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Research Article

Distributed Consensus-Based Robust Adaptive Formation Control for Nonholonomic Mobile Robots with Partial Known Dynamics

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This paper investigates the distributed consensus-based robust adaptive formation control for nonholonomic mobile robots with partially known dynamics. Firstly, multirobot formation control problem has been converted into a state consensus problem. Secondly, the practical control strategies, which incorporate the distributed kinematic controllers and the robust adaptive torque controllers, are designed for solving the formation control problem. Thirdly, the specified reference trajectory for the geometric centroid of the formation is assumed as the trajectory of a virtual leader, whose information is available to only a subset of the followers. Finally, numerical results are provided to illustrate the effectiveness of the proposed control approaches.

1. Introduction

In the past decades, cooperative control of multiple mobile robots has been receiving significant attention owing to many potential advantages of such systems over single robot. In fact, multirobot cooperative control means a group of mobile robots working cooperatively that can achieve great benefits including low cost, greater flexibility, adaptability to unknown environments, and robustness [1–4]. In the field of cooperative control, formation control has received a lot of attention from the researchers for its potential applications such as surveillance-and-security, object transportation, object manipulation, search-and-rescue, intelligent transportation systems, and exploration. The formation control means the problem of controlling the relative position and orientation of mobile robots in a group according to some desired pattern for executing a given task.

Various control approaches have been proposed in the literature for mobile robot formations, including leader-follower approach [5–10], behavior-based approach [11–13], virtual-structure approach [14–18], artificial potential approach [19–22], and graph theory [23, 24]. The main idea behind these approaches is to find suitable velocity control inputs to stabilize the closed-loop system. In the literature, formation control for multiple nonholonomic mobile robots, just simply consider the kinematic model by ignoring the robot dynamics. To design the control inputs to guarantee the stability of the closed-loop system, it is assumed that there is "perfect velocity tracking." Reference [25] proposed an error-based tracking model and designed a stable kinematic tracking controller for the nonholonomic mobile robot. Reference [26] presents a kinematic controller based on the receding-horizon leader-follower (RH-LF) control framework to solve the formation problem of multiple nonholonomic mobile robots. Reference [27] studied the tracking control problem for nonholonomic mobile robots with limited information of a desired of trajectory. Reference [28] proposed a kinematic controller for the distributed consensus-based formation control. However, the perfect velocity tracking assumption does not hold in practice, and the dynamics of robot should not be ignored and practical control strategies accounting for both the kinematic and dynamic affect should be implemented [29–31]. In [32], the decentralized cooperative robust controllers are proposed for the formation control of a group of wheeled mobile robots with dynamics. In [31], an adaptive tracking controller for the dynamic model with unknown parameters was designed for a nonholonomic mobile robot by using an adaptive backstepping approach. Though these works consider the dynamics of the mobile robot, the dynamics of the mobile robot do not have the friction and bounded disturbance. It is well known that friction plays a central, controlling role in a rich variety of physical systems. Therefore, the friction term and bounded disturbance term should not be ignored and practical control strategies accounting for the friction term and bounded disturbance term should be implemented in practice.

Motivated by the above discussions, this paper investigates the distributed consensus-based robust adaptive formation control for nonholonomic mobile robots with partial known dynamics. The contribution of this paper is given as follows. Firstly, a variable transformation is given to convert the formation control problem into a state consensus problem. Then, the distributed consensus-based kinematic controllers are developed to make a group of robots asymptotically converge to a desired geometric pattern. In this paper, the specified reference trajectory for the geometric centroid of the formation is assumed as the trajectory of a virtual leader whose information is available to only a subset of the followers. Also the followers are assumed to have only local interaction with their neighbors. It is well known in practice that the perfect knowledge of dynamic model of the wheeled mobile robot is unattainable, and it is almost impossible to obtain exact values of the parameters of the mobile robot. Therefore, this paper considers that the dynamics of the mobile robot is partial known, in which there exist some unknown factors that will affect the robust trajectory tracking of the system. Then the corresponding robust adaptive torque controllers for mobile robots are developed for guaranteeing the robust velocity tracking, and the corresponding sufficient conditions are obtained for a group of nonholonomic mobile robots asymptotically converge to a desired geometric pattern with its centroid moving along the specified reference trajectory. The rigorous proofs are given by using graph theory, matrix theory, and Lyapunov theory. Finally, simulation examples illustrate the effectiveness of the proposed controllers. Compared with existing works in the literature, the current paper has the following advantages. Firstly, the relative distance and angular for each robot with its leader are not required to be known that is different from the traditional leader-follower approach [5, 6, 8, 9, 33]. Secondly, in contrast to that only kinematic control models considered in [26, 27, 34–36], the controllers designed in this paper are based on both the kinematic and dynamic models of robots. Moreover, the dynamics of wheeled mobile robots with possible uncertainty are considered. Thirdly, in contrast to that complete knowledge of the dynamics needed in [32, 37], only partial knowledge of the dynamics is needed. Fourthly, the control laws proposed in this paper are distributed. It is not necessary to know the global information for each robot. In fact, each robot can obtain information only from its neighbors.

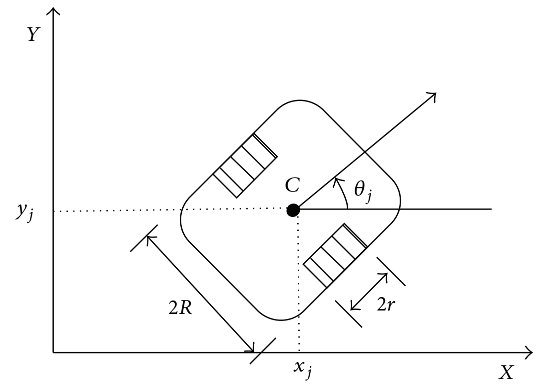


FIGURE 1: Model of a differential wheeled mobile robot.

The remainder of this paper is organized as follows. Section 2 introduces some preliminaries and gives the problem formulation. Section 3 and Section 4 present some new results on distributed formation control problem for multiple nonholonomic mobile robots. Simulations results are provided to verify the theoretical analysis in Section 5. Section 6 concludes this article.

2. Background

In this section, the model of nonholonomic wheeled mobile robot is first briefly presented. Then some notations for graph theory and nonsmooth analysis are introduced. Finally, the problem description is given.

2.1. Dynamics of Nonholonomic Wheeled Mobile Robot. Consider a multirobot system consisting of m nonholonomic wheeled mobile robots indexed by $1, 2, \ldots, m$. The nonholonomic mobile robot is shown in Figure 1. The kinematic model and dynamic model of the mobile robot j can be described as follows [38]:

$$\dot{q}_j = S(q_j)\bar{v}_j, \quad j = 1, \ldots, m, \tag{1}$$

$$\begin{aligned} M_j(q_j)\ddot{q}_j + C_j(q_j, \dot{q}_j)\dot{q}_j + F_j(\dot{q}_j) + G_j(q_j) + \tau_{dj} \\ = B_j(q_j)\tau_j - A_j^T(q_j)\lambda_j, \end{aligned} \tag{2}$$

where $q_j = [x_j, y_j, \theta_j]$ is the coordinates of the mobile robot j, x_j, y_j, and θ_j are the position and orientation of the mobile robot. v_j and w_j are the linear velocity and angular velocity, respectively, and $\bar{v}_j = [v_j, w_j]^T$. $S(q_j)$ is the Jacobian matrix, and $S(q_j) = \begin{bmatrix} \cos\theta_j & 0 \\ \sin\theta_j & 0 \\ 0 & 1 \end{bmatrix}$. $M_j(q_j) \in \mathbb{R}^{3\times 3}$ is a symmetric positive definite inertia matrix, $C_j(q_j, \dot{q}_j) \in \mathbb{R}^{3\times 3}$ is the bounded centripetal and coriolis matrix, $F_j(\dot{q}_j) \in \mathbb{R}^{3\times 1}$ denotes surface friction, $G_j(q_j) \in \mathbb{R}^{3\times 1}$ is the gravitational vector, and τ_{dj} denotes bounded unknown disturbances including unstructured unmodeled dynamics. $B_j(q_j) \in \mathbb{R}^{3\times 2}$ is the input transformation matrix, $\tau_j \in \mathbb{R}^{2\times 1}$ is the control torque vector,

$A_j(q) \in \mathbb{R}^{1\times 3}$ is the matrix associated with the constraints, and $\lambda_j \in \mathbb{R}^{1\times 1}$ is the vector of constraint forces.

The dynamic model (2) has the following properties [38].

Property 1. The inertia matrix $M_j(q_j)$ is symmetric positive definite and satisfies the following inequality:

$$m_1 \|q_j\|^2 \le q_j^T M_j(q_j) q_j \le m_2 \|q_j\|^2, \quad q_j \in \mathbb{R}^3, \tag{3}$$

where m_1, m_2 are positive constants, and $\|\cdot\|$ is the standard Euclidean norm.

Property 2. $\dot{M}_j(q_j) - 2C_j(q_j, \dot{q}_j)$ is skew symmetric; that is to say,

$$\xi^T \left[\frac{1}{2}\dot{M}_j(q_j) - C_j(q_j, \dot{q}_j)\right] \xi = 0. \quad \forall \xi \in \mathbb{R}^3. \tag{4}$$

2.2. Graph Theory. The communication topology among robots is presented by a weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a vertex set $\mathcal{V} = \{v_1, \ldots, v_m\}$, an edges set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij})_{m\times m}$. Here, each node v_i in $\mathcal{V}$ represents a robot i, and each edge $(v_i, v_j) \in \mathcal{E}$ in a weighted undirected graph represents an information link from robot j to robot i, which means that the robots i and j can receive information from each other. The weighted adjacency matrix $\mathcal{A}$ of a digraph $\mathcal{G}$ is defined $a_{jj} = 0$ for any $\mathcal{V}_j \in \mathcal{V}$; that is, self-edges are not allowed, $a_{ji} > 0$ if $(v_j, v_i) \in \mathcal{E}$, $a_{ji} = 0$ otherwise, where a_{ji} is the weight of the link (v_j, v_i). Note that here $a_{ji} = a_{ij}$, $\forall j \ne i$, since $(v_j, v_i) \in \mathcal{E}$ implies $(v_i, v_j) \in \mathcal{E}$. We can say that v_j is a neighbor vertex of v_i, if $(v_j, v_i) \in \mathcal{E}$. The neighbor set of node j is defined as

$$\mathcal{N}_j = \{v_i \in \mathcal{V} : a_{ji} \ne 0\} = \{v_i \in \mathcal{V} : (j, i) \in \mathcal{E}\}. \tag{5}$$

A path in the undirected graph $\mathcal{G}$ is a sequence of edges in the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots$, where $v_{i_j} \in \mathcal{V}$. We call an undirected graph $\mathcal{G}$ connected if for any different nodes v_i and v_j in $\mathcal{V}$ there exists an undirected path.

The Laplacian matrix $L = (l_{ji})_{m\times m}$ associated with $\mathcal{A}$ for graph $\mathcal{G}$ is defined as $l_{ji} = -a_{ji}$ for $j \ne i$, and $l_{jj} = \sum_{i=1, i\ne j}^{m} a_{ji}$, $j, i \in \{1, \ldots, m\}$. For an undirected graph, L is symmetric positive semidefinite.

Lemma 1 (Chung [39]). *Assume that $\mathcal{G}$ is a weighted undirected graph with Laplacian matrix L; then $\mathcal{G}$ is connected if and only if the matrix L has an eigenvalue zero with multiplicity* 1 *and corresponding eigenvector* **1**, *and all other eigenvalues are positive.*

2.3. Nonsmooth Analysis. In what follows, some elements from nonsmooth analysis will be presented. Consider a vector differential equation with a discontinuous right-hand side as

$$\dot{x} = f(t, x), \tag{6}$$

where $f(t, x)$ is measurable and essentially locally bounded. The vector function $x(\cdot)$ is called a Filippov solution [40] of (6) if $x(\cdot)$ is absolutely continuous and satisfies

$$\dot{x} \in \mathcal{K}[f](t, x) \tag{7}$$

almost everywhere where

$$\mathcal{K}[f](t, x) \equiv \overline{\mathrm{co}} \left\{ \lim_{x_i \to x} f(x_i) \mid x_i \notin \Omega_v \right\}, \tag{8}$$

where Ω_v denotes the set of measure zero that contains the set of points where f is not differentiable and $\overline{\mathrm{co}}$ denotes the convex closure.

Lemma 2 (see [40]). *The Filippov set-value map has the following useful properties.*

(1) *Consistency: if $f : \mathbb{R}^d \to \mathbb{R}^m$ is continuous at $x \in \mathbb{R}^d$, then*

$$\mathcal{K}[f](x) = \{f(x)\}. \tag{9}$$

(2) *Sum Rule: if function $f_1, f_2 : \mathbb{R}^d \to \mathbb{R}^m$ are locally bounded at $x \in \mathbb{R}^d$, then*

$$\mathcal{K}[f_1 + f_2](x) \subseteq \mathcal{K}[f_1](x) + \mathcal{K}[f_2](x). \tag{10}$$

Moreover, if either f_1 or f_2 is continuous at x, then equality holds.

Lyapunov theorems have been extended to nonsmooth systems in [41]. The following chain rule provides a calculus for the time derivative of the energy function in the nonsmooth case.

Definition 3 (see [42]). Let $V(x)$ be a locally Lipschitz continuous function. The generalized gradient of $V(x)$ is given by

$$\partial V(x) \triangleq \mathrm{co}\left\{\lim \nabla V(x) \mid x_i \longrightarrow x,\ x_i \in \Omega_v \cap \overline{N}\right\}, \tag{11}$$

where co denotes the convex hull, Ω_v is the set of Lebesgue measure zero, where ∇V does not exist, and $\overline{N}$ is an arbitrary set of zero measure.

In this paper, the candidate Lyapunov function V we use is smooth and hence regular, while its generalized gradient is a singleton which is equal to its usual gradient everywhere in the state space: $\partial V(x) = \{\nabla V(x)\}$.

Definition 4 (see [40]). Consider the vector differential equation (6), a set-valued map $\mathcal{K} : \mathbb{R}^d \to \mathcal{B}(\mathbb{R})$, the set-valued Lie derivative of V with respect to (6) is defined as

$$\dot{\tilde{V}} \triangleq \bigcap_{\xi \in \partial V} \xi^T \mathcal{K}[f](t, x). \tag{12}$$

In what follows, we introduce a Lyapunov stability theorem in terms of the set-valued map $\dot{\tilde{V}}$.

Lemma 5 (see [41]). *For* (6), *let $f(t, x)$ be locally essentially bounded and $0 \in \mathcal{K}[f](t, 0)$ in a region $Q \supset \{t \mid t_0 \le t \le \infty\} \times \{x \in \mathbb{R}^d \mid \|x\| < r\}$, where $r > 0$. Also, let $V : \mathbb{R}^d \to \mathbb{R}$ be a regular function satisfying*

$$V(t, 0) = 0, \quad 0 < V_1(\|x\|) \le V(t, x) \le V_2(\|x\|), \quad \textit{for } x \ne 0, \tag{13}$$

in Q for some V_1 and V_2 belonging to class $\mathscr{K}$. If there exists a class $\mathscr{K}$ function $w(\cdot)$ in Q such that the set-valued Lie derivative of $V(x)$ satisfies

$$\max \dot{\tilde{V}}(t,x) \le -w(x) < 0, \quad \text{for } x \ne 0, \tag{14}$$

then the solution $x \equiv 0$ is asymptotically stable.

2.4. Problem Formulation. In this paper, the desired geometric pattern $\mathscr{F}$ of m mobile robots is described by the orthogonal coordinates (p_{jx}, p_{jy}) as follows:

$$\sum_{j=1}^{m} p_{jx} = p_{0x}, \qquad \sum_{j=1}^{m} p_{jy} = p_{0y}, \tag{15}$$

where (p_{0x}, p_{0y}) denotes the center of $\mathscr{F}$. Without loss of generality, assume that $p_{0x} = 0$, $p_{0y} = 0$.

The objective of this paper is to design the control inputs v_j and w_j for the wheeled nonholonomic mobile robot j using its states $(q_j, \dot{q}_j)$ and (p_{jx}, p_{jy}) as well as its neighbors' states $(q_i, \dot{q}_i)$ and (p_{ix}, p_{iy}) for $i \in \mathscr{N}_j$, such that

(a) the group of mobile robots converges to the desired formation $\mathscr{F}$;

(b) each robot in group converges to the desired orientation θ_0;

(c) the geometric centroid of the formation converges to the desired reference trajectory (x_0, y_0);

that is to say,

$$\lim_{t\to\infty} \begin{bmatrix} x_j - x_i \\ y_j - y_i \end{bmatrix} = \begin{bmatrix} p_{jx} - p_{ix} \\ p_{jy} - p_{iy} \end{bmatrix}, \tag{16}$$

$$\lim_{t\to\infty} \left(\theta_j - \theta_0\right) = 0, \tag{17}$$

$$\lim_{t\to\infty} \left(\sum_{j=1}^{m} \frac{x_j}{m} - x_0 \right) = 0, \qquad \lim_{t\to\infty} \left(\sum_{j=1}^{m} \frac{y_j}{m} - y_0 \right) = 0, \tag{18}$$

where (x_0, y_0, θ_0) can be considered as the posture of a virtual leader 0, which does not have to be an actual robot but is specified by

$$\dot{x}_0 = v_0 \cos\theta_0, \qquad \dot{y}_0 = v_0 \sin\theta_0, \qquad \dot{\theta}_0 = w_0. \tag{19}$$

Hereafter, the m robots in system (1) are called followers.

The connection weight between robot j and the virtual leader 0 is described by $B = \operatorname{diag}\{b_1, b_2, \ldots, b_m\}$, in which $b_j > 0$ if robot j can obtain information from the virtual leader 0, $b_j = 0$ otherwise. Note that if the undirected graph $\mathscr{G}$ is connected, it then follows that the matrix $L + B = L + \operatorname{diag}\{b_1, \ldots, b_m\}$ and the matrix $\mathscr{M} = \operatorname{diag}\{L+B, L+B\}$ are symmetric positive definite.

In this paper, the following assumptions are needed for achieving our control objective.

Assumption 6. The θ_j for $(0 \le j \le m)$ is bounded, w_j for $(0 \le j \le m)$ is persistently exciting, and $|w_j| \le w_{\max}$.

Remark 7. w_j is persistently exciting, which means that w_j does not converge to 0. The assumption is because of the fact that the wheeled mobile robot system is nonholonomic.

Assumption 8. There exists at least one follower which can obtain information from the virtual leader.

Remark 9. Note from Assumption 8 that all follower robots do not need to obtain the information from the virtual leader; that is to say, the desired reference trajectory is not required to available for each robot, which is different from the existing works in [35, 43].

The following notations will be used throughout this paper. Let I_m denote the $m \times m$ identity matrix, $0_{m\times m}$ denote the $m \times m$ zero matrix, and $\mathbf{1}_m = [1, 1, \ldots, 1]^T \in R^m$ ($\mathbf{1}$ for short, when there is no confusion). $\lambda_{\min}(\mathscr{M})$ and $\lambda_{\max}(\mathscr{M})$ are the smallest and the largest eigenvalues of the matrix $\mathscr{M}$, respectively.

3. Distributed Control Algorithm

To achieve the control objective (16)–(18), the following transformation is defined to convert the formation control problem for multiple nonholonomic mobile robots into a state consensus problem:

$$\begin{aligned} z_{1j} &= \theta_j, \\ z_{2j} &= \left(x_j - p_{jx}\right)\cos\theta_j + \left(y_j - p_{jy}\right)\sin\theta_j \\ &\quad + k_0 \operatorname{sign}\left(u_{1j}\right) z_{3j}, \\ z_{3j} &= \left(x_j - p_{jx}\right)\sin\theta_j - \left(y_j - p_{jy}\right)\cos\theta_j, \\ u_{1j} &= w_j, \\ u_{2j} &= v_j - \left(1 + k_0^2\right) u_{1j} z_{3j} + k_0 \left|u_{1j}\right| z_{2j}, \end{aligned} \tag{20}$$

where u_{1j} and u_{2j} are control inputs, $0 \le j \le m$, $k_0 > 0$, and $\operatorname{sign}(\cdot)$ is the signum function. The definitions in (20) yield the following dynamic system as

$$\begin{aligned} \dot{z}_{1j} &= u_{1j}, \\ \dot{z}_{2j} &= u_{2j}, \\ \dot{z}_{3j} &= u_{1j} z_{2j} - k_0 \left|u_{1j}\right| z_{3j}. \end{aligned} \tag{21}$$

Then the control objective is changed to design u_{1j} and u_{2j} such that the following equations are satisfied:

$$\lim_{t\to\infty} \left(z_{1j} - z_{10}\right) = 0, \tag{22}$$

$$\lim_{t\to\infty} \left(z_{2j} - z_{20}\right) = 0, \tag{23}$$

$$\lim_{t\to\infty} \left(z_{3j} - z_{30}\right) = 0, \tag{24}$$

$$\lim_{t\to\infty} \left(u_{1j} - u_{10}\right) = 0. \tag{25}$$

Lemma 10. *If* (22)–(25) *hold for* $0 \leq j \leq m$, *then the* m *mobile robots can converge to the formation pattern* $\mathcal{F}$*; that is,* (16)–(18) *can be satisfied.*

Proof. Due to the fact that it is similar to the proof of Lemma 3.1 in [28], it is therefore omitted. □

In practice, it is well known that the dynamics model of the wheel mobile robot may have unknown dynamical parameters and bounded unknown disturbances, which will affect the robust trajectory tracking of the system; that is to say, the "perfect velocity tracking" for robot may not hold. Hence, the following desired control inputs for the mobile robot j are proposed in this paper as

$$\begin{aligned} u_{1jr} = {} & u_{10} - \alpha \sum_{i\in\mathcal{N}_j} a_{ji}\left(z_{1j} - z_{1i}\right) - \alpha b_j\left(z_{1j} - z_{10}\right) \\ & - \beta \operatorname{sign}\left(\sum_{i\in\mathcal{N}_j} a_{ji}\left(z_{1j} - z_{1i}\right) + b_j\left(z_{1j} - z_{10}\right)\right), \end{aligned} \tag{26}$$

$$\begin{aligned} u_{2jr} = {} & -\alpha \sum_{i\in\mathcal{N}_j} a_{ji}\left(z_{2j} - z_{2i}\right) - \alpha b_j\left(z_{2j} - z_{20}\right) \\ & - \beta \operatorname{sign}\left(\sum_{i\in\mathcal{N}_j} a_{ji}\left(z_{2j} - z_{2i}\right) + b_j\left(z_{2j} - z_{20}\right)\right), \end{aligned} \tag{27}$$

where $j = 1, \ldots, m$, b_j is a positive constant if the virtual leader's position is available to the follower j, and $b_j = 0$ otherwise, $|\dot{z}_{20}| \leq \kappa$, κ is a positive constant, α is a nonnegative constant, and β is a positive constant and satisfies $\beta > \kappa$.

Define the auxiliary velocity tracking error as

$$\tilde{u}_j = \begin{bmatrix} \tilde{u}_{wj} \\ \tilde{u}_{vj} \end{bmatrix} = u_{jr} - u_j = \begin{bmatrix} u_{1jr} \\ u_{2jr} \end{bmatrix} - \begin{bmatrix} u_{1j} \\ u_{2j} \end{bmatrix}, \tag{28}$$

where $u_{jr} = [u_{1jr}, u_{2jr}]^T$ and $u_j = [u_{1j}, u_{2j}]^T$. Then the dynamic system (21) becomes in the following form:

$$\dot{z}_{1j} = u_{1jr} - \tilde{u}_{wj}, \tag{29}$$

$$\dot{z}_{2j} = u_{2jr} - \tilde{u}_{vj}, \tag{30}$$

$$\dot{z}_{3j} = \left(u_{1jr} - \tilde{u}_{wj}\right) z_{2j} - k_0 \left|u_{1jr} - \tilde{u}_{wj}\right| z_{3j}. \tag{31}$$

Substitute (26) into the dynamic system (29) and (30). Then the closed-loop system (29) and (30) can be written as

$$\begin{aligned} \dot{z}_{1*} = {} & -\alpha (L + B) z_{1*} + \alpha B \mathbf{1}_m z_{10} \\ & - \beta \operatorname{sign}\left((L + B) z_{1*} - B \mathbf{1}_m z_{10}\right) + \mathbf{1}_m u_{10} - \hat{u}_w, \\ \dot{z}_{2*} = {} & -\alpha (L + B) z_{2*} + \alpha B \mathbf{1}_m z_{20} \\ & - \beta \operatorname{sign}\left((L + B) z_{2*} - B \mathbf{1}_m z_{20}\right) - \hat{u}_v, \end{aligned} \tag{32}$$

where $z_{1*} = [z_{11}, \ldots, z_{1m}]^T$ and $z_{2*} = [z_{21}, \ldots, z_{2m}]^T$, $\hat{u}_w = [\tilde{u}_{w1}, \ldots, \tilde{u}_{wm}]^T$, and $\hat{u}_v = [\tilde{u}_{v1}, \ldots, \tilde{u}_{vm}]^T$. Let $\tilde{z}_{1*} = z_{1*} - \mathbf{1}_m z_{10}$ and $\tilde{z}_{2*} = z_{2*} - \mathbf{1}_m z_{20}$. Then

$$\begin{aligned} \dot{\tilde{z}}_{1*} &= -\alpha (L + B) \tilde{z}_{1*} - \beta \operatorname{sign}\left((L + B) \tilde{z}_{1*}\right) - \hat{u}_w, \\ \dot{\tilde{z}}_{2*} &= -\alpha (L + B) \tilde{z}_{1*} - \beta \operatorname{sign}\left((L + B) \tilde{z}_{2*}\right) - \mathbf{1}_m \dot{z}_{20} - \hat{u}_v, \end{aligned} \tag{33}$$

where the fact that $L\mathbf{1}_m z_{10} = 0$ has been applied. Let $Z = [z_{1*}, z_{2*}]^T = [Z_1, \ldots, Z_{2m}]^T$, $\tilde{Z} = [\tilde{z}_{1*}, \tilde{z}_{2*}]^T = [\tilde{Z}_1, \ldots, \tilde{Z}_{2m}]^T$, and $\mathbf{f}_0 = [\underbrace{0, \ldots, 0}_{m}, \underbrace{z_{20}, \ldots, z_{20}}_{m}]^T$. Hence, the error dynamic system (33) can be rewritten in a vector form as

$$\dot{\tilde{Z}} = -\alpha \mathcal{M} \tilde{Z} - \beta \operatorname{sign}\left(\mathcal{M} \tilde{Z}\right) - \dot{\mathbf{f}}_0 - \hat{u}, \tag{34}$$

where $\hat{u} = [\hat{u}_w, \hat{u}_v]^T = [\tilde{u}_{w1}, \ldots, \tilde{u}_{wm}, \tilde{u}_{v1}, \ldots, \tilde{u}_{vm}]^T$.

4. Adaptive Dynamic Controller Design

4.1. Robot Model and Its Properties. According to (20) and the definition of $\bar{v}_j$ in Section 2.2, it is easy to obtain that

$$\bar{v}_j = \begin{bmatrix} 1 & 0 \\ A & 1 \end{bmatrix} u_j, \tag{35}$$

where $A = (1 + k_0^2) - k_0 \operatorname{sign}(u_{1j}) z_{2j}$. Then, it follows from (1) that we have

$$\dot{q}_j = s(q)_j \bar{v}_j = S\left(q_j\right) \begin{bmatrix} 1 & 0 \\ A & 1 \end{bmatrix} u_j = \hat{S}\left(q_j\right) u_j, \tag{36}$$

where

$$\hat{S} = \begin{bmatrix} \left(1 + k_0^2\right) z_{3j} \cos\theta_j + k_0 \operatorname{sign}\left(u_{1j}\right) z_{2j} & \cos\theta_j \\ \left(1 + k_0^2\right) z_{3j} \cos\theta_j + k_0 \operatorname{sign}\left(u_{1j}\right) z_{2j} & \cos\theta_j \\ 1 & 0 \end{bmatrix}. \tag{37}$$

Hence, the dynamics (2) of the mobile robot can be rewritten as follows:

$$\begin{aligned} & \hat{S}^T M_j \hat{S} \dot{u}_j + \hat{S}^T \left(M_j \dot{\hat{S}} + C_j \hat{S}\right) u_j + \hat{S}^T F_j + \hat{S}^T G_j \\ & \quad = \hat{S}^T B \tau_j - \hat{S}^T \tau_{dj}, \quad j = 1, \ldots, m; \end{aligned} \tag{38}$$

that is,

$$\overline{M}_j \dot{u}_j + \overline{C}_j u_j + \overline{F}_j + \overline{G}_j = \bar{\tau}_j - \bar{\tau}_{dj}, \quad j = 1, \ldots, m, \tag{39}$$

where $\overline{M}_j = \hat{S}^T M_j \hat{S}$ is a symmetric positive definite inertia matrix. $\overline{C}_j = \hat{S}^T (M_j \dot{\hat{S}} + C_j \hat{S})$ is the centripetal and coriolis matrix, $\overline{G}_j = \hat{S}^T G_j$ is the gravitation vector, $\overline{G}_j = 0$. $\overline{F}_j = \hat{S}^T F_j$ is the surface friction, $\bar{\tau}_{dj} = \hat{S}^T \tau_{dj}$ denotes the bounded unknown disturbances including unstructured unmodeled dynamics, and $\bar{\tau}_j = \hat{S}^T B \tau_j$ is the input vector.

Similar to the Properties 1 and 2 in Section 2.2, (39) has the following properties.

Property 3. The inertia matrix $\overline{M}_j(q_j)$ is symmetric positive definite.

Proof. It is easy to verify the result, and it is therefore omitted here. □

Property 4. The matrix $\dot{\overline{M}}_j - 2\overline{C}_j$ is skew symmetric.

Proof. The derivative of the inertia matrix and the centripetal and coriolis matrix are given by

$$\dot{\overline{M}}_j(q_j) = \dot{\widehat{S}}^T M_j + \widehat{S}^T \dot{M}_j \widehat{S} + \widehat{S}^T M_j \dot{\widehat{S}}, \qquad 2\overline{C}_j = 2\widehat{S}^T\left(M_j\dot{\widehat{S}} + C_j\widehat{S}\right). \tag{40}$$

Since $\dot{M}_j - 2C_j$ is skew symmetric and M_j is symmetric positive definite, it follows that

$$\begin{aligned}\dot{\overline{M}}_j - 2\overline{C}_j &= \dot{\widehat{S}}^T M_j\widehat{S} + \widehat{S}^T\dot{M}_j\widehat{S} + \widehat{S}^T M_j\dot{\widehat{S}} - 2\widehat{S}^T\left(M_j\dot{\widehat{S}} + C_j\widehat{S}\right)\\ &= \dot{\widehat{S}}^T M_j\widehat{S} - \widehat{S}^T M_j\dot{\widehat{S}} + \widehat{S}^T\left(\dot{M}_j - 2C_j\right)\widehat{S}\\ &= \widehat{S}^T\left(\dot{M}_j - 2C_j\right)\widehat{S}.\end{aligned} \tag{41}$$

Hence, the matrix $\dot{\overline{M}}_j - 2\overline{C}_j$ is skew symmetric. □

4.2. Controller Design. Taking the derivative of (28) and multiplying by the inertia matrix $\overline{M}_j$ to both sides of (28) give

$$\begin{aligned}\overline{M}_j\dot{\widetilde{u}}_j &= \overline{M}_j\dot{u}_{jr} - \overline{M}_j\dot{u}_j\\ &= -\overline{C}_j\widetilde{u}_j - \overline{\tau}_j + f_j\left(u_{jr}, \dot{u}_{jr}\right) + w_j(t), \quad j = 1,\dots,m,\end{aligned} \tag{42}$$

where $f_j(u_{jr}, \dot{u}_{jr}) = \overline{M}_j\dot{u}_{jr} + \overline{C}_j u_{jr}$ is composed of known quantities and the disturbance term is

$$w_j(t) = \Delta_j + \overline{\tau}_{dj}, \quad j = 1,\dots,m, \tag{43}$$

with Δ_j representing any model uncertainties and unmodeled dynamics and $\overline{\tau}_{dj}$ being the unknown bounded disturbance which could represent any inaccurately modeled dynamics.

Lemma 11 (bounds on the disturbance term, [30]). *The disturbance term $w_j(t)$ is bounded according to*

$$\left\|w_j(t)\right\| \le C_0 + C_1\left\|\widetilde{u}_j\right\| + C_2\left\|\widetilde{u}_j\right\|^2 = Y_j\Theta_j, \tag{44}$$

with C_0, C_1, C_2 depending on the terms like the disturbance bound, the changes in the mass of the robot due to payload, and friction coefficients with Y_j being a known regression vector.

When the robot dynamics are partially known, the torque control algorithm for the dynamics system (42) is designed to be

$$\overline{\tau}_j = K_j\widetilde{u}_j + f_j\left(u_{jr}, \dot{u}_{jr}\right) + \mu_{jr}, \tag{45}$$

where K_j is a symmetric positive-definite matrix defined by $K_j = k_j I_2$ with k_j being a positive gain constant and $I_2 \in \mathbb{R}^{2\times 2}$ being the identity matrix. The nonlinear term μ_{jr} is an adaptive robustifying term and is defined as [44]

$$\mu_{jr} = \frac{\widetilde{u}_j\left(Y_j\widehat{\Theta}_j\right)^2}{\left(Y_j\widehat{\Theta}_j\right)\left\|\widetilde{u}_j\right\| + \delta_j}, \qquad \dot{\delta}_j = -\gamma_j\delta_j, \quad \delta_j(0) = C_\delta > 0, \tag{46}$$

where γ_j and C_δ are positive design constants, $Y_j\widehat{\Theta}_j$ is the adaptive estimate of the known function $Y_j\Theta_j$, $\widehat{\Theta}_j$ is the estimate of Θ_j, and the parameter turning law for the estimate $\widehat{\Theta}_j$ is defined as

$$\dot{\widehat{\Theta}}_j = \Gamma_j Y_j\left\|\widetilde{u}_j\right\|, \tag{47}$$

with Γ_j being a symmetric and positive definite matrix. Let $\widetilde{\Theta}_j$ be the estimation error of the parameter turning law, and $\widetilde{\Theta}_j = \Theta_j - \widehat{\Theta}_j$. It then follows that $\dot{\widetilde{\Theta}}_j = -\dot{\widehat{\Theta}}_j$.

Substituting (45) into (42) and writing it in a vector form give

$$\overline{M}(q)\dot{\widetilde{u}} + \overline{C}(q,\dot{q})\widetilde{u} = -K\widetilde{u} - \mu_r + w(t), \tag{48}$$

where $\overline{M}(q)$, $\overline{C}(q,\dot{q})$, and K are the block diagonal matrices of $\overline{M}_j(q_j)$, $\overline{C}_j(q_j,\dot{q}_j)$, and K_j, respectively, $\mu_r = [\mu_{1r},\dots,\mu_{mr}]^T$, $w(t) = [w_1,\dots,w_m]^T$ with $\overline{\tau}_d = [\overline{\tau}_{d1},\dots,\overline{\tau}_{dm}]^T$ and $\Delta = [\Delta_1,\dots,\Delta_m]^T$ under (43).

Theorem 12. *Suppose that the communication graph $\mathscr{G}$ is connected, Assumption 8 is satisfied, the velocity controllers for (29) and (30) are, respectively, designed by (26) and (27), and the torque control input for the dynamics system (42) is designed by (45), if the control gains are chosen as $\alpha > 1/2\lambda_{\max}(\mathscr{M})$, $\beta > \kappa$, and $k_{\max} > \lambda_{\max}(\mathscr{M})/2$, where $k_{\max} = \max\{k_1, k_2,\dots,k_m\}$; then, for $1 \le j \le m$, the errors $\widetilde{z}_{1j} = 0$, $\widetilde{z}_{2j} = 0$, $\widetilde{u}_{wj} = 0$, and $\widetilde{u}_{vj} = 0$ are globally asymptotically stable.*

Proof. Choose the Lyapunov candidate as

$$V = V_1 + V_2, \tag{49}$$

where V_1 and V_2 are chosen as

$$V_1 = \frac{1}{2}\widetilde{Z}^T\mathscr{M}\widetilde{Z}, \qquad V_2 = \frac{1}{2}\widetilde{u}^T\overline{M}\widetilde{u} + \frac{1}{2}\widetilde{\Theta}^T\Gamma^{-1}\widetilde{\Theta} + \frac{\delta_1}{\gamma_1}, \tag{50}$$

with $\widetilde{\Theta} = [\widetilde{\Theta}_1,\dots,\widetilde{\Theta}_m]^T$ and Γ being the block diagonal matrices of Γ_j. Using the properties of $\mathscr{K}[\cdot]$, the set-valued Lie derivative of V can be obtained as follows:

$$\begin{aligned}\dot{\widetilde{V}} &= \mathscr{K}[V_1 + V_2] \subseteq \mathscr{K}[V_1] + \mathscr{K}[V_2]\\ &= \dot{\widetilde{V}}_1 + \dot{\widetilde{V}}_2.\end{aligned} \tag{51}$$

Since V_2 is continuous, it follows from Lemma 2 that the equality (51) holds.

According to Definition 4, the set-valued Lie derivative of V_1 is given as

$$\dot{\tilde{V}} \triangleq \bigcap_{\xi\in\partial V(\widetilde{Z})} \xi^T \mathscr{K}\left[-\alpha\mathscr{M}\widetilde{Z} - \beta\,\text{sign}\left(\mathscr{M}\widetilde{Z}\right) - \dot{\mathbf{f}}_0 - \hat{u}\right], \quad (52)$$

where $\partial V(\widetilde{Z})$ is the generalized gradient of V at $\widetilde{Z}$. Because V is continuously differentiable with respect to $\widetilde{Z}$, $\partial V(\widetilde{Z}) = \{M\widetilde{Z}\}$, which is a singleton. Therefore, it follows that

$$\begin{aligned}\dot{\tilde{V}}\left(\widetilde{Z}\right) &= \mathscr{K}\left[-\alpha\widetilde{Z}^T\mathscr{M}^2\widetilde{Z} - \beta\widetilde{Z}^T\mathscr{M}\,\text{sgn}\left(\mathscr{M}\widetilde{Z}\right)\right. \\ &\qquad \left. - \widetilde{Z}^T\mathscr{M}\dot{\mathbf{f}}_0 - \widetilde{Z}^T\mathscr{M}\hat{u}\right] \\ &= \left\{-\alpha\widetilde{Z}^T\mathscr{M}^2\widetilde{Z} - \beta\widetilde{Z}^T\mathscr{M}\,\text{sgn}\left(\mathscr{M}\widetilde{Z}\right)\right. \\ &\qquad \left. -\widetilde{Z}^T\mathscr{M}\dot{\mathbf{f}}_0 - \widetilde{Z}^T\mathscr{M}\hat{u}\right\},\end{aligned} \quad (53)$$

where the fact that $x^T\,\text{sign}(x) = \|x\|_1$ has been used. By Lemma 2 and [42], if f is continuous, then $\mathscr{K}[f] = \{f\}$. Note that the set-valued Lie derivative $\dot{\tilde{V}}$ is a singleton, whose only element is actually $\dot{V}$. Therefore, it follows that

$$\begin{aligned}\max\dot{\tilde{V}} = \dot{V} &\le -\alpha\widetilde{Z}^T\mathscr{M}^2\widetilde{Z} - (\beta-\kappa)\left\|\widetilde{Z}^T\mathscr{M}\right\|_1 - \widetilde{Z}^T\mathscr{M}\hat{u} \\ &\le -\alpha\widetilde{Z}^T\mathscr{M}^2\widetilde{Z} - (\beta-\kappa)\left\|\widetilde{Z}^T\mathscr{M}\right\|_1 \\ &\quad + \frac{\lambda_{\max}(\mathscr{M})}{2}\left(\left\|\widetilde{Z}\right\|_2^2 + \|\hat{u}\|_2^2\right),\end{aligned} \quad (54)$$

where $\beta \ge \kappa$ and α is positive. It is easy to verify that $\mathscr{M}^2$ is symmetric positive definite.

Since V_2 is continuous, it follows that the set-valued Lie derivative of V_2 satisfies $\max\dot{\tilde{V}}_2 = \dot{V}_2$. Hence, we have

$$\begin{aligned}\dot{V}_2 &= \tilde{u}^T\overline{M}\dot{\tilde{u}} + \frac{1}{2}\tilde{u}^T\dot{\overline{M}}\tilde{u} + \widetilde{\Theta}^T\Gamma^{-1}\dot{\widetilde{\Theta}} + \frac{\dot{\delta}_1}{\gamma_1} \\ &= \tilde{u}^T\left\{-\overline{C}\tilde{u} - K\tilde{u} - \mu_r + w(t)\right\} \\ &\quad + \frac{1}{2}\tilde{u}^T\dot{\overline{M}}\tilde{u} + \widetilde{\Theta}^T\Gamma^{-1}\dot{\widetilde{\Theta}} - \delta_1 \\ &= -\tilde{u}^TK\tilde{u} + \tilde{u}^T\left(\frac{1}{2}\dot{\overline{M}} - \overline{C}\right)\tilde{u} - \tilde{u}^T\mu_r \\ &\quad + \tilde{u}^Tw(t) + \dot{\widetilde{\Theta}}^T\Gamma^{-1}\widetilde{\Theta} - \delta_1.\end{aligned} \quad (55)$$

Since the matrix $(\dot{\overline{M}}_j - 2\overline{C}_j)$ is skew symmetric, we have

$$\tilde{u}_j^T\left[\frac{1}{2}\dot{\overline{M}}_j(q) - \overline{C}_j(q,\dot{q})\right]\tilde{u}_j = 0. \quad (56)$$

Let $g_j = -\delta_j - \tilde{u}_j^T\mu_{rj} + \dot{\widetilde{\Theta}}^T\Gamma^{-1}\widetilde{\Theta} + \tilde{u}_j^Tw_j(t)$, $(1 \le j \le m)$. Substituting the robustifying term (46) and the disturbance (43) into g_j gives

$$\begin{aligned}g_j &\le -\delta_j - \frac{\left\|\tilde{u}_j\right\|^2\left(Y_j\widehat{\Theta}_j\right)^2}{\left(Y_j\widehat{\Theta}_j\right)\left\|\tilde{u}_j\right\| + \delta_j} + \left\|\tilde{u}_j\right\|Y_j\widehat{\Theta}_j \\ &\le -\delta_j + \frac{\delta_j\left\|\tilde{u}_j\right\|\left(Y_j\widehat{\Theta}_j\right)}{\left(Y_j\widehat{\Theta}_j\right)\left\|\tilde{u}_j\right\| + \delta_j} \\ &\le -\delta_j\left(1 - \frac{\left\|\tilde{u}_j\right\|\left(Y_j\widehat{\Theta}_j\right)}{\left(Y_j\widehat{\Theta}_j\right)\left\|\tilde{u}_j\right\| + \delta_j}\right) \\ &\le 0.\end{aligned} \quad (57)$$

Hence, it can be obtained that

$$\tilde{u}^T\mu_r + \tilde{u}^T\left(\epsilon + \tau_d\right) + \dot{\widetilde{\Theta}}^T\Gamma\widetilde{\Theta} + \frac{\dot{\delta}_1}{\gamma_1} \le 0. \quad (58)$$

Substituting (58) into (55) gives the following inequality:

$$\begin{aligned}\dot{V}_2 &\le -\tilde{u}^TK\tilde{u} + \tilde{u}^Tw(t) - \tilde{u}^T\mu_r - \|\tilde{u}\|Y\widetilde{\Theta} - \delta_1 \\ &\le -\tilde{u}^TK\tilde{u}.\end{aligned} \quad (59)$$

Now, substituting (54) and (59) into the set-valued Lie derivative $\dot{\tilde{V}}$ reveals

$$\begin{aligned}\max\dot{\tilde{V}} = \dot{V} \le\ & -\alpha\widetilde{Z}^T\mathscr{M}^2\widetilde{Z} - (\beta-\kappa)\left\|\widetilde{Z}^T\mathscr{M}\right\|_1 \\ & + \frac{\lambda_{\max}(\mathscr{M})}{2}\left(\left\|\widetilde{Z}\right\|_2^2 + \|\hat{u}\|_2^2\right) - \tilde{u}^TK\tilde{u} \\ =\ & -\alpha\widetilde{Z}^T\mathscr{M}^2\widetilde{Z} - (\beta-\kappa)\left\|\widetilde{Z}^T\mathscr{M}\right\|_1 \\ & + \frac{\lambda_{\max}(\mathscr{M})}{2}\left(\left\|\widetilde{Z}\right\|_2^2 + \|\tilde{u}\|_2^2\right) - \tilde{u}^TK\tilde{u} \\ \le\ & -\alpha\lambda_{\max}^2(\mathscr{M})\left\|\widetilde{Z}\right\|_2^2 - (\beta-\kappa)\left\|\widetilde{Z}^T\mathscr{M}\right\|_1 \\ & + \frac{\lambda_{\max}(\mathscr{M})}{2}\left(\left\|\widetilde{Z}\right\|_2^2 + \|\tilde{u}\|_2^2\right) - k_{\max}\|\tilde{u}\|_2^2 \\ \le\ & -\left(\alpha\lambda_{\max}^2(\mathscr{M}) - \frac{\lambda_{\max}(\mathscr{M})}{2}\right)\left\|\widetilde{Z}\right\|_2^2 - (\beta-\kappa) \\ & \times\left\|\widetilde{Z}^T\mathscr{M}\right\|_1 - \left(k_{\max} - \frac{\lambda_{\max}(\mathscr{M})}{2}\right)\|\tilde{u}\|_2^2.\end{aligned} \quad (60)$$

Therefore, $\max\dot{\tilde{V}} \le 0$ as $\dot{z}_{20} \le \kappa$, $\alpha > 1/2\lambda_{\max}(\mathscr{M})$, $\beta > \kappa$ and $k_{\max} > \lambda_{\max}(\mathscr{M})/2$. It then follows from Lemma 5 that $\tilde{u} \to 0$ and $\widetilde{Z} \to 0$ as $t \to \infty$; that is, $\tilde{z}_{1j} \to 0$, $\tilde{z}_{2j} \to 0$, $\tilde{u}_{wj} \to 0$, and $\tilde{u}_{vj} \to 0$ as $t \to \infty$. Therefore, the errors $\tilde{z}_{1j} = 0$, $\tilde{z}_{2j} = 0$, $\tilde{u}_{wj} = 0$, and $\tilde{u}_{vj} = 0$ are globally asymptotically stable. This proof is completed. □

Remark 13. From Theorem 12, we have proved that the variables z_{1j} $(1 \le j \le m)$ and z_{2j} $(1 \le j \le m)$, respectively, converge to z_{10} and z_{20} globally asymptotically under the proposed control laws (26), (27), and (45). In Theorem 14, we will prove that z_{3j} asymptotically converges to z_{30} under the control laws (26), (27), and (45).

Theorem 14. *Suppose that the communication graph $\mathcal{G}$ is connected, Assumption 8 is satisfied, the velocity controllers for* (29) *and* (30) *are, respectively, designed by* (26) *and* (27), *and the torque control input for the dynamics system* (42) *is designed by* (45). *If z_{1j} and z_{2j} asymptotically converge to z_{10} and z_{20}, then z_{3j} also asymptotically converges to z_{30}.*

Proof. Let $\tilde{z}_{3j} = z_{3j} - z_{30}$. Take the derivative of $\tilde{z}_{3j}$ as

$$\begin{aligned}\dot{\tilde{z}}_{3j} &= \dot{z}_{3j} - \dot{z}_{30} \\ &= -k_0 \left|u_{1j}\right| \tilde{z}_{3j} + u_{1j}\tilde{z}_{2j} + \left(u_{1j} - u_{10}\right) z_{20} \\ &\quad - k_0 \left(\left|u_{1j}\right| - \left|u_{10}\right|\right) z_{30} \\ &= -k_0 \left|u_{1j}\right| \tilde{z}_{3j} + x_2(t),\end{aligned} \tag{61}$$

where $x_2(t) = u_{1j}\tilde{z}_{2j} + (u_{1j} - u_{10})z_{20} - k_0(|u_{1j}| - |u_{10}|)z_{30}$. The solution of the differential equation (61) is given as follows:

$$\tilde{z}_{3j}(t) = e^{\int_0^t -k_0|u_{1j}|d\tau}\tilde{z}_{3j}(0) + \int_0^t e^{\int_\tau^t -k_0|u_{1j}|d\nu} x_2(\tau)\, d\tau. \tag{62}$$

According to Theorem 12, $\tilde{z}_{2j}$ asymptotically converges to zero, and u_{1j} asymptotically converges to u_{10}. It then follows from the definition of $x_2(t)$ that $x_2(t)$ also asymptotically converges to zero. Hence, according to the definition of asymptotic stable, for an arbitrary positive value $\sigma > 0$, $o > 0$ exists; when the $|x_2(0)| < o$, it has $|x_2(t)| < \sigma$.

From Assumption 6, the u_{1j} is bounded, and $u_{1j} = w_j$, Hence, $|u_{1j}| \le w_{\max}$.

The solution of the differential equation (62) satisfies the inequality

$$\begin{aligned}\tilde{z}_{3j}(t) &= e^{\int_0^t -k_0|u_{1j}|d\tau}\tilde{z}_{3j}(0) + \int_0^t e^{\int_\tau^t -k_0|u_{1j}|d\nu} x_2(\tau)\, d\tau \\ &\le e^{-k_0 w_{\max} t}\tilde{z}_{3j}(0) + \int_0^t e^{-k_0 w_{\max}(t-\tau)} x_2(\tau)\, d\tau \\ &\le e^{-k_0 w_{\max} t}\tilde{z}_{3j}(0) + e^{-k_0 w_{\max} t}\int_0^t e^{k_0 w_{\max}\tau} x_2(\tau)\, d\tau \\ &\le e^{-k_0 w_{\max} t}\tilde{z}_{3j}(0) + \frac{\sigma k_0 w_{\max} - \sigma k_0 w_{\max} e^{-k_0 w_{\max} t}}{k_j w_{\max}} \\ &= \sigma + e^{-k_0 w_{\max} t}\left(\tilde{z}_{3j}(0) - \sigma\right).\end{aligned} \tag{63}$$

Hence, when $t \to +\infty$, $|\tilde{z}_{3j}(t)| \le \sigma$. Since σ is an arbitrary positive value, from the definition of asymptotic stable, the $\tilde{z}_{3j}(t)$ is asymptotic stable at the neighborhood of origin. This proof is completed. □

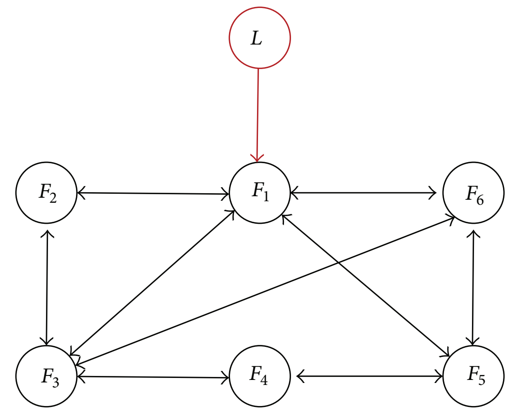


FIGURE 2: Communication graph among mobile robots.

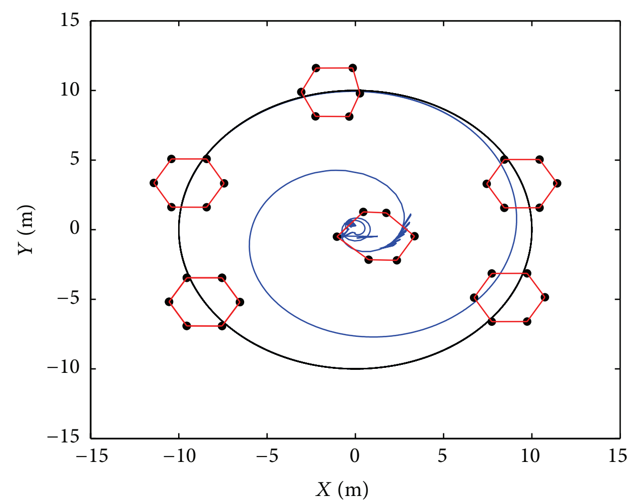


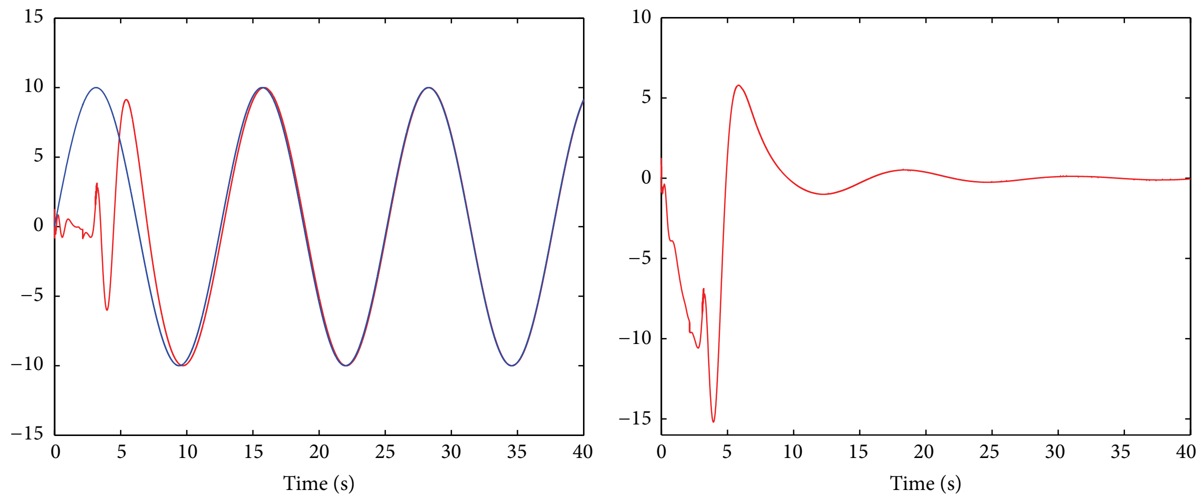
FIGURE 3: Path of the six robots' centroid (blue line), the desired trajectory of the centroid of the robots (black line), and the formation of the six robots at several moments under the distributed kinematic controller (26), (27), and the torque controller (45).

Remark 15. From Theorems 12 and 14, our control objectives (22)–(25) hold under the distributed kinematic controller (26) and the torque controller (45). Therefore, from Lemma 10, the m mobile robots converge to the formation pattern $\mathcal{F}$; that is, (16)–(18) are satisfied.

5. Simulation

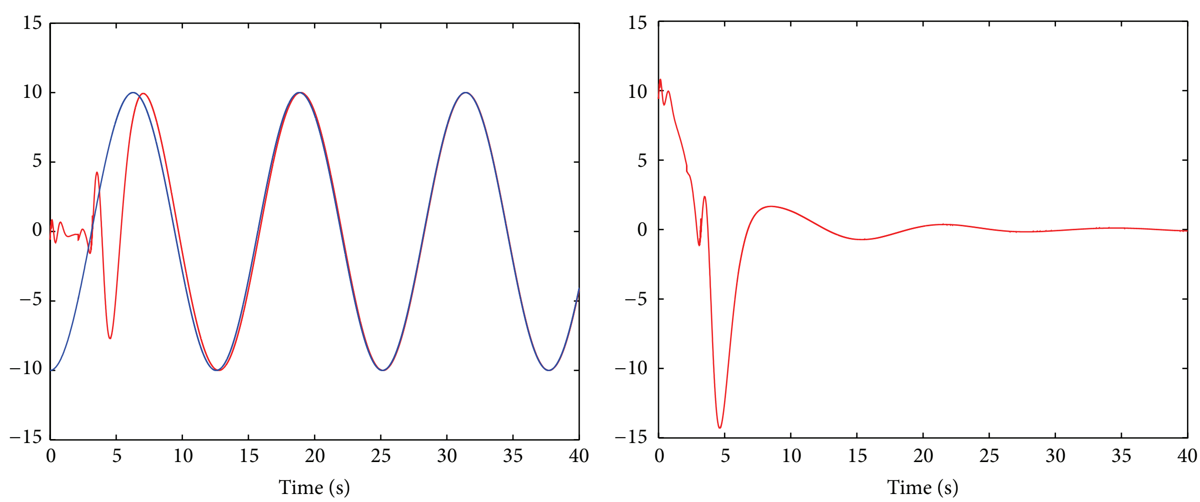
In this section, some simulations results will be provided to demonstrate the effectiveness of some theoretical results of the previous sections. Consider a multiple mobile robot system with six followers denoted by F_1–F_6 and one virtual leader denoted by L, respectively. The communication graph of the multiple mobile robot system is shown in Figure 2.

For simplicity, in this simulation we suppose that $a_{ij} = 1$ if robot i can receive information from robot j, $a_{ij} = 0$ otherwise; $b_j = 1$ if the virtual leader's information is available to the follower j, and $b_j = 0$ otherwise, where $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, m\}$.



(a) (b)

FIGURE 4: (a) The trajectories of x_0 (blue line) and the centroid of x_i $(1 \leq i \leq 6)$ (red line); (b) the position error between x_0 and the centroid of x_i.



(a) (b)

FIGURE 5: (a) The trajectories of y_0 (blue line) and the centroid of y_i $(1 \leq i \leq 6)$ (red line); (b) the position error between y_0 and the centroid of y_i.

The desired formation geometric pattern $\mathcal{F}$ is defined by orthogonal coordinates as $(p_{1x}, p_{1y}) = (2, 0)$, $(p_{2x}, p_{2y}) = (1, \sqrt{3})$, $(p_{3x}, p_{3y}) = (-1, \sqrt{3})$, $(p_{4x}, p_{4y}) = (-2, 0)$, $(p_{5x}, p_{5y}) = (-1, -\sqrt{3})$, and $(p_{6x}, p_{6y}) = (1, -\sqrt{3})$. The reference trajectory of the virtual leader is chosen as

$$x_0 = 10 \sin\left(\frac{t}{2}\right), \quad y_0 = -10 \cos\left(\frac{t}{2}\right). \tag{64}$$

The control gain parameters are chosen as $\alpha = 10$, $\beta = 0.99$, $k_0 = 2$. For $1 \leq j \leq 6$, $\delta_j(0) = 30$, $\gamma_j = 0.5$, $\Gamma_j = \left[\begin{smallmatrix} 0.001 & 0 \\ 0 & 0.001 \end{smallmatrix}\right]$, $K_j = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$. The parameters for each robot are considered as the mass $\widehat{m} = 5\,\text{kg}$ and the moment of inertia $I = 3\,\text{kg}{\cdot}\text{m}^2$. The unmodeled dynamics are introduced in the form of friction as

$$\overline{F}_j = \begin{bmatrix} a_{j1} \operatorname{sign}\left(u_{2j}\right) + a_{j2} u_{2j} \\ a_{j3} \operatorname{sign}\left(u_{1j}\right) + a_{j4} u_{1j} \end{bmatrix}. \tag{65}$$

The disturbance is introduced as $\overline{\tau}_{dj} = 2 \sin(2t) \cos(5t)$.

5.1. Verification of Formation Control Based on Robust Adaptive Techniques. In this simulation, Figure 3 shows the trajectory of virtual leader (black line), the trajectory of the six followers' centroid (blue line), and the formation positions and pattern of the six followers at several moments. We could see from Figure 3 that the six robots converge to the desired

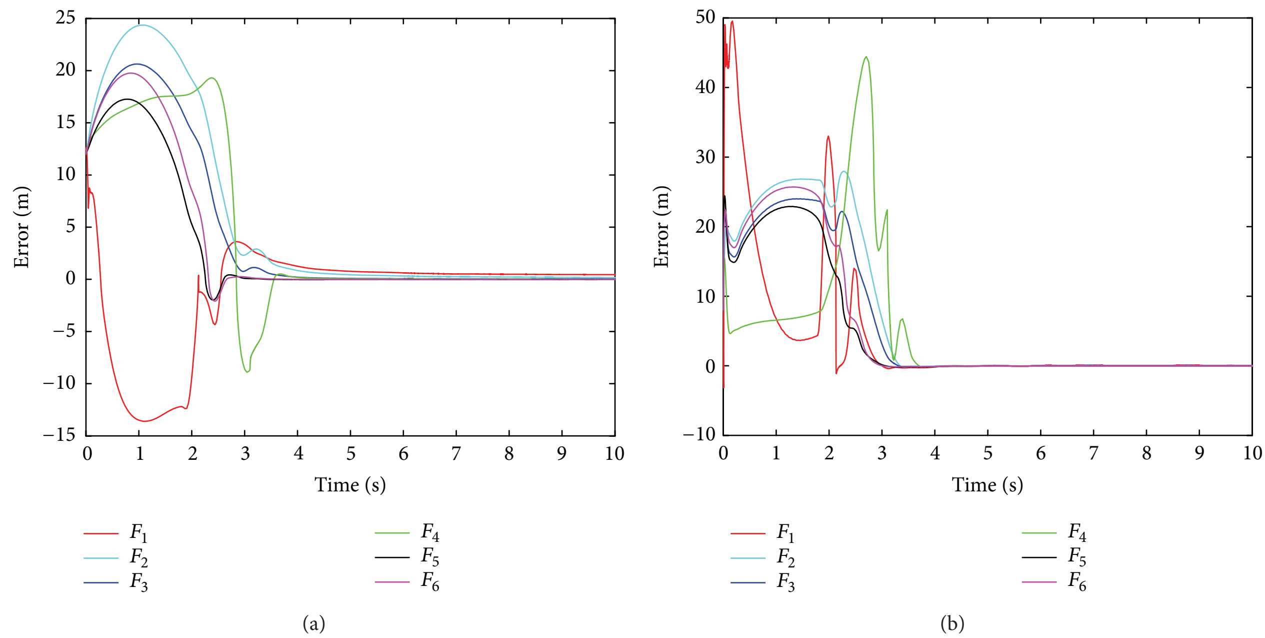


Figure 6: (a) The tracking error $\tilde{u}_{wi}$ for $(1 \leq i \leq 6)$ using the torque controller (45); (b) the tracking error $\tilde{u}_{vi}$ for $(1 \leq i \leq 6)$ using the torque controller (45).

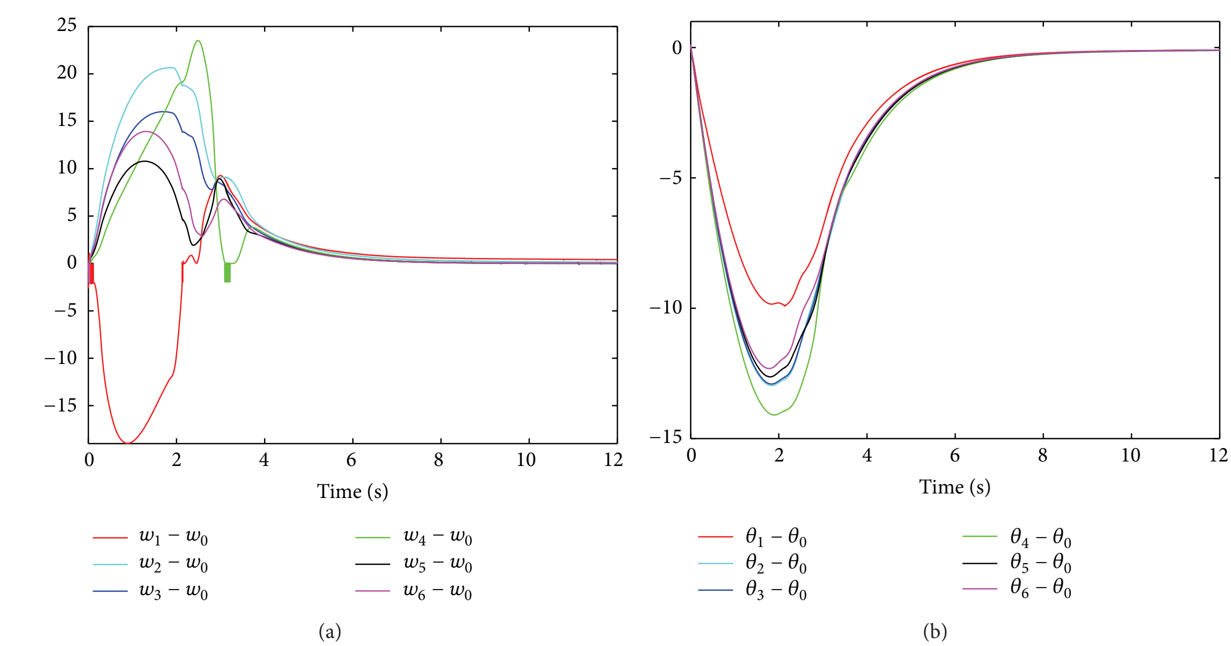


Figure 7: (a) Response of the centroid of $w_i - w_0$ for $1 \leq i \leq 6$; (b) response of the centroid of $\theta_i - \theta_0$ for $1 \leq i \leq 6$.

geometry pattern under the proposed controllers (27), (26), and (45); that is to say, (16) has been verified.

Figure 4(a) shows the trajectories of x_0 (blue line) and the centroid of x_i $(1 \leq i \leq 6)$ (red line), and Figure 4(b) shows the position error between x_0 and the centroid of x_i. Figure 5(a) shows the trajectories of y_0 (blue line) and the centroid of y_i $(1 \leq i \leq 6)$ (red line), and Figure 5(b) shows the position error between y_0 and the centroid of y_i. We could see, from Figures 4 and 5, the trajectory of the formation geometric centroid converges to the trajectory of virtual leader; that is to say, (18) has been verified. Figure 6 shows the tracking error $\tilde{u}_{wi}$ for $(1 \leq i \leq 6)$ and $\tilde{u}_{vi}$ for $(1 \leq i \leq 6)$ under the torque controller (45). From Figure 6, $\tilde{u}_{wi}$ and $\tilde{u}_{vi}$, respectively, converge to zeros. The perfect tracking of velocity and angular velocity has been guaranteed. Figure 7, respectively, shows the angular velocity tracking errors $w_i - w_0$ and the orientation tracking errors $\theta_i - \theta_0$ between follower F_i $(1 \leq i \leq 6)$ and virtual leader. It can be seen from Figure 7 that $w_i - w_0$ and $\theta_i - \theta_0$ converge to zero over time; that is, (17) and (25) have been verified.

6. Conclusion

In this paper, the distributed consensus-based robust adaptive formation control problem for nonholonomic mobile robots with partial known dynamics has been investigated, in which the dynamics model of the wheeled mobile robot has the friction term and bounded disturbance term in the dynamic model. The partial knowledge of the mobile robot dynamics has been assumed to be available. Then an asymptotically stable torque controller has been proposed by using robust adaptive control techniques to account for unmolded dynamics and bounded disturbances.

As further extensions of this study, there still exist a number of topics for future works. In practice, formations have to avoid obstacles and need to admit changes in the formation speed and strong deformations in formation shape. The obstacles avoidance problem for formation control of nonholonomic mobile robots will be considered. In addition, the formation control is assumed to be noiseless in this paper. However, it is inevitable in reality. Hence, in the future it is necessary to investigate the formation control problem with measurement noise. Finally, it is well known that most operations in mobile robots systems are naturally delayed. Moreover, it has been observed from numerical experiments that formation control algorithms without considering time delays may lead to unexpected instability. Hence, in future, we may consider the formation control with time-varying delays.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] Z. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*, Springer, London, UK, 2010.

[2] M. Ghasemi and S. G. Nersesov, "Sliding mode cooperative control for multirobot systems: a finite-time approach," *Mathematical Problems in Engineering*, vol. 2013, Article ID 450201, 16 pages, 2013.

[3] H. G. Tanner and J. L. Piovesan, "Randomized receding horizon navigation," *IEEE Transactions on Automatic Control*, vol. 55, no. 11, pp. 2640–2644, 2010.

[4] M. Lei, S. Zhou, X. Yang, and G. Yin, "Complex formation control of large-scale intelligent autonomous vehicles," *Mathematical Problems in Engineering*, vol. 2012, Article ID 241916, 19 pages, 2012.

[5] J. P. Desai, J. P. Ostrowski, and V. Kumar, "Modeling and control of formations of nonholonomic mobile robots," *IEEE Transactions on Robotics and Automation*, vol. 17, no. 6, pp. 905–908, 2001.

[6] A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, "A vision-based formation control framework," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 813–825, 2002.

[7] G. L. Mariottini, F. Morbidi, D. Prattichizzo, G. J. Pappas, and K. Daniilidis, "Leader-follower formations: uncalibrated vision-based localization and control," in *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA '07)*, pp. 2403–2408, April 2007.

[8] L. Consolini, F. Morbidi, D. Prattichizzo, and M. Tosques, "Leader-follower formation control of nonholonomic mobile robots with input constraints," *Automatica*, vol. 44, no. 5, pp. 1343–1349, 2008.

[9] Z. Peng, G. Wen, and A. Rahmani, "Leader-follower formation control of multiple nonholonomic robots based on backstepping," in *Proceedings of the 28th Annual ACM Symposium on Applied Computing (SAC '13)*, pp. 211–216, ACM, Coimbra, Portugal, 2013.

[10] Z.-J. Tang, T.-Z. Huang, J.-P. Hu, and J.-L. Shao, "Leader-following consensus in networks of agents with nonuniform time-varying delays," *Mathematical Problems in Engineering*, vol. 2012, Article ID 848942, 14 pages, 2012.

[11] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, 1998.

[12] J. R. T. Lawton, R. W. Beard, and B. J. Young, "A decentralized approach to formation maneuvers," *IEEE Transactions on Robotics and Automation*, vol. 19, no. 6, pp. 933–941, 2003.

[13] A. Brunete, M. Hernando, E. Gambao, and J. E. Torres, "A behaviour-based control architecture for heterogeneous modular, multi-configurable, chained micro-robots," *Robotics and Autonomous Systems*, vol. 60, no. 12, pp. 1607–1624, 2012.

[14] K.-H. Tan and M. A. Lewis, "Virtual structures for high-precision cooperative mobile robotic control," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS '96)*, vol. 1, pp. 132–139, November 1996.

[15] M. Egerstedt and X. Hu, "Formation constrained multi-agent control," *IEEE Transactions on Robotics and Automation*, vol. 17, no. 6, pp. 947–951, 2001.

[16] M. A. Lewis and K.-H. Tan, "High precision formation control of mobile robots using virtual structures," *Autonomous Robots*, vol. 4, no. 4, pp. 387–403, 1997.

[17] J. Ghommam, H. Mehrjerdi, M. Saad, and F. Mnif, "Formation path following control of unicycle-type mobile robots," *Robotics and Autonomous Systems*, vol. 58, no. 5, pp. 727–736, 2010.

[18] K. D. Do and J. Pan, "Nonlinear formation control of unicycle-type mobile robots," *Robotics and Autonomous Systems*, vol. 55, no. 3, pp. 191–204, 2007.

[19] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[20] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Stable flocking of mobile agents—part I: fixed topology," in *Proceedings of*

the 42nd IEEE Conference on Decision and Control, pp. 2010–2015, December 2003.

[21] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Stable flocking of mobile agents—part II: dynamic topology," in *Proceedings of the 42nd IEEE Conference on Decision and Control*, pp. 2016–2021, December 2003.

[22] H. Shi, L. Wang, and T. Chu, "Flocking of multi-agent systems with a dynamic virtual leader," *International Journal of Control*, vol. 82, no. 1, pp. 43–58, 2009.

[23] J. A. Fax and R. M. Murray, "Graph laplacians and stabilization of vehicle formations," in *Proceedings of the 15th IFAC World Congress*, pp. 238–288, 2002.

[24] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465–1476, 2004.

[25] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for an autonomous mobile robot," in *Proceedings of the IEEE International Conference on Robotics and Automation*, vol. 1, pp. 384–389, May 1990.

[26] J. Chen, D. Sun, J. Yang, and H. Chen, "Leader-follower formation control of multiple non-holonomic mobile robots incorporating a receding-horizon scheme," *The International Journal of Robotics Research*, vol. 29, no. 6, pp. 727–747, 2010.

[27] W. Dong, "Tracking control of multiple-wheeled mobile robots with limited information of a desired trajectory," *IEEE Transactions on Robotics*, vol. 28, no. 1, pp. 262–268, 2012.

[28] Z. Peng, G. Wen, A. Rahmani, and Y. Yu, "Distributed consensus-based formation control for multiple nonholonomic mobile robots with a specified reference trajectory," *International Journal of Systems Science*, 2013.

[29] I. Zohar, A. Ailon, and R. Rabinovici, "Mobile robot characterized by dynamic and kinematic equations and actuator dynamics: trajectory tracking and related application," *Robotics and Autonomous Systems*, vol. 59, no. 6, pp. 343–353, 2011.

[30] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot using neural networks," *IEEE Transactions on Neural Networks*, vol. 9, no. 4, pp. 589–600, 1998.

[31] T. Fukao, H. Nakagawa, and N. Adachi, "Adaptive tracking control of a nonholonomic mobile robot," *IEEE Transactions on Robotics and Automation*, vol. 16, no. 5, pp. 609–615, 2000.

[32] W. Dong, "Robust formation control of multiple wheeled mobile robots," *Journal of Intelligent & Robotic Systems*, vol. 62, no. 3-4, pp. 547–565, 2011.

[33] Z. Peng, G. Wen, A. Rahmani, and Y. Yu, "Leader-follower formation control of nonholonomic mobile robots based on a bioinspired neurodynamic based approach," *Robotics and Autonomous Systems*, vol. 61, pp. 988–996, 2013.

[34] T. Dierks and S. Jagannathan, "Neural network control of mobile robot formations using RISE feedback," *IEEE Transactions on Systems, Man, and Cybernetics B: Cybernetics*, vol. 39, no. 2, pp. 332–347, 2009.

[35] W. Dong and J. A. Farrell, "Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty," *Automatica*, vol. 45, no. 3, pp. 706–710, 2009.

[36] W. Dong, "Flocking of multiple mobile robots based on backstepping," *IEEE Transactions on Systems, Man, and Cybernetics B: Cybernetics*, vol. 41, no. 2, pp. 414–424, 2011.

[37] Z. Meng, Z. Lin, and W. Ren, "Leader-follower swarm tracking for networked Lagrange systems," *Systems and Control Letters*, vol. 61, no. 1, pp. 117–126, 2012.

[38] F. L. Lewis, C. T. Abdallah, and D. M. Dawson, *Control of Robot Manipulators*, vol. 92, MacMillan, New York, NY, USA, 1993.

[39] F. R. K. Chung, *Spectral Graph Theory*, vol. 92 of *CBMS Regional Conference Series in Mathematics*, American Mathematical Society, Providence, RI, USA, 1997.

[40] J. Cortés, "Discontinuous dynamical systems," *IEEE Control Systems*, vol. 28, no. 3, pp. 36–73, 2008.

[41] D. Shevitz and B. Paden, "Lyapunov stability theory of nonsmooth systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 9, pp. 1910–1914, 1994.

[42] B. E. Paden and S. S. Sastry, "A calculus for computing Filippov's differential inclusion with application to the variable structure control of robot manipulators," *IEEE Transactions on Circuits and Systems*, vol. 34, no. 1, pp. 73–82, 1987.

[43] W. Dong and J. A. Farrell, "Cooperative control of multiple nonholonomic mobile agents," *IEEE Transactions on Automatic Control*, vol. 53, no. 6, pp. 1434–1448, 2008.

[44] C. M. Kwan, D. M. Dawson, and F. L. Lewis, "Robust adaptive control of robots using neural network: global tracking stability," in *Proceedings of the 34th IEEE Conference on Decision and Control*, vol. 2, pp. 1846–1851, December 1995.