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Research Article

Supply Chain Coordination under Stock- and Price-Dependent Selling Rates under Declining Market

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We explore coordination issues of a two-echelon supply chain, consisting of a distributor and a retailer. The effect of revenue-sharing contract mechanism is examined under stock-time-price-sensitive demand rate. First, we investigate relationships between distributor and retailer under noncooperative distributor-Stackelberg games. Then we establish analytically that revenue sharing contact is able to coordinate the system and leads to the win-win outcomes. Finally, numerical examples are presented to compare results between the different models.

1. Introduction

In the last few decades, extensive researches have been performed in the area of supply chain coordination. Numerous studies have offered various contractual forms of alliance to enhance joint performance of supply chain partners. This includes buybacks [1–3], quantity discount [4–6], revenue sharing [7, 8], two part tariffs [9], quantity flexibility contracts [10], and target-level sales rebates [11], to coordinate the decisions of supply chain partners. In this paper, we mainly consider a specific type of supply chain contract, namely, the revenue-sharing contract, which is adopted to coordinate a decentralized supply chain. The revenue-sharing contract mechanism can be identified by two parameters, namely, the wholesale price and percentage of the revenue-shared between the supply chain entities [12]. The performance of revenue sharing contract has been examined on standard newsvendor problem, and it has been observed that this coordination mechanism perfectly coordinates the system.

Warren and Peers [13] have reported that after the adoption of revenue-sharing contract Blockbuster's market share of video rental has increased from 24% in 1997 to 40% in 2002. Cachon and Lariviere [7] have provided an analysis of these contracts in a more general setting and demonstrate that revenue-sharing contracts coordinate the supply chain and arbitrarily allocate its profit to the two parties. Gerchak et al. [8] study the revenue sharing contracts in a decentralized Stackelberg setting in which the video rental channel and the studio make independent decisions. Wang et al. [14] have studied effect of revenue-sharing contract in a supply chain with fuzzy demand. Qin and Yang [15] have discussed effect of revenue sharing contract when supplier offers price discount to the retailer. Yao et al. [16] investigated a revenue-sharing contract for coordinating a supply chain comprising one manufacturer and two competing retailers under classic newsvendor problem model framework. Lu et al. [17] have used revenue contract to analyze mobile service supply chain system. However, most of the models cited above relate to the coordination issues of supply chain and are based on deterministic price-sensitive or stochastic newsvendor setting.

According to Levin et al. [18] at times, the presence of inventory has a motivational effect on people around it. It is a common belief that large piles of goods displayed in a departmental store leads the customers to buy more. Urban [19] also observed the phenomenon and stated that we often see mass displays of items in stores that are used as psychic stock to stimulate sales of some retail items. Store window displays or stocks in shelf are regarded as a key instrument of a retailer's communication and visual merchandising strategy. Window display serves two main purposes: to identify the store and its product (e.g., promotion, merchandise, and fashion) and to induce consumers to have shopping attitudes. By showing fashional or seasonal goods, a store can show that it is contemporary. Chang et al. [20] mentioned that an increase in shelf space for an item induces more customers to buy it. To explore this, in the last three decades, the variability of inventory level-dependent demand rate on the analysis of inventory system was described by researchers like Silver and Meal [21], Silver and Peterson [22], Baker and Urban [23], Datta and Pal [24], Padmanabhan and Vrat [25], Chang et al. [20], Panda et al. [26, 27], Chang et al. [28], Sana and Chaudhuri [29], and others. There is a vast literature on inventory level-dependent demand and its overview can be found in the review article by Urban [30]. Items like fashion apparel, hi-tech product parts, periodicals, Christmas accessories, seasonal fruits, fashionable garments, and so forth have limited sales season and become outdated at the end of the season. The demand of such products is sensitive to time, on-hand stock as well as price. Recently, Sana and Chaudhuri [31], Das Roy et al. [32], Sana [33], and Sana [34] discussed effect of pricing on EOQ. To cope up with pricing effect in stock-dependent demands several authors [35–37] considered stock-price-dependent demand pattern whereas to explore the effect of time in stock-dependent demand Valliathal and Uthayakumar [38] have studied inventory model for price and time-dependent demand. Sana [39] discussed effect of pricing on time-varying demand. Wang and Grachak [40] have developed a two-echelon supply chain model where end customers demand is initial stock level-dependent. Recently Zhou et al. [41] have proposed a supply chain model where demand is sensitive to instantaneous inventory level. They have used profit sharing mechanism and quantity discount contract to coordinate the supply chain. They have assumed selling price is exogenous. As a result they have discussed coordination issues based on order quantity. However, very few authors have incorporated stock-price-time-dependent demand into supply chain and discussed coordination issue. In this paper, coordination issue is studied when the demand of the retailer's end is influenced by stock-time and unit selling price-sensitive demand rate. It is established that revenue sharing contract mechanism coordinates the system with respect to distributor, and it is shown that this mechanism coordinates the system perfectly by arbitrarily allocation of profit and leads to win-win situation for the system. The rest of the paper is organized as follows. In Section 3, the results are obtained for the centralized and decentralized scenarios. These results are used in subsequent analysis for revenue sharing mechanism. In Section 4, numerical example is presented to illustrate the development of the model. Finally, Section 5 deals with summary and some concluding remarks.

2. Mathematical Modeling

A two-stage supply chain is considered with a single distributor and a retailer, which operates for a single product. Demand rate $D(t, I(t), p)$, at retailer's end is a multivariable function of time, price, and on-hand stock level. The functional form of demand is

$$D(I(t), t, p) = Ae^{-bt} + \gamma I(t) - \beta p. \tag{2.1}$$

A (>0) is the initial demand rate, b (>0) is the time sensitive parameter of demand, that is, demand decreases with respect to time. γ (>0) reflects the elasticity of the demand rate with respect to the inventory level. $I(t)$ represent instantaneous level of inventory. β (>0) is the price sensitive parameter. p is the unit end selling price of the product. This type of demand is quite appropriate. Visual merchandising, until recently just called merchandising, is the activity of promoting the sale of goods in retail outlets. This is a mechanism to stimulate and engage stock display to encourage the sale of a product. Demand of fashion apparel, electronic goods, and so forth is sensitive to time as well as price. As time progresses, demand of the product decreases and glamorous display of the product entices its demand. If we substitute $b = 0$, then the demand will convert to instantaneous inventory level and price dependent. If we consider $\gamma = 0$, then it turns into price-sensitive declining demand. The retailer can allocate sufficient shelf space to display all items ordered from the distributor. The distributor follows the lot-for-lot policy, which is a common assumption in the literature on channel coordination. A_1 and A_2 are, respectively, fixed ordering cost of the retailer and the distributor. Unit cost of the distributor and retailer are c_d and c_r, respectively. h is the holding cost per unit quantity per unit time for the retailer. The retailer replenishes Q amounts of inventory from the distributor at the beginning of her replenishment cycle L, which is assumed to be known. This assumption is well documented in the inventory literature [6, 42]. Under this situation, the governing differential equations for the retailer are

$$\frac{dI(t)}{dt} = -Ae^{-bt} - \gamma I(t) + \beta p. \tag{2.2}$$

Solving the above equation with the condition $I(L) = 0$, we get

$$I(t) = \frac{A}{\gamma - b}\left(e^{(\gamma-b)L}e^{-\gamma t} - e^{-bt}\right) - \frac{\beta p}{\gamma}\left(e^{\gamma(L-t)} - 1\right). \tag{2.3}$$

Using the condition $I(0) = Q$, we get total order quantity of the retailer as,

$$Q = \frac{A}{\gamma - b}\left(e^{(\gamma-b)L} - 1\right) - \frac{\beta p}{\gamma}\left(e^{\gamma L} - 1\right). \tag{2.4}$$

If HC denotes the total holding cost within time interval $[0, L]$, then HC can be expressed as

$$\mathrm{HC} = h\left[\frac{Ae^{(\gamma-b)L}}{\gamma-b} - \frac{\beta p e^{\gamma L}}{\gamma}\right]\left(\frac{1-e^{-\gamma L}}{\gamma}\right) - \left[\frac{A}{\gamma-b}\left(\frac{1-e^{-bL}}{b}\right) - \frac{\beta p}{\gamma}L\right]. \tag{2.5}$$

The total profit of the retailer for entire planning horizon is $\pi_r(p) = pQ - c_r Q - h\mathrm{HC} - A_1$. On simplification,

$$\begin{aligned}\pi_r = {} & (p-c_r)\left[\frac{A}{\gamma-b}\left(e^{(\gamma-b)L}-1\right) - \frac{\beta p}{\gamma}\left(e^{\gamma L}-1\right)\right] - A_1 \\ & - h\left[\frac{Ae^{(\gamma-b)L}}{\gamma-b}\left(\frac{\left(1-e^{-\gamma L}\right)}{L} - \frac{1-e^{-bL}}{b}\right) - \frac{\beta p}{\gamma}\left(\frac{e^{L\gamma}-1}{\gamma} - L\right)\right].\end{aligned} \tag{2.6}$$

The total profit of the distributor for entire planning horizon is

$$\pi_d(c_r) = (c_r - c_d)\left[\frac{A}{\gamma-b}\left(e^{(\gamma-b)L}-1\right) - \frac{\beta p}{\gamma}\left(e^{\gamma L}-1\right)\right] - A_2. \tag{2.7}$$

From (2.6) and (2.7), it follows that the total profit for the total supply chain is obtained by $\pi_c = \pi_r(p) + \pi_d(c_r)$. On simplification we have

$$\begin{aligned}\pi_c = {} & (p-c_d)\left[\frac{A}{\gamma-b}\left(e^{(\gamma-b)L}-1\right) - \frac{\beta p}{\gamma}\left(e^{\gamma L}-1\right)\right] \\ & - h\left[\frac{A}{\gamma-b}\left(\frac{\left(1-e^{-\gamma L}\right)}{L}\left(e^{(\gamma-b)L}\right) - \frac{1-e^{-bL}}{b}\right) - \frac{\beta p}{\gamma}\left(\frac{e^{L\gamma}-1}{\gamma} - L\right)\right] - A_1 - A_2.\end{aligned} \tag{2.8}$$

Under the so-called centralized scenario all decisions are assumed to be made by a single entity, that is, the total supply chain profit obtained by (2.8) is to be optimized. The necessary condition for optimality $d\pi_c(p)/dp = 0$ yields

$$p = \frac{(A/(\gamma-b))\left(e^{(\gamma-b)L}-1\right)}{(2\beta/\gamma)\left(e^{\gamma L}-1\right)} + \frac{h\left(\left(\left(e^{\gamma L}-1\right)/\gamma\right)-L\right)}{2\left(e^{\gamma L}-1\right)} + \frac{c_d}{2} = p_c \quad \text{(say)}. \tag{2.9}$$

Moreover, $d^2\pi_c/dp^2 = -2\beta L < 0$. This leads to the following observation.

Observation 1. In a centralized scenario, total supply chain profit function is always concave.

Substituting optimal value of p from (2.9) in (2.8) we get total supply chain profit

$$\pi_c = \frac{1}{(4\beta/\gamma)(e^{\gamma L}-1)}\left[\frac{A}{\gamma-b}\left(e^{(\gamma-b)L}-1\right)+\frac{h\beta}{\gamma}\left(\frac{e^{\gamma L}-1}{\gamma}-L\right)-\frac{c_d\beta}{\gamma}\left(e^{\gamma L}-1\right)\right]^2 + \frac{c_d h\beta}{\gamma}\left(\frac{e^{\gamma L}-1}{\gamma}-L\right)-h\left[\frac{A}{\gamma-b}\left(e^{(\gamma-b)L}\frac{1-e^{-\gamma L}}{L}-\frac{1-e^{-bL}}{b}\right)\right]-A_1-A_2. \tag{2.10}$$

And optimal order quantity in integrated system is

$$Q_c = \frac{A}{2(\gamma-b)}\left(e^{(\gamma-b)L}-1\right)-\frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L}-1}{\gamma}-L\right)-\frac{c_d\beta}{2\gamma}\left(e^{\gamma L}-1\right). \tag{2.11}$$

Now consider the decentralized scenario in which it is assumed that there is no coordination, that is, all entities act independently and take decisions that maximize their respective profits. Although there are various types of approaches to analyze this situation, in this paper it is assumed that optimization takes place in a Stackelberg sequence. First the distributor is considered as Stackelberg leader, that is, the objective of the distributor is to design her move in such a way that her profit is maximized after considering all possible moves the retailer can apply. Then, for a given wholesale price c_r of the distributor, the retailer first determines her optimal decision. Now, the necessary condition for optimal profit function of the retailer obtained in (2.6), is $d\pi_r(p)/dp = 0$, yields

$$p = \frac{(A/(\gamma-b))(e^{(\gamma-b)L}-1)}{(2\beta/\gamma)(e^{\gamma L}-1)}+\frac{h(((e^{\gamma L}-1)/\gamma)-L)}{2(e^{\gamma L}-1)}+\frac{c_r}{2}. \tag{2.12}$$

Moreover, $d^2\pi_r/dp^2 = -2\beta L < 0$. This leads to the following observation.

Observation 2. In a decentralized scenario, total profit function of the retailer is always concave for the distributor's given wholesale price.

For the optimal price of the retailer, the distributor profit function is

$$\pi_d = (c_r - c_d)\left[\frac{A}{2(\gamma-b)}\left(e^{(\gamma-b)L}-1\right)-\frac{\beta c_r}{2\gamma}\left(e^{\gamma L}-1\right)-\frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L}-1}{\gamma}-L\right)\right]-A_2. \tag{2.13}$$

The necessary condition $d\pi_d(c_r)/dc_r = 0$ for maximization of $\pi_d(c_r)$ yields

$$c_r = \frac{A(e^{(\gamma-b)L}-1)}{2(\gamma-b)(\beta/\gamma)(e^{\gamma L}-1)}+\frac{c_d}{2}-\frac{h(((e^{\gamma L}-1)/\gamma)-L)}{2(e^{\gamma L}-1)}. \tag{2.14}$$

Moreover, $d^2\pi_d/dc_r{}^2 = -\beta L < 0$. This leads to the following observation.

Observation 3. In a decentralized scenario, total profit function of the distributor is always concave.

Substituting optimal value of c_r, we have optimal selling price in decentralized system as

$$p = \frac{(3A/(\gamma - b))(e^{(\gamma-b)L} - 1)}{(4\beta/\gamma)(e^{\gamma L} - 1)} + \frac{h(((e^{\gamma L} - 1)/\gamma) - L)}{2(e^{\gamma L} - 1)} + \frac{c_d}{2} = p_d \quad (\text{say}). \tag{2.15}$$

Hence, in a decentralized system, optimal profits for the retailer and distributor are

$$\begin{aligned}\pi_{rdc} = {} & \frac{1}{(4\beta/\gamma)(e^{L\gamma} - 1)}\left[\frac{A}{2(\gamma - b)}\left(e^{(\gamma-b)L} - 1\right) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L} - 1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}\left(e^{\gamma L} - 1\right)\right]^2 - A_1 \\ & + \frac{Ah(((e^{\gamma L} - 1)/\gamma) - L)}{(\gamma - b)(e^{L\gamma} - 1)}\left(e^{(\gamma-b)L} - 1\right) - \frac{hA}{\gamma - b}\left(e^{(\gamma-b)L}\frac{(1 - e^{-\gamma L})}{\gamma} - \frac{1 - e^{-bL}}{b}\right),\end{aligned} \tag{2.16}$$

$$\pi_{ddc} = \frac{1}{(2\beta/\gamma)(e^{L\gamma} - 1)}\left[\frac{A}{2(\gamma - b)}\left(e^{(\gamma-b)L} - 1\right) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L} - 1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}\left(e^{\gamma L} - 1\right)\right]^2 - A_2. \tag{2.17}$$

Now difference between total supply chain profit under centralized and decentralized scenario is

$$\begin{aligned}& \pi_c - (\pi_{rdc} + \pi_{ddc}) \\ & = (p_c - p_d)\left[\frac{A}{\gamma - b}\left(e^{(\gamma-b)L} - 1\right) - \frac{\beta}{\gamma}\left(e^{L\gamma} - 1\right)(p_c + p_d) + \frac{c_d\beta}{\gamma}\left(e^{L\gamma} - 1\right) + \frac{h\beta}{\gamma}\left(\frac{(e^{L\gamma} - 1)}{\gamma} - L\right)\right].\end{aligned} \tag{2.18}$$

Since

$$(p_c + p_d)\frac{\beta}{\gamma}\left(e^{\gamma L} - 1\right) = \frac{5}{4}\frac{A}{\gamma - b}\left(e^{(\gamma-b)L} - 1\right) + \frac{3h\beta}{4\gamma}\left(\frac{e^{\gamma L} - 1}{\gamma} - L\right) + \frac{3\beta c_d}{4\gamma}\left(e^{L\gamma} - 1\right), \tag{2.19}$$

on simplification,

$$\pi_c - (\pi_{rdc} + \pi_{ddc}) = \left[\frac{A}{4(\gamma - b)}\left(e^{(b-\gamma)L} - 1\right) - \frac{c_d\beta}{4\gamma}\left(e^{L\gamma} - 1\right) - \frac{h\beta}{4\gamma}\left(\frac{e^{L\gamma} - 1}{r} - L\right)\right]^2 > 0, \tag{2.20}$$

that is, $\pi_c > (\pi_{rddc} + \pi_{dddc})$, this leads to the following observation.

Observation 4. Under stock-price-time sensitive demand rate, total supply chain profit for the centralized scenario is higher than total profit obtained in decentralized scenario.

Optimal supply chain profit in the integrated system is greater in comparison to that of the decentralized framework, that is, we get suboptimal solution. To avoid such situations, coordination contract mechanisms are proposed by several practitioners. In this paper, revenue sharing contact mechanism is applied to check whether it coordinates the system or not. Revenue sharing contact is governed by wholesale price c_r and the percentage ρ $(0 < \rho < 1)$ of the revenue of the retailer that is shared with the distributor. In newsvendor framework, Cachon and Lariviere [7] have mentioned that the position $c_r < c_d$ ensures channel coordination and ρ determines the distribution of the channel profit between the retailer and the distributor. Under the revenue sharing contract the retailer profit function turns into

$$\begin{aligned}\pi_{rrs} = {}& (\rho p - c_r)\left[\frac{A}{\gamma - b}\left(e^{(\gamma-b)L} - 1\right) - \frac{\beta p}{\gamma}\left(e^{\gamma L} - 1\right)\right] - A_1 \\ & - h\left[\frac{A}{\gamma - b}\left(\frac{e^{(\gamma-b)L}(1 - e^{-\gamma L})}{L} - \frac{1 - e^{-bL}}{b}\right) - \frac{\beta p}{\gamma}\left(\frac{e^{L\gamma} - 1}{\gamma} - L\right)\right].\end{aligned} \tag{2.21}$$

Here our object is to optimize $\pi_{rrs}(p)$, which is a function of a single variable p. The necessary condition $d\pi_{rrs}(p)/dp = 0$, for maximization of $\pi_{rrs}(p)$ yields

$$p = \frac{(A/(\gamma - b))(e^{(\gamma-b)L} - 1)}{(2\beta\gamma/\gamma)(e^{\gamma L} - 1)} + \frac{c_r}{2\rho} + \frac{h(((e^{b\gamma} - 1)/\gamma) - L)}{2\rho(e^{\gamma L} - 1)} = {p_r}^* \quad (\text{say}). \tag{2.22}$$

For perfect coordination, if distributor increases her order quantity to the order quantity obtained in integrated system and reduces her selling price, only then the optimal system profit, which is higher than the decentralized total profit, is achievable. Comparing (2.9) and (2.22), we get unit price of the distributor as,

$$c_r = \rho c_d + (\rho - 1)\frac{h(((e^{\gamma L} - 1)/\gamma) - L)}{(e^{\gamma L} - 1)} = {c_r}^* \quad (\text{say}). \tag{2.23}$$

Substituting the optimal values of ${p_r}^*$ and ${c_r}^*$ obtained form (2.22) and (2.23) in (2.6) and (2.7), we get optimal profit share for the retailer and distributor under revenue sharing contract as

$$\begin{aligned}\pi_{rrs} = {}& \frac{\rho}{(\beta/\gamma)(e^{L\gamma} - 1)}\left[\frac{A}{2(\gamma - b)}\left(e^{(\gamma-b)L} - 1\right) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L} - 1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}(e^{\gamma L} - 1)\right]^2 - A_1 \\ & + \frac{Ah(((e^{\gamma L} - 1)/\gamma) - L)}{(\gamma - b)(e^{L\gamma} - 1)}\left(e^{(\gamma-b)L} - 1\right) - \frac{hA}{\gamma - b}\left(\frac{e^{(\gamma-b)L}(1 - e^{-\gamma L})}{\gamma} - \frac{1 - e^{-bL}}{b}\right),\end{aligned} \tag{2.24}$$

$$\pi_{drs} = \frac{(1 - \rho)}{(\beta/\gamma)(e^{L\gamma} - 1)}\left[\frac{A}{2(\gamma - b)}\left(e^{(\gamma-b)L} - 1\right) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L} - 1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}\left(e^{\gamma L} - 1\right)\right]^2 - A_2. \tag{2.25}$$

Since the coordination mechanism is acceptable to the retailer as well as distributor if their profits in coordinated system are equal or higher than the profits, which they achieve in the decentralized scenarios, that is, if $\pi_{rrs} \geq \pi_{rdc}$ and $\pi_{drs} \geq \pi_{ddc}$. Comparing (2.17) and (2.25) we have $\pi_{drs} \geq \pi_{ddc}$ if $(1-\rho) \geq 1/2 \Rightarrow 1/2 \geq \rho$. Again from (2.16) and (2.24) we have $\pi_{rrs} \geq \pi_{rdc}$ if $\rho/4 \geq 1/16 \Rightarrow \rho \geq 1/4$.

Observation 5. Under a stock-price-time-sensitive demand rate, any value of the revenue sharing component $\rho \in [1/4, 1/2]$ coordinates the system perfectly and leads to a win-win outcome for the retailer as well as distributor when distributor is Stackelberg leader.

From the above observations, it turns out that if the revenue sharing fraction falls in the interval $[1/4, 1/2]$ when distributor is the Stackelberg leader, then both parties can agree to operate jointly. But, as mentioned by Cachon and Lariviere [7] for revenue sharing contract, the final choice of revenue sharing fraction which is based on the profit split in the monetary space will reach through bargaining between two parties. Several authors provide some serious thought based on information sharing between supply chain parties. They have modeled various realistic scenarios on supply chain management based on symmetric and asymmetric information between supply chain entities. The models are based on the fact that the retailers setup/purchase costs or holding cost may be unknown to the distributor. But in this paper it is found that revenue sharing fraction ρ is independent of cost structure of both the parties involved in the system.

3. Model for Stock- and Price-Sensitive Product

If the product is sensitive to stock and price only, in that case if we substitute $b \to 0$ in (2.15) and (2.16), we obtain optimal profits for decentralized system for the retailer and distributor as

$$\begin{aligned}
\pi_{rdc} &= \frac{1}{(4\beta/\gamma)(e^{L\gamma}-1)}\left[\frac{A}{2\gamma}\left(e^{\gamma L}-1\right) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L}-1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}\left(e^{\gamma L}-1\right)\right]^2 - A_1 \\
&\quad + \frac{Ah}{\gamma}\left(\frac{e^{\gamma L}-1}{\gamma} - L\right) - \frac{hA}{\gamma}\left(\frac{(e^{\gamma L}-1)}{\gamma} - L\right), \\
\pi_{ddc} &= \frac{1}{(2\beta/\gamma)(e^{L\gamma}-1)}\left[\frac{A}{2\gamma}\left(e^{\gamma L}-1\right) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L}-1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}\left(e^{\gamma L}-1\right)\right]^2 - A_2.
\end{aligned} \tag{3.1}$$

Similarly from (2.24) and (2.25), we get the optimal profit in revenue sharing system as

$$\begin{aligned}
\pi_{rrs} &= \frac{\rho}{(\beta/\gamma)(e^{L\gamma}-1)}\left[\frac{A}{2\gamma}(e^{\gamma L}-1) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L}-1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}(e^{\gamma L}-1)\right]^2 - A_1 \\
&\quad + \frac{Ah}{\gamma}\left(\frac{e^{\gamma L}-1}{\gamma} - L\right) - \frac{hA}{\gamma}\left(\frac{(e^{\gamma L}-1)}{\gamma} - L\right), \\
\pi_{drs} &= \frac{(1-\rho)}{(\beta/\gamma)(e^{L\gamma}-1)}\left[\frac{A}{2\gamma}(e^{\gamma L}-1) - \frac{h\beta}{2\gamma}\left(\frac{e^{\gamma L}-1}{\gamma} - L\right) - \frac{c_d\beta}{2\gamma}(e^{\gamma L}-1)\right]^2 - A_2.
\end{aligned} \tag{3.2}$$

Observation 6. Under stock-price-sensitive demand rate, any value of the revenue sharing component $\rho \in [1/4, 1/2]$ coordinates the system perfectly and leads to a win-win outcome for the retailer as well as distributor when distributor is Stackelberg leader.

4. Model for Time- and Price-Sensitive Product

If the product is sensitive to time and price only, in that case if we substitute $\gamma \to 0$ in (2.15) and (2.16), we obtain optimal profit in decentralized system for the retailer and distributor as

$$\pi_{rdc} = \frac{1}{4\beta L}\left[\frac{A}{2b}\left(1-e^{-bL}\right) - \frac{h\beta L}{4} - \frac{c_d\beta L}{2}\right]^2 + \frac{AhL}{2b}\left(1-e^{-bL}\right) - \frac{hA}{b}\left(\frac{1-e^{-bL}}{b} - e^{-bL}L\right) - A_1,$$
$$\pi_{ddc} = \frac{1}{2\beta L}\left[\frac{A}{2b}\left(1-e^{-bL}\right) - \frac{h\beta L^2}{4} - \frac{c_d\beta L}{2}\right]^2 - A_2. \tag{4.1}$$

Similarly from (2.24) and (2.25), we get the optimal profit in revenue sharing system as

$$\pi_{rdc} = \frac{\rho}{\beta L}\left[\frac{A}{2b}\left(1-e^{-bL}\right) - \frac{h\beta L}{4} - \frac{c_d\beta L}{2}\right]^2 + \frac{AhL}{2b}\left(1-e^{-bL}\right) - \frac{hA}{b}\left(\frac{1-e^{-bL}}{b} - e^{-bL}L\right) - A_1,$$
$$\pi_{ddc} = \frac{(1-\rho)}{\beta L}\left[\frac{A}{2b}\left(1-e^{-bL}\right) - \frac{h\beta L^2}{4} - \frac{c_d\beta L}{2}\right]^2 - A_2. \tag{4.2}$$

Observation 7. Under time-price-sensitive demand rate, any value of the revenue sharing component $\rho \in [1/4, 1/2]$ coordinates the system perfectly and leads to a win-win outcome for the retailer as well as distributor when distributor is Stackelberg leader.

5. Numerical Example

In this section, we provide numerical examples to illustrate the above discussion. Consider the parameters values $L = 7.0$ weeks, $A = 80.0$, $b = 0.01$ unit, $\beta = 0.5$, $c_d = \$50$ per unit, $h = \$0.01$ per unit time, $\gamma = 0.5$. $A_1 = \$5000.0$, and $A_2 = \$15000.0$. Since any value of $\rho \in [1/4, 1/2]$ leads to a win-win situation, we consider that both parties agree to implement revenue sharing contract with $\rho = 0.4$. The corresponding optimal solutions are given in Table 1.

From Table 1, it can be concluded that both retailer and distributor could gain higher profit for the proposed revenue sharing contract compared to the decentralized distributor Stackleberg game. It is also observed that the proposed revenue sharing contract coordinates the system perfectly.

In Figures 1 and 2, we examine the effect of change of parameters values for revenue sharing contract with $\rho = 0.4$. We consider A, b, γ, β, h, and c_d one at a time, keeping others

Table 1: Optimal order quantity and price of retailer and distributor under various setting.

	Q	p	c_r	π_d	π_r	$\pi_d + \pi_r$
Stackelberg game	817.87	126.42	100.99	26656.52	15828.42	42484.96
Integrated system	1635.74	100.95	—	—	—	63313.25
Revenue sharing contact	1635.74	100.95	19.98	34987.81	28325.44	63313.25

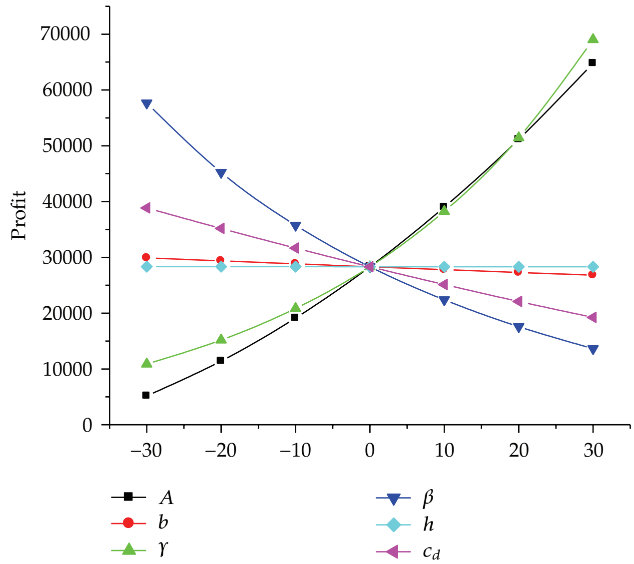

Figure 1: Behavior of retailer's profit functions with respect to percentage of change in parameters value.

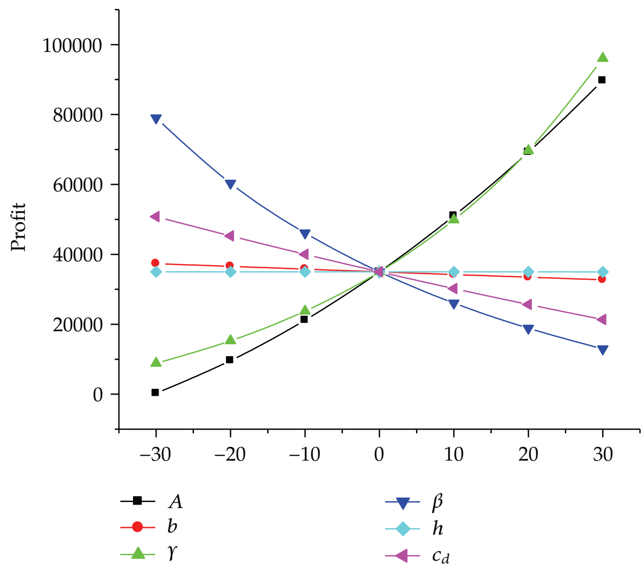

Figure 2: Behavior of distributor's profit functions with respect to percentage of change in parameters value.

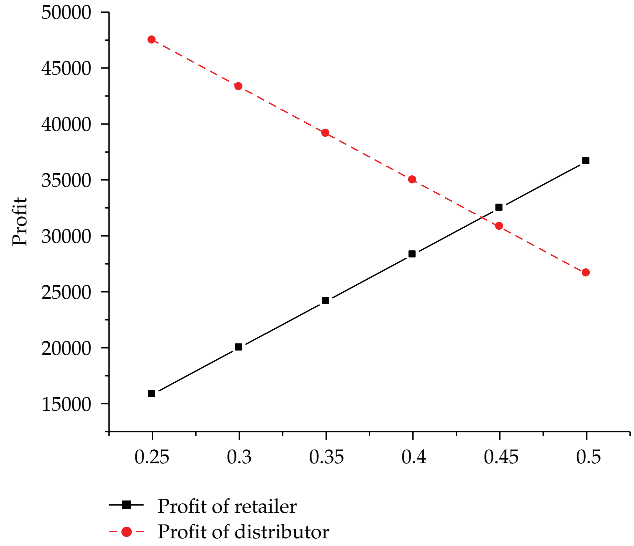

Figure 3: Behavior of retailer's and distributor's profit functions with respect to ρ.

unchanged. It is observed that the profit functions of retailer and distributor are high sensitive to the change of parameter value A, γ, moderately sensitive with respect to the parameters value β, c_d, and least sensitive to the change of parameters value h and b. As A and γ are demand parameters, increment or decrement of value has high influence on demand. Since β is price sensitive parameter of demand, if β increases then customers are reluctant to buy more. As a result the demand drops and retailer orders less quantity and sets low price to enhance demand and both parties conceive lower profit margins.

Behavior of distributor's and retailer's profit depicted in Figure 3 for $\rho \in [1/4, 1/2]$, retailers profit function increases as ρ increases and reverse trends is observed for distributor. The profit functions are equal at $\rho^* = 0.4231$, consequently retailers want to set the value of ρ above ρ^* and distributors want to settle the value below ρ^*. In such situation both, that is, if the distributor and retailer agree to share the revenue in the range $\rho \in (1/4, 1/2)$ then not only the supply chain will be coordinated but also win-win out comes for all members of the chain would be achievable.

6. Summary and Concluding Remarks

In this paper, a two-stage supply chain is studied that consists of a retailer and a distributor when downstream retailer faces stock-time- and price-dependent demand. As noted by several previous authors, price, time, and on hand stock are the significant factors influencing product demand. But no one has tried evaluating the effect of the above factors simultaneously under supply chain environment. In these paper we make an effort to discuss these issues simultaneously. Under symmetric information, a revenue-sharing contract is proposed to coordinate the behavior of the two partners in the supply chain so that the system's profit is maximized. Analytical study reveals that the revenue sharing contract coordinate the system and provides win-win situation. It is observed that revenue sharing contract coordinates the system when demand is only time-price- or stock-price-sensitive. The obtained range of revenue sharing fraction is independent of cost structure of retailer as well as distributor. Thus, the format of retailer's or distributor's profit structure does not affect the range of cost-sharing fraction which coordinates the system.

Several possible extensions of the present model could constitute future research endeavors in this field. Since the demand of the product is stock dependent and seasonal, hence to generate maximum effect of on hand stock at the introductory stage here, we have assumed the retailer will replenish her stock at the beginning of the season. As a consequence demand of the product as well as holding cost of the retailer will increase. An important extension of the paper is to check whether multireplenishment strategy is suitable for the supply chain. If the distributor provides multireplenishment opportunity then the demand of the product will be reduced due to cutback of effect of on hand stock and there will be additional costs to be incorporated by the distributor to carry inventory. It might be also interesting to investigate the effect response to multisupplier and multiretailer. The model proposed here may also be extended by introducing shortage and partial backlogging, deterioration, and so forth.

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