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Research Article

Existence and Global Stability of a Periodic Solution for Discrete-Time Cellular Neural Networks

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A novel sufficient condition is developed to obtain the discrete-time analogues of cellular neural network (CNN) with periodic coefficients in the three-dimensional space. Existence and global stability of a periodic solution for the discrete-time cellular neural network (DT-CNN) are analysed by utilizing continuation theorem of coincidence degree theory and Lyapunov stability theory, respectively. In addition, an illustrative numerical example is presented to verify the effectiveness of the proposed results.

1. Introduction

Cellular neural networks (CNNs) are the basis of both discrete-time cellular neural networks (DT-CNNs) [1] and the cellular neural networks universal machine (CNNs-UM). The dynamical behaviour of Chua and Yang cellular neural network (CY-CNN) is given by the state equation

$$C\frac{\mathrm{d}x_{ij}}{\mathrm{d}t} = -\frac{1}{R}x_{ij} + \sum_{C(k,l)\in Nr(ij)} A_{kl}y_{kl} + \sum_{C(k,l)\in Nr(ij)} B_{kl}u_{kl} + I_{ij},$$
$$y_{ij} = f(x_{ij}) = \frac{1}{2}\left(\left|x_{ij}+1\right| - \left|x_{ij}-1\right|\right), \quad i = 1,\dots,m,\ j = 1,\dots,n, \tag{1.1}$$

where I, u, y, and x denotes input bias, input, output, and state variable of each cell, respectively. $N_r(ij)$ is the t-neighbourhood of cell $C(k,l)$ as $N_r(ij) = \{C(k,l) \mid \max\{|k-i|, |l-j|\} \le r\}$, i and j denote the position of the cell in the network, k and l denote the position of the neighbour cell relative to the cell in consideration. B is the nonlinear weights template matrices for input feedback and A is the corresponding template matrices for the outputs of neighbour cells. Non-linearity means that templates can change over time.

A large number of cellular neural networks (CNNs) models have appeared in the literature [2–4], and these models differ in cell complexity, parameterization, cell dynamics, and network topology. Various generalizations of cellular neural networks have attracted attention of scientific community due to their promising potential for tasks of classification, associative memory, parallel computation [5–9], pattern recognition, computer vision, and solving any optimization problem [10–13]. Such applications rely on the existence of equilibrium points and the qualitative properties of cellular neural networks.

Discrete-time cellular neural networks (DT-CNNs) have been studied both in theory and applications. Previous results introduced many properties of DT-CNN in the two dimensional plane. For instance, [14] has been successfully applied to investigate the discrete-time analogues of cellular neural network (CNN) with variable coefficients in the two-dimensional plane. However, three-dimensional structure is more accurate, specific, and closer to real structures of CNN. Based on the above discussion, this paper proposes some effective results of DT-CNN in the three-dimensional space.

Motivated by the constructing of continuous system (1.1), the discrete analogue of the system (1.1) is considered as follows:

$$X_{ij}[n+1] = e^{-h}X_{ij}[n] + \left(1-e^{-h}\right)\left(\sum_{C(k,l)\in Nr(ij)} A_{kl}Y_{kl}[n] + \sum_{C(k,l)\in Nr(ij)} B_{kl}U_{kl} + I_{ij}\right), \quad \forall n \in Z_0^+. \tag{1.2}$$

For any $h > 0$, the discrete-time analogues (1.2) converge to the continuous-time system (1.1) will be provided. Without loss of generality, (1.2) can be substituted in the DT-CNNs model:

$$\begin{aligned} x_{ij}[n+1] &= e^{-h}x_{ij}[n] + \left(1-e^{-h}\right)\sum_{C\left(k_{ijh}l_{ijh}\right)\in Nr\left(k_{ijh}l_{ijh}\right)}\left(A_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + B_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right) \\ &\quad + I_{ij}[n], \quad \forall n \in Z_0^+, \\ x_{ij}[n] &= \varphi_{ij}[n], \quad \forall n \in Z_0^- = \{0,-1,-2,\ldots\}. \end{aligned} \tag{1.3}$$

Then, the spatial structure with respect to (1.3) is shown in Figure 1, where $r = \max_{C(k_{ijh}l_{ijh})\in N_r(ijh)}(|x_{k_{ijh}l_{ijh}} - \partial\Omega|)$, $\Omega \triangleq \{x \in \mathcal{X} \subset N_r(ijh), \|x\| < \Theta\}$, $N_r(ijh)$ is the r-neighbourhood of a cell $C(k,l) = C_{k_{ijh}l_{ijh}}$, and Θ will be denoted by the proof of Theorem 3.1 in Section 3.

The rest of the paper is organized as follows: in Section 2, system description and preliminaries are developed in detail and some definitions, assumptions, and lemmas are stated. Section 3 gives sufficient conditions for a periodic solution for DT-CNN in three-dimensional space by utilizing continuation theorem of coincidence degree theory. Section 4 proposes global stability of a periodic solution for the DT-CNN. A numerical simulation is given to show correctness of our analysis in Section 5 and concluded in Section 6.

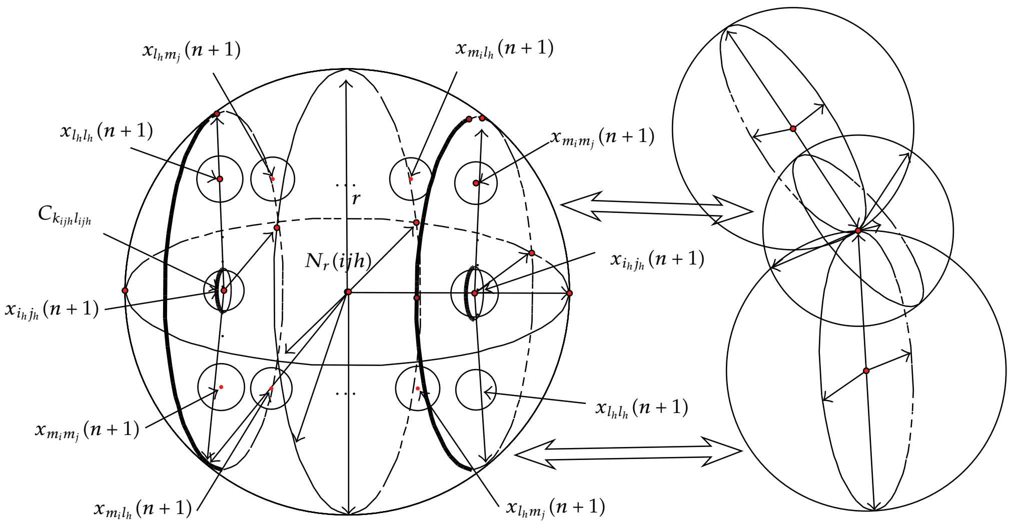

Figure 1: Spatial structure with respect to (1.3).

2. System Preliminaries and Description

Consider the following model which is equivalent to the (1.3):

$$\begin{aligned}
x_{ij}[n+1] &= \alpha(h)x_{ij}[n] + \beta(h)\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left(A_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + B_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right) + I_{ij}[n], \quad \forall n \in Z_0^+, \\
x_{ij}[n] &= \varphi_{ij}[n], \quad \forall n \in Z_0^- = \{0,-1,-2,\ldots\}, \\
k_{ijh} &= k(i,j,h) \in N^+, \quad l_{ijh} = l(i,j,h) \in N^+, \quad i = 1,2,\ldots,m_i, \\
& j = 1,2,\ldots,m_j, \quad k = 1,2,\ldots,m_k,
\end{aligned} \tag{2.1}$$

where $\alpha(h) = e^{-h}$, $\beta(h) = 1 - \alpha(h)$, for all $h > 0$, $\varphi_{ij}(n)$ are $N(h)$-periodic sequences, that is, $\varphi_{ij}(n) = \varphi_{ij}(n + N(h))$.

Throughout the paper, the following definitions and lemmas will be introduced.

Definition 2.1 (Fredholm operator). Let $\mathcal{X}$ and $\mathcal{Y}$ be a Banach space, an operator L is called Fredholm operator if L is a bounded linear operator between $\mathcal{X}$ and $\mathcal{Y}$ whose kernel and cokernel are finite-dimensional and whose range is closed. Equivalently, an operator $L : \mathcal{X} \to \mathcal{Y}$ is Fredholm if it is invertible modulo compact operator, that is, if there exists a bounded linear operator $S : \mathcal{Y} \to \mathcal{X}$ such that $Id_{\mathcal{X}} - SL$, $Id_{\mathcal{Y}} - LS$ are compact operators on $\mathcal{X}$ and $\mathcal{Y}$, respectively, where $Id_{\mathcal{X}}$ and $Id_{\mathcal{Y}}$ are the identity operator.

Definition 2.2 (L-compact). An operator N will be called L-compact on $\overline{\Omega}$ if the open bounded set $QN(\overline{\Omega})$ is bounded and $K_p(I-Q)N : \overline{\Omega} \to \mathcal{X}$ is compact, where K_p is the inverse operator of N. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

The index of a Fredholm operator is $\operatorname{ind} L = \dim \operatorname{Ker} L$–codim $\operatorname{Im} L$, then operator L will be called a Fredholm operator of index zero if $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in $\mathcal{Y}$. Then a following abstract equation in Banach space $\mathcal{X}$ is defined by

$$Lx = \lambda N x. \tag{2.2}$$

Let $L : \operatorname{Dom} L \subset \mathcal{X} \to \mathcal{Y}$ be linear operator, and $N : \mathcal{X} \to \mathcal{Y}$ be a continuous operator. If L is a Fredholm operator of index zero, there must exist continuous projectors $P : \mathcal{X} \to \mathcal{X}$ and $Q : \mathcal{Y} \to \mathcal{Y}$, such that:

$$\begin{gathered} P : \mathcal{X} \cap \operatorname{Dom} L \longrightarrow \operatorname{Ker} L, \qquad \operatorname{Ker} L = \operatorname{Im} P, \\ Q : \mathcal{Y} \longrightarrow \mathcal{Y} / \operatorname{Im} L, \qquad \operatorname{Im} L = \operatorname{Ker} Q. \end{gathered} \tag{2.3}$$

In other words, $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : \operatorname{Dom} L \cap \operatorname{Ker} P \to \operatorname{Im} L$ is invertible, and the inverse of the operator L is denoted by K_p.

Lemma 2.3 (Gaines and Mawhin [15]). *Let $\mathcal{X}$ be a Banach space, L be a Fredholm operator of index zero, and let $N : \overline{\Omega} \to \mathcal{X}$ be L-compact on $\overline{\Omega}$, $\Omega \subset \mathcal{X}$, where Ω is an open bounded set, suppose:*

$$\begin{aligned} &(\text{i})\ Lx \neq \lambda N x, \quad \text{for any } (x, \lambda) \in (\partial\Omega \cap \operatorname{Dom} L) \times (0,1); \\ &(\text{ii})\ QNx \neq 0, \quad \text{for any } x \in \partial\Omega \cap \operatorname{Ker} L; \\ &(\text{iii})\ \deg(JQN, \Omega \cap \operatorname{Ker} L, 0) \neq 0. \end{aligned} \tag{2.4}$$

Then $Lx = Nx$ has at least one solution in $\operatorname{Dom} L \cap \overline{\Omega}$.

Lemma 2.4. *If a and b are some certain nonnegative vectors, then there exists a positive constant β, such that $ab \le (\beta/2)a^2 + (1/2\beta)b^2$.*

Proof. Assuming a and b are some certain non-negative vectors, β is a positive constant, then

$$2ab = 2a\sqrt{\beta}\left(\sqrt{\beta}\right)^{-1} b \le \beta a^2 + \frac{1}{\beta} b^2 \Longrightarrow ab \le \frac{\beta}{2} a^2 + \frac{1}{2\beta} b^2. \tag{2.5}$$

Thus, the proof of Lemma 2.4 is completed. □

Assumption 2.5. $A_{k_{ijh} l_{ijh}}, B_{k_{ijh} l_{ijh}}, I_{ij}$ ($i = 1, \ldots, m_i$, $j = 1, \ldots, m_j$, $h = 1, \ldots, m_h$) are N-periodic sequence of Z_0^+. For the sake of convenience, we use the following notations:

$\|x\|_{\mathbb{R}^2}^2 = \sum_{j=1}^{m_j}\sum_{i=1}^{m_i}\max_{n\in I_N}|x_{ij}[n]|^2$. For each operator $P : N_r(ijh) \to N_r(ijh)$ and any $s = s(u,v,w)$, $t = t(i,j,h) \in N_r(ijh)$, such that:

$$\begin{aligned} |P(s)-P(t)| &\le \left(|m_i||u-i|^2 + |m_j|\left|v-j\right|^2 + |m_h||w-h|^2\right)^{1/2} \\ &\le 2m_{ijh}\times\max\{\text{dist}(s,o_{ijh}),\text{dist}(t,o_{ijh})\} \le 2m_{ijh}r, \end{aligned} \tag{2.6}$$

where o_{ijh} is the spherical centre of $N_r(ijh)$ with a radius length r, $m_{ijh} = \max\{|m_i|,|m_j|,|m_h|\}$, then it is easy to obtain: $|P(s)-P(t)| \le \|P\||(s-t)| \le 2m_{ijh}r < +\infty$.

Assumption 2.6. There is a positive constant $C_{y_{kl}}$, such that $|y_{k_{ijh}l_{ijh}}[x_1] - y_{k_{ijh}l_{ijh}}[x_2]| \le C_{y_{kl}}|x_1 - x_2|$, for all $x_1 \ne x_2 \in R$.

3. Existence of a Periodic Solution with respect to (2.1)

In many cases, many proposed results are not ideal and therefore it is necessary to formulate a novel and effective result for DT-CNN in the three-dimensional space. Can we obtain the result about the existence and stability of a periodic solution for DT-CNN in three-dimensional space? This is the topic we wish to address in this paper. The aim of the present work is to develop a strategy to determine the existence and global stability of a periodic solution with respect to (2.1) in the three-dimensional space. Consequently, we processed with the following result.

Theorem 3.1. *Suppose that Assumptions 2.5 and 2.6 hold, and the following condition holds:*

$$\sum_{h=1}^{m_h}\left|\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right|^2 - \gamma_{ij} > 0, \tag{3.1}$$

where $\gamma_{ij} = \sum_{h=1}^{m_h}|\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}|^2 - \widetilde{I}_{ij}^2$, $i = 1,\ldots,m_i$, $j = 1,\ldots,m_j$, $n \in I_N = \{0,1,\ldots,N-1\}$, *then* (2.1) *has at least one N-periodic solution.*

Proof. In this section, by means of using Mawhin's continuation theorem of coincidence degree theory, we will study the existence of at least one periodic solution with respect to (2.1), for convenience, some following notations will be used:

$$I_N = \{0,1,\ldots,N-1\}, \qquad \underline{f} = \min_{n\in I_N}\{|f(n)|\}, \qquad \overline{f} = \max_{n\in I_N}\{|f(n)|\}, \tag{3.2}$$

where $f(n)$ is any function. Let $\mathcal{X} = \mathcal{Y} = x[n] = \{(x_{11}[n],\ldots,x_{1m_j}[n],\ldots,x_{m_i1}[n],\ldots, x_{m_im_j}[n])^T : x_{ij}[n] = x_{ij}[n+N] \in R_+^{m_im_j}, N \in N^+, i = 1,\ldots,m_i, j = 1,\ldots,m_j\}$, and $y^N \subset \mathcal{X} = \mathcal{Y}$ be the subspace of all N-periodic sequence; equip it with the norm $\|x\|_{\mathbb{R}^2}^2 = \sum_{j=1}^{m_j}\sum_{i=1}^{m_i}\max_{n\in I_N}|x_{ij}[n]|^2$. For any $\varepsilon > 0$, $\{x_{i_m}\}_{i_m=1}^{m_i} \subset N_r(ijh)$, there exists $N(\varepsilon) > 0$ and $\varepsilon > 0$, such that $i_m > N(\varepsilon) \Rightarrow d(x_{i_m}, x_{i_m+1}) \le d(x_{i_m}, o_{ijh}) + d(x_{i_m+1}, o_{ijh}) < \varepsilon$. Thus, $\{x_{i_m}\}_{i_m=1}^{m_i}$ is a Cauchy sequence in $N_r(ijh)$ and o_{ijh} is the spherical centre of $N_r(ijh)$, $d(x,y) = \max\{|x-y| : x \in X\}$. By utilizing the meaning of $N_r(ijh)$ and Bolzano-Weierstrass theorem (Each bounded sequence in R^n has a convergent subsequence, here $R^{m_im_j} \subset R^n$, $\dim(R^{m_im_j}) < +\infty$), it is easy to know that $(\mathcal{X}, \|\cdot\|)$ is a Banach space.

Set

$$Nx = \left(\widehat{x}_{11}[n], \dots, \widehat{x}_{1m_j}[n], \widehat{x}_{21}[n], \dots, \widehat{x}_{2m_j}[n], \widehat{x}_{m_i1}[n], \dots, \widehat{x}_{m_im_j}[n]\right)^T,$$
$$Lx = \left(\Delta x_{11}[n], \dots, \Delta x_{1m_j}[n], \Delta x_{21}[n], \dots, \Delta x_{2m_j}[n], \Delta x_{m_i1}[n], \dots, \Delta x_{m_im_j}[n]\right)^T, \tag{3.3}$$
$$Px = Qx = \left(\widetilde{x}_{11}[n], \dots, \widetilde{x}_{1m_j}[n], \widetilde{x}_{21}[n], \dots, \widetilde{x}_{2m_j}[n], \widetilde{x}_{m_i1}[n], \dots, \widetilde{x}_{m_im_j}[n]\right)^T,$$

that is

$$x[n] = \left(x_{11}[n], \dots, x_{1m_j}[n], \dots, x_{m_i1}[n], \dots, x_{m_im_j}[n]\right)^T \in \mathcal{X},$$
$$\operatorname{Ker} L = \left\{x = \{x[n]\} \in y^N \subset \mathcal{X} : x[n] = c \in R^{m_im_j}, n \in I_N\right\}, \tag{3.4}$$
$$\operatorname{Im} L = \left\{x = \{x[n]\} \in y^N \subset \mathcal{X} : \sum_{n=0}^{N-1} x_{ij}[n] = 0, i = 1, \dots, m_i, j = 1, \dots, m_j, n \in I_N\right\},$$

where $\Delta x_{ij}[n] = x_{ij}[n+1] - x_{ij}[n] = -\beta(h)[x_{ij}[n] - \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}(A_{k_{ijh}l_{ijh}} y_{k_{ijh}l_{ijh}}[n] + B_{k_{ijh}l_{ijh}} u_{k_{ijh}l_{ijh}})] + I_{ij}[n], \widehat{x}_{ij}[n] = -\beta(h)[x_{ij}[n] - \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}(A_{k_{ijh}l_{ijh}} y_{k_{ijh}l_{ijh}}[n] + B_{k_{ijh}l_{ijh}} u_{k_{ijh}l_{ijh}})] + I_{ij}[n], \widetilde{x} = (1/N)\sum_{n=0}^{N-1} x[n]$, for all $n \in I_N$. Then we will learn that $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$, it is easy to prove that L is a bounded linear operator, P and Q are two continuous operators such that $\operatorname{Ker} L = \operatorname{Im} P$, $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$, and $K_p|_{\operatorname{Im} L} : \operatorname{Im} L \to \operatorname{Dom} L \cap \operatorname{Ker} P$, that is

$$K_p(x[n]) = \sum_{s=0}^{N-1} x(s) - \frac{1}{N}\sum_{s=1}^{N}\sum_{t=1}^{s-1} x(t), \quad n \in I_N, \tag{3.5}$$

that is

$$QNx = \frac{\beta(h)}{1+\beta(h)} \times \begin{pmatrix} \sum_{h=1}^{m_h}\left(\widetilde{A}_{k_{11h}l_{11h}} y_{k_{11h}l_{11h}}[n] + \widetilde{B}_{k_{11h}l_{11h}} u_{k_{11h}l_{11h}}\right) + \left(1 + (\beta(h))^{-1}\right)\widetilde{I}_{11} \\ \vdots \\ \sum_{h=1}^{m_h}\left(\widetilde{A}_{k_{1m_jh}l_{1m_jh}} y_{k_{1m_jh}l_{1m_jh}}[n] + \widetilde{B}_{k_{1m_jh}l_{1m_jh}} u_{k_{1m_jh}l_{1m_jh}}\right) + \left(1 + (\beta(h))^{-1}\right)\widetilde{I}_{1m_j} \\ \vdots \\ \sum_{h=1}^{m_h}\left(\widetilde{A}_{k_{m_i1h}l_{m_i1h}} y_{k_{m_i1h}l_{m_i1h}}[n] + \widetilde{B}_{k_{m_i1h}l_{m_i1h}} u_{k_{m_i1h}l_{m_i1h}}\right) + \left(1 + (\beta(h))^{-1}\right)\widetilde{I}_{m_i1} \\ \vdots \\ \sum_{h=1}^{m_h}\left(\widetilde{A}_{k_{m_im_jh}l_{m_im_jh}} y_{k_{m_im_jh}l_{m_im_jh}}[n] + \widetilde{B}_{k_{m_im_jh}l_{m_im_jh}} u_{k_{m_im_jh}l_{m_im_jh}}\right) + \left(1 + (\beta(h))^{-1}\right)\widetilde{I}_{m_im_j} \end{pmatrix}_{n \in I_N},$$

$$
\begin{aligned}
&K_p(I-Q)Nx \\
&\quad = -\beta(h) \\
&\quad \times \left(\begin{array}{l}
\delta \sum_{n=0}^{N-1} \left(\left[x_{11}[n] - \sum_{h=1}^{m_h} \left(A_{k_{11h}l_{11h}} y_{k_{11h}l_{11h}}[n] + B_{k_{11h}l_{11h}} u_{k_{11h}l_{11h}} \right) \right] + (\beta(h))^{-1} I_{11}[n] \right) \\
-\frac{1}{N} \sum_{s=1}^{N} \sum_{t=1}^{s-1} \left(\left[x_{11}[t] - \sum_{h=1}^{m_h} \left(A_{k_{11h}l_{11h}} y_{k_{11h}l_{11h}}[t] + B_{k_{11h}l_{11h}} u_{k_{11h}l_{11h}} \right) \right] + (\beta(h))^{-1} I_{11}[n] \right) \\
\vdots \\
\delta \sum_{n=0}^{N-1} \left(\left[x_{m_i m_j}[n] - \sum_{h=1}^{m_h} \left(A_{k_{m_i m_j h} l_{m_i m_j h}} y_{k_{m_i m_j h} l_{m_i m_j h}}[n] + B_{k_{m_i m_j h} l_{m_i m_j h}} u_{k_{m_i m_j h} l_{m_i m_j h}} \right) \right] \right. \\
\qquad \left. + (\beta(h))^{-1} I_{k_{m_i m_j h} l_{m_i m_j h}}[n] \right) \\
-\frac{1}{N} \sum_{s=1}^{N} \sum_{t=1}^{s-1} \left(\left[x_{m_i m_j}[t] - \sum_{h=1}^{m_h} \left(A_{k_{m_i m_j h} l_{m_i m_j h}} y_{k_{m_i m_j h} l_{m_i m_j h}}[t] + B_{k_{m_i m_j h} l_{m_i m_j h}} u_{k_{m_i m_j h} l_{m_i m_j h}} \right) \right] \right. \\
\qquad \left. + (\beta(h))^{-1} I_{k_{m_i m_j h} l_{m_i m_j h}}[n] \right)
\end{array} \right)_{n \in I_N},
\end{aligned}
\tag{3.6}
$$

where $\delta(\beta(h), n, N)$ is a constant, which is only depended on variables h, n, and N.

Obviously, employing the Lebesgue's convergence theorem, we can easily learn that $QN(\overline{\Omega})$ is bounded, $K_p(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega \subset \mathcal{X} \subset N_r(ijh)$ by using Ascoli-Arzela's theorem (A subset $\mathcal{F}$ of $C(\mathcal{X})$ is compact if and only if it is closed, bounded and equi-continuous). Thus, N is L-compact on a closed set $\overline{\Omega}$ with any open bounded set $\Omega \subset \mathcal{X} \subset N_r(ijh)$.

Suppose that $x[n] = (x_{11}[n], \ldots, x_{1m_j}[n], \ldots, x_{m_i 1}[n], \ldots, x_{m_i m_j}[n])^T \in \mathcal{X}$ is a solution with respect to (2.1), for certain $\lambda \in (0,1)$. Then the following equation can be derived by (2.2):

$$
\begin{aligned}
\Delta x_{ij}[n] &= x_{ij}[n+1] - x_{ij}[n] \\
&= -\lambda \left\{ \beta(h) \left[x_{ij}[n] - \sum_{h=1}^{m_h} \sum_{j=1}^{m_j} \left(A_{k_{ijh}l_{ijh}} y_{k_{ijh}l_{ijh}}[n] + B_{k_{ijh}l_{ijh}} u_{k_{ijh}l_{ijh}} \right) \right] + I_{ij}[n] \right\}, \quad \forall n \in I_N.
\end{aligned}
\tag{3.7}
$$

Then, the following results can be derived by utilizing (3.7):

$$
\begin{aligned}
\max_{n \in I_N} \left| x_{ij}[n] \right| &= \max_{n \in I_N} \left| x_{ij}[n+1] \right| \\
&\leq \max_{n \in I_N} \left[(1 - \lambda\beta(h)) \left| x_{ij}[n] \right| + \lambda\beta(h) \right. \\
&\qquad \left. \times \sum_{h=1}^{m_h} \sum_{j=1}^{m_j} \left(\left| A_{k_{ijh}l_{ijh}} y_{k_{ijh}l_{ijh}}[n] \right| + \left| B_{k_{ijh}l_{ijh}} u_{k_{ijh}l_{ijh}} \right| \right) + \left| \lambda I_{ij}[n] \right| \right]
\end{aligned}
$$

$$\leq \max_{n\in I_N}(1-\lambda\beta(h))^n\left|\varphi_{ij}[0]\right| + (\lambda(1-\alpha(h)))^n \times \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left(\left|\overline{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[0]\right|\right) + \lambda^n\overline{I}_{ij} + \sigma_{ij}$$
$$\leq \max_{n\in I_N}\left\{\left|\varphi_{ij}[0]\right| + (1-\alpha(h))^n\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\overline{A}_{k_{ijh}l_{ijh}}\right\} + \sigma_{ij} < +\infty, \tag{3.8}$$

where $\sigma_{ij} = \max_{n\in I_N}(1-\alpha(h))\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}|\overline{B}_{k_{ijh}l_{ijh}}\overline{u}_{k_{ijh}l_{ijh}}| + \overline{I}_{ij}$, for all $n \in I_N$. Therefore, the solution with respect to (2.1) is bounded for certain $\lambda \in (0,1)$. In other words,

$$\max_{n\in I_N}\left|x_{ij}[n]\right| \leq \max_{n\in I_N}\left\{\left|\varphi_{ij}[0]\right| + (1-\alpha(h))^n\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left(\left|\overline{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[0]\right|\right) + (1-\alpha(h))\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left|\overline{B}_{k_{ijh}l_{ijh}}\overline{u}_{k_{ijh}l_{ijh}}\right| + \overline{I}_{ij}\right\} \triangleq \Theta_{ij}. \tag{3.9}$$

Then the open bounded set Ω is presented as follows:

$$\Omega \triangleq \left\{x \in \mathcal{X} \subset N_r(ijh), \|x\| < \sum_{i=1}^{m_i}\sum_{j=1}^{m_j}\Theta_{ij}\right\}. \tag{3.10}$$

Thus $Lx \neq \lambda Nx$ for any $(x,\lambda) \in (\partial\Omega \cap \operatorname{Dom} L) \times (0,1)$, the Ω satisfies condition (i) in Lemma 2.3.

In Figure 2, the nonlinear weights template matrices B and the boundary of Ω are shown, respectively. Then for any two dimensional plane of any spherical neighbourhood is denoted. Thus, for any $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap R^{m_i m_j}$, $\operatorname{Ker} L = \{x = \{x[n]\} \in y^N \subset \mathcal{X} : x[n] = c \in R^{m_i m_j}, n \in I_N\}$, it is easy to learn that x is a constant vector in $R^{m_i m_j}$ with $\|x\| = \Theta$; Thus, we have

$$QNx = \left(QNx_{11}, \ldots, QNx_{1m_j}, \ldots, QNx_{m_i 1}, \ldots, QNx_{m_i m_j}\right)^T, \tag{3.11}$$

where $QNx_{ij}[n] = (\beta(h)/(1+\beta(h)))\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}(\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + \widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}) + \widetilde{I}_{ij}[n]$, for all $n \in I_N$. Furthermore, we can calculate the bound of QNx as follows:

$$\|QNx\|^2 = \sum_{j=1}^{m_j}\sum_{i=1}^{m_i}\max_{n\in I_N}\left|x_{ij}[n]\right|^2$$
$$= \sum_{j=1}^{m_j}\sum_{i=1}^{m_i}\max_{n\in I_N}\left|\frac{\beta(h)}{1+\beta(h)}\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left(\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + \widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right) + \widetilde{I}_{ij}[n]\right|^2$$

$$\geq \sum_{j=1}^{m_j}\sum_{i=1}^{m_i}\left|\frac{\beta(h)}{1+\beta(h)}\right|^2 \times \max_{n\in I_N}\left\{\sum_{h=1}^{m_h}\left|\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right|^2 + \widetilde{I}_{ij}^2[n] - \sum_{h=1}^{m_h}\left|\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}\right|^2\right\}$$
$$\geq \sum_{j=1}^{m_j}\sum_{i=1}^{m_i}\left(\max_{n\in I_N}\left(\sum_{h=1}^{m_h}\left|\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right|^2 - \gamma_{ij}\right)\right) > 0, \tag{3.12}$$

where $\gamma_{ij} = \sum_{h=1}^{m_h}|\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}|^2 - \widetilde{I}_{ij}^2$, $i = 1,\dots,m_i$, $j = 1,\dots,m_j$, $n \in I_N$. Thus for any $x = \partial\Omega \cap \operatorname{Ker} L$, $QNx \neq 0$, this proves the condition (ii) in Lemma 2.3.

In order to prove the condition (iii) is satisfied with respect to (2.1), we only need to prove that $\deg(JQN, \Omega\cap\operatorname{Ker} L, 0) \neq 0$. Define $\Phi : \operatorname{Dom} L \to \mathcal{X}$ by

$$\Phi\left(x_{11},\dots,x_{1m_j},\dots,x_{m_i1},\dots,x_{m_im_j}\right)^T = \left(\widetilde{x}_{11},\dots,\widetilde{x}_{1m_j},\dots,\widetilde{x}_{m_i1},\dots,\widetilde{x}_{m_im_j}\right)^T, \tag{3.13}$$

where $\widetilde{x}_{ij}[n] = (\beta(h)/(1+\beta(h)))\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}(\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + \widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}) + \widetilde{I}_{ij}[n]$, for all $n \in I_N$.

Now we will prove that $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap R^{m_im_j}$, $\Phi(x_{11},\dots,x_{1m_j},\dots,x_{m_i1},\dots,x_{m_im_j})^T \neq (0,0,\dots,0)^T$. If this is not true, then $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap R^{m_im_j}$, $\Phi(x_{11},\dots,x_{1m_j},\dots,x_{m_i1},\dots,x_{m_im_j})^T = (0,0,\dots,0)^T$, thus, for constant vector $x \in \partial\Omega$, we have:

$$\widetilde{x}_{ij}[n] = \frac{\beta(h)}{1+\beta(h)}\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left(\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + \widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right) + \widetilde{I}_{ij}[n] = 0. \tag{3.14}$$

Equivalently, (3.14) can be written as the following form:

$$\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left(\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + \widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right)$$
$$= -\frac{1+\beta(h)}{\beta(h)}\widetilde{I}_{ij}[n] \tag{3.15}$$
$$\Longrightarrow \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] = -\left(\frac{1+\beta(h)}{\beta(h)}\widetilde{I}_{ij}[n] + \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right).$$

Combining (3.12) and (3.15), the following results are obtained:

$$\max_{n\in I_N}\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left|\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}\right| = \max_{n\in I_N}\left|\frac{1+\beta(h)}{\beta(h)}\widetilde{I}_{ij} + \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right|. \tag{3.16}$$

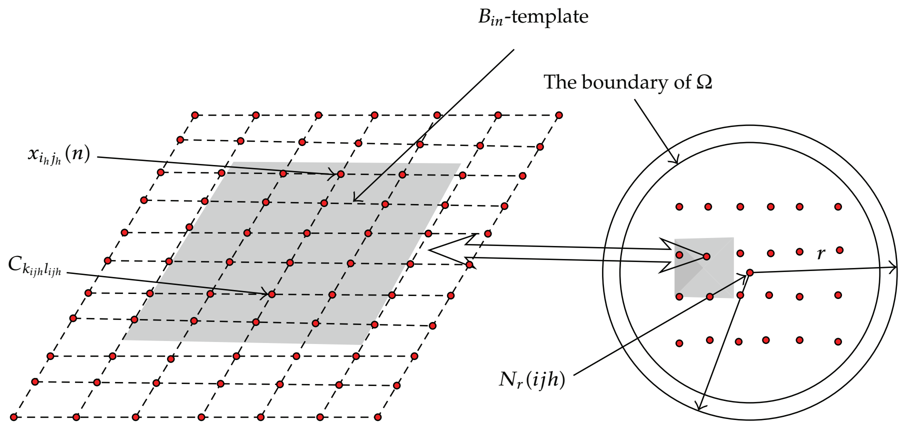

Figure 2: Input B-template and the boundary of Ω.

Thus, the following result is derived by calculating the (3.16):

$$\begin{aligned}
&\left|\frac{1+\beta(h)}{\beta(h)}\widetilde{I}_{ij}+\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right|-\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left|\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right| \\
&\quad\le\left|\frac{1+\beta(h)}{\beta(h)}\widetilde{I}_{ij}\right|<\left|\widetilde{I}_{ij}\right| \\
&\quad<\max_{n\in I_N}\left\{\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left|\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right|+\widetilde{I}_{ij}\right\}.
\end{aligned}\tag{3.17}$$

Obviously, (3.17) is a contradiction since $((1+\beta(h))/\beta(h))>1$, then for any $x=\partial\Omega\cap\operatorname{Ker}L$, $\operatorname{Ker}L=\operatorname{Im}P$, $Px=(\widetilde{x}_{11}[n],\dots,\widetilde{x}_{1m_j}[n],\widetilde{x}_{m_i1}[n],\dots,\widetilde{x}_{m_im_j}[n])^T\neq(0,\dots,0)^T$, $\Phi(x_{11},\dots,x_{1m_j},\dots,x_{m_i1},\dots,x_{m_im_j})^T\neq(0,\dots,0)^T$. Thus, $\deg(JQN,\Omega\cap\operatorname{Ker}L,0)\neq0$. Therefore, (2.1) has at least one N-periodic solution, thus the proof of Theorem 3.1 is completed. □

Corollary 3.2. *Suppose that Assumptions 2.5 and 2.6 hold, and the following condition holds:*

$$\sum_{h=1}^{m_h}\left|\widetilde{A}_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}\right|^2-\eta_{ij}>0,\tag{3.18}$$

where $\eta_{ij}=\sum_{h=1}^{m_h}|\widetilde{B}_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}|^2-\widetilde{I}_{ij}^2$, $i=1,\dots,m_i$, $j=1,\dots,m_j$, $n\in I_N$, *then* (2.1) *has at least one N-periodic solution.*

Proof. Similar to the proof of Theorem 3.1, so it is omitted. □

4. Globally Stability of a Periodic Solution with respect to (2.1)

The existence of a periodic solution for the system (2.1) is derived in the Theorem 3.1. Then global stability of a periodic solution with respect to (2.1) in the three-dimensional space is presented in the following.

Theorem 4.1. *Suppose that Assumptions 2.5 and 2.6 hold, and the following condition holds:*

$$\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\overline{A}_{k_{ijh}l_{ijh}} \leq \frac{1}{C_{y_{kl}} m_h m_j}, \tag{4.1}$$

where $i = 1, 2, \ldots, m_i$, $C_{y_{kl}}$,m_j *and* m_h *are positive constants, then the periodic solution with respect to* (2.1) *is global stability.*

Proof. It follows from the Theorem 3.1 that (2.1) has at least a periodic solution, without loss of generality, the periodic solution can be described by:

$$x^*[n] = \left(x^*_{11}[n], \ldots, x^*_{1m_j}[n], \ldots, x^*_{m_i 1}[n], \ldots, x^*_{m_i m_j}[n]\right)^T \in \mathcal{X}. \tag{4.2}$$

Then we can define the following formula:

$$u_{ij}[n] = \left|x_{ij}[n] - x^*_{ij}[n]\right|, \quad i = 1, \ldots, m_i, j = 1, \ldots, m_j. \tag{4.3}$$

Now, we show that the a periodic solution $x^*[n]$ is globally stable, and the following inequality is obtained by utilizing (2.1) and (4.3):

$$\begin{aligned}
u_{ij}[n+1] &= \left|x_{ij}[n+1] - x^*_{ij}[n+1]\right| \\
&\leq \left\{\alpha(h)\left|x_{ij}[n] - x^*_{ij}[n]\right| + C_{y_{kl}}\beta(h)\sum_{h=1}^{m_h}\sum_{j=1}^{m_j} A_{k_{ijh}l_{ijh}}\left|y_{k_{ijh}l_{ijh}}[n] - y^*_{k_{ijh}l_{ijh}}[n]\right|\right\} \\
&\leq \left\{\alpha(h)\left[\left|x_{ij}[n] - x_{o_{ijh}}[n]\right| + \left|x_{o_{ijh}}[n] - x^*_{ij}[n]\right|\right] + C_{y_{kl}}\beta(h)\sum_{h=1}^{m_h}\sum_{j=1}^{m_j} A_{k_{ijh}l_{ijh}}\right. \\
&\qquad \left.\times\left[\left|x_{k_{ijh}l_{ijh}}[n] - x_{o_{ijh}}[n]\right| + \left|x_{o_{ijh}}[n] - x^*_{k_{ijh}l_{ijh}}[n]\right|\right]\right\} \\
&\leq 2r\left(\alpha(h) + C_{y_{kl}}\beta(h) m_h m_j \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\overline{A}_{k_{ijh}l_{ijh}}\right) < +\infty.
\end{aligned} \tag{4.4}$$

We design the following Lyapunov-type sequence $V[n]$ by

$$V[n] = \sum_{i=1}^{m_i}\sum_{j=1}^{m_j}\left|x_{ij}[n] - x_{ij}^*[n]\right|. \tag{4.5}$$

Then, we can calculate the $\Delta V[n]$ by combining (2.1) and (4.5):

$$\begin{aligned}
\Delta V[n] &= V[n+1] - V[n] \\
&= \sum_{i=1}^{m_i}\sum_{j=1}^{m_j}\left(\left|x_{ij}[n+1] - x_{ij}^*[n+1]\right| - \left|x_{ij}[n] - x_{ij}^*[n]\right|\right) \\
&\le \sum_{i=1}^{m_i}\sum_{j=1}^{m_j}\left\{-\beta(h)\left|x_{ij}[n] - x_{ij}^*[n]\right| + C_{y_{kl}}\beta(h)\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}A_{k_{ijh}l_{ijh}}\left|y_{k_{ijh}l_{ijh}}[n] - y_{k_{ijh}l_{ijh}}^*[n]\right|\right\} \\
&\le \sum_{h=1}^{m_h}\sum_{j=1}^{m_j}2r\beta(h)\left(C_{y_{kl}}m_h m_j\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\overline{A}_{k_{ijh}l_{ijh}} - 1\right) \le 0.
\end{aligned} \tag{4.6}$$

Thus, it is easy to obtain $V[n] \le V[0]$ by the meaning of the (4.6), and furthermore,

$$\begin{aligned}
&\sum_{i=1}^{m_i}\sum_{j=1}^{m_j}\left|x_{ij}[n] - x_{ij}^*[n]\right| \le \beta(h)\left\{C_{y_{kl}}m_h m_j\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\overline{A}_{k_{ijh}l_{ijh}} - 1\right\}\sup_{s\in Z_0^-}d[s] \\
&\quad\Longrightarrow \sum_{i=1}^{m_i}\sum_{j=1}^{m_j}\left|x_{ij}[n] - x_{ij}^*[n]\right| \le \beta(h)\left\{C_{y_{kl}}m_h m_j\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\overline{A}_{k_{ijh}l_{ijh}} - 1\right\}\sup_{s\in Z_0^-}d[s] \le 0 \\
&\quad\Longrightarrow x_{ij}[n] = x_{ij}^*[n], \quad n \in I_N = \{0, 1, \ldots, N-1\},
\end{aligned} \tag{4.7}$$

where $d[s] = \max_{s\in Z_0^-}\{|x_{k_{ijh}l_{ijh}}[s] - x_{k_{ijh}l_{ijh}}^*[s]|,\ |x_{ij}[s] - x_{ij}^*[s]|\}$. Obviously, from the proof of Theorem 4.1, the globally stable of a periodic solution with respect to (2.1) is derived. Then, existence and global stability of a periodic solution for DT-CNNs are obtained by utilizing the conditions of the proposed theorems in an arbitrary diameter plane of a convex space. Thus the proof of Theorem 4.1 is completed. □

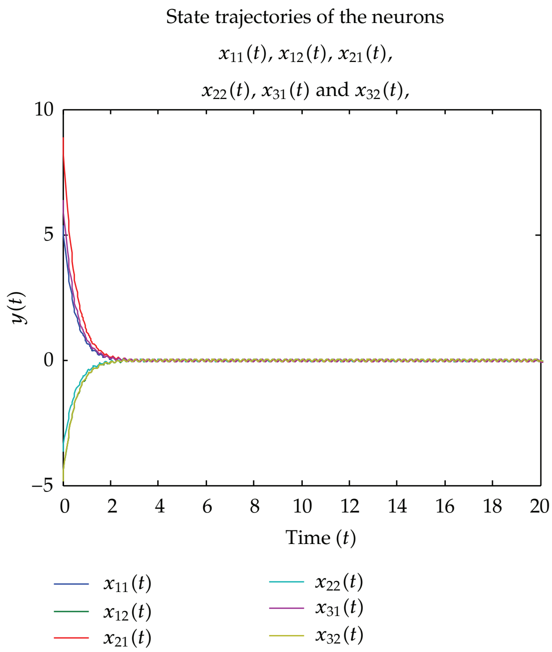

Figure 3: State trajectories of the neurons x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}.

5. Numerical Simulation

In this section, we give an example to show the effectiveness and improvement of the derived results. Consider the following continuous cellular neural networks:

$$C\frac{\mathrm{d}x_{ij}}{\mathrm{d}t} = -\frac{1}{R}x_{ij} + \sum_{C(k,l)\in N_r(ij)} A_{kl}y_{kl} + \sum_{C(k,l)\in N_r(ij)} B_{kl}u_{kl} + I_{ij},$$
$$y_{ij} = f(x_{ij}) = \frac{1}{2}\left(\left|x_{ij}+1\right| - \left|x_{ij}-1\right|\right), \quad i = 1,2,3,\ j = 1,2, \tag{5.1}$$

for $t > 0$, where $C = 1, R = 1/2, y_{kl} = (1/2)(|x_{ij}+1| - |x_{ij}-1|), A_{k_{111}l_{111}} = A_{k_{311}l_{311}} = A_{k_{321}l_{321}} = 0.1\sin(8\pi t), A_{k_{121}l_{121}} = A_{k_{211}l_{211}} = A_{k_{221}l_{221}} = 0.1\cos(8\pi t), B_{k_{i11}l_{i11}}u_{k_{i11}l_{i11}} = -0.06 + 0.08\sin(8\pi t), B_{k_{i21}l_{i21}}u_{k_{i21}l_{i21}} = -0.02 + 0.06\cos(8\pi t), I_{i1} = \sin(8\pi t), I_{i2} = \cos(8\pi t), i = 1,2,3, x[0] = (5.386, -4.836, 8.863, -3.683, 6.386, -4.836)^T$. Then, state trajectories of $x_{ij}(i = 1,2,3, j = 1,2)$ are denoted in Figure 3.

From Figure 3, it is easy to know that a $(1/4)$-periodic solution of the continuous cellular neural networks is globally stable. Compared to the system (5.1), we design the discrete-time analogue of the continuous cellular neural network as follows:

$$x_{ij}[n+1] = \alpha(h)x_{ij}[n] + \beta(h)\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\left(A_{k_{ijh}l_{ijh}}y_{k_{ijh}l_{ijh}}[n] + B_{k_{ijh}l_{ijh}}u_{k_{ijh}l_{ijh}}\right)$$
$$+ I_{ij}[n], \quad n \in Z_0^+, \quad i = 1,2,3,\ j = 1,2,$$

$$
\begin{aligned}
x[0] &= \left(x_{11}[0], x_{12}[0], x_{21}[0], x_{22}[0], x_{31}[0], x_{32}[0]\right)^{T} \\
&= (5.386, -4.836, 8.863, -3.683, 6.386, -4.836)^{T},
\end{aligned}
\tag{5.2}
$$

for $h > 0$, by using Assumptions 2.5 and 2.6 in Section 2, each variable is denoted as:

$$
\begin{gathered}
\alpha(h) = e^{-h}, \qquad \beta(h) = 1 - e^{-h}, \\
A_{k_{111}l_{111}} = A_{k_{311}l_{311}} = A_{k_{321}l_{321}} = \frac{1}{20}\left(1 - e^{-2h}\right)\sin(8\pi nh), \\
y_{k_{ijh}l_{ijh}}[n] = \frac{1}{2}\left(\left|x_{k_{ijh}l_{ijh}}[n] + 1\right| - \left|x_{k_{ijh}l_{ijh}}[n] - 1\right|\right), \\
A_{k_{121}l_{121}} = A_{k_{211}l_{211}} = A_{k_{221}l_{221}} = \frac{1}{20}\left(1 - e^{-2h}\right)\sin(8\pi nh), \qquad I_{i1} = \frac{1}{20}\left(1 - e^{-2h}\right)\sin(8\pi nh), \\
I_{i2} = \frac{1}{20}\left(1 - e^{-2h}\right)\cos(8\pi nh), \qquad B_{k_{i11}l_{i11}} u_{k_{i11}l_{i11}} = -0.02 + 0.08\sin(8\pi nh), \\
i = 1, 2, 3, \quad B_{k_{i21}l_{i21}} u_{k_{i21}l_{i21}} = -0.08 + 0.06\cos(8\pi nh), \quad i = 1, 2, 3.
\end{gathered}
\tag{5.3}
$$

The derived results of this paper are verified by the following steps.

(1) According to the illustrations of the neighbourhood distance r for cell $C(k, l) = C_{k_{ijh}l_{ijh}}$ which is given by $N_{\mathrm{r}}(ijh)$ function, and by (3.9) and (3.10), the exact values of distance r and Ω are illustrated as:

$$
\begin{gathered}
\Omega \triangleq \left\{ x \in \mathcal{X} \subset N_r(ijh), \|x\| < \sum_{i=1}^{3}\sum_{j=1}^{2}\Theta_{ij} \right\}, \\
\max_{n\in I_N}\left|x_{ij}[n]\right| \leq \max_{n\in I_N}\left\{ \left|\varphi_{ij}[0]\right| + (1 - \alpha(h))^{n}\sum_{h=1}^{1}\sum_{j=1}^{2}\left(\left|\overline{A}_{k_{ijh}l_{ijh}} y_{k_{ijh}l_{ijh}}[0]\right|\right) \right. \\
\left. + (1 - \alpha(h))\sum_{h=1}^{1}\sum_{j=1}^{2}\left|\overline{B}_{k_{ijh}l_{ijh}}\overline{u}_{k_{ijh}l_{ijh}}\right| + \overline{I}_{ij} \right\} \triangleq \Theta_{ij}, \quad i = 1, 2, 3.
\end{gathered}
\tag{5.4}
$$

Then, $\Theta_{ij}(i = 1, 2, 3, \ j = 1, 2)$ is calculated below,

$$
\begin{aligned}
\max_{n\in I_N}\left|x_{1j}[n]\right| &\leq \max_{n\in I_N}\left\{ \left|\varphi_{1j}[0]\right| + (1 - \alpha(h))^{n}\sum_{h=1}^{1}\sum_{j=1}^{2}\left(\left|\overline{A}_{k_{1jh}l_{1jh}} y_{k_{1jh}l_{1jh}}[0]\right|\right) \right. \\
&\qquad \left. + (1 - \alpha(h))\sum_{h=1}^{1}\sum_{j=1}^{2}\left|\overline{B}_{k_{1jh}l_{1jh}}\overline{u}_{k_{1jh}l_{1jh}}\right| + \overline{I}_{1j} \right\} \\
&\leq 5.386 + 4.836 + 0.05 \times 4 + 0.24 = 10.662 \triangleq \Theta_{1j}, \quad j = 1, 2,
\end{aligned}
$$

$$\max_{n\in I_N}\left|x_{2j}[n]\right| \le \max_{n\in I_N}\left\{\left|\varphi_{2j}[0]\right| + (1-\alpha(h))^n\sum_{h=1}^{1}\sum_{j=1}^{2}\left(\left|\overline{A}_{k_{2jh}l_{2jh}}y_{k_{2jh}l_{2jh}}[0]\right|\right)\right.$$
$$\left. +(1-\alpha(h))\sum_{h=1}^{1}\sum_{j=1}^{2}\left|\overline{B}_{k_{2jh}l_{2jh}}\overline{u}_{k_{2jh}l_{2jh}}\right| + \overline{I}_{2j}\right\}$$
$$\le 8.863 + 3.683 + 0.05\times 4 + 0.24 = 12.986 \triangleq \Theta_{2j}, \quad j=1,2,$$
$$\max_{n\in I_N}\left|x_{3j}[n]\right| \le \max_{n\in I_N}\left\{\left|\varphi_{3j}[0]\right| + (1-\alpha(h))^n\sum_{h=1}^{1}\sum_{j=1}^{2}\left(\left|\overline{A}_{k_{3jh}l_{3jh}}y_{k_{3jh}l_{3jh}}[0]\right|\right)\right.$$
$$\left. +\ (1-\alpha(h))\sum_{h=1}^{1}\sum_{j=1}^{2}\left|\overline{B}_{k_{3jh}l_{3jh}}\overline{u}_{k_{3jh}l_{3jh}}\right| + \overline{I}_{3j}\right\}$$
$$\le 6.386 + 4.836 + 0.05\ \times\ 4 + 0.24 = 11.662 \triangleq \Theta_{3j}, \quad j=1,2. \tag{5.5}$$

Thus, the subset Ω of function $N_r(ijh)$ is derived by the following:

$$\Omega \triangleq \left\{x \in \mathcal{X} \subset N_{35.31}(ijh), \|x\| < \sum_{i=1}^{3}\sum_{j=1}^{2}\Theta_{ij} = 35.31\right\}. \tag{5.6}$$

(2) We will verify the condition of Theorem 3.1 if we want to utilize Theorem 4.1. After strictly calculating the condition of Theorem 3.1, it is easy to obtain that the function $g[n] = |\widetilde{B}_{k_{ij1}l_{ij1}}u_{k_{ij1}l_{ij1}}|^2 - |\widetilde{A}_{k_{ij1}l_{ij1}}y_{k_{ij1}l_{ij1}}|^2 + \widetilde{I}_{ij}^2 \in R^+$, $i=1,2,3$, $j=1,2$, $n\in I_N$; therefore, the condition of the Theorem 3.1 is critically satisfied as well.

(3) According to (4.1), the condition of the Theorem 4.1 will be derived as follows:

$$\sum_{h=1}^{m_h}\sum_{j=1}^{m_j}\overline{A}_{k_{ijh}l_{ijh}} < \frac{3}{10} < \frac{1}{1\ \times\ 1\times 2} = \frac{1}{C_{y_{kl}}m_h m_j}. \tag{5.7}$$

Then state trajectories of neurons x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32} are shown in Figures 4 and 5.

From Figures 4 and 5, we can learn that all the periodic solution converges to a unique a $1/4h$-periodic solution, then the DT-CNN (2.1) has a globally stable $1/4h$-periodic solution. Thus, all conditions of Theorems 3.1 and 4.1 are strictly satisfied; therefore all conditions of proposed theorems are critically verified.

6. Conclusions

Existence and global stability are important dynamical properties in CNN. In this paper, we consider the discrete-time analogues of CNN with periodic coefficients and obtain some new

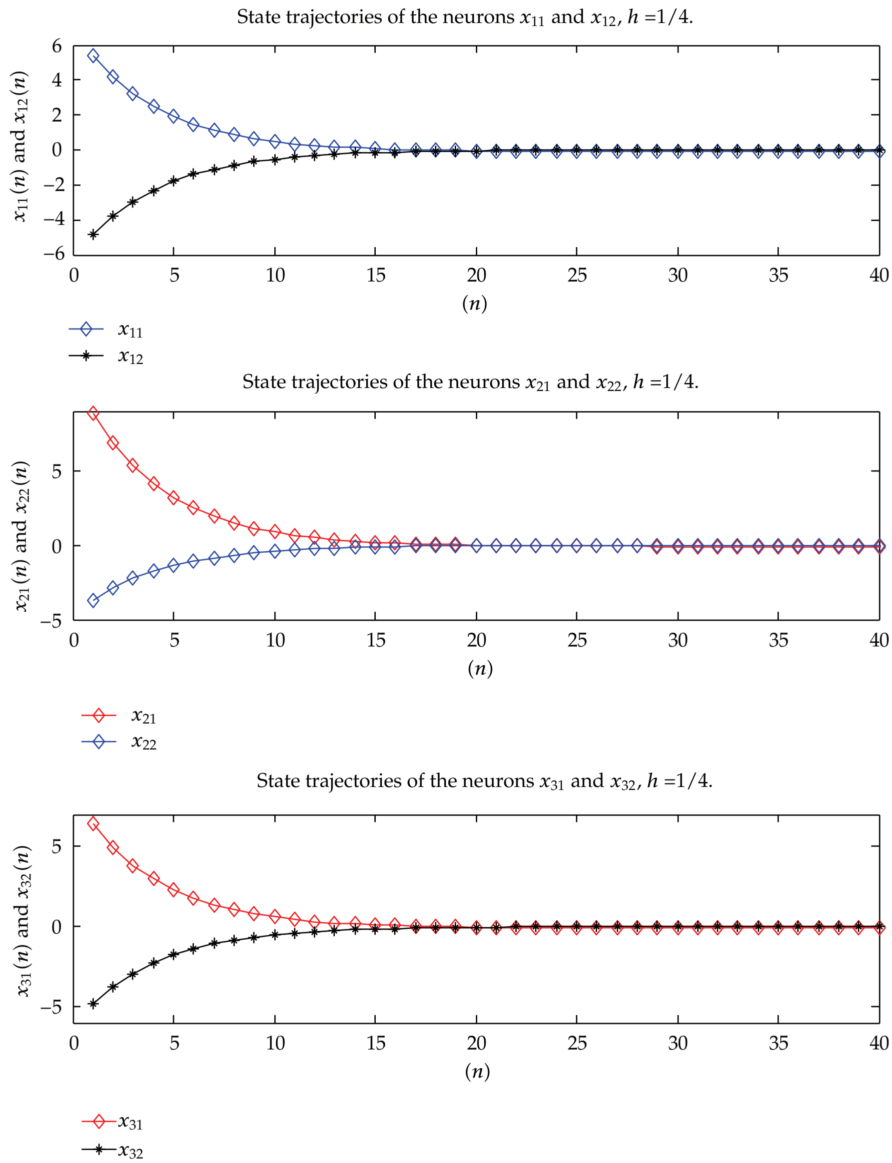

Figure 4: State trajectories of neurons $x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}(h = 1/4)$.

results for the DT-CNN in the three-dimensional space. Comparisons between our results and the previous results have also been made. And it has been demonstrated that our criteria are more general and effective than those reported in the literature.

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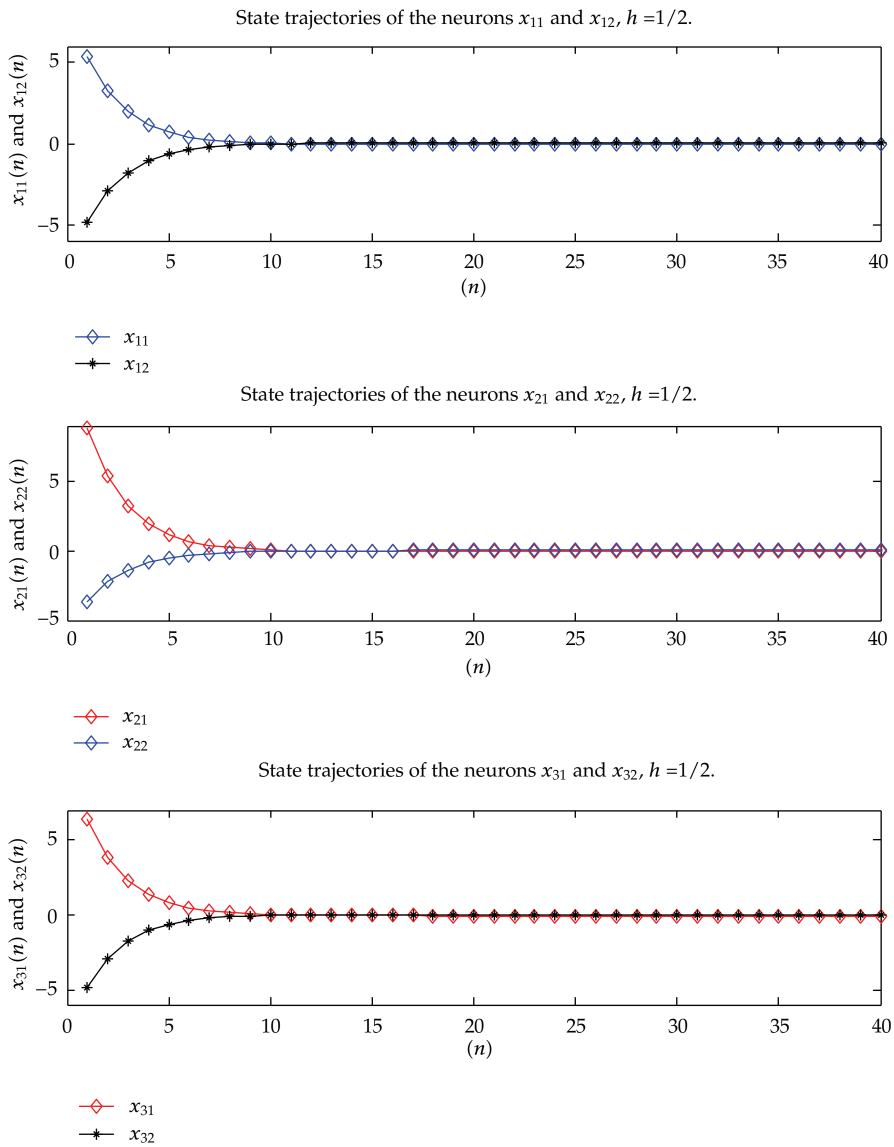

Figure 5: State trajectories of neurons x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, $x_{32}(h = 1/2)$.

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