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Research Article

Mean Square Synchronization of Stochastic Nonlinear Delayed Coupled Complex Networks

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We investigate the problem of adaptive mean square synchronization for nonlinear delayed coupled complex networks with stochastic perturbation. Based on the LaSalle invariance principle and the properties of the Weiner process, the controller and adaptive laws are designed to ensure achieving stochastic synchronization and topology identification of complex networks. Sufficient conditions are given to ensure the complex networks to be mean square synchronization. Furthermore, numerical simulations are also given to demonstrate the effectiveness of the proposed scheme.

1. Introduction

As is known to all, complex networks are shown to exist ubiquitously in the nature world [1]. Among various behaviors of complex networks, synchronization is a significant and interesting phenomenon. It has been demonstrated that many real-world problems have close relationships with synchronization [2–4].

Recently, as signals transmitted between subsystems of complex networks are unavoidably subjected to stochastic perturbations from the environment, which may cause information contained in these signals to be lost, stochastic model has played an important role in many scientific and engineering applications and stochastic systems have received increasing attention. Many fundamental results about stochastic systems have been studied [5–14]. Moreover, in some circumstances, this simplification that networks are coupled linearly does not match satisfactorily the peculiarities of real networks. In many practical problems, it often happens that the states of the system cannot be observed directly. Instead, we can only observe the states of the system with nonlinear coupled, which means that the coupling scheme is nonlinear [15–20]. In addition, in practical situations, there exists much uncertain information in complex networks, such as the topological structures, for example, genomic coexpression networks, energy network, biological neural networks, and so on. The accurate topological structures of these complex networks are often difficult to know in the real world. So, identification of the topology is also an important issue in the research of the complex networks. In [21–27], a great many results have been reported about parameter identification for complex dynamical networks.

To the best of our knowledge, the topological identification of complex networks with multidelayed coupling and nonlinear stochastic effects are seldom discussed. Motivated by the previous discussions, the aim of this paper is to discuss topological identification of a general nonlinear multidelayed coupled complex dynamical network with stochastic effects. We derive some criteria and design controller which ensure topology identification of stochastic nonlinear delayed coupled complex networks.

The rest of this work is organized as follows. Section 2 gives the problem formulation. Section 3 gives some theoretical analyses. Section 4 gives illustrative example. Section 5 gives the conclusions of the paper.

2. Problem Formulation

The nonlinear delayed coupled complex networks can be described by stochastic effects:

$$dx^i(t) = \left[f\left(x^i(t)\right) + \sum_{j=1}^{N} a_{ij}\left(h\left(x^j(t)\right) + h\left(x^i(t)\right)\right) + \sum_{j=1}^{N} b_{ij}\left(h\left(x^j(t-\tau_1)\right) + h\left(x^i(t-\tau_2)\right)\right)\right] dt + \varphi\left(t, x^i(t)\right) d\omega(t), \tag{1}$$

where $x^i = (x_1, x_2, \dots, x_n)^T \in R^n$, $f : R^n \to R^n$ standing for the activity of an individual subsystem is a vector value function. $A = (a_{ij})_{N\times N} \in R^{N\times N}$ and $B = (b_{ij})_{N\times N} \in R^{N\times N}$ are the coupling matrix, and a_{ij} and b_{ij} are the weight or coupling strength. If there exists a link from node i to j $(i \neq j)$, then $a_{ij} \neq 0$ and $b_{ij} \neq 0$. Otherwise, $a_{ij} = 0$ and $b_{ij} = 0$. $\varphi : R^n \times R^+ \to R^{n\times m}$ is the noise intensity function, ω are arrays of appropriate dimensional Brownian motions, and $E[d\omega] = 0$, $E\{[d\omega]^2\} = dt$, $E[\cdot]$ is the mathematical expectation.

Remark 1. In fact, the stochastically coupled complex networks model considered in this paper could be more general. The stochastic coupling term is introduced to model the array of coupled complex networks, which can not only reflect more realistic dynamical behaviors of the networks, but also make it possible to simulate more complicate dynamical behaviors. For example, the nonlinear stochastic coupling can be the general stochastic coupling or the unknown general stochastic coupling. Moreover, the coupled system (1) itself can be a general complex network.

Assumption 2. There exists nonnegative constant μ, such that

$$\frac{1}{2}\operatorname{Tr}\left(\varphi^T\left(t, e^i(t)\right)\varphi\left(t, e^i(t)\right)\right) \le \mu^i\left(e^i(t)\right)^T e^i(t) \le \mu\left(e^i(t)\right)^T e^i(t), \tag{2}$$

where μ^i denotes nonnegative constant, $\mu = \max_{1\le i\le N}\{\mu^i\}$, $e^i(t) = y^i(t) - x^i(t)$, and $\operatorname{Tr}(\cdot)$ denotes the trace of matrix.

Assumption 3. Assume that f is Lipschitz with respect to its argument; that is,

$$\left| f\left(t, y^i(t)\right) - f\left(t, x^i(t)\right)\right| \le \lambda e^i(t). \tag{3}$$

Consequently, one can consider

$$f\left(t, y^i(t)\right) - f\left(t, x^i(t)\right) = M\left(x^i(t), y^i(t)\right) e^i(t), \tag{4}$$

where $\|M(x^i, y^i)\| \le \lambda$ with its elements dependent on x^i and y^i, $e^i(t) = y^i(t) - x^i(t)$, λ be positive constant, $\|\cdot\|$ stands for L^2 norm.

Assumption 4. The nonlinear function $h(\cdot)$ satisfies $\gamma_1 \le (h(z_1) - h(z_2))/(z_1 - z_2) \le \gamma_2$ for some $\gamma_1 > 0, \gamma_2 > 0$, all $z_1, z_2 \in R (z_1 \neq z_2)$.

Lemma 5 (see [28]). *Given any vectors x, y of appropriate dimensions and a positive definite matrix $P > 0$ with compatible dimensions, then the following inequality holds:*

$$2x^T y \le x^T P y + y^T P^{-1} y. \tag{5}$$

Remark 6. For studying the convergence of random process, instead of the standard Euclidian norm, the mean square norm, L^2 norm, is used which is defined as

$$\|e(t)\| = \left(E\left[e^T(t)\, e(t)\right]\right)^{1/2}. \tag{6}$$

3. Synchronization Criterion

In order to achieve topology identification and synchronization, if system (1) is considered as the drive system with state variable denoted by $x^i(t)$, we construct a response system as follows:

$$dy^i(t) = \left[f\left(y^i(t)\right) + \sum_{j=1}^{N} \hat{a}_{ij}\left(h\left(y^j(t)\right) + h\left(y^i(t)\right)\right) + \sum_{j=1}^{N} \hat{b}_{ij}\left(h\left(y^j(t-\tau_1)\right) + h\left(y^i(t-\tau_2)\right)\right) + u^i(t)\right] dt + \varphi\left(t, y^i(t)\right) d\omega(t), \tag{7}$$

where $y^i = (y_1, y_2, \dots, y_n)^T \in R^n$, $u^i(t)$ is the adaptive controller, $\hat{a}_{ij}$ and $\hat{b}_{ij}$ are the estimation of the weight a_{ij} and b_{ij}.

Definition 7. The systems (1) and (7) are said to achieve stochastic synchronization in the mean square sense if

$$\lim_{t\to\infty} E\left\{\left\|e^i(t)\right\|^2\right\} = \lim_{t\to\infty} E\left\{\left\|y^i(t) - x^i(t)\right\|^2\right\} = 0, \tag{8}$$

where $e^i(t) = (e_1(t), e_2(t), \dots, e_n(t))^T$, $\|\cdot\|$ stands for L^2 norm.

Let the synchronization errors $e^i(t) = y^i(t) - x^i(t)$; then we have the following error dynamical equations:

$$de^i(t) = \left[f\left(y^i(t)\right) - f\left(x^i(t)\right) + \sum_{j=1}^{N} \hat{a}_{ij}\left(h\left(y^j(t)\right) + h\left(y^i(t)\right)\right) + \sum_{j=1}^{N} \hat{b}_{ij}\left(h\left(y^j(t-\tau_1)\right) + h\left(y^i(t-\tau_2)\right)\right) - \sum_{j=1}^{N} a_{ij}\left(h\left(x^j(t)\right) + h\left(x^i(t)\right)\right)\right.$$

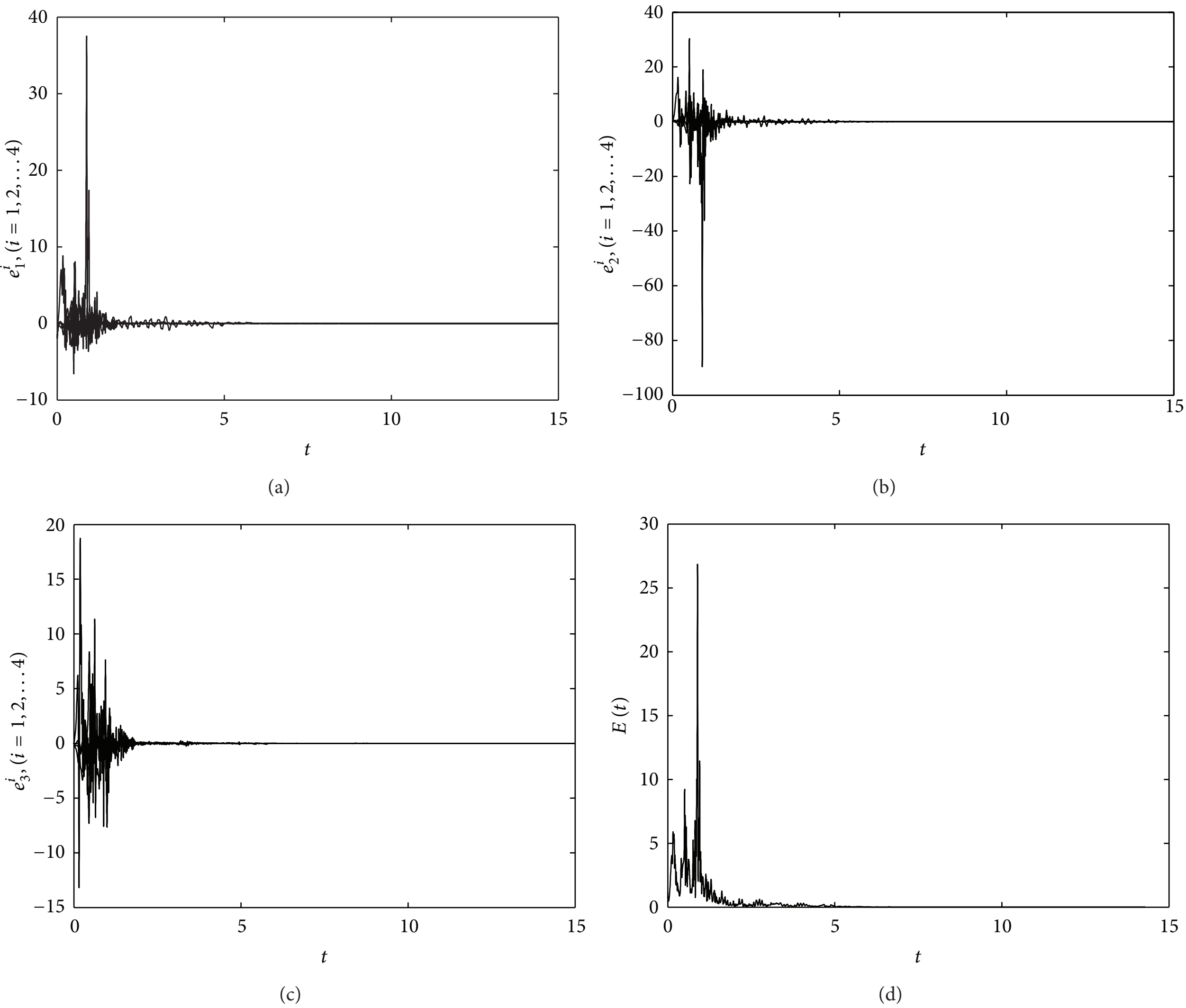

Figure 1: Synchronization errors $e^i(t)$ and $E(t)$ of network.

$$- \sum_{j=1}^{N} b_{ij} \left(h \left(x^j \left(t - \tau_1 \right) \right) + h \left(x^i \left(t - \tau_2 \right) \right) \right)$$

$$\left. + u^i (t) \right] dt + \varphi \left(t, e^i \right) d\omega (t) , \qquad (9)$$

where $\varphi(t, e^i(t)) = \varphi(t, y^i(t)) - \varphi(t, x^i(t))$.

If we let $\widetilde{a}_{ij}(t) = a_{ij} - \widehat{a}_{ij}(t)$ and $\widetilde{b}_{ij}(t) = b_{ij} - \widehat{b}_{ij}(t)$, then we can derive the following result.

Theorem 8. *Under Assumptions 2–4, $\alpha_1 = \max(a_{ij})$, $\alpha_2 = \max(a_{ij}^2)$, $\beta = \max(b_{ij})$, $1 \leq i, j \leq N$. The controlled complex dynamical networks* (7) *synchronizes to the complex dynamical networks* (1) *using the following update laws of the coupling strengths:*

$$u^i (t) = -r^i (t) e^i (t) , \qquad (10)$$

$$\dot{r}^i (t) = \theta_i \left(e^i (t) \right)^T e^i (t) , \qquad (11)$$

$$\dot{\widehat{a}}_{ij} (t) = - \left(e^i (t) \right)^T \left(h \left(y^j (t) \right) + h \left(y^i (t) \right) \right), \qquad (12)$$

$$\dot{\widehat{b}}_{ij} (t) = - \left(e^i (t) \right)^T \left(h \left(y^j \left(t - \tau_1 \right) \right) + h \left(y^i \left(t - \tau_2 \right) \right) \right), \qquad (13)$$

where θ_i is positive constant and $i, j = 1, 2, \ldots, N$.

Proof. We choose a nonnegative function as

$$V (t) = \frac{1}{2} \sum_{i=1}^{N} \left\| e^i (t) \right\|^2 + \frac{1}{2} \sum_{i=1}^{N} \frac{1}{\theta^i} \left\| r^i (t) - k \right\|^2$$

$$+ \sum_{j=1}^{N} \int_{t-\tau_1}^{t} \left\| e^i (s) \right\|^2 ds$$

$$+ \sum_{j=1}^{N} \int_{t-\tau_2}^{t} \left\| e^i (s) \right\|^2 ds + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left\| \widetilde{a}_{ij} (t) \right\|^2$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left\| \widetilde{b}_{ij} (t) \right\|^2$$

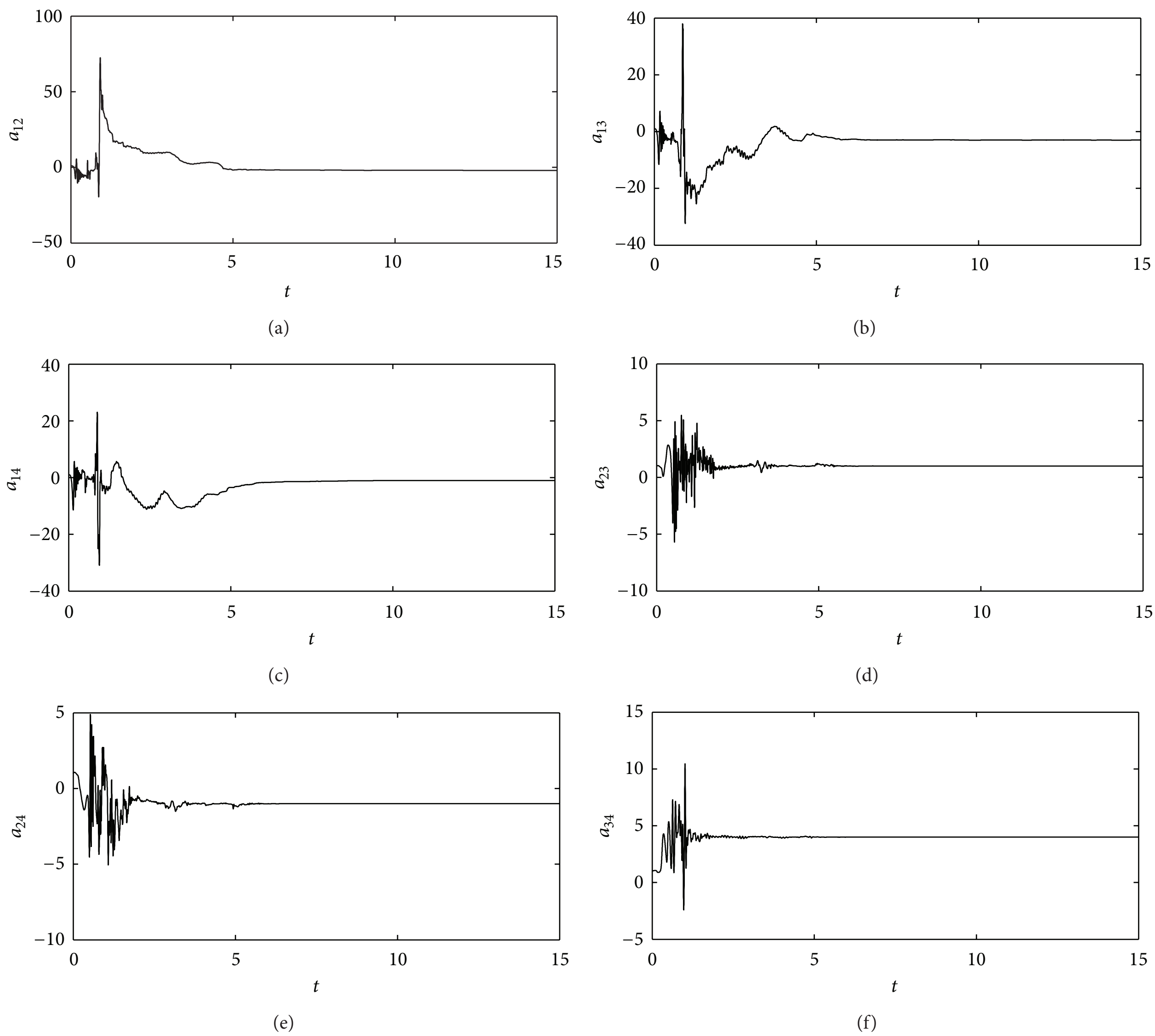

Figure 2: Identification of network structure (some elements of matrix $\widehat{A} = (\widehat{a}_{ij})_{4\times 4}$ are displayed).

$$
\begin{aligned}
&= E\left\{\frac{1}{2}\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{\theta^{i}}\left(r^{i}(t)-k\right)^{2}\right. \\
&\quad + \sum_{j=1}^{N}\int_{t-\tau_1}^{t}\left(e^{i}(s)\right)^{T}e^{i}(s)\,ds \\
&\quad + \sum_{j=1}^{N}\int_{t-\tau_2}^{t}\left(e^{i}(s)\right)^{T}e^{i}(s)\,ds \\
&\quad \left. + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{a}_{ij}^{2}(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{b}_{ij}^{2}(t)\right\},
\end{aligned}
\tag{14}
$$

where $k \geq \lambda + \alpha_1\gamma_2 + \alpha_2\gamma_2^2 + 2 + 2\beta\gamma_2^2 + \mu$.

By Itô's differential rule, the stochastic derivative of V along trajectories of error systems (9) can be obtained as follows:

$$
dV(t) = LV(t)\,dt + \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\left[\varphi\left(t, e^{i}\right)d\omega(t)\right], \tag{15}
$$

where the weak infinitesimal operator L is given by

$$
\begin{aligned}
LV(t) = \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\Bigg[&\left(f\left(y^{i}(t)\right) - f\left(x^{i}(t)\right)\right) \\
&+ \sum_{j=1}^{N}\widehat{a}_{ij}(t)\left(h\left(y^{j}(t)\right) + h\left(y^{i}(t)\right)\right)
\end{aligned}
$$

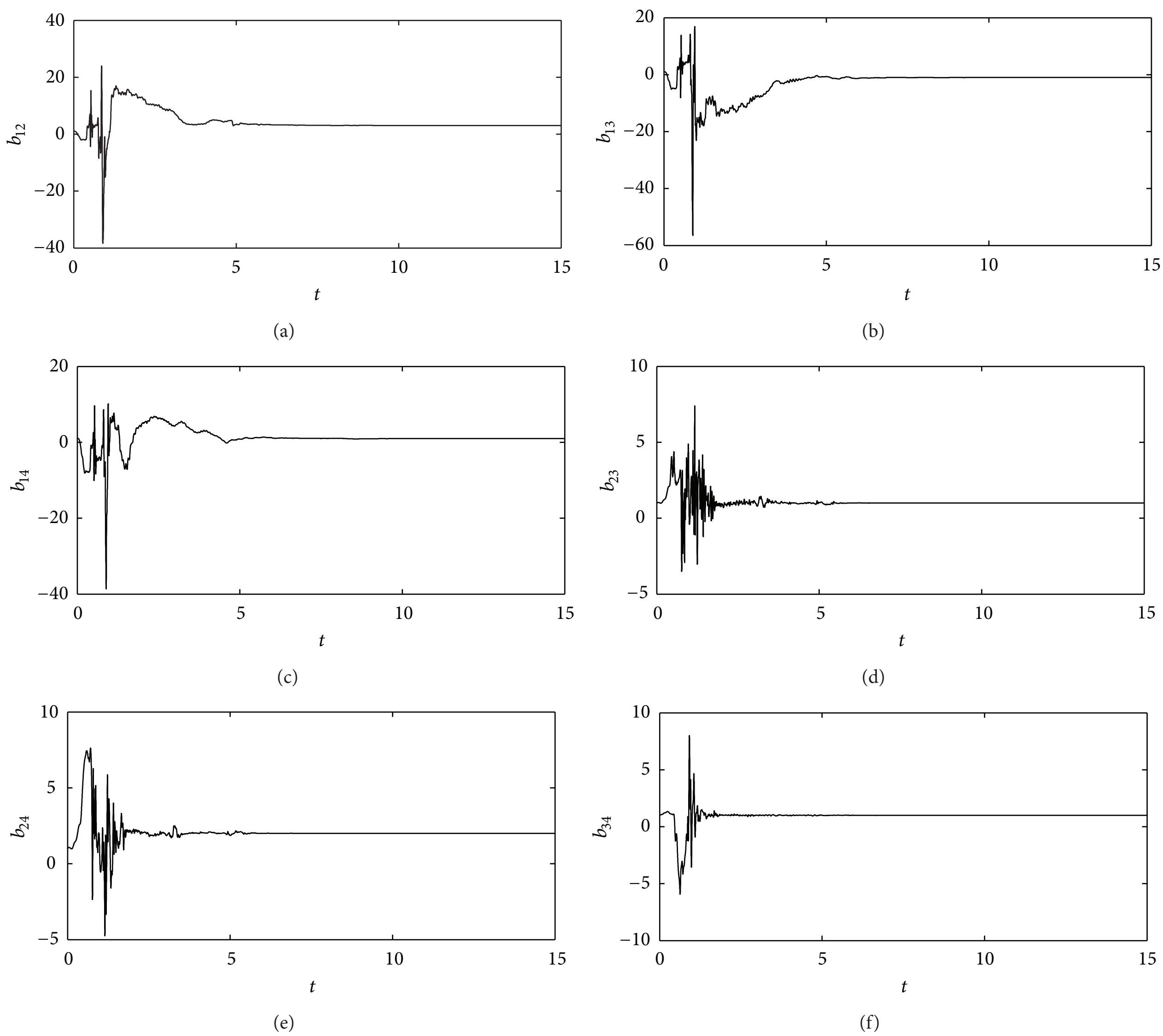

Figure 3: Identification of network structure (some elements of matrix $\widehat{B} = (\widehat{b}_{ij})_{4\times 4}$ are displayed).

$$
\begin{aligned}
&+ \sum_{j=1}^{N} \widehat{b}_{ij}(t)\left(h\left(y^j(t-\tau_1)\right)\right. \\
&\qquad \left.+ h\left(y^i(t-\tau_2)\right)\right) \\
&- r^i(t)e^i(t) - \sum_{j=1}^{N} a_{ij}\left(h\left(x^j(t)\right)\right. \\
&\qquad \left.+ h\left(x^i(t)\right)\right) \\
&- \sum_{j=1}^{N} b_{ij}\left(h\left(x^j(t-\tau_1)\right)\right. \\
&\qquad \left.\left.+ h\left(x^i(t-\tau_2)\right)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{i=1}^{N}\left(r^i(t)-k\right)\left(e^i(t)\right)^T e^i(t) \\
&+ \sum_{i=1}^{N}\left[\left(e^i(t)\right)^T e^i(t) - \left(e^i(t-\tau_1)\right)^T e^i(t-\tau_1)\right] \\
&+ \sum_{i=1}^{N}\left[\left(e^i(t)\right)^T e^i(t) - \left(e^i(t-\tau_2)\right)^T e^i(t-\tau_2)\right] \\
&+ \sum_{i=1}^{N}\sum_{j=1}^{N} \widetilde{a}_{ij}(t)\left(e^i(t)\right)^T\left(h\left(y^j(t)\right)+h\left(y^i(t)\right)\right) \\
&+ \sum_{i=1}^{N}\sum_{j=1}^{N} \widetilde{b}_{ij}(t)\left(e^i(t)\right)^T \\
&\qquad \times\left(h\left(y^j(t-\tau_1)\right)+h\left(y^i(t-\tau_2)\right)\right) \\
&+ \sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^T\left(t,e^i(t)\right)\varphi\left(t,e^i(t)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\Bigg[\left(f\left(y^{i}(t)\right)-f\left(x^{i}(t)\right)\right) \\
&\qquad + \sum_{j=1}^{N}a_{ij}\left(h\left(y^{j}(t)\right)+h\left(y^{i}(t)\right)\right) \\
&\qquad + \sum_{j=1}^{N}b_{ij}\left(h\left(y^{j}(t-\tau_{1})\right) + h\left(y^{i}(t-\tau_{2})\right)\right) \\
&\qquad - r^{i}(t)e^{i}(t) \\
&\qquad - \sum_{j=1}^{N}a_{ij}\left(h\left(x^{j}(t)\right)+h\left(x^{i}(t)\right)\right) \\
&\qquad - \sum_{j=1}^{N}b_{ij}\left(h\left(x^{j}(t-\tau_{1})\right) + h\left(x^{i}(t-\tau_{2})\right)\right)\Bigg] \\
&\quad + \sum_{i=1}^{N}\left(r^{i}(t)-k\right)\left(e^{i}(t)\right)^{T}e^{i}(t) \\
&\quad + \sum_{i=1}^{N}\left[\left(e_{i}(t)\right)^{T}e_{i}(t)-\left(e_{i}(t-\tau_{1})\right)^{T}e_{i}(t-\tau_{1})\right] \\
&\quad + \sum_{i=1}^{N}\left[\left(e_{i}(t)\right)^{T}e_{i}(t)-\left(e_{i}(t-\tau_{2})\right)^{T}e_{i}(t-\tau_{2})\right] \\
&\quad + \sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right] \\
&\le \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\Bigg[M\left(x^{i}(t)\right)e^{i}(t) \\
&\qquad + \sum_{j=1}^{N}a_{ij}\left(\gamma_{2}e^{j}(t)+\gamma_{2}e^{i}(t)\right) \\
&\qquad - r^{i}(t)e^{i}(t) \\
&\qquad + \sum_{j=1}^{N}b_{ij}\left(\gamma_{2}e^{j}(t-\tau_{1}) + \gamma_{2}e^{i}(t-\tau_{2})\right)\Bigg] \\
&\quad + \sum_{i=1}^{N}\left(r^{i}(t)-k\right)\left(e^{i}(t)\right)^{T}e^{i}(t) \\
&\quad + \sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_{1})\right)^{T}e^{i}(t-\tau_{1})\right] \\
&\quad + \sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_{2})\right)^{T}e^{i}(t-\tau_{2})\right] \\
&\quad + \sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right] \\
&= \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}M\left(x^{i}(t)\right)e^{i}(t) - \sum_{i=1}^{N}k\left(e^{i}(t)\right)^{T}e^{i}(t) \\
&\quad + \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\Bigg[\sum_{j=1}^{N}a_{ij}\left(\gamma_{2}e^{j}(t)+\gamma_{2}e^{i}(t)\right) \\
&\qquad + \sum_{j=1}^{N}b_{ij}\left(\gamma_{2}e^{j}(t-\tau_{1}) + \gamma_{2}e^{i}(t-\tau_{2})\right)\Bigg] \\
&\quad + \sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_{1})\right)^{T}e^{i}(t-\tau_{1})\right] \\
&\quad + \sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_{2})\right)^{T}e^{i}(t-\tau_{2})\right] \\
&\quad + \sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right] \\
&= \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}M\left(x^{i}(t)\right)e^{i}(t) - \sum_{i=1}^{N}k\left(e^{i}(t)\right)^{T}e^{i}(t) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left(e^{i}(t)\right)^{T}a_{ij}\gamma_{2}e^{j}(t) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left(e^{i}(t)\right)^{T}a_{ij}\gamma_{2}e^{i}(t) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left(e^{i}(t)\right)^{T}b_{ij}\gamma_{2}e^{j}(t-\tau_{1}) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left(e^{i}(t)\right)^{T}b_{ij}\gamma_{2}e^{i}(t-\tau_{2}) \\
&\quad + \sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_{1})\right)^{T}e^{i}(t-\tau_{1})\right] \\
&\quad + \sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_{2})\right)^{T}e^{i}(t-\tau_{2})\right] \\
&\quad + \sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right] \\
&\le \sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}M\left(x^{i}(t)\right)e^{i}(t) - \sum_{i=1}^{N}k\left(e^{i}(t)\right)^{T}e^{i}(t) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left(e^{i}(t)\right)^{T}a_{ij}\gamma_{2}e^{i}(t) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{2}^{2}a_{ij}^{2}\left(e^{i}(t)\right)^{T}e^{i}(t)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{N}\left(e^{j}(t)\right)^{T}e^{j}(t)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_2^2 b_{ij}^2\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{j=1}^{N}\left(e^{j}(t-\tau_1)\right)^{T}e^{j}(t-\tau_1)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_2^2 b_{ij}^2\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{j=1}^{N}\left(e^{j}(t-\tau_2)\right)^{T}e^{j}(t-\tau_2)\\
&+\sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_1)\right)^{T}e^{i}(t-\tau_1)\right]\\
&+\sum_{i=1}^{N}\left[\left(e^{i}(t)\right)^{T}e^{i}(t)-\left(e^{i}(t-\tau_2)\right)^{T}e^{i}(t-\tau_2)\right]\\
&+\sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right]\\
={}&\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}M\left(x^{i}(t)\right)e^{i}(t)-\sum_{i=1}^{N}k\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(e^{i}(t)\right)^{T}a_{ij}\gamma_2 e^{i}(t)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_2^2 a_{ij}^2\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{j=1}^{N}\left(e^{j}(t)\right)^{T}e^{j}(t)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_2^2 b_{ij}^2\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_2^2 b_{ij}^2\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)+\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right]\\
\le{}&\sum_{i=1}^{N}\lambda\left(e^{i}(t)\right)^{T}e^{i}(t)-\sum_{i=1}^{N}k\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\alpha_1\gamma_2 e^{i}(t)\\
&+\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\alpha_2\gamma_2^2 e^{i}(t)\\
&+\sum_{j=1}^{N}\left(e^{j}(t)\right)^{T}e^{j}(t)\\
&+2\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}\beta\gamma_2^2 e^{i}(t)\\
&+\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right]\\
={}&-\left[k-\lambda-\alpha_1\gamma_2-\alpha_2\gamma_2^2-3-2\beta\gamma_2^2\right]\\
&\times\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\frac{1}{2}\operatorname{Tr}\left[\varphi^{T}\left(t,e^{i}(t)\right)\varphi\left(t,e^{i}(t)\right)\right]\\
\le{}&-\left[k-\lambda-\alpha_1\gamma_2-\alpha_2\gamma_2^2-3-2\beta\gamma_2^2\right]\\
&\times\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)\\
&+\sum_{i=1}^{N}\mu\left(e^{i}(t)\right)^{T}e^{i}(t)\\
={}&-\left[k-\lambda-\alpha_1\gamma_2-\alpha_2\gamma_2^2-3-2\beta\gamma_2^2-\mu\right]\\
&\times\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)\le-\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t).
\end{aligned}
\tag{16}
$$

Based on the LaSalle invariance principle of stochastic differential equation, which was developed in [29], we have $e^{i}(t)\to 0$, which in turn illustrates that $E\|e^{i}(t)\|^{2}\to 0$, and at the same time $r^{i}(t)\to k$, $\widehat{a}_{ij}(t)\to a_{ij}$ and $\widehat{b}_{ij}(t)\to b_{ij}$. This completes the proof. □

If we let the coupling matrix $A=0$ or $B=0$, we will get two simple conditions:

$$
\begin{aligned}
dx^{i}(t)&\\
={}&\left[f\left(x^{i}(t)\right)+\sum_{j=1}^{N}b_{ij}\left(h\left(x^{j}(t-\tau_1)\right)+h\left(x^{i}(t-\tau_2)\right)\right)\right]dt\\
&+\varphi\left(t,x^{i}\right)d\omega(t),
\end{aligned}
\tag{17}
$$

$$\begin{aligned} dx^i(t) &= \left[f\left(x^i(t)\right) + \sum_{j=1}^{N} a_{ij}\left(h\left(x^j(t)\right) + h\left(x^i(t)\right)\right) \right] dt \\ &\quad + \varphi\left(t, x^i\right) d\omega(t). \end{aligned} \tag{18}$$

For the simple cases, we have the following corollaries.

Corollary 9. *Under Assumptions 2–4, the drive system* (17) *and the response system* (19) *will achieve synchronization under the adaptive controllers* (10)-(11) *and the adaptive gains* (13)*:*

$$\begin{aligned} dy^i(t) &= \left[f\left(y^i(t)\right) + \sum_{j=1}^{N} \hat{b}_{ij}\left(h\left(y^j(t-\tau_1)\right) + h\left(y^i(t-\tau_2)\right)\right) \right. \\ &\quad \left. + u^i(t) \right] dt + \varphi\left(t, y^i(t)\right) d\omega(t). \end{aligned} \tag{19}$$

Corollary 10. *Under Assumptions 2–4, the drive system* (18) *and the response system* (20) *will achieve synchronization under the adaptive controllers* (10)-(11) *and the adaptive gains* (12)*:*

$$\begin{aligned} dy^i(t) &= \left[f\left(y^i(t)\right) + \sum_{j=1}^{N} \hat{a}_{ij}\left(h\left(y^j(t)\right) + h\left(y^i(t)\right)\right) + u^i(t) \right] dt \\ &\quad + \varphi\left(t, y^i(t)\right) d\omega(t). \end{aligned} \tag{20}$$

The proofs of Corollaries 9 and 10 follow directly from Theorem 8 and be omitted here.

4. Illustrative Example

In this section, numerical simulations are presented to verify the theoretical result obtained in previous sections. We consider chaotic Lü oscillators as nodes of the uncoupled network. A single Lü oscillator is described by [30]

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right), \\ \dot{x}_2 &= c x_2 - x_1 x_3, \\ \dot{x}_3 &= -b x_3 + x_1 x_2, \end{aligned} \tag{21}$$

where $a = 36$, $c = 20$, $b = 3$.

We rewrite (21) as follows:

$$\dot{x} = M\left(x_1\right) x, \tag{22}$$

where $x = (x_1, x_2, x_3)^T$, and

$$M\left(x_1\right) = \begin{pmatrix} -a & a & 0 \\ 0 & c & -x_1 \\ 0 & x_1 & -b \end{pmatrix}. \tag{23}$$

According to Section 3, we show that the network with four nodes described by

$$\begin{aligned} dx^i(t) &= \left[f\left(x^i(t)\right) + \sum_{j=1}^{4} a_{ij}\left(h\left(x^j(t)\right) + h\left(x^i(t)\right)\right) \right. \\ &\quad \left. + \sum_{j=1}^{4} b_{ij}\left(h\left(x^j(t-\tau_1)\right) + h\left(x^i(t-\tau_2)\right)\right) \right] dt \\ &\quad + \varphi\left(t, x^i(t)\right) d\omega(t), \\ dy^i(t) &= \left[f\left(y^i(t)\right) + \sum_{j=1}^{4} \hat{a}_{ij}\left(h\left(y^j(t)\right) + h\left(y^i(t)\right)\right) \right. \\ &\quad + \sum_{j=1}^{4} \hat{b}_{ij}\left(h\left(y^j(t-\tau_1)\right) + h\left(y^i(t-\tau_2)\right)\right) \\ &\quad \left. - r^i(t) e^i(t) \right] dt + \varphi\left(t, y^i(t)\right) d\omega(t), \end{aligned} \tag{24}$$

where

$$\begin{aligned} \dot{r}^i(t) &= \theta_i \left(e^i(t)\right)^T e^i(t), \\ \dot{\hat{a}}_{ij} &= -\left(e^i(t)\right)^T \left(h\left(y^j(t)\right) + h\left(y^i(t)\right)\right), \\ \dot{\hat{b}}_{ij} &= -\left(e^i(t)\right)^T \left(h\left(y^j(t-\tau_1)\right) + h\left(y^i(t-\tau_2)\right)\right), \\ &\quad 1 \le i, j \le 4. \end{aligned} \tag{25}$$

In numerical simulation, let

$$A = \begin{pmatrix} 6 & -2 & -3 & -1 \\ -2 & 2 & 1 & -1 \\ -3 & 1 & -2 & 4 \\ -1 & -1 & 4 & -2 \end{pmatrix},$$

$$B=\begin{pmatrix} -3 & 3 & -1 & 1 \\ 3 & -6 & 1 & 2 \\ -1 & 1 & -1 & 1 \\ 1 & 2 & 1 & -4 \end{pmatrix}, \tag{26}$$

the nonlinear function $h(z) = z + \sin z$ which satisfies $y_1 \le (h(z_1) - h(z_2))/(z_1 - z_2) \le y_2$ with $y_1 = 0, y_2 = 2$ the parameters are given as follows: $\theta_i = 1, \tau_1 = 0.1, \tau_2 = 0.15$, the initial values are $r^i(0) = i$, $\hat{a}_{ij}(0) = 1, \hat{b}_{ij}(0) = 1$, $x^i(0) = (1+i, 1+i, i)^T$, $y^i(0) = (-1+i, 1+i, i)^T$. The noise intensity are chosen as $\varphi(t, x^i(t)) = 0.01x^i(t)$, $\varphi(t, y^i(t)) = 0.01y^i(t)$. Figure 1 shows the variance of the synchronization errors. We introduce the quantity $(t) = \sqrt{\sum_{i=1}^{N} \|y^i(t) - x^i(t)\|^2/N}$, which is used to measure the quality of the control process. It is obvious that when $E(t)$ no longer increases, two networks achieve synchronization. Figures 2 and 3 show the identification of the network structure. It is very clear that the identification of the network structure is very successful. All numerical simulations illustrate the effectiveness of Theorem 8.

Numerical simulations of Corollaries 9 and 10 can be illustrated in a similar way as shown in Theorem 8. Thus, we leave out numerical simulations here.

5. Conclusion

We have proposed an adaptive feedback control approach to identify the uncertain network topological structure of the nonlinear coupled stochastic complex dynamical networks with two differently delayed coupling. Several useful identification criteria have been attained. Numerical simulations have been given to verify the effectiveness of the proposed adaptive identification schemes.

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