Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 734271, 7 pages
http://dx.doi.org/10.1155/2013/734271

Research Article

Extended $\mathscr{H}_\infty$ Estimation for Two-Dimensional Markov Jump Systems under Asynchronous Switching

Rui Zhang,[1,2] Ying Zhang,[3] Yan Zhao,[3] Jingsheng Liao,[1,2] and Baopu Li[1,2]

[1] *Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences, Shenzhen 518055, China*
[2] *The Chinese University of Hong Kong, Hong Kong*
[3] *Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, China*

Correspondence should be addressed to Ying Zhang; zhangyinghit@126.com

Received 11 January 2013; Revised 29 March 2013; Accepted 1 April 2013

Academic Editor: Engang Tian

This paper is concerned with the problem of designing $\mathscr{H}_\infty$ filters for a class of two-dimensional (2D) Markov jump systems under asynchronous switching. The problem under consideration is primarily motivated by a realistic situation that the switching of candidate filters may have a lag to the switching of system modes. Different from conventional techniques, by a suitable augmentation, the jumping process of the error system is represented by a two-component Markov chain. Then, the extended transition probabilities are provided for the error system. A stochastic Lyapunov function approach is proposed for the design of desired filters that ensure a prescribed $\mathscr{H}_\infty$ performance for admissible asynchronous switching. Finally, a numerical example is given to illustrate the effectiveness of the developed method.

1. Introduction

Networked systems have been an active topic that has attracted increasing attention of researchers [1–3]. Recently, 2D systems have gained substantial research interests due to their potential applications in many engineering fields, especially in the areas of linear iterative circuit networks, which have been extensively studied in encoding, decoding networks for linear codes, and image processing, and so forth, [4]. The challenges associated with 2D systems are their structural and dynamical complexities, and thus numerous methods have been conducted for the analysis and synthesis of such systems [5–7]. In the context of state estimation, various filtering problems for 2D systems have been widely investigated. According to the types of noise signals and performance criteria, many important filtering approaches have been proposed in the literature including the minimum mean-square state estimation [8, 9], $\mathscr{H}_\infty$ filtering [10–17], and $l_2 - l_\infty$ filtering [17].

As is well known, Markov jump linear systems, which were first introduced in [18], have the ability to capture the abrupt changes that appear in the system structure or its parameters. Markov jump systems can be regarded as a kind of multimodal systems in which the transitions among different modes are governed by a Markov chain taking values in a finite set. The past few decades have witnessed a significant progress on various aspects of Markov jump systems. The practical motivations as well as theoretical results on analysis and design for such systems can be found in several references, for example, in [19–22] for 1D case and [7, 13] for 2D case.

In particular, a lot of efforts have been focused on the filtering problem of Markov jump systems because of its importance in theory and practice [13, 22, 23]. The existing results can be broadly classified into two categories: mode-dependent and mode-independent filtering. With an adaptation sense, the mode-dependent filter design has gained popularity owing to its less conservatism. However, most of the mode-dependent results are based on the critical assumption that the switches of filters are strictly synchronized with those of the system modes. In practical case, it is inevitable that operations related to identifying the system modes and specifying the matched filter will take time, and thus mismatch of the modes between the filter and the system generally exists. Most recently, research results concerning asynchronous filtering problems have been derived [16, 24]. In spite of these developments, the problems of asynchronous filtering for 2D Markov jump systems are not fully resolved.

In this paper, we focus on 2D systems described by the Roesser model subject to Markovian jump parameters. We address the issue of designing filters such that an upper bound on $\mathscr{H}_\infty$ norm of the estimation error system for admissible asynchronous switching is minimized. By a suitable augmentation, a novel approach is adopted to model the jumping process of the error system. The $\mathscr{H}_\infty$ analysis result is derived, followed by a stochastic parameter-dependent approach. The asynchronous filter is then designed such that the error system is mean-square asymptotically stable and has a prescribed $\mathscr{H}_\infty$ performance level. It is shown from the derived results that the effect of the asynchronous behavior in reality can be comprehensively understood for 2D Markov jump systems.

Notations. The notation used throughout the paper is fairly standard. The superscript T stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space, and $P > 0$ means that P is real symmetrical and positive definite. $\mathbb{E}\{\cdot\}$ stands for the expectation operation. $l_2\{[0,\infty),[0,\infty)\}$ is the space of square summable sequences on $\{[0,\infty),[0,\infty)\}$. In symmetrical block matrices expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry and stand diag$\{\cdots\}$ for a block diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation

Consider the following class of 2D Markov jump systems:

$$\begin{aligned}\begin{bmatrix} x^h_{i+1,j} \\ x^v_{i,j+1}\end{bmatrix} &= A\left(r_{i,j}\right)\begin{bmatrix} x^h_{i,j} \\ x^v_{i,j}\end{bmatrix} + B\left(r_{i,j}\right) w_{i,j},\\ y_{i,j} &= C\left(r_{i,j}\right)\begin{bmatrix} x^h_{i,j} \\ x^v_{i,j}\end{bmatrix} + D\left(r_{i,j}\right) w_{i,j},\\ z_{i,j} &= H\left(r_{i,j}\right)\begin{bmatrix} x^h_{i,j} \\ x^v_{i,j}\end{bmatrix} + L\left(r_{i,j}\right) w_{i,j},\end{aligned} \tag{1}$$

where $x^h_{i,j} \in \mathbb{R}^{n_1}$, $x^v_{i,j} \in \mathbb{R}^{n_2}$ represent the horizontal and vertical states, respectively, $y_{i,j} \in \mathbb{R}^m$ is the measured output, $z_{i,j} \in \mathbb{R}^p$ is the objective signal to be estimated, and $w_{i,j} \in \mathbb{R}^q$ is the noise signal which belongs to $l_2\{[0,\infty),[0,\infty)\}$. $A(r_{i,j})$, $B(r_{i,j})$, $C(r_{i,j})$, $D(r_{i,j})$, $H(r_{i,j})$, and $L(r_{i,j})$ are appropriately dimensioned real valued system matrices. These matrices are functions of $r_{i,j}$, which is a discrete-time, discrete-state Markov chain taking values in a finite set $\mathscr{T} = \{1, 2, \ldots, g\}$ with transition probabilities

$$\begin{aligned} p_{mn} &= \Pr\left\{r_{i+1,j} = n \mid r_{i,j} = m\right\}\\ &= \Pr\left\{r_{i,j+1} = n \mid r_{i,j} = m\right\},\end{aligned} \tag{2}$$

where $p_{mn} \geq 0$ and $\sum_{n=1}^{g} p_{mn} = 1$, for all $m \in \mathscr{T}$. To simplify the notation, the system matrices are denoted by $\mathscr{S}_m = (A_m, B_m, C_m, D_m, H_m, L_m)$, when $r_{i,j} = m \in \mathscr{T}$.

For the asynchronous phenomenon under consideration, that is, the switches of filter gains may not coincide precisely with those of system modes, we are interested in estimating the objective signal $z_{i,j}$ by a filter with the structure as follows:

$$\begin{aligned}\begin{bmatrix} \hat{x}^h_{i+1,j} \\ \hat{x}^v_{i,j+1}\end{bmatrix} &= \left[\tau_k A_{fz} + \left(1-\tau_k\right) A_{fm}\right]\begin{bmatrix} \hat{x}^h_{i,j} \\ \hat{x}^v_{i,j}\end{bmatrix}\\ &\quad + \left[\tau_k B_{fz} + \left(1-\tau_k\right) B_{fm}\right] y_{i,j},\\ \hat{z}_{i,j} &= \left[\tau_k C_{fz} + \left(1-\tau_k\right) C_{fm}\right]\begin{bmatrix} \hat{x}^h_{i,j} \\ \hat{x}^v_{i,j}\end{bmatrix}\\ &\quad + \left[\tau_k D_{fz} + \left(1-\tau_k\right) D_{fm}\right] y_{i,j},\end{aligned} \tag{3}$$

where $\hat{x}^h_{i,j} \in \mathbb{R}^{n_1}$, $\hat{x}^v_{i,j} \in \mathbb{R}^{n_2}$ are the filter states, $\hat{z}_{i,j} \in \mathbb{R}^p$ is the estimation of $z_{i,j}$, $\mathscr{F}_m = (A_{fm}, B_{fm}, C_{fm}, D_{fm})$ and $\mathscr{F}_z = (A_{fz}, B_{fz}, C_{fz}, D_{fz})$ are the filter gains corresponding to the current and previous stage, respectively, for all $m, z \in \mathscr{T}$, and τ_k is a Bernoulli distributed white sequence specified by

$$\begin{aligned}\Pr\left\{\tau_k = 1\right\} &= \mathbb{E}\left\{\tau_k\right\} = \phi,\\ \Pr\left\{\tau_k = 0\right\} &= 1 - \mathbb{E}\left\{\tau_k\right\} = 1 - \phi.\end{aligned} \tag{4}$$

In addition, τ_k and $r_{i,j}$ are mutually independent.

In view of (1) and (3), the estimation error $\bar{z}_{i,j} = z_{i,j} - \hat{z}_{i,j}$ can be represented by the following model:

$$\begin{aligned}\begin{bmatrix} \bar{x}^h_{i+1,j} \\ \bar{x}^v_{i,j+1}\end{bmatrix} &= \bar{A}_{m,l}\begin{bmatrix} \bar{x}^h_{i,j} \\ \bar{x}^v_{i,j}\end{bmatrix} + \bar{B}_{m,l} w_{i,j},\\ \bar{z}_{i,j} &= \bar{C}_{m,l}\begin{bmatrix} \bar{x}^h_{i,j} \\ \bar{x}^v_{i,j}\end{bmatrix} + \bar{D}_{m,l} w_{i,j},\end{aligned} \tag{5}$$

where $\bar{x}^h_{i,j} = [x^{hT}_{i,j} \quad \hat{x}^{hT}_{i,j}]^T$, $\bar{x}^v_{i,j} = [x^{vT}_{i,j} \quad \hat{x}^{vT}_{i,j}]^T$ and

$$\begin{aligned}\bar{A}_{m,l} &= \Omega^T\begin{bmatrix} A_m & 0 \\ B_{fl}C_m & A_{fl}\end{bmatrix}\Omega, \qquad \bar{B}_{m,l} = \Omega^T\begin{bmatrix} B_m \\ B_{fl}D_m\end{bmatrix},\\ \bar{C}_{m,l} &= \begin{bmatrix} H_m - D_{fl}C_m & -C_{fl}\end{bmatrix}\Omega, \qquad \bar{D}_{m,l} = L_m - D_{fl}D_m,\\ \Omega &= \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I\end{bmatrix}, \quad \forall m, l \in \mathscr{T}.\end{aligned} \tag{6}$$

Accordingly, the jumping process $\{\mathscr{S}_m, \mathscr{F}_l\}$ in the error system (5) forms a two-component Markov chain $\bar{r}_{i,j}$ on

$\mathcal{T} \times \mathcal{T}$ with the extended transition probabilities $\overline{p}_{(m,l)(n,u)} = \Pr\{\mathcal{S}_n, \mathcal{F}_u \mid \mathcal{S}_m, \mathcal{F}_l\}$ given by

$$\overline{p}_{(m,l)(n,u)} = \begin{cases} p_{mn}, & u = n,\ u = l \\ \phi p_{mn}, & u \neq n,\ u = l \\ (1-\phi)\, p_{mn}, & u = n,\ u \neq l \\ 0, & u \neq n,\ u \neq l. \end{cases} \quad (7)$$

Remark 1. An important feature of the augmented system (5) lies in the fact that a two-component Markov chain with g^2 modes is considered for the jumping process, which constitutes the most significant distinction from previous results on asynchronous switching. The scalar ϕ in extended transition probabilities (7) indicates the mismatch degree of the modes between the filter and the system.

The following definitions are needed in formulating the considered problem. For more details refer to [13] and the references therein.

Definition 2. The system (5) is said to be mean-square asymptotically stable if for $w_{i,j} = 0$ and bounded boundary conditions, the following holds:

$$\lim_{i+j\to\infty} \mathbb{E}\left\{\left\|\overline{x}_{i,j}\right\|^2\right\} = 0, \quad (8)$$

where $\overline{x}_{i,j} = [\overline{x}^{hT}_{i,j} \quad \overline{x}^{vT}_{i,j}]^T$.

Definition 3. Given a scalar $\gamma > 0$, the system in (5) is said to be mean-square asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level γ, if it is mean-square asymptotically stable, and under zero initial conditions satisfies

$$\left\|\overline{z}_{i,j}\right\|_E < \gamma \left\|w_{i,j}\right\|_2, \quad (9)$$

for all nonzero $w_{i,j}$, where

$$\left\|\overline{z}_{i,j}\right\|_E = \sqrt{\mathbb{E}\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\|\overline{z}_{i,j}\right\|^2\right\}},$$
$$\left\|w_{i,j}\right\|_2 = \sqrt{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\|w_{i,j}\right\|^2}. \quad (10)$$

Then, the estimation problem of interest is stated as follows: given $\gamma > 0$, design a filter of the form in (3) such that the error system in (5) with extended transition probabilities (7) is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ disturbance attenuation level γ.

3. Main Results

In this section, we will show the procedure to design the asynchronous $\mathcal{H}_\infty$ filter, which guarantees that the error system is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ disturbance attenuation level γ. We first provide a new analysis result to check if the $\mathcal{H}_\infty$ norm of the error system is bounded by the asynchronous filter. The corresponding analysis result is formulated in the following theorem.

Theorem 4. *Consider the system* (5) *with extended transition probabilities* (7)*, and let $\gamma > 0$ be a given constant. If there exist matrices $P_{m,l} = \operatorname{diag}\{P^h_{m,l}, P^v_{m,l}\} > 0$, for all $(m,l) \in \mathcal{T} \times \mathcal{T}$, such that*

$$\begin{bmatrix} -\mathcal{P}_{n,u} & 0 & \mathcal{P}_{n,u}\overline{A}_{m,l} & \mathcal{P}_{n,u}\overline{B}_{m,l} \\ * & -I & \overline{C}_{m,l} & \overline{D}_{m,l} \\ * & * & -P_{m,l} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (11)$$

where $\mathcal{P}_{n,u} = \sum_{(n,u)\in\mathcal{T}\times\mathcal{T}} \overline{p}_{(m,l)(n,u)} P_{n,u}$, then the system in (5) *is mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance index γ.*

Proof. First, we handle the stochastic stability of the system in (5) with $w_{i,j} \equiv 0$. Construct the following index:

$$\begin{aligned}\mathcal{L}_{i,j} = \mathbb{E}\Big\{&\overline{x}^{hT}_{i+1,j} P^h\left(\overline{r}_{i+1,j}\right)\overline{x}^h_{i+1,j} + \overline{x}^{vT}_{i,j+1} P^v\left(\overline{r}_{i,j+1}\right)\overline{x}^v_{i,j+1} \\ &- \overline{x}^T_{i,j} P\left(\overline{r}_{i,j}\right)\overline{x}_{i,j} \mid \overline{x}_{i,j}, \overline{r}_{i,j} = (m,l)\Big\},\end{aligned} \quad (12)$$

where $P(\overline{r}_{i,j}) = \operatorname{diag}\{P^h(\overline{r}_{i,j}), P^v(\overline{r}_{i,j})\}$, which is represented as $P_{m,l}$ when $\overline{r}_{i,j} = (m,l)$. Then, along the evolution of the system (5) with $w_{i,j} \equiv 0$, it follows that

$$\mathcal{L}_{i,j} = \overline{x}^T_{i,j}\left\{\overline{A}^T_{m,l}\sum_{(n,u)\in\mathcal{T}\times\mathcal{T}} \overline{p}_{(m,l)(n,u)} P_{n,u}\overline{A}_{m,l} - P_{m,l}\right\}\overline{x}_{i,j}. \quad (13)$$

By Schur's complement, (11) guarantees $\mathcal{L}_{i,j} < 0$, which implies that

$$\lim_{i+j\to\infty}\mathbb{E}\left\{\left\|\overline{x}_{i,j}\right\|^2\right\} = 0. \quad (14)$$

Thus, the system is mean-square asymptotically stable; see [13] for more details.

Next, to establish the $\mathcal{H}_\infty$ performance for the system, we consider the following index:

$$\begin{aligned}\mathcal{J}_{i,j} = \mathbb{E}\Big\{&\overline{x}^{hT}_{i+1,j} P^h\left(\overline{r}_{i+1,j}\right)\overline{x}^h_{i+1,j} + \overline{x}^{vT}_{i,j+1} P^v\left(\overline{r}_{i,j+1}\right)\overline{x}^v_{i,j+1} \\ &- \overline{x}^T_{i,j} P\left(\overline{r}_{i,j}\right)\overline{x}_{i,j} + \overline{z}^T_{i,j}\overline{z}_{i,j} - \gamma^2 w^T_{i,j} w_{i,j} \mid \\ &\overline{x}_{i,j}, \overline{r}_{i,j} = (m,l)\Big\}.\end{aligned} \quad (15)$$

It is inferred from (15) that

$$\mathcal{J}_{i,j} = \zeta^T_{i,j}\Phi_{m,l}\zeta_{i,j}, \quad (16)$$

where $\zeta_{i,j} = [\overline{x}^T_{i,j} \quad w^T_{i,j}]^T$ and

$$\begin{aligned}\Phi_{m,l} = &\begin{bmatrix}\overline{A}^T_{m,l} \\ \overline{B}^T_{m,l}\end{bmatrix}\sum_{(n,u)\in\mathcal{T}\times\mathcal{T}} \overline{p}_{(m,l)(n,u)} P_{n,u}\left[\overline{A}_{m,l} \quad \overline{B}_{m,l}\right] \\ &+ \begin{bmatrix}\overline{C}^T_{m,l} \\ \overline{D}^T_{m,l}\end{bmatrix}\left[\overline{C}_{m,l} \quad \overline{D}_{m,l}\right] - \begin{bmatrix}P_{m,l} & 0 \\ 0 & \gamma^2 I\end{bmatrix}.\end{aligned} \quad (17)$$

Applying Schur's complement again, (11) yields $\mathcal{J}_{i,j} < 0$, which means that $\|\overline{z}_{i,j}\|_E < \gamma\|w_{i,j}\|_2$. Hence, the system is mean-square asymptotically stable and has a prescribed $\mathscr{H}_\infty$ disturbance attenuation performance. The proof is completed. □

Remark 5. It is worth stressing that the two-component Markov chain adopted in (5) makes the analysis criterion more concise and clear. It is shown that only condition (11) is necessary to test whether the system (5) has a prescribed disturbance attenuation level in the $\mathscr{H}_\infty$ sense for a given asynchronous filter. The analysis result can be used to clarify the relationship between the asynchronous switching and the $\mathscr{H}_\infty$ performance level.

The developments, in the above, lead to the asynchronous filter design result in the next theorem.

Theorem 6. *The system in* (5) *with extended transition probabilities* (7) *is mean-square asymptotically stable and has a prescribed $\mathscr{H}_\infty$ performance index $\gamma > 0$, if there exist matrices $P_{1m,l} = \operatorname{diag}\{P^h_{1m,l}, P^v_{1m,l}\} > 0$, $P_{3m,l} = \operatorname{diag}\{P^h_{3m,l}, P^v_{3m,l}\} > 0$, $P_{2m,l} = \operatorname{diag}\{P^h_{2m,l}, P^v_{2m,l}\}$, X_l, S_l, U_l, and $\overline{A}_{fl}, \overline{B}_{fl}, \overline{C}_{fl}, \overline{D}_{fl}$, for all $(m,l) \in \mathcal{T}\times\mathcal{T}$, such that*

$$\begin{bmatrix} \Sigma^1_{m,l} & \Sigma^2_{m,l} & 0 & \Sigma^4_{m,l} & X_l^T B_m + \overline{B}_{fl} D_m \\ * & \Sigma^3_{m,l} & 0 & \Sigma^5_{m,l} & S_l^T B_m + \overline{B}_{fl} D_m \\ * & * & -I & \Sigma^6_{m,l} & L_m - \overline{D}_{fl} D_m \\ * & * & * & \Sigma^7_{m,l} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{18}$$

where

$$\begin{aligned} \Sigma^1_{m,l} &= \sum_{(n,u)\in\mathcal{T}\times\mathcal{T}} \overline{p}_{(m,l)(n,u)} P_{1n,u} - X_l^T - X_l, \\ \Sigma^2_{m,l} &= \sum_{(n,u)\in\mathcal{T}\times\mathcal{T}} \overline{p}_{(m,l)(n,u)} P_{2n,u} - U_l^T - S_l, \\ \Sigma^3_{m,l} &= \sum_{(n,u)\in\mathcal{T}\times\mathcal{T}} \overline{p}_{(m,l)(n,u)} P_{3n,u} - U_l^T - U_l, \\ \Sigma^4_{m,l} &= \begin{bmatrix} X_l^T A_m + \overline{B}_{fl} C_m & \overline{A}_{fl} \end{bmatrix}, \\ \Sigma^5_{m,l} &= \begin{bmatrix} S_l^T A_m + \overline{B}_{fl} C_m & \overline{A}_{fl} \end{bmatrix}, \\ \Sigma^6_{m,l} &= \begin{bmatrix} H_m - \overline{D}_{fl} C_m & -\overline{C}_{fl} \end{bmatrix}, \\ \Sigma^7_{m,l} &= \begin{bmatrix} -P_{1m,l} & -P_{2m,l} \\ * & -P_{3m,l} \end{bmatrix}. \end{aligned} \tag{19}$$

In this case, the admissible filter gains are given by

$$A_{fl} = U_l^{-T}\overline{A}_{fl}, \quad B_{fl} = U_l^{-T}\overline{B}_{fl}, \quad C_{fl} = \overline{C}_{fl}, \quad D_{fl} = \overline{D}_{fl}. \tag{20}$$

Proof. First, for a matrix G_l, for all $l \in \mathcal{T}$, it is verified from the fact $(G_l - \mathscr{P}_{n,u})^T(\mathscr{P}_{n,u})^{-1}(G_l - \mathscr{P}_{n,u}) \geq 0$ that $\mathscr{P}_{n,u} - G_l^T - G_l \geq -G_l^T(\mathscr{P}_{n,u})^{-1}G_l$. Then, we know that

$$\begin{bmatrix} \mathscr{P}_{n,u} - G_l^T - G_l & 0 & G_l^T\overline{A}_{m,l} & G_l^T\overline{B}_{m,l} \\ * & -I & \overline{C}_{m,l} & \overline{D}_{m,l} \\ * & * & -P_{m,l} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{21}$$

leads to (11). Thus, consider the system in (5), and assume that the matrices $P^h_{m,l}$, $P^v_{m,l}$, G_l in (21) have the following forms:

$$\begin{gathered} P^h_{m,l} = \begin{bmatrix} P^h_{1m,l} & P^h_{2m,l} \\ * & P^h_{3m,l} \end{bmatrix}, \qquad P^v_{m,l} = \begin{bmatrix} P^v_{1m,l} & P^v_{2m,l} \\ * & P^v_{3m,l} \end{bmatrix}, \\ G_l = \begin{bmatrix} G_{1l} & G_{2l} \\ G_{4l} & G_{3l} \end{bmatrix}, \qquad G_{il} = \begin{bmatrix} X_{il} & S_{il} \\ U_{il} & U_{il} \end{bmatrix}, \quad i = 1,\ldots,4. \end{gathered} \tag{22}$$

Notice that

$$\begin{gathered} \Omega P_{m,l}\Omega^T = \begin{bmatrix} P_{1m,l} & P_{2m,l} \\ * & P_{3m,l} \end{bmatrix}, \qquad \Omega G_l \Omega^T = \begin{bmatrix} X_l & S_l \\ U_l & U_l \end{bmatrix}, \\ \Omega G_l^T \overline{A}_{m,l}\Omega^T = \begin{bmatrix} X_l^T A_m + \overline{B}_{fl} C_m & \overline{A}_{fl} \\ S_l^T A_m + \overline{B}_{fl} C_m & \overline{A}_{fl} \end{bmatrix}, \\ \Omega G_l^T \overline{B}_{m,l} = \begin{bmatrix} X_l^T B_m + \overline{B}_{fl} D_m \\ S_l^T B_m + \overline{B}_{fl} D_m \end{bmatrix}, \\ \overline{C}_{m,l}\Omega^T = \begin{bmatrix} H_m - \overline{D}_{fl} C_m & -\overline{C}_{fl} \end{bmatrix}, \\ \overline{D}_{m,l} = L_m - \overline{D}_{fl} D_m. \end{gathered} \tag{23}$$

Furthermore, define matrix variables as

$$\begin{gathered} \overline{A}_{fl} = U_l^T A_{fl}, \qquad \overline{B}_{fl} = U_l^T B_{fl}, \\ \overline{C}_{fl} = C_{fl}, \qquad \overline{D}_{fl} = D_{fl}. \end{gathered} \tag{24}$$

It is obvious from the inequality in (18) that (11) in Theorem 4 is satisfied with the filter matrices as in (24) and $\overline{A}_{m,l}$, $\overline{B}_{m,l}$, $\overline{C}_{m,l}$, $\overline{D}_{m,l}$ in (5). Hence, it follows from Theorem 4 that the system (5) with extended transition probabilities (7) is mean-square asymptotically stable and has a prescribed $\mathscr{H}_\infty$ performance. Meanwhile, the filter gains in (20) follow immediately from (24). This completes the proof. □

Remark 7. Theorem 6 provides a sufficient condition for designing an admissible $\mathscr{H}_\infty$ filter of the form in (3) for the system (1) under asynchronous switching. Compared with the well-known $\mathscr{H}_\infty$ filtering results of 1D Markov jump systems, such as [23], the structure of matrix variables in Theorem 6 is slightly involved due to the structural complexity of the 2D system (1).

Remark 8. By solving the convex problem formulated in Theorem 6, the $\mathscr{H}_\infty$ performance γ can be optimized in terms of the feasibility of the corresponding condition. The result in Theorem 6 indicates that the different ϕ in (7) brings about the different optimal γ achieved for the system (5). Thus, the effect of asynchronous switching can be readily known by comparing the $\mathscr{H}_\infty$ performance indexes, which will be shown in the next section.

4. A Numerical Example

In this section, an example is provided to demonstrate the merits of our main result. Consider a 2D Markov jump system with three operation modes as follows:

$$A_1 = \begin{bmatrix} 0.2 & 0 \\ 1 & 0.1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.8 & 0 \\ 1 & 0.6 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.5 & 0 \\ 1 & 0.9 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.2 & 1 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.8 & 1 \end{bmatrix}, \qquad C_3 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \tag{25}$$
$$B_1 = B_2 = B_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$D_1 = D_2 = D_3 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$H_1 = H_2 = H_3 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

The transition probability matrix is given by

$$p = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.6 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}. \tag{26}$$

Moreover, ϕ is set as 0.35.

For this system, the optimized $\mathscr{H}_\infty$ performance level is 2.8202 via the method in [16]. By using the filter design method in Theorem 6, the minimum $\mathscr{H}_\infty$ cost is obtained $\gamma^* = 2.1694$ as well as the resulting filter gain matrices

$$A_{f1} = \begin{bmatrix} 0.4496 & -0.2009 \\ 0.4245 & 0.0413 \end{bmatrix}, \qquad B_{f1} = \begin{bmatrix} -0.2091 \\ -0.1126 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} -0.1020 & -0.8002 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} 0.3607 & 0.0281 \\ 0.8642 & -0.0670 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -0.0999 \\ -0.6389 \end{bmatrix}, \tag{27}$$
$$C_{f2} = \begin{bmatrix} -0.0442 & -0.9759 \end{bmatrix},$$
$$A_{f3} = \begin{bmatrix} 0.2914 & 0.0082 \\ 0.5848 & -0.0314 \end{bmatrix}, \qquad B_{f3} = \begin{bmatrix} -0.1127 \\ -0.8919 \end{bmatrix},$$
$$C_{f3} = \begin{bmatrix} -0.0234 & -1.0341 \end{bmatrix}.$$

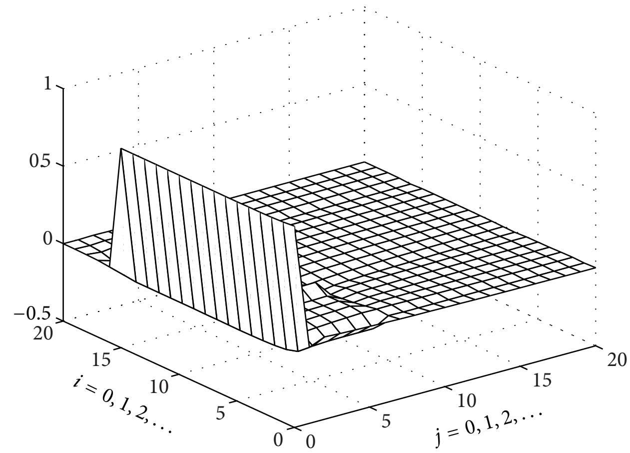

Figure 1: Filtering error $\bar{z}_{i,j}$ in Case I.

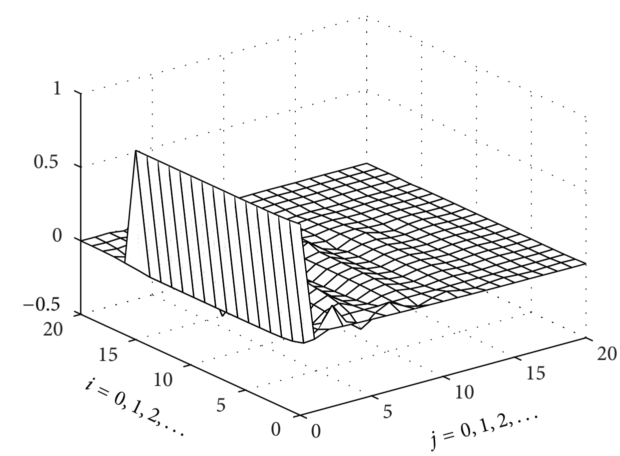

Figure 2: Filtering error $\bar{z}_{i,j}$ in Case II.

In addition, let the system initial condition $x^v_{i,0} = 0.8$, $0 \le i \le 15$, and select the following noise signals:

$$\text{Case I:} w_{i,j} = \begin{cases} 0.1, & 1 \le i, j \le 5, \\ 0, & \text{otherwise}, \end{cases}$$
$$\text{Case II:} w_{i,j} = \begin{cases} 0.5e^{-0.25j} \sin(0.5\pi j), & 1 \le i \le 10,\ j \ge 1, \\ 0, & \text{otherwise}. \end{cases} \tag{28}$$

By applying the obtained filter, the filtering errors are depicted in Figures 1 and 2, respectively, from which we can see that $\bar{z}_{i,j}$ converge to zero. The effectiveness of the designed filter is apparent.

Figure 3 shows the minimum γ versus ϕ for the design method in Theorem 6. It is observed from the curve's trend of the asynchronous design that the lower the probability ϕ is, the better the performance γ is. In this sense, the proposed method reveals a compromise between the asynchronous switching and the performance benefit. Thus, by comparing the $\mathscr{H}_\infty$ performance indexes achieved in Figure 3,

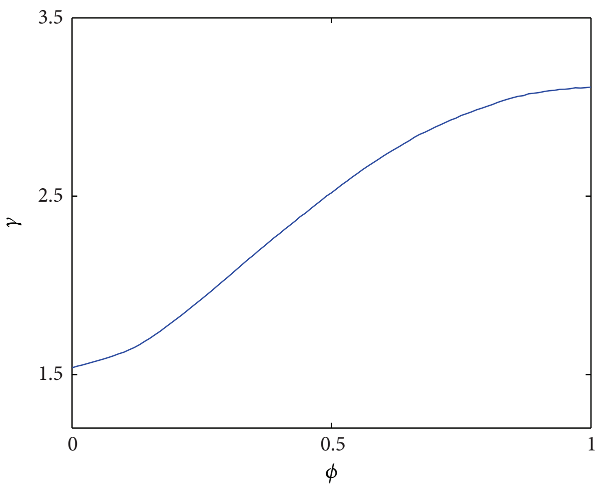

Figure 3: Optimized $\mathscr{H}_\infty$ performance.

the effect of the asynchronous switching can be comprehensively understood just as mentioned before.

5. Conclusion

This paper is dedicated to the problem of $\mathscr{H}_\infty$ estimation for a class of 2D Markov jump systems under asynchronous switching. A novel method has been proposed to model the process of the error system. The existence condition of asynchronous $\mathscr{H}_\infty$ filters has been derived to ensure the stochastic stability and $\mathscr{H}_\infty$ disturbance attenuation level of the error system. It is also shown from the obtained conditions that the effect of the asynchronous phenomenon can be easily known for 2D Markov jump systems. A numerical example is given to demonstrate the validity and the merits of the proposed theoretical results.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (61105038), by the Specialized Research Fund for the Doctoral Program of Higher Education (20122302120069), by the Basic Research Plan in Shenzhen City (JCYJ20120613135212389 and ZYC201105170376A), by the Fundamental Research Funds for the Central Universities (HIT.NSRIF.2011129), by the Guangdong Innovative Research Team Program (201001D0104648280), by the Guangzhou Nansha Research Funds (201201015), and by the Guangdong Foshan Shunde Research Funds (2012YSQ7).

References

[1] E. Tian, D. Yue, T. C. Yang, Z. Gu, and G. Lu, "TS fuzzy model-based robust stabilization for networked control systems with probabilistic sensor and actuator failure," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 3, pp. 553–561, 2011.

[2] Z. Wang, J. Liang, and Y. Liu, "Mathematical problems for complex networks," *Mathematical Problems in Engineering*, vol. 2012, Article ID 934680, 5 pages, 2012.

[3] E. Tian, D. Yue, and C. Peng, "Brief Paper: reliable control for networked control systems with probabilistic sensors and actuators faults," *IET Control Theory & Applications*, vol. 4, no. 8, pp. 1478–1488, 2010.

[4] C. Du, L. Xie, and C. Zhang, *$\mathscr{H}_\infty$ Control and Filtering of Two-Dimensional Systems*, Lecture Notes in Control and Information Sciences, Springer, Berlin, Germany, 2002.

[5] Z. Y. Feng, L. Xu, M. Wu, and Y. He, "Delay-dependent robust stability and stabilisation of uncertain two-dimensional discrete systems with time-varying delays," *IET Control Theory & Applications*, vol. 4, no. 10, pp. 1959–1971, 2010.

[6] L. Xie, C. Du, C. Zhang, and Y. C. Soh, "$\mathscr{H}_\infty$ deconvolution filtering of 2-D digital systems," *IEEE Transactions on Signal Processing*, vol. 50, no. 9, pp. 2319–2332, 2002.

[7] H. Gao, J. Lam, S. Xu, and C. Wang, "Stabilization and $\mathscr{H}_\infty$ control of two-dimensional Markovian jump systems," *IMA Journal of Mathematical Control and Information*, vol. 21, no. 4, pp. 377–392, 2004.

[8] M. Šebek, "Polynomial solution of 2-D Kalman-Bucy filtering problem," *IEEE Transactions on Automatic Control*, vol. 37, no. 10, pp. 1530–1533, 1992.

[9] J. W. Woods and C. H. Radewan, "Kalman filtering in two dimensions," *IEEE Transactions on Information Theory*, vol. 23, no. 4, pp. 473–482, 1977.

[10] N. T. Hoang, H. D. Tuan, T. Q. Nguyen, and S. Hosoe, "Robust mixed generalized $\mathscr{H}_2/\mathscr{H}_\infty$ filtering of 2-D nonlinear fractional transformation systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 12, pp. 4697–4706, 2005.

[11] S. Xu, J. Lam, Y. Zou, Z. Lin, and W. Paszke, "Robust $\mathscr{H}_\infty$ filtering for uncertain 2-D continuous systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 5, pp. 1731–1738, 2005.

[12] C. E. de Souza, L. Xie, and D. F. Coutinho, "Robust filtering for 2-D discrete-time linear systems with convex-bounded parameter uncertainty," *Automatica*, vol. 46, no. 4, pp. 673–681, 2010.

[13] L. Wu, P. Shi, H. Gao, and C. Wang, "$\mathscr{H}_\infty$ filtering for 2D Markovian jump systems," *Automatica*, vol. 44, no. 7, pp. 1849–1858, 2008.

[14] H. Xu and Y. Zou, "Robust $\mathscr{H}_\infty$ filtering for uncertain two-dimensional discrete systems with state-varying delays," *International Journal of Control, Automation and Systems*, vol. 8, no. 4, pp. 720–726, 2010.

[15] R. Zhang, Y. Zhang, C. Hu, M. Q.-H. Meng, and Q. He, "Delay-range-dependent $\mathscr{H}_\infty$ filtering for two-dimensional Markovian jump systems with interval delays," *IET Control Theory & Applications*, vol. 5, no. 18, pp. 2191–2199, 2011.

[16] R. Zhang, Y. Zhang, C. Hu, M. Q.-H. Meng, and Q. He, "Asynchronous $\mathscr{H}_\infty$ filtering for a class of two-dimensional Markov jump systems," *IET Control Theory & Applications*, vol. 6, no. 7, pp. 979–984, 2012.

[17] L. Wu, Z. Wang, H. Gao, and C. Wang, "$\mathscr{H}_\infty$ and $l_2 - l_\infty$ filtering for two-dimensional linear parameter-varying systems," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 12, pp. 1129–1154, 2007.

[18] N. N. Krasovskiĭ and È. A. Lidskiĭ, "Analytical design of controllers in systems with random attributes. I. Statement of the problem, method of solving," *Automation and Remote Control*, vol. 22, pp. 1021–1025, 1961.

[19] M. Liu, D. W. C. Ho, and Y. Niu, "Stabilization of Markovian jump linear system over networks with random communication delay," *Automatica*, vol. 45, no. 2, pp. 416–421, 2009.

[20] L. Wu, X. Su, and P. Shi, "Sliding mode control with bounded $\mathscr{L}_2$ gain performance of Markovian jump singular time-delay systems," *Automatica*, vol. 48, no. 8, pp. 1929–1933, 2012.

[21] Z. Wang, Y. Liu, and X. Liu, "Exponential stabilization of a class of stochastic system with Markovian jump parameters and mode-dependent mixed time-delays," *IEEE Transactions on Automatic Control*, vol. 55, no. 7, pp. 1656–1662, 2010.

[22] C. E. de Souza, A. Trofino, and K. A. Barbosa, "Mode-independent $\mathscr{H}_\infty$ filters for Markovian jump linear systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 11, pp. 1837–1841, 2006.

[23] L. Zhang and E.-K. Boukas, "Mode-dependent $\mathscr{H}_\infty$ filtering for discrete-time Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 6, pp. 1462–1467, 2009.

[24] L. Zhang, N. Cui, M. Liu, and Y. Zhao, "Asynchronous filtering of discrete-time switched linear systems with average dwell time," *IEEE Transactions on Circuits and Systems. I. Regular Papers*, vol. 58, no. 5, pp. 1109–1118, 2011.