Research Article

Stabilization of an Underactuated Surface Vessel Based on Adaptive Sliding Mode and Backstepping Control

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The paper studied controlling problem of an underactuated surface vessel with unknown interferences. It proved that the control problem of underactuated surface vessel can be transformed into the stabilization analysis of two small subsystems. This controller was designed by backstepping method and adaptive sliding mode, was suitable for solving the problem of the control of higher systems, can keep the system global asymptotic stability, and can inhibit unknown interference, and boundary layer can weaken the buffeting generated by sliding mode. The unknown interference was estimated by adaptive function. Finally, the simulation results are given to demonstrate the effectiveness of the proposed control laws.

1. Introduction

With the rapid development of the modern shipping industry, the study about the controller of the underactuated surface vessel has received extensive attention. Since stabilization control problem of the underactuated surface vessel belongs to the field of nonlinear control, nonlinear control theory is the basic foundation of the underactuated stabilization control theory [1]. Underactuated ship system is a nonholonomic system, and the nonholonomic system which does not meet the sufficient condition of Brockett system linear approximation is not controllable; so, stabilization control of underactuated systems cannot be continuous and smooth [2]. Similarly, it cannot use output feedback or smooth state feedback on a linear system, and the linear control method is not satisfactory to the stabilization control of underactuated system.

So far, underactuated ship stabilization control problem is more studied without interferences, just like [3–7] for the case of noninterference. But for a bounded environment disturbance (or unknown disturbances) and the uncertainty of the ship systems, underactuated stabilization controller is few, except for [8], but from the research results of them shows that the antidisturbance (or uncertainty) ability of the designed controller is less than ideal. Backstepping

and sliding mode control method is the combination of backstepping and sliding mode control and takes full advantages of their own characteristics in [9]. Boundedness and convergence can keep the equilibrium point; sliding mode control system can match perturbation and disturbance, and when using backstepping under less restrictive conditions, it can ensure that the closed-loop system has strong robustness. The controller exist sliding mode; so, it will cause system chattering and make control input change severely, and it cannot meet the requirements of the actual system, but boundary layer method can solve it. Adaptive function is used to cope with approximation of uncertain functions. The paper studies underactuated surface vessel, which under the conditions of bounded variable environmental interference effect. A new adaptive sliding mode and backstepping control method is designed to deal with the underactuated ship stabilization problem, and according to the experimental simulation, the designed controller is effective.

2. Model of an Underactuated Surface Vessel

We consider an underactuated surface vessel with simplified dynamics off-diagonal terms of the linear and nonlinear

damping matrices which are ignored. Based on [9], the dynamics and kinematics of an underactuated surface vessel are described as follows:

$$\dot{x} = u\cos(\psi) - v\sin(\psi),$$

$$\dot{y} = u\sin(\psi) + v\cos(\psi),$$

$$\dot{\psi} = r,$$

$$\dot{u} = \frac{m_{22}}{m_{11}}vr - \frac{d_{11}}{m_{11}}u + \frac{1}{m_{11}}\tau_1 + \frac{1}{m_{11}}\tau_{w1},$$ (1)

$$\dot{v} = -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v + \frac{1}{m_{22}}\tau_{w2},$$

$$\dot{r} = \frac{m_{11} - m_{22}}{m_{33}}uv - \frac{d_{33}}{m_{33}}r + \frac{1}{m_{33}}\tau_3 + \frac{1}{m_{33}}\tau_{w3},$$

where $[x \quad y \quad \psi]^T$ denote position and orientation in inertial frame, $[u \quad v \quad r]^T$ denote linear velocities in surge sway and angular velocity in yaw, m_{11}, m_{22}, m_{33} are the constants of inertia matrix, and d_{11}, d_{22}, d_{33} are the constants of damping matrix, and we consider the case that the surface vessel has no side thruster; so, the control input can be described as $[\tau_1 \quad 0 \quad \tau_3]^T$, and $[\tau_{w1} \quad \tau_{w2} \quad \tau_{w3}]^T$ are the environmental disturbances in the body-fixed frame.

3. Controller Design

3.1. Coordinate and Feedback Transformation.
In order to design the controller conveniently, the underactuated ships model will be converted to a suitable form. Based on [10], the coordinate and input transformations are as follows:

$$z_1 = x\cos\psi + y\sin\psi,$$

$$z_2 = -x\sin\psi + y\cos\psi,$$

$$z_3 = \psi,$$

$$z_4 = -\frac{m_{11}}{d_{22}}u - z_1,$$

$$z_5 = v,$$ (2)

$$z_6 = r,$$

$$u_1 = \left(\frac{d_{11}}{d_{22}} - 1\right)u - z_2 z_6 - \frac{\tau_1 + \tau_{w1}}{d_{22}},$$

$$u_2 = \frac{m_{11} - m_{22}}{m_{33}}uv - \frac{d_{33}}{m_{33}}r + \frac{\tau_3 + \tau_{w3}}{m_{33}}.$$

By using (2), we can transform the differential equation as follows:

$$\dot{z}_1 = -\frac{d_{22}}{m_{11}}z_1 - \frac{d_{22}}{m_{11}}z_4 + z_2 z_6 - \frac{m_{11}}{d_{22}}z_5 z_6,$$ (3)

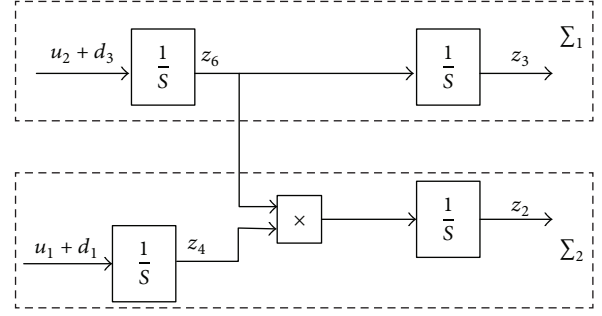$$\dot{z}_5 = -\frac{d_{22}}{m_{22}}z_5 + \frac{d_{22}}{m_{22}}z_6(z_1 + z_4),$$

FIGURE 1: Block diagram of the system of (4).

$$\dot{z}_2 = z_4 z_6 + \frac{\tau_{w2}}{d_{22}},$$

$$\dot{z}_3 = z_6,$$ (4)

$$\dot{z}_4 = u_1 + d_1,$$

$$\dot{z}_6 = u_2 + d_3,$$

where $d_1 = \tau_{w1}/d_{22}$, and $d_3 = -\tau_{w3}/m_{33}$.

Remark 1. Because state transformation (2) is a global diffeomorphism transformation, so $\lim_{t \to \infty} z_i = 0$ for $i = 1, 2, \ldots, 6$ is convergent by exponential form, and thus we imply that $\lim_{t \to \infty}(\eta, V) = 0$.

Proposition 2. $\lim_{t \to \infty} z_i = 0, i = 1, 2, \ldots, 6,$ *is globally exponential stable if* $\lim_{t \to \infty} z_i = 0, i = 2, 3, 4, 6,$ *is globally exponential stable.*

Proof. The similar proof has been given in [1]. □

According to Proposition 2, we know that in order to make the stabilization of underactuated surface vessels from the initial position to the origin, the system only needs to make transformation (4). It can be seen as strict feedback nonlinear subsystem, and the cascade system consists of two components which are shown in Figure 1 as follows:

$$\Sigma_1 : \begin{cases} \dot{z}_3 = z_6, \\ \dot{z}_6 = u_2 + d_3, \end{cases}$$ (5)

$$\Sigma_2 : \begin{cases} \dot{z}_2 = z_4 z_6 + \dfrac{\tau_{w2}}{d_{22}}, \\ \dot{z}_4 = u_1 + d_1. \end{cases}$$ (6)

3.2. Controller Design.
Figure 1 shows that the controller of underactuated surface vessels can be transformed into two single-input and single-output systems. So, we only need to design the controller to satisfy the fact that u_1 and u_2 can be controlled in three directions of the ship.

3.2.1. Design the Control Law of System Σ_1.
Firstly, we consider the subsystem z_3 and define Lyapunov function

$V_1(z_3) = (1/2)z_3^2$, and the time derivative of $V_1(z_3)$ is as follows:

$$\dot{V}_1(z_3) = z_3\dot{z}_3 = z_3 z_6. \tag{7}$$

We take z_6 as a virtual control input of z_3 subsystem and define $z_6 = e_1 + \alpha_1(z_3)$. In order to ensure that the subsystem z_3 is asymptotically stable when $e_1 = 0$, we choose $\alpha_1(z_3) = -k_1 z_3$, where $\alpha_1(0) = 0$, and $k_1 > 0$. Then, we can get equations as follows:

$$\dot{e}_1 = \dot{z}_6 - \dot{\alpha}_1(z_3) = u_2 + d_3 + k_1 z_6,$$

$$\dot{V}_1(z_3) = z_3 z_6 = -k_1 z_3^2 + z_3 e_1 \le 0. \tag{8}$$

Secondly, we consider the whole system

$$\dot{z}_3 = e_1 - k_1 z_3,$$

$$\dot{e}_1 = u_2 + d_3 + k_1 z_6. \tag{9}$$

We can assume the uncertain parameters d_3 exists the upper bound and choose a linear sliding mode switch function

$$\sigma_1 = c_1 z_3 + e_1, \quad c_1 > 0. \tag{10}$$

Define the Lyapunov function $V_2(z_3, z_6) = (1/2)\sigma_1^2 + V_1(z_3)$; the time derivative of $V_2(z_3, z_6)$ becomes

$$\dot{V}_2(z_3, z_6) = -k_1 z_3^2 + z_3 e_1$$
$$+ \sigma_1 \left[c_1(e_1 - k_1 z_3) + u_2 + d_3 + k_1 z_6 \right]. \tag{11}$$

Choose the control law

$$u_2 = -c_1(e_1 - k_1 z_3) - k_1 z_6$$
$$- \bar{d}_3 \operatorname{sgn}(e_1) - h_1(\sigma_1 + \beta_1 \operatorname{sgn}(\sigma_1)), \tag{12}$$

where $\operatorname{sgn}(x)$ is the sign function, $h_1 \ge 0$, $\beta_1 \ge 0$, and $\bar{d}_3$ is the upper bound of uncertain parameter d_3. Combining (12) into (11), we can obtain

$$\dot{V}_2(z_3, z_6) = -k_1 z_3^2 + z_3 e_1 - h_1 \sigma_1^2 - h_1 \beta_1 |\sigma_1| + d_3 \sigma_1 - \bar{d}_3 |\sigma_1|$$
$$\le -k_1 z_3^2 + z_3 e_1 - h_1 \sigma_1^2 - h_1 \beta_1 |\sigma_1| + (d_3 - \bar{d}_3) |\sigma_1|$$
$$\le -k_1 z_3^2 + z_3 e_1 - h_1 \sigma_1^2 - h_1 \beta_1 |\sigma_1| \le 0. \tag{13}$$

Finally, because the upper bound of the general object is difficult to predict, and in order to determine the uncertainties of the upper bound, we use the adaptive algorithm to estimate it. $\bar{d}_3$ is the upper bound of uncertain parameter d_3. And its adaptive rate is $\dot{\bar{d}}_3 = \gamma_1 \sigma_1$.

3.2.2. Design the Control Law of System Σ_2. Firstly, consider the subsystem z_2, and define Lyapunov function $V_3(z_2) = (1/2)z_2^2$. We see z_4 as a virtual control input of subsystem z_2

and define $z_4 = -k_2 z_2 \operatorname{sgn}(z_6)$. The time derivative of $V_3(z_2)$ is as follows:

$$\dot{V}_3(z_2) = -k_2 z_2^2 z_6 \operatorname{sgn}(z_6) = -k_2 z_2^2 |z_6| \le 0, \quad k_2 > 0. \tag{14}$$

Assuming that lateral disturbance force decrease with time, and $|\tau_{w2}/d_{22}| \le |z_2 z_6|$. In this paper, we can make $\tau_{w2} = 0$; so, z_2 is asymptotically convergent to zero when $z_6(0) \ne 0$ (or $z_3(0) \ne 0$). In order to make z_2 asymptotically converging to zero, we can adjust the k_2 (and k_1) such that the convergence rate of z_2 is greater than the convergence rate of z_6 (or z_3).

Define $z_4 = e_2 + \alpha_2(z_2)$. In order to ensure that the subsystem z_2 is asymptotically stable when $e_2 = 0$, we choose $\alpha_2(z_2) = -k_2 z_2 \operatorname{sgn}(z_6)$, where $\alpha_2(0) = 0$. Then, we can get equations as follows:

$$\dot{e}_2 = \dot{z}_4 - \dot{\alpha}_2(z_2) = u_1 + d_1 + k_2 z_4 z_6 \operatorname{sgn}(z_6),$$

$$\dot{V}_3(z_2) = z_2 z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6)) \le 0. \tag{15}$$

Secondly, consider the whole system

$$\dot{z}_2 = z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6)),$$

$$\dot{e}_2 = u_1 + d_1 + k_2 z_4 z_6 \operatorname{sgn}(z_6). \tag{16}$$

We assume the uncertain parameters d_1 exists the upper bound. Choose a linear sliding mode switch function $\sigma_2 = c_2 z_2 + e_2$, $c_2 > 0$. Define the Lyapunov function $V_4(z_2, z_4) = (1/2)\sigma_2^2 + V_3(z_2)$; the time derivative of $V_4(z_2, z_4)$ is as follows:

$$\dot{V}_4(z_2, z_4) = z_2 z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6))$$
$$+ \sigma_2 [c_2 z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6))] \tag{17}$$
$$+ \sigma_2 [u_1 + d_1 + k_2 z_4 z_6 \operatorname{sgn}(z_6)].$$

Choose the control law

$$u_1 = -c_2 z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6)) - k_2 z_4 z_6 \operatorname{sgn}(z_6)$$
$$- \bar{d}_1 \operatorname{sgn}(e_2) - h_2(\sigma_2 + \beta_2 \operatorname{sgn}(\sigma_2)), \tag{18}$$

where $h_2 \ge 0$, $\beta_2 \ge 0$, and $\bar{d}_1$ is the upper bound of uncertain parameter d_1.

Combining (18) into (17), we can obtain

$$\dot{V}_4(z_2, z_4) = z_2 z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6)) - h_2 \sigma_2^2$$
$$- h_2 \beta_2 |\sigma_2| + d_1 \sigma_2 - \bar{d}_1 |\sigma_2|$$
$$\le z_2 z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6)) - h_2 \sigma_2^2$$
$$- h_2 \beta_2 |\sigma_2| - (\bar{d}_1 - d_1) |\sigma_2|$$
$$\le z_2 z_6 (e_2 - k_2 z_2 \operatorname{sgn}(z_6)) - h_2 \sigma_2^2 - h_2 \beta_2 |\sigma_2| \le 0. \tag{19}$$

Finally, we use the adaptive algorithm to estimate $\bar{d}_3$. And its adaptive rate is $\dot{\bar{d}}_3 = \gamma_1 \sigma_1$.

3.3. Stability Analysis. Firstly, we should analyze the stability of the system Σ_1, defined as follows:

$$Q_1 = \begin{bmatrix} k_1 + h_1 c_1^2 & h_1 c_1 - \dfrac{1}{2} \\ h_1 c_1 - \dfrac{1}{2} & h_1 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} z_3 & e_1 \end{bmatrix}^T. \quad (20)$$

We can get

$$|Q_1| = h_1 \left(k_1 + h_1 c_1^2\right) - \left(h_1 c_1 - \frac{1}{2}\right)^2 = h_1 \left(k_1 + c_1\right) - \frac{1}{4}, \quad (21)$$

$$W_1^T Q_1 W_1 = \begin{bmatrix} z_3 & e_1 \end{bmatrix} \begin{bmatrix} k_1 + h_1 c_1^2 & h_1 c_1 - \dfrac{1}{2} \\ h_1 c_1 - \dfrac{1}{2} & h_1 \end{bmatrix} \begin{bmatrix} z_3 \\ e_1 \end{bmatrix}$$

$$= z_3^2 \left(k_1 + h_1 c_1^2\right) + 2 e_1 z_3 \left(h_1 c_1 - \frac{1}{2}\right) + e_1^2 h_1$$

$$= k_1 z_3^2 + h_1 c_1^2 z_3^2 + 2 h_1 c_1 e_1 z_3 - e_1 z_3 + e_1^2 h_1. \quad (22)$$

So, (13) can be written as follows:

$$\dot{V}_2\left(z_3, z_6\right) \leq -W_1^T Q_1 W_1 - h_1 \beta_1 \left|\sigma_1\right|. \quad (23)$$

From (21), choosing the adaptive value of k_1, h_1, and c_1, we can make $|Q_1| > 0$; then, $\dot{V}_2(z_3, z_6) \leq 0$. The wide range of Lyapunov asymptotic stability theorem shows that the system Σ_1 is stable.

Secondly, system Σ_1 and system Σ_2 use the same method; so, the stability of the system Σ_2 can refer to system Σ_1.

Proposition 3. *The system shown in* (1) *can be controlled by the input of* (12) *and* (18)*, and a wide range of asymptotically stabilization can be obtained by* (13) *and* (19)*.*

Proof. The certification process has been given in the design steps. □

From field of the theory, the motion choosing sliding mode is that parameter variations and external interference have nothing to do with the systems; so, the robustness of the system using sliding mode controller is better than the general control system. But in fact, sliding mode control can cause the system chattering for the reason of noncontinuous switch; so, the boundary layer method will be introduced to weaken this vibration. Saturation function is used to replace the ideal relay functions in the appropriate boundary layer; in other words, sat (s) is used instead of sgn (s) as follows:

$$\text{sat}\,(s) = \begin{cases} +1 & s > \Delta, \\ ks & |s| \leq \Delta, \\ -1 & s < -\Delta, \end{cases} \quad (24)$$

where Δ is the thickness of boundary layer; k is a constant. And $\Delta = 0.33$; $k = 1$.
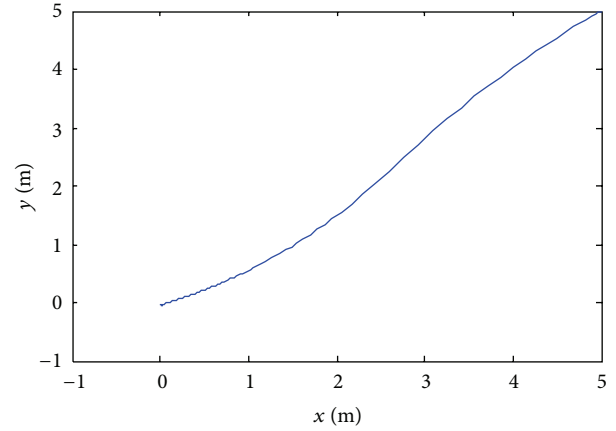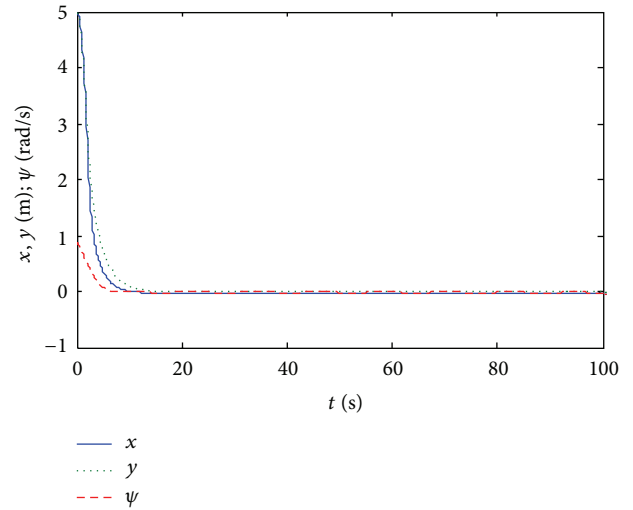

FIGURE 2: Curve of the ship's trajectory.

FIGURE 3: Curve of ship position and heading angle variables.

4. Simulation

In order to verify that the controller is able to calm from the initial point to the origin, consider an underactuated surface vessel with the model parameters in [11], and assume the initial condition to be as follows: $m_{11} = 200$ kg, $m_{22} = 250$ kg, $m_{33} = 80$ kg m^2, $d_{11} = 70$ kg s^{-1}, $d_{22} = 100$ kg s^{-1}, $d_{33} = 50$ kg m^2 s^{-1}, and $[x_0, y_0, \psi_0, u_0, v_0, r_0]^T = [5$ m, 5 m, $(\pi/4)$ rad, 0 m/s, 0 m/s, 0.5 rad/s$]^T$.

Take the disturbances as follows: $\tau_{w1} = 50 + 10\sin((\pi/6)t)$ N, and $\tau_{w2} = 0$ N, $\tau_{w3} = 10 + 10\sin((\pi/6)t)$ N · m.

In order to make ship at low speed operation, the longitudinal limited value of thrust is ± 400 N, and torque limit value is ± 200 N·m. The parameters of the control law are designed with the following: $k_1 = 5$, $c_1 = 8$, $h_1 = 0.2$, $\beta_1 = 0.1$, $k_2 = 10$, $c_2 = 0.1$, $h_2 = 6$, $\beta_2 = 1.8$, $\gamma_1 = 0.05$, and $\gamma_2 = 0.08$. Simulation results are shown as follows.

From Figure 2, we can see the trajectory of the system to be better when there exist disturbances. Designed controller can make the underactuated surface vessel stabilization. From Figures 3 and 4, although the response time is short,
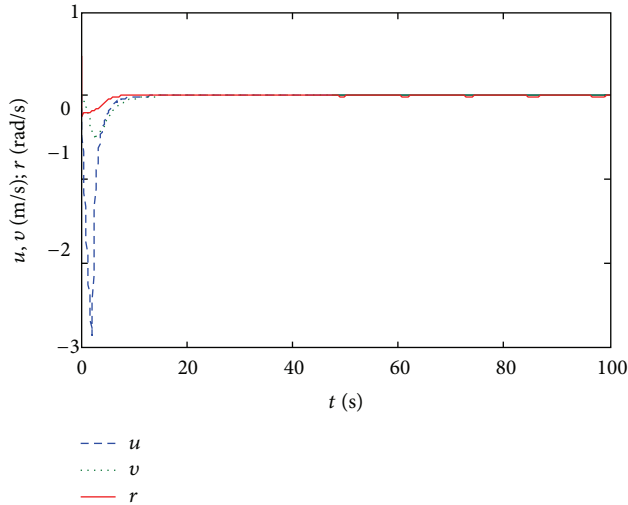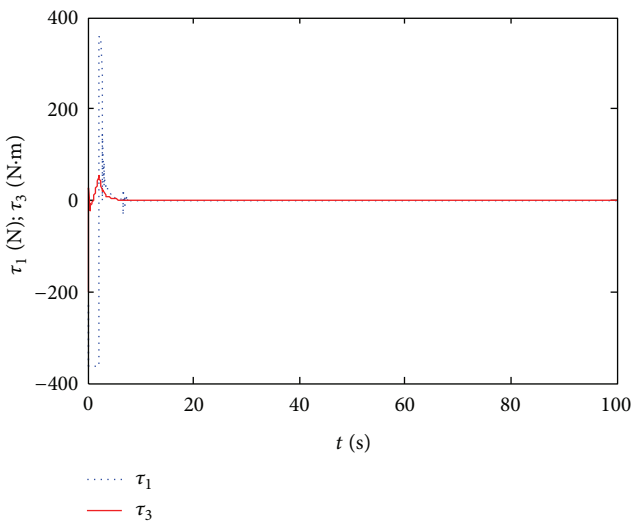
FIGURE 4: Curve of ship speed.

FIGURE 5: Curve of the longitudinal force and yawing moment.

designed controller can make all the states of the system stable to the origin. From Figure 5, because of introduction of the boundary layer approach to weaken the chattering phenomenon of the control force and moment, there is no change when the system is balanced, and it can save the power as well.

5. Conclusions

In this paper, underactuated surface vessels use the combination of adaptive sliding mode and backstepping to design the controller with the consideration of environmental disturbance and use the boundary layer method to solve the chattering problem. From the analysis of stabilization, the system is state global stabilization to the origin. The designed controller overcomes the problems of anti-to perturbation poor difficulties of the other controllers [8]; so, the designed controller improves the robustness of the system.

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