(x_t) = &\dot{x}^T(t)\left(\left(\frac{\tau_m^2}{4}\right)Z_1 + (\tau_M-\tau_m)^2 Z_2 + \left(\frac{\tau_M^2}{4}\right)Z_3\right) \\ &\times \dot{x}(t) - \frac{\tau_m}{2}\int_{-\tau_m/2}^{0} \dot{x}^T(s) Z_1 \dot{x}(s)\,ds - (\tau_M - \tau_m) \\ &\times \int_{-\tau_M}^{-\tau_m} \dot{x}^T(s) Z_2 \dot{x}(s)\,ds \\ &- \frac{\tau_M}{2}\int_{-\tau_M/2}^{0} \dot{x}^T(s) Z_3 \dot{x}(s)\,ds.\end{aligned} \tag{19}$$

By utilizing Lemma 1, we obtain

$$\begin{aligned}&-\frac{\tau_m}{2}\int_{t-(\tau_m/2)}^{t} \dot{x}^T(s) Z_1 \dot{x}(s)\,ds \\ &\leq \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_m}{2}\right)\end{bmatrix}^T \begin{bmatrix} -Z_1 & Z_1 \\ * & -Z_1\end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_m}{2}\right)\end{bmatrix}, \\ &-\frac{\tau_M}{2}\int_{t-(\tau_M/2)}^{t} \dot{x}^T(s) Z_3 \dot{x}(s)\,ds \\ &\leq \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_M}{2}\right)\end{bmatrix}^T \begin{bmatrix} -Z_3 & Z_3 \\ * & -Z_3\end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau_M}{2}\right)\end{bmatrix}.\end{aligned} \tag{20}$$

Define $\alpha = (d(t)-\tau_m)/(\tau_M-\tau_m)$, $\beta = (\tau_M - d(t))/(\tau_M - \tau_m)$; by the reciprocally convex combination in Lemma 3, the following inequality holds:

$$\begin{aligned}&-\begin{bmatrix} \sqrt{\frac{\beta}{\alpha}}\left(x(t-\tau_m) - x(t-d(t))\right) \\ -\sqrt{\frac{\alpha}{\beta}}\left(x(t-d(t)) - x(t-\tau_M)\right)\end{bmatrix}^T \begin{bmatrix} Z_2 & S_{12} \\ * & Z_2\end{bmatrix} \\ &\times \begin{bmatrix} \sqrt{\frac{\beta}{\alpha}}\left(x(t-\tau_m) - x(t-d(t))\right) \\ -\sqrt{\frac{\alpha}{\beta}}\left(x(t-d(t)) - x(t-\tau_M)\right)\end{bmatrix} < 0.\end{aligned} \tag{21}$$

Due to $\tau_m \leq d(t) \leq \tau_M$, according to Lemma 1 and inequalities (21), we have

$$\begin{aligned}&-(\tau_M-\tau_m)\int_{t-\tau_M}^{t-\tau_m} \dot{x}^T(s) Z_2 \dot{x}(s)\,ds \\ &= -(\tau_M-\tau_m)\int_{t-d(t)}^{t-\tau_m} \dot{x}^T(s) Z_2 \dot{x}(s)\,ds \\ &\quad -(\tau_M-\tau_m)\int_{t-\tau_M}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s)\,ds \\ &\leq -\frac{\tau_M-\tau_m}{d(t)-\tau_m}\begin{bmatrix} x(t-\tau_m) \\ x(t-d(t))\end{bmatrix}^T \begin{bmatrix} Z_2 & -Z_2 \\ * & Z_2\end{bmatrix} \begin{bmatrix} x(t-\tau_m) \\ x(t-d(t))\end{bmatrix} \\ &\quad -\frac{\tau_M-\tau_m}{\tau_M-d(t)}\begin{bmatrix} x(t-d(t)) \\ x(t-\tau_M)\end{bmatrix}^T \begin{bmatrix} Z_2 & -Z_2 \\ * & Z_2\end{bmatrix} \begin{bmatrix} x(t-d(t)) \\ x(t-\tau_M)\end{bmatrix}\end{aligned}$$

$$\le -\begin{bmatrix} x(t-\tau_m) - x(t-d(t)) \\ x(t-d(t)) - x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} Z_2 & S_{12} \\ * & Z_2 \end{bmatrix} \times \begin{bmatrix} x(t-\tau_m) - x(t-d(t)) \\ x(t-d(t)) - x(t-\tau_M) \end{bmatrix}$$
$$= \zeta^T(x_t) \begin{bmatrix} -2Z_2 + S_{12} + S_{12}^T & Z_2 - S_{12} & Z_2 - S_{12} \\ * & -Z_2 & S_{12} \\ * & * & -Z_2 \end{bmatrix} \zeta(x_t), \quad (22)$$

where

$$\zeta^T(x_t) = \left[x^T(t-d(t)) \;\; x^T(t-\tau_m) \;\; x^T(t-\tau_M)\right]. \quad (23)$$

Based on Lemma 2, an upper bound of $\dot{V}_4(x_t)$ can be estimated as

$$\begin{aligned}
\dot{V}_4(x_t) &= \frac{\tau_m^4}{64}\dot{x}^T(t)R_1\dot{x}(t) - \frac{\tau_m^2}{8} \times \int_{t-(\tau_m/2)}^{t}\int_s^t \dot{x}^T(u)R_1\dot{x}(u)\,du\,ds \\
&\quad + \frac{\tau_M^4}{64}\dot{x}^T(t)R_3\dot{x}(t) - \frac{\tau_M^2}{8} \times \int_{t-(\tau_M/2)}^{t}\int_s^t \dot{x}^T(u)R_3\dot{x}(u)\,du\,ds \\
&\quad + \frac{(\tau_M-\tau_m)^4}{4}\dot{x}^T(t)R_2\dot{x}(t) - \frac{(\tau_M-\tau_m)^2}{2} \times \int_{t-\tau_M}^{t-\tau_m}\int_s^t \dot{x}^T(u)R_2\dot{x}(u)\,du\,ds \\
&\le \frac{\tau_m^4}{64}\dot{x}^T(t)R_1\dot{x}(t) + \frac{\tau_M^4}{64}\dot{x}^T(t)R_3\dot{x}(t) \\
&\quad - \left(\frac{\tau_m}{2}x(t) - \int_{t-(\tau_m/2)}^{t}x(s)\,ds\right)^T \times R_1\left(\frac{\tau_m}{2}x(t) - \int_{t-(\tau_m/2)}^{t}x(s)\,ds\right) \\
&\quad - \left(\frac{\tau_M}{2}x(t) - \int_{t-(\tau_M/2)}^{t}x(s)\,ds\right)^T \times R_3\left(\frac{\tau_M}{2}x(t) - \int_{t-(\tau_M/2)}^{t}x(s)\,ds\right) \\
&\quad + \frac{(\tau_M-\tau_m)^4}{4}\dot{x}^T(t)R_2\dot{x}(t) \\
&\quad - \left((\tau_M-\tau_m)x(t) - \int_{t-\tau_M}^{t-\tau_m}x(s)\,ds\right)^T \times R_2\left((\tau_M-\tau_m)x(t) - \int_{t-\tau_M}^{t-\tau_m}x(s)\,ds\right).
\end{aligned} \quad (24)$$

From (16)–(24), we have

$$\begin{aligned}
\dot{V}(x_t) \le \xi^T(x_t)\Bigg(&\frac{\tau_m^4}{64}\Psi^T R_1\Psi + \frac{(\tau_M-\tau_m)^4}{4}\Psi^T R_2\Psi \\
&+ \frac{\tau_M^4}{64}\Psi^T R_3\Psi + \frac{\tau_m^2}{4}\Psi^T Z_1\Psi \\
&+ (\tau_M-\tau_m)^2\Psi^T Z_2\Psi \\
&+ \frac{\tau_M^2}{4}\Psi^T Z_3\Psi + \Theta_{22}\Bigg)\xi(x_t).
\end{aligned} \quad (25)$$

If $\Theta < 0$, then $\dot{V}(x_t)$ is negatively defined, based on the Lyapunov theory, we can conclude that NCS (5) is asymptotically stable. □

Remark 5. The robust stability criteria presented in Theorem 4 are for the nominal system. However, it is easy to further extend Theorem 4 to uncertain systems, where the systems matrices A and A_d contain parameter uncertainties.

Remark 6. To reduce the conservatism of stability criteria, the Lyapunov-Krasovskii functional contains some triple-integral terms in [12]. However, the new Lyapunov-Krasovskii functional in our paper which not only contains some triple-integral terms but also divides the lower and upper bounds of the delay into two equal segments, respectively, is proposed. The results will be less conservative.

Remark 7. Unlike [2], $-(\tau_M - \tau_m)\int_{t-\tau_M}^{t-\tau}\dot{x}^T(s)Z_2\dot{x}(s)ds$ is enlarged as $-(\tau_M - \tau)\int_{t-\tau_M}^{t-\tau}\dot{x}^T(s)Z_2\dot{x}(s)ds$ which may lead to conservation. In this paper, $-(\tau - \tau_m)\int_{t-\tau_M}^{t-\tau}\dot{x}^T(s)Z_2\dot{x}(s)ds$ is retained as well. Meanwhile, we deal with the integral terms of quadratic quantities by virtue of the reciprocally convex combination in Lemma 3; the obtained results are improved.

Remark 8. The relationship between the time-varying delay and its lower bound and upper bound is taken into account. As a result, fewer useful information is ignored in the derivative of the Lyapunov-Krasovskii functional.

Remark 9. The delay-departioning method in this paper reduces the conservativeness considerably, which arises from the LMI analysis of the interval $[\tau_m, \tau_M]$. But when the interval $[\tau_m, \tau_M]$ is reduced, that is, when $\tau_m \to \tau_M$, the benefits of analysis method are shortened. The new strategy is to further make use of the information of the delay lower bound through departioning of the delay interval $[0, \tau_m]$ into N equidistant subintervals.

4. Numerical Example

Example 10. Consider the following NCS [10]:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t). \quad (26)$$

Table 1: Admissible upper bound τ_M for various τ_m.

τ_m	0.00	0.01	0.05	0.10
Jiang and Han [10]	0.941	0.942	0.948	0.952
Zhang et al. [8]	1.003	1.024	1.026	1.027
Theorem 4	1.113	1.114	1.116	1.119

Table 2: Admissible upper bound τ_M for various τ_m.

τ_m	1	2	3	4
Shao [11]	1.617	2.480	3.389	4.325
Sun et al. [12]	1.620	2.488	3.403	4.342
Theorem 4	1.765	2.606	3.502	4.429

Assume that the state feedback gain matrix $K = [-3.75 \quad -11.5]$ when we do not consider the lower bound of the delay, that is, $\tau_m = 0.00$, by applying Theorem 4, the maximum upper bound of delay obtained is $\tau_M = 1.113$, while in [10], $\tau_M = 0.941$, and the maximum allowable value is $\tau_M = 1.003$ in [8]. A more detailed comparison for different values of τ_m is provided in Table 1. As shown in the table, it can be seen that our results are less conservative than the ones in [8, 10].

Example 11. Consider the NCS (5) with the following parameters [11]:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \qquad A_d = BK = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}. \qquad (27)$$

Table 2 lists the maximum allowable upper bound of τ_M with respect to different conditions of τ_m along with some existing results from the literature. From Table 2, we can conclude that the criterion derived in this paper presents superior results.

5. Conclusion

The problem of robust stability is addressed for a class of NCSs with network-induced delay and packet dropout. Improved and simplified stability criterion is established without involving any model transformation or free weighting matrices. The simulation results indicate that the criterion derived in this paper can exhibit better performance compared to that in the existing literature.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by the National Nature Science Foundation of China (Grant no. 61203136 and Grant no. 61304064), the Natural Science Foundation Hunan Province of China (Grant no. 11JJ2038), and the Construct Program for the Key Discipline of electrical engineering in Hunan Province.

References

[1] G. C. Walsh, H. Ye, and L. G. Bushnell, "Stability analysis of networked control systems," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 3, pp. 438–446, 2002.

[2] X. F. Jiang, Q.-L. Han, S. R. Liu, and A. K. Xue, "A new H_∞ stabilization criterion for networked control systems," *IEEE Transactions on Automatic Control*, vol. 53, no. 4, pp. 1025–1032, 2008.

[3] H. J. Gao, T. W. Chen, and J. Lam, "A new delay system approach to network-based control," *Automatica*, vol. 44, no. 1, pp. 39–52, 2008.

[4] H. G. Zhang, G. T. Hui, and Y. C. Wang, "Stabilization of networked control systems with piecewise constant generalized sampled-data hold function," *International Journal of Innovative Computing, Information & Control*, vol. 9, no. 3, pp. 1159–1170, 2013.

[5] M. Yu, L. Wang, T. G. Chu, and F. Hao, "Stabilization of networked control systems with data packet dropout and transmission delays: continuous-time case," *European Journal of Control*, vol. 11, no. 1, pp. 40–49, 2005.

[6] H. Bentez-Prez and A. Bentez-Prez, "Networked control systems design considering scheduling restrictions and local faults using local state estimation," *International Journal of Innovative Computing, Information & Control*, vol. 9, no. 8, pp. 3225–3239, 2013.

[7] D. Yue, Q.-L. Han, and J. Lam, "Network-based robust H_∞ control of systems with uncertainty," *Automatica*, vol. 41, no. 6, pp. 999–1007, 2005.

[8] X. X. Zhang, X. Jiang, and Q.-L. Han, "An improved stability criterion of networked control systems," in *Proceedings of the American Control Conference (ACC '10)*, pp. 586–589, July 2010.

[9] C. Peng, D. Yue, E. Tian, and Z. Gu, "A delay distribution based stability analysis and synthesis approach for networked control systems," *Journal of the Franklin Institute*, vol. 346, no. 4, pp. 349–365, 2009.

[10] X. Jiang and Q.-L. Han, "Delay-dependent robust stability for uncertain linear systems with interval time-varying delay," *Automatica*, vol. 42, no. 6, pp. 1059–1065, 2006.

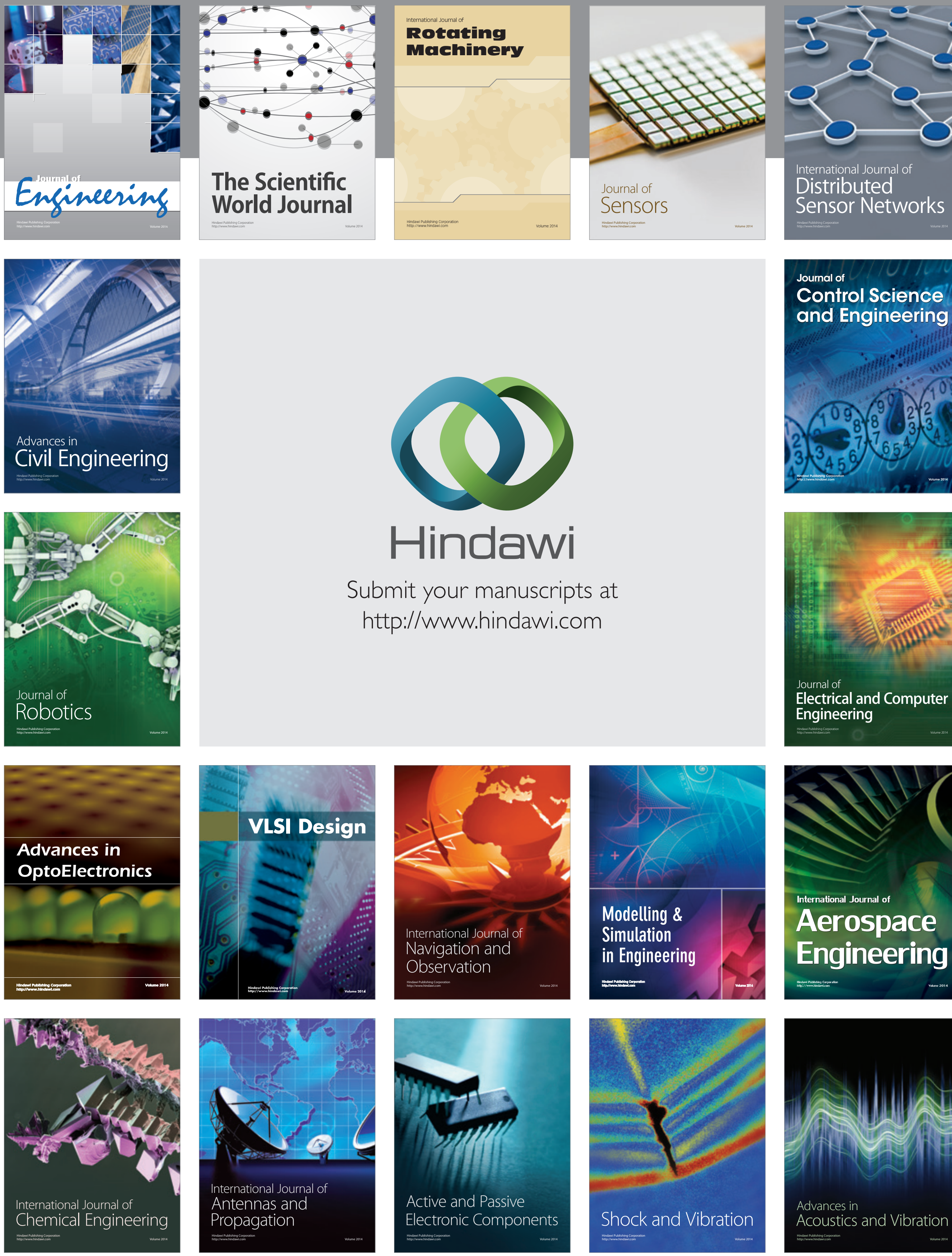
[11] H. Shao, "New delay-dependent stability criteria for systems with interval delay," *Automatica*, vol. 45, no. 3, pp. 744–749, 2009.

[12] J. Sun, G. P. Liu, J. Chen, and D. Rees, "Improved delay-range-dependent stability criteria for linear systems with time-varying delays," *Automatica*, vol. 46, no. 2, pp. 466–470, 2010.

[13] K. Gu, "An integral inequality in the stability problem of time-delay systems," in *Proceedings of the 39th IEEE Confernce on Decision and Control*, pp. 2805–2810, December 2000.

[14] J. Sun, G. P. Liu, and J. Chen, "Delay-dependent stability and stabilization of neutral time-delay systems," *International Journal of Robust and Nonlinear Control*, vol. 19, no. 12, pp. 1364–1375, 2009.

[15] P. G. Park, J. W. Ko, and C. K. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.


Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
International Journal of Distributed Sensor Networks
Advances in Civil Engineering
Journal of Control Science and Engineering
Hindawi
Submit your manuscripts at
http://www.hindawi.com
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration