Eur. Phys. J. C (2015) 75:327
DOI 10.1140/epjc/s10052-015-3549-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Green's functions in perturbative quantum gravity

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Received: 6 April 2015 / Accepted: 28 June 2015 / Published online: 17 July 2015

Abstract We show that the Green's functions in a non-linear gauge in the theory of perturbative quantum gravity is expressed as a series in terms of those in linear gauges. This formulation also holds for operator Green's functions. We further derive the explicit relation between the Green's functions in the theory of perturbative quantum gravity in a pair of arbitrary gauges. This process involves some sort of modified FFBRST transformations which are derivable from infinitesimal field-dependent BRST transformations.

1 Introduction

Since its inception, general relativity has had many striking similarities to gauge theories. For instance, both involve the idea of local symmetry and therefore share a number of formal properties. Moreover, consistent quantum gauge theories are well established but as yet no satisfactory quantum field theory of gravity has been found. The structures of the Lagrangians of these theories are rather different. The Yang–Mills Lagrangian contains only up to four-point interactions, while the Einstein–Hilbert Lagrangian contains infinitely many interactions. Despite these differences, string theory provides us with sufficient reasons for claiming that gravity and gauge theories can, in fact, be unified. For example, the Maldacena conjecture [1,2] relates the weak coupling limit of a gravity theory to a strong coupling limit of a special supersymmetric gauge field theory. With this similarity, the gauge theories are allowed to be used directly as a resource for computations in perturbative quantum gravity.

The perturbative quantum gravity as a gauge theory is a subject of extensive research [3–5]. For example, the mode analysis and Ward identities for a ghost propagator for perturbative quantum gravity have been demonstrated [6]. The Feynman rules and propagator for gravity in the physically interesting cases of inflation have been analyzed [7]. The propagator for a gauge theory exists only after fixing a gauge. For instance, the Landau and Curci–Ferrari type gauges have their common uses in perturbation theory [8,9]. Being gauge-fixed, the theory loses their local gauge invariance. However, it possesses the rather different fermionic rigid BRST invariance [10,11].

The BRST symmetry and the associated concept of BRST cohomology provide the most used covariant quantization method for constrained systems such as gauge and string theories [12,13]. The BRST and the anti-BRST symmetries for perturbative quantum gravity in flat spacetime have also been investigated [14–16], which was summarized by Nakanishi and Ojima [17]. Recently, the BRST formulation for the perturbative quantum gravity in general curved spacetime has also been analyzed [18–20]. The usual infinitesimal BRST transformation has been generalized by allowing the parameter to be finite and field-dependent [21]. This FFBRST enjoys the properties of the usual BRST except that it does not leave the path integral measure invariant. For the FFBRST transformations have been found several applications in gauge field theories in flat spacetime [21–41] as well as in curved spacetime [42,43]. The FFBRST formulation to connect the Green's function of Yang–Mills theory in a set of two otherwise unrelated gauge choices has been established [45]. Nevertheless, the FFBRST formulation to connect Green's functions has not been developed so far in the context of perturbative quantum gravity. The development of a FFBRST formulation to connect Green's functions in perturbative quantum gravity is the goal of the present investigation.

In this paper, we discuss the usual FFBRST transformation in perturbative quantum gravity to connect the linear and non-linear gauges of the theory. Further, we establish a connection between arbitrary Green's functions (or operator Green's functions) in two sets of gauges for the theory of perturbative quantum gravity. In view of their extreme importance, we choose these to be the linear (Landau) and non-linear (Curci–Ferrari) type gauges. Here we find that

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to connect the Green's functions of the theory rather than a connection of gauges we require a different FFBRST transformation. Finally, we establish a compact result expressing an arbitrary Green's function or operator Green's function in non-linear gauges with a closed expression involving similar Green's functions in Landau gauges.

This paper is presented as follows. In Sect. 2, we present the usual FFBRST transformation for a general gauge theory. In Sect. 3, we recapitulate the FFBRST transformation to connect the linear and non-linear gauges in linearized gravity. In Sect. 4, we demonstrate a similar FFBRST transformation to connect the Green's functions of the perturbative quantum gravity by a compact formula. In the last section, we summarize the results motivated by future developments.

2 The usual FFBRST transformations

In this subsection, we recapitulate the FFBRST transformation for the general gauge theory in general curved spacetime [46]. For this purpose, we first write the usual BRST transformation,

$$\delta_b\phi(x) = s\phi(x)\delta\Lambda, \tag{1}$$

where $\delta\Lambda$ is an infinitesimal and field-independent Grassmann parameter and $\phi(x)$ is the generic notation of fields $(h, c, \bar{c}, b)$ involved in the theory of quantum gravity. One observes for a BRST transformation that its basic properties do not depend on whether the parameter $\delta\Lambda$ is (i) finite or infinitesimal, (ii) field-dependent or not, as long as it is anticommuting and spacetime independent. This renders us the freedom to make the parameter $\delta\Lambda$ finite and field-dependent without affecting its basic features. The first step toward this goal is to make the infinitesimal parameter field-dependent by interpolating a continuous parameter, κ $(0 \leq \kappa \leq 1)$, in the theory. The fields, $\phi(x, \kappa)$, depend on κ; $\phi(x, \kappa = 0) = \phi(x)$ is for the initial fields and $\phi(x, \kappa = 1) = \phi'(x)$ is for the transformed fields.

The infinitesimal field-dependent BRST transformation is defined by [21]

$$\mathrm{d}\phi(x, \kappa) = s[\phi(x)]\Theta'[\phi(\kappa)]\mathrm{d}\kappa, \tag{2}$$

where $\Theta'[\phi(\kappa)]\mathrm{d}\kappa$ is the infinitesimal but field-dependent parameter. The FFBRST transformation is then obtained by integrating this infinitesimal transformation from $\kappa = 0$ to $\kappa = 1$, as follows:

$$\phi' \equiv \phi(x, \kappa = 1) = \phi(x, \kappa = 0) + s[\phi(x)]\Theta[\phi], \tag{3}$$

where

$$\Theta[\phi] = \Theta'[\phi]\frac{\exp f[\phi] - 1}{f[\phi]} \tag{4}$$

is the finite field-dependent parameter and $f[\phi]$ is given by

$$f[\phi] = \sum_i \int \mathrm{d}^4x \frac{\delta\Theta'[\phi]}{\delta\phi_i(x)} s_b\phi_i(x). \tag{5}$$

The resulting FFBRST transformation leaves the effective action invariant but the functional integral changes non-trivially under it [21]. Now we compute the Jacobian of the path integral measure under the FFBRST transformation.

We first define the Jacobian of the path integral measure under such transformations with an arbitrary finite field-dependent parameter, $\Theta[\phi(x)]$, by

$$\mathcal{D}\phi' = J(\kappa)\mathcal{D}\phi(\kappa). \tag{6}$$

The Jacobian, $J(\kappa)$, can be replaced within the functional integral as

$$J(\kappa) \to \exp[iS_1[\phi(x, \kappa), \kappa]], \tag{7}$$

where $S_1[\phi(x), \kappa]$ is a local functional of the fields if and only if the following condition is satisfied [21]:

$$\int \mathcal{D}\phi \left[\frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}\kappa} - i\frac{\mathrm{d}S_1[\phi(x, \kappa), \kappa]}{\mathrm{d}\kappa}\right] \mathrm{e}^{i(S_L[\phi]+S_1[\phi,\kappa])} = 0. \tag{8}$$

The infinitesimal change in the Jacobian $J(\kappa)$ is addressed with the following formula [21]:

$$\frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}\kappa} = -\int \mathrm{d}^4y \left[\pm s\phi(y, \kappa)\frac{\delta\Theta'[\phi]}{\delta\phi(y, \kappa)}\right], \tag{9}$$

where the + sign is used for bosonic fields ϕ and the − sign is used for fermionic fields ϕ.

Recently, exactly similar FFBRST transformations have also been considered and a general Jacobian has been calculated explicitly in terms of the general finite parameter Θ [47].

3 The FFBRST transformation in perturbative quantum gravity: preliminaries

In this section we consider perturbative quantum gravity in the framework of the FFBRST transformation. In particular we analyze the perturbative quantum gravity in linear and non-linear gauges. Then we generalize the BRST transformation by making the transformation finite and field-dependent. Furthermore, we establish the connection between these two gauges using the FFBRST transformation [46].

3.1 The linearized quantum gravity

Let us start by writing the classical Lagrangian density for gravity in general curved spacetime,

$$\mathcal{L}_c = \sqrt{-g}(R - 2\lambda), \tag{10}$$

where R is the Ricci scalar curvature and λ is a cosmological constant. Here units are settled in such a manner that $16\pi G = 1$. In the weak approximation the full metric g^f_{ab} can be written as a sum of the fixed metric of background spacetime g_{ab} and the small perturbations around it, denoted by h_{ab}. This fluctuation is considered as a quantum field that needs to be quantized. Therefore, numerically

$$g^f_{ab} = g_{ab} + h_{ab}. \tag{11}$$

Incorporating such a decomposition, the Lagrangian density given in (10) described in terms of h_{ab} remains invariant under the following coordinate transformation:

$$\delta_\Lambda h_{ab} = \nabla_a \Lambda_b + \nabla_b \Lambda_a + \pounds_{(\Lambda)} h_{ab}, \tag{12}$$

where the Lie derivative of h_{ab} with respect to the vector field Λ_a is defined by

$$\pounds_{(\Lambda)} h_{ab} = \Lambda^c \nabla_c h_{ab} + h_{ac} \nabla_b \Lambda^c + h_{cb} \nabla_a \Lambda^c. \tag{13}$$

The gauge invariance reflects the redundancy in physical degrees of freedom. Such a redundancy in gauge degrees of freedom produces constraints in the canonical quantization and leads to divergences in the generating functional. In order to fix the redundancy we choose the following gauge-fixing condition satisfied by the quantum field:

$$G[h]_a = (\nabla^b h_{ab} - \beta \nabla_a h) = 0, \tag{14}$$

where the parameter $\beta \neq 1$. This is so because $\beta = 1$ leads to vanishing conjugate momentum corresponding to h_{00} and therefore the generating functional diverges. This gauge-fixing condition on the quantum level comes about by adding the following term in the classical action:

$$\mathcal{L}_{gf} = \sqrt{-g}[i b^a (\nabla^b h_{ab} - \beta \nabla_a h)]. \tag{15}$$

The induced (Faddeev–Popov) ghost term is then defined by

$$\mathcal{L}_{gh} = \sqrt{-g}\bar{c}^a M_{ab} c^b, \tag{16}$$

where the Faddeev–Popov matrix operator M_{ab} has the following expression:

$$\begin{aligned} M_{ab} = {} & i\nabla_c[\delta^c_b \nabla_a + g_{ab}\nabla^c - 2\beta\delta^c_a \nabla_b + \nabla_b h^c_a \\ & -h_{ab}\nabla^c - h^c_b \nabla_a - \beta g^c_a g^{ef}(\nabla_b h_{ef} + h_{eb}\nabla_f + h_{fb}\nabla_e)]. \end{aligned} \tag{17}$$

Henceforth, the effective action for perturbative quantum gravity in curved spacetime dimensions (in linear gauge) reads

$$S_L = \int d^4x (\mathcal{L}_c + \mathcal{L}_{gf} + \mathcal{L}_{gh}), \tag{18}$$

which is invariant under the following BRST transformations:

$$\begin{aligned} & s h_{ab} = (\nabla_a c_b + \nabla_b c_a + \pounds_{(c)} h_{ab}), \\ & s c^a = -c_b \nabla^b c^a,\ s\bar{c}^a = b^a,\ s b^a = 0. \end{aligned} \tag{19}$$

Here we observe that the gauge-fixing and the ghost parts of the effective Lagrangian density are BRST-exact. Therefore,

$$\begin{aligned} \mathcal{L}_g &= \mathcal{L}_{gf} + \mathcal{L}_{gh}, \\ &= i s\sqrt{-g}[\bar{c}^a(\nabla^b h_{ab} - \beta\nabla_a h)], \\ &= s\Psi. \end{aligned} \tag{20}$$

The gauge-fixed fermion (Ψ) then has the expression

$$\Psi = i\sqrt{-g}[\bar{c}^a(\nabla^b h_{ab} - \beta\nabla_a h)]. \tag{21}$$

However, the gauge-fixing and ghost terms in the non-linear Curci–Ferrari gauge condition are written

$$\begin{aligned} \mathcal{L}'_g = {} & \mathcal{L}'_{gf} + \mathcal{L}'_{gh}, \\ = {} & \sqrt{-g}\Big[i b^a(\nabla^b h_{ab} - \beta\nabla_a h) - i\bar{c}^b \nabla_b c^a (\nabla^c h_{ac} \\ & -\beta\nabla_a h) + \bar{c}^a M_{ab} c^b + \frac{\alpha}{2} b^b \nabla_b \bar{c}^a c_a \\ & -\frac{\alpha}{2}\bar{c}^c \nabla_c c^b \nabla_b \bar{c}^a c_a - \frac{\alpha}{2}\bar{b}^b \nabla_b b^a c_a \\ & -\frac{\alpha}{2}\bar{c}^b \nabla_b \bar{c}^a c_d \nabla^d c_a - \frac{\alpha}{2} b_a b^a + \alpha\bar{c}^a b^b \nabla_b c_a \\ & +\alpha\bar{c}^a \bar{c}^b c^d \nabla_b \nabla_d c_a \Big], \end{aligned} \tag{22}$$

where α is a gauge parameter. For instance, the effective action, having such gauge-fixing and Faddeev–Popov ghost terms, in a non-linear gauge is given by

$$S_{NL} = \int d^4x (\mathcal{L}_c + \mathcal{L}'_g), \tag{23}$$

which remains unchanged under the following BRST transformations:

$$\begin{aligned} s\, h_{ab} &= \nabla_a c_b + \nabla_b c_a + \pounds_{(c)} h_{ab}, \\ s\, c^a &= -\, c_b \nabla^b c^a, \\ s\, \bar{c}^a &= b^a - \bar{c}^b \nabla_b c^a, \\ s\, b^a &= -\, b^b \nabla_b c^a - \bar{c}^b c^d \nabla_b \nabla_d c^a. \end{aligned} \tag{24}$$

3.2 FFBRST transformation for linear to non-linear gauge

We construct the FFBRST transformation for perturbative quantum gravity utilizing the BRST transformation (19) as follows:

$$\begin{aligned} f\,h_{ab} &= (\nabla_a c_b + \nabla_b c_a + \pounds_{(c)} h_{ab})\ \Theta[\phi], \\ f\,c^a &= -c_b \nabla^b c^a\ \Theta[\phi], \\ f\,\bar{c}^a &= b^a\ \Theta[\phi], \\ f\,b^a &= 0, \end{aligned} \tag{25}$$

where $\Theta[\phi]$ is an arbitrary finite field-dependent parameter. To establish the connection between the Landau and the (non-linear) Curci–Ferrari gauge we opt for the finite field-dependent parameter constructed from the following infinitesimal field-dependent parameter:

$$\Theta'[\phi]=i\frac{\alpha}{2}\sqrt{-g}\int \mathrm{d}^4y\ \left(\bar{c}_b\nabla^b\bar{c}^a c_a - \bar{c}^a b_a - \bar{c}^a\bar{c}_b\nabla^b c_a\right). \tag{26}$$

Exploiting Eqs. (9) and (26) we calculate the change in Jacobian as

$$\begin{aligned} \frac{1}{J(\kappa)}\frac{\mathrm{d}J(\kappa)}{\mathrm{d}\kappa} &= -i\frac{\alpha}{2}\sqrt{-g}\int d^4x\left[-b_b\nabla^b\bar{c}^a c_a \right. \\ &+\bar{c}^d\nabla_d c_b\nabla^b\bar{c}^a c_a + \bar{c}_b\nabla^b b^a c_a + \bar{c}_b\nabla^b\bar{c}^a c_d\nabla^d c_a \\ &\left.+b_a b^a - 2\bar{c}^a b_b\nabla^b c_a - 2\bar{c}^a\bar{c}_b c_d\nabla^b\nabla^d c_a\right]. \end{aligned} \tag{27}$$

The local functional S_1 in the expression (7) is written by

$$\begin{aligned} S_1[\phi(\kappa),\kappa] &= \int \mathrm{d}^4x\left[\xi_1 b_b\nabla^b\bar{c}^a c_a + \xi_2\bar{c}^d\nabla_d c_b\nabla^b\bar{c}^a c_a \right. \\ &+\xi_3\bar{c}_b\nabla^b b^a c_a + \xi_4\bar{c}_b\nabla^b\bar{c}^a c_d\nabla^d c_a \\ &\left.+\xi_5 b_a b^a + \xi_6\bar{c}^a b_b\nabla^b c_a + \xi_7\bar{c}^a\bar{c}_b c_d\nabla^b\nabla^d c_a\right], \end{aligned} \tag{28}$$

where the parameters $\xi_i (i = 1, 2, \ldots, 7)$ depend explicitly on the parameter κ as follows [46]:

$$\begin{aligned} &\xi_1 = \frac{\alpha}{2}\sqrt{-g}\kappa,\ \ \xi_2 = -\frac{\alpha}{2}\sqrt{-g}\kappa,\ \ \xi_3 = -\frac{\alpha}{2}\sqrt{-g}\kappa, \\ &\xi_4 = -\frac{\alpha}{2}\sqrt{-g}\kappa, \\ &\xi_5 = -\frac{\alpha}{2}\sqrt{-g}\kappa,\ \ \xi_6 = \alpha\sqrt{-g}\kappa,\ \ \xi_7 = \alpha\sqrt{-g}\kappa. \end{aligned} \tag{29}$$

With these identifications of $\xi_i(\kappa)$ the expression of S_1 becomes

$$\begin{aligned} S_1[\phi(\kappa),\kappa] &= \kappa\int \mathrm{d}^4x\sqrt{-g}\left[\frac{\alpha}{2}b_b\nabla^b\bar{c}^a c_a - \frac{\alpha}{2}\bar{c}^d\nabla_d c_b\nabla^b\bar{c}^a c_a \right. \\ &-\frac{\alpha}{2}\bar{c}_b\nabla^b b^a c_a - \frac{\alpha}{2}\bar{c}_b\nabla^b\bar{c}^a c_d\nabla^d c_a \\ &\left.-\frac{\alpha}{2}b_a b^a + \alpha\bar{c}^a b_b\nabla^b c_a + \alpha\bar{c}^a\bar{c}_b c_d\nabla^b\nabla^d c_a\right]. \end{aligned} \tag{30}$$

Therefore, the FFBRST transformation (25) changes the effective action in a functional integration as

$$\begin{aligned} S_L + S_1(\kappa = 1) &= \int \mathrm{d}^4x\left[\mathcal{L}_c + i\sqrt{-g}b^a(\nabla^b h_{ab} - \beta\nabla_a h) \right. \\ &+\sqrt{-g}\bar{c}^a M_{ab}c^b \\ &+\frac{\alpha}{2}\sqrt{-g}b_b\nabla^b\bar{c}^a c_a - \frac{\alpha}{2}\sqrt{-g}\bar{c}^d\nabla_d c_b\nabla^b\bar{c}^a c_a \\ &-\frac{\alpha}{2}\sqrt{-g}\bar{c}_b\nabla^b b^a c_a - \frac{\alpha}{2}\sqrt{-g}\bar{c}_b\nabla^b\bar{c}^a c_d\nabla^d c_a \\ &-\frac{\alpha}{2}\sqrt{-g}b_a b^a + \alpha\sqrt{-g}\bar{c}^a b_b\nabla^b c_a \\ &\left.+\alpha\sqrt{-g}\bar{c}^a\bar{c}_b c_d\nabla^b\nabla^d c_a\right]. \end{aligned} \tag{31}$$

After performing a shift in the Nakanishi–Lautrup field by $\bar{c}^b\nabla_b c^a$, the above expression reduces to

$$\begin{aligned} S_L + S_1(\kappa = 1) = \int \mathrm{d}^4x &\left[\mathcal{L}_c + i\sqrt{-g}b^a(\nabla^b h_{ab} - \beta\nabla_a h)\right. \\ &-i\sqrt{-g}\bar{c}^b\nabla_b c^a(\nabla^b h_{ab} - \beta\nabla_a h) \\ &\left.+\sqrt{-g}\bar{c}^a M_{ab}c^b\right], = S_{NL}, \end{aligned} \tag{32}$$

which is nothing but the effective action for perturbative quantum gravity in a Landau gauge.

4 Relation between Green's function for linear and non-linear gauges

In this section, we establish a procedure for a FFBRST transformation that transforms the generating functional (Green's function) in one kind of a gauge choice to the generating functional in another kind of a gauge choice. For this purpose we define the generating functional for perturbative quantum gravity in a linear gauge,

$$W_L = \int \mathcal{D}\phi\ \mathrm{e}^{iS_L[\phi]}, \tag{33}$$

which transforms under a FFBRST transformation $\phi'(x) = \phi(x) + s\phi\Theta[\phi]$ defined in (25) as follows:

$$W_{NL} = \int \mathcal{D}\phi'\ \mathrm{e}^{iS_L[\phi']} = W_L. \tag{34}$$

Now, we want to implement this transformation to connect the Green's functions in the two gauges for quantum gravity theory. According to the standard procedure, n-point Green's functions in a non-linear gauge under the FFBRST transformation transform as

$$\begin{aligned} G^{NL}_{i_1,\ldots,i_n} &= \int \mathcal{D}\phi'\prod_{r=1}^{n}\phi'_{i_r}\mathrm{e}^{iS_{NL}[\phi']}, \\ &= \int \mathcal{D}\phi\prod_{r=1}^{n}(\phi_{i_r} + s_{i_r}\phi\Theta[\phi])\mathrm{e}^{iS_L[\phi']}, \\ &= G^L_{i_1,\ldots,i_n} + \Delta G^L_{i_1,\ldots,i_n}, \end{aligned} \tag{35}$$

where $\Delta G^L_{i_1,\ldots,i_n}$ refers to the difference between the n-point Green's functions in the two sets of gauges. This may involve additional vertices corresponding to insertions of operators $s_{i_r}\phi$. But it seems technically incorrect for the following reasons.

A priori, it is not obvious that if condition (8) (for replacing the Jacobian to e^{iS_1}) holds for quantum gravity; then an equation modified to include an arbitrary operator $\mathcal{O}[\phi]$ of type

$$\int \mathcal{D}\phi\mathcal{O}[\phi]\left[\frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}\kappa}-i\frac{\mathrm{d}S_1[\phi(x,\kappa),\kappa]}{\mathrm{d}\kappa}\right] \mathrm{e}^{i(S_L[\phi]+S_1[\phi,\kappa])}=0 \tag{36}$$

would also hold. Of course it does not hold in general for the reason discussed in [45]. For this reason, to connect the Green's functions for the two type of gauges we need an elaborate treatment of the FFBRST transformation.

We begin with a general Green's function in non-linear gauge defined by

$$G=\int \mathcal{D}\phi'\mathcal{O}[\phi']\mathrm{e}^{iS_{NL}[\phi']}, \tag{37}$$

where $\mathcal{O}[\phi']$ is an arbitrary operator. Therefore, (37) covers both the arbitrary operator Green's functions and the arbitrary ordinary Green's functions. Specifically, for $\mathcal{O}_1[\phi']=h'_{ab}h'_{cd}$, Eq. (37) describes the gauge graviton propagator, however, for $\mathcal{O}_2[\phi']=h'_{ab}\bar{c}'^c c'_c$ it describes the 3-point propagator. We want to express the Green's function (G) of perturbative gravity entirely in terms of the linear type gauge Green's functions (and possibly involving vertices from $s\phi$). So we define

$$G(\kappa)=\int \mathcal{D}\phi\mathcal{O}[\phi(\kappa),\kappa]\mathrm{e}^{i(S_L[\phi]+S_1[\phi,\kappa])}, \tag{38}$$

where the form of operator $\mathcal{O}[\phi(\kappa),\kappa]$ demands

$$\frac{\mathrm{d}G}{\mathrm{d}\kappa}=0. \tag{39}$$

Under a FFBRST transformation ($\kappa=1$), it reflects that

$$G(1)=\int \mathcal{D}\phi'\mathcal{O}[\phi',1]\mathrm{e}^{iS_{NL}[\phi']}, \tag{40}$$

which coincides with (37), whereas at $\kappa=0$ this reads

$$G(0)=\int \mathcal{D}\phi\mathcal{O}[\phi,0]\mathrm{e}^{iS_L[\phi]}, \tag{41}$$

and is numerically equal to (40). Now, we need to determine the form of $\mathcal{O}[\phi(\kappa),\kappa]$ in (38) so that the condition (39) gets satisfied. For this purpose, we perform the field transformation from $\phi(\kappa)$ to $\phi(\kappa+d\kappa)$ through an infinitesimal field-dependent BRST transformation defined in (2), which leads to

$$\begin{aligned}
G(\kappa)&=\int \mathcal{D}\phi(\kappa+\mathrm{d}\kappa)\frac{J(\kappa+\mathrm{d}\kappa)}{J(\kappa)}\\
&\quad\times\left(\mathcal{O}[\phi(\kappa+\mathrm{d}\kappa),\kappa+\mathrm{d}\kappa]-s\phi\Theta'\frac{\delta\mathcal{O}}{\delta\phi}\mathrm{d}\kappa+\frac{\partial\mathcal{O}}{\partial\kappa}\mathrm{d}\kappa\right)\\
&\quad\times\left(1-i\frac{\mathrm{d}S_1}{\mathrm{d}\kappa}\mathrm{d}\kappa\right)\mathrm{e}^{iS_L[\phi(\kappa+\mathrm{d}\kappa)]+iS_1[\phi(\kappa+\mathrm{d}\kappa),\kappa+\mathrm{d}\kappa]},\\
&=\int \mathcal{D}\phi(\kappa+\mathrm{d}\kappa)\left(1+\frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}\kappa}\mathrm{d}\kappa\right)\\
&\quad\times\left(\mathcal{O}[\phi(\kappa+\mathrm{d}\kappa),\kappa+\mathrm{d}\kappa]-s\phi\Theta'\frac{\delta\mathcal{O}}{\delta\phi}\mathrm{d}\kappa+\frac{\partial\mathcal{O}}{\partial\kappa}\mathrm{d}\kappa\right)\\
&\quad\times\left(1-i\frac{\mathrm{d}S_1}{\mathrm{d}\kappa}\mathrm{d}\kappa\right)\mathrm{e}^{iS_L[\phi(\kappa+\mathrm{d}\kappa)]+iS_1[\phi(\kappa+\mathrm{d}\kappa),\kappa+\mathrm{d}\kappa]},\\
&=G(\kappa+\mathrm{d}\kappa),
\end{aligned} \tag{42}$$

if and only if

$$\int \mathcal{D}\phi(\kappa)\left(\left[\frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}\kappa}-i\frac{\mathrm{d}S_1}{\mathrm{d}\kappa}\right]\mathcal{O}[\phi(\kappa),\kappa]-s\phi\Theta'\frac{\delta\mathcal{O}}{\delta\phi}+\frac{\partial\mathcal{O}}{\partial\kappa}\right) \mathrm{e}^{iS_L[\phi(\kappa)]+iS_1[\phi(\kappa),\kappa]}=0. \tag{43}$$

So we get precisely the correct expression (43) for replacing the Jacobian of the path integral measure in the Green's function of quantum gravity as e^{iS_1} instead of the incorrect one (36).

Exploiting the information of above expression, the required condition for the κ-independence of G is

$$\begin{aligned}
&\int \mathcal{D}\phi(\kappa)e^{iS_L[\phi(\kappa)]+iS_1[\phi(\kappa),\kappa]}\left(\frac{\partial\mathcal{O}}{\partial\kappa}+\int(\nabla_a c_b+\nabla_b c_a\right.\\
&\quad+\pounds_{(c)}h_{ab})\Theta'\frac{\delta\mathcal{O}}{\delta h_{ab}}-\int c_b\nabla^b c_a\Theta'\frac{\delta\mathcal{O}}{\delta c_a}\\
&\quad\left.+\int\left[b_a-\kappa\bar{c}^b\nabla_b c^a\right]\Theta'\frac{\delta\mathcal{O}}{\delta\bar{c}_a}\right)=0.
\end{aligned} \tag{44}$$

Now, we construct the operator $\mathcal{O}$ to satisfy

$$\begin{aligned}
&\frac{\partial\mathcal{O}}{\partial\kappa}+\int(\nabla_a c_b+\nabla_b c_a+\pounds_{(c)}h_{ab})\Theta'\frac{\delta\mathcal{O}}{\delta h_{ab}}\\
&\quad-\int c_b\nabla^b c_a\Theta'\frac{\delta\mathcal{O}}{\delta c_a}+\int\left[b_a-\kappa\bar{c}^b\nabla_b c_a\right]\Theta'\frac{\delta\mathcal{O}}{\delta\bar{c}_a}=0.
\end{aligned} \tag{45}$$

Then condition (44) automatically is satisfied. Now, we consider a new set of fields $(\tilde{h}_{ab},\tilde{c}_a,\tilde{\bar{c}}_a,\tilde{b}_a)$ having the following infinitesimal field-dependent BRST transformation:

$$\begin{aligned}
\frac{\delta\tilde{h}_{ab}}{\delta\kappa}&=(\nabla_a\tilde{c}_b+\nabla_b\tilde{c}_a+\pounds_{(\tilde{c})}\tilde{h}_{ab})\ \Theta'[\tilde{\phi}],\\
\frac{\delta\tilde{c}^a}{\delta\kappa}&=-\tilde{c}_b\nabla^b\tilde{c}^a\ \Theta'[\tilde{\phi}],\\
\frac{\delta\tilde{\bar{c}}^a}{\delta\kappa}&=\tilde{B}^a\ \Theta'[\tilde{\phi}],\\
\frac{\delta\tilde{B}^a}{\delta\kappa}&=0,
\end{aligned} \tag{46}$$

where $\tilde{B}^a = \tilde{b}^a - \kappa\tilde{\bar{c}}^b\nabla_b\tilde{c}^a$. These new fields satisfy the following boundary condition: $\phi\tilde{(}1) = \phi(1)$. The condition (45) for $\mathcal{O}[\tilde{\phi}(\kappa), \kappa]$ instead of $\mathcal{O}[\phi(\kappa), \kappa]$ reads

$$\frac{\mathrm{d}\mathcal{O}[\tilde{\phi}(\kappa), \kappa]}{\mathrm{d}\kappa} = 0. \tag{47}$$

Now utilizing $\mathcal{O}[\tilde{\phi}(1), 1] = \mathcal{O}[\phi(1), 1] = \mathcal{O}[\phi']$ we obtain

$$\mathcal{O}[\tilde{\phi}(\kappa), \kappa] = \mathcal{O}[\phi'], \tag{48}$$

which tells us how the operator $\mathcal{O}[\phi(\kappa), \kappa]$ evolves. To derive the FFBRST transformation corresponding to (46), we first define the modification in f of (5) as follows:

$$f[\tilde{\phi}, \kappa] = f_1[\tilde{\phi}] + \kappa f_2[\tilde{\phi}]. \tag{49}$$

Therefore,

$$\frac{\mathrm{d}\Theta'[\tilde{\phi}(\kappa)]}{\mathrm{d}\kappa} = (f_1[\tilde{\phi}] + \kappa f_2[\tilde{\phi}])\Theta'[\tilde{\phi}(\kappa)]. \tag{50}$$

Performing an integration from 0 to κ, we have

$$\Theta'[\tilde{\phi}(\kappa)] = \Theta[\phi] \exp\left(\kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]\right). \tag{51}$$

Similarly, integrating (46) we get the FFBRST transformation, written compactly as

$$\begin{aligned} \phi' &= \phi + \left[(\tilde{\delta}_1[\phi] + \tilde{\delta}_2[\phi]) \int \mathrm{d}\kappa \exp\right. \\ &\quad \times \left.\left(\kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]\right)\right] \Theta'[\phi], \\ &= \phi + \delta\phi[\phi]. \end{aligned} \tag{52}$$

Now we apply the FFBRST transformation (52) on the Green's function in non-linear gauge (37)

$$\begin{aligned} G &= \int \mathcal{D}\phi' \mathcal{O}[\phi'] \mathrm{e}^{iS_{NL}[\phi']}, \\ &= \int \mathcal{D}\phi \mathcal{O}[\phi + +\delta\phi[\phi]] \mathrm{e}^{iS_L[\phi]}, \\ &= \int \mathcal{D}\phi \mathcal{O}[\phi] \mathrm{e}^{iS_L[\phi]} \\ &\quad + \int \mathcal{D}\phi \left[(\tilde{\delta}_1[\phi] + \tilde{\delta}_2[\phi])\right. \\ &\quad \times \left.\int \mathrm{d}\kappa \exp\left(\kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]\right)\right] \Theta'[\phi] \frac{\delta\mathcal{O}[\phi]}{\delta\phi} \mathrm{e}^{iS_L[\phi]}. \end{aligned} \tag{53}$$

Further, it can be written by

$$\begin{aligned} \langle\mathcal{O}\rangle_{NL} &= \langle\mathcal{O}\rangle_L + \int_0^1 \mathrm{d}\kappa \int \mathcal{D}\phi (\tilde{\delta}_1[\phi] \\ &\quad + \tilde{\delta}_2[\phi])\Theta'[\phi] \frac{\delta\mathcal{O}[\phi]}{\delta\phi} \mathrm{e}^{iS_M}, \end{aligned} \tag{54}$$

where $iS_M == iS_L + \kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]$. In this way, we establish the connection between the Green's function in two gauges in perturbative quantum gravity.

5 Concluding remarks

In this work, unlike the usual FFBRST transformation we have demonstrated a different FFBRST transformation in the case of perturbative quantum gravity to relate the arbitrary Green's functions of the theory corresponding to two different gauges. For concreteness, we have considered the linear and the non-linear gauges from the point of view of their common usage in gravity theory. The Green's functions in a non-linear gauge in the theory of perturbative quantum gravity is expressed as a series in terms of those in linear gauges. In this context we have shown the remarkable difference between the modified FFBRST transformation and the usual one. Further, being related to the usual FFBRST formulation, this modified FFBRST transformation is obtained by integration of (46). We hope that the final result, put in a simple form, will be very useful from a computational point of view in the theory of perturbative quantum gravity.

Funded by SCOAP^3.

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