Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2015, Article ID 104378, 10 pages
http://dx.doi.org/10.1155/2015/104378

Research Article

$B_c \to BP, BV$ Decays with the QCD Factorization Approach

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Received 7 January 2015; Accepted 17 March 2015

Academic Editor: Michal Kreps

We studied the nonleptonic $B_c \to BP, BV$ decays with the QCD factorization approach. It is found that the Cabibbo favored processes of $B_c \to B_s\pi, B_s\rho, B_u\overline{K}$ are the promising decay channels with branching ratio larger than 1%, which should be observed earlier by the LHCb collaboration.

1. Introduction

The B_c meson is the ground pseudoscalar meson of the $\bar{b}c$ system [1]. Compared with the heavy unflavored charmonium $c\bar{c}$ and bottomonium $b\bar{b}$, the B_c meson is unique in some respects. (1) Heavy quarkonia could be created in the parton-parton process $ij \to Q\overline{Q}$ at the order of α_s^2 (where $ij = gg$ or $q\bar{q}$, $Q = b, c$), while the production probability for the B_c meson is at least at the order of α_s^4 via $ij \to B_c^{(*)+} + b\bar{c}$, where the gluon-gluon fusion mechanism is dominant at Tevatron and LHC [2]. The B_c meson is difficult to produce experimentally, but it was observed for the first time via the semileptonic decay mode $B_c \to J/\psi\ell\nu$ in $p\bar{p}$ collisions by the CDF collaboration in 1998 [3, 4], which showed the realistic possibility of experimental study of the B_c meson. One of the best measurements on the mass and lifetime of the B_c meson is reported recently by the LHCb collaboration, $m_{B_c} = 6276.28 \pm 1.44 \pm 0.36$ MeV [5] and $\tau_{B_c} = 513.4 \pm 11.0 \pm 5.7$ fs [6]. With the running of the LHC, the B_c meson has a promising prospect. It is estimated that one could expect some 10^{10} B_c events at the high-luminosity LHC experiments per year [7, 8]. The studies on the B_c meson have entered a new precision era. (2) For charmonium and bottomonium, the strong and electromagnetic interactions are mainly responsible for annihilation of the $Q\overline{Q}$ quark pair into final states. The B_c meson, carrying nonzero flavor number $B = C = \pm 1$ and lying below the BD meson pair threshold, can decay only via the weak interaction, which offers an ideal sample to investigate the weak decay mechanism of heavy flavors that is inaccessible to both charmonium and bottomonium. The B_c weak decay provides great opportunities to investigate the perturbative and nonperturbative QCD, final state interactions, and so forth.

With respect to the heavy-light $B_{u,d,s}$ mesons, the doubly heavy B_c meson has rich decay channels because of its relatively large mass and that both b and c quarks can decay individually. The decay processes of the B_c meson can be divided into the following three classes [2, 9–11]: (1) the c quark decays with the b quark as a spectator; (2) the b quark decays with the c quark as a spectator; (3) the b and c quarks annihilate into a virtual W boson, with the ratios of ~70%, 20% and 10%, respectively [2]. Up to now, the experimental evidences of pure annihilation decay mode [class (3)] are still nothing. The $b \to c$ transition, belonging to the class (2), offers a well-constructed experimental structure of charmonium at the Tevatron and LHC. Although the detection of the c quark decay is very challenging to experimentalists, the clear signal of the $B_c \to B_s\pi$ decay is presented by the LHCb group using the $B_s \to D_s\pi$ and $B_s \to J/\psi\phi$ channels with statistical significance of 7.7σ and 6.1σ, respectively [12].

Anticipating the forthcoming accurate measurements on the B_c meson at hadron colliders and the lion's share of the B_c

decay width from the c quark decay [31–33], many theoretical papers were devoted to the study of the $B_c \to BP, BV$ decays (where P and V denote the $SU(3)$ ground pseudoscalar and vector mesons, resp.), such as [17, 34–37] with the BSW model [38, 39] or IGSW model [40], [18, 19, 41] based on the Bethe-Salpeter (BS) equation, [20–24, 42] with potential models, [25] with constituent quark model, [26–28] with QCD sum rules, [43] with the quark diagram scheme, [29, 30] with the perturbative QCD approach (pQCD) [44–49], and so on. The previous predictions on the branching ratios for the $B_c \to BP, BV$ decays are collected in Table 3. The discrepancies of previous investigations arise mainly from the different model assumptions. Recently, several phenomenological methods have been fully developed to cope with the hadronic matrix elements and successfully applied to the nonleptonic B decay, such as the pQCD approach [44–49] based on the k_T factorization scheme, the soft-collinear effective theory [50–57] and the QCD-improved factorization (QCDF) approach [58–63] based on the collinear approximation and power countering rules in the heavy quark limits. In this paper, we will study the $B_c \to BP, BV$ decays with the QCDF approach to provide a ready reference to the existing and upcoming experiments.

This paper is organized as follows. In Section 2, we will present the theoretical framework and the amplitudes for the $B_c \to BP, BV$ decays within the QCDF framework. Section 3 is devoted to numerical results and discussion. Finally, Section 4 is our summation.

2. Theoretical Framework

2.1. The Effective Hamiltonian. The low energy effective Hamiltonian responsible for the nonleptonic bottom-conserving $B_c \to BP, BV$ decays constructed by means of the operator product expansion and the renormalization group (RG) method is usually written in terms of the four-quark interactions [64, 65]. Consider

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \Bigg\{ V_{ub}V_{cb}^{*} \left[C_1^a(\mu) Q_1^a(\mu) + C_2^a(\mu) Q_2^a(\mu)\right] + \sum_{q_1,q_2=d,s} V_{uq_1}V_{cq_2}^{*} \left[C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu)\right] + \sum_{q_3=d,s} V_{uq_3}V_{cq_3}^{*} \sum_{k=3}^{10} C_k(\mu) Q_k(\mu) \Bigg\} + \text{h.c.}, \quad (1)$$

where the Fermi coupling constant $G_F \simeq 1.166 \times 10^{-5}\ \text{GeV}^{-2}$ [1]; $Q_{1,2}$, $Q_{1,2}^a$, and $Q_{3,\dots,10}$ are the relevant local tree, annihilation, and penguin four-quark operators, respectively, which govern the decays in question. The Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{uq_i}V_{cq_j}^{*}$ and Wilson coefficients C_i describe the coupling strength for a given operator.

Using the unitarity of the CKM matrix, there is a large cancellation of the CKM factors

$$V_{ud}V_{cd}^{*} + V_{us}V_{cs}^{*} = -V_{ub}V_{cb}^{*} \sim \mathcal{O}\left(\lambda^5\right), \quad (2)$$

where the Wolfenstein parameter $\lambda = \sin\theta_c = 0.225\ 37(61)$ [1] and θ_c is the Cabibbo angle. Hence, compared with the tree contributions, the contributions of annihilation and penguin operators are strongly suppressed by the CKM factor. If the CP-violating asymmetries that are expected to be very small due to the small weak phase difference for c quark decay are prescinded from the present consideration, then the penguin and annihilation contributions could be safely neglected. The local tree operators $Q_{1,2}$ in (1) are expressed as follows:

$$\begin{aligned} Q_1 &= \left[\bar{q}_{2,\alpha}\gamma_\mu (1-\gamma_5) c_\alpha\right]\left[\bar{u}_\beta \gamma^\mu (1-\gamma_5) q_{1,\beta}\right], \\ Q_2 &= \left[\bar{q}_{2,\alpha}\gamma_\mu (1-\gamma_5) c_\beta\right]\left[\bar{u}_\beta \gamma^\mu (1-\gamma_5) q_{1,\alpha}\right], \end{aligned} \quad (3)$$

where α and β are the $SU(3)$ color indices.

The Wilson coefficients $C_i(\mu)$ summarize the physics contributions from scales higher than μ. They are calculable with the RG improved perturbation theory and have properly been evaluated to the next-to-leading order (NLO) [64, 65]. They can be evolved from a higher scale $\mu \sim \mathcal{O}(m_W)$ down to a characteristic scale $\mu \sim \mathcal{O}(m_c)$ with the functions including the flavor thresholds [64, 65]

$$\vec{C}(\mu) = U_4(\mu, m_b) M(m_b) U_5(m_b, m_W) \vec{C}(m_W), \quad (4)$$

where $U_f(\mu_f, \mu_i)$ is the RG evolution matrix converting coefficients from the scale μ_i to μ_f, and $M(\mu)$ is the quark threshold matching matrix. The expressions of $U_f(\mu_f, \mu_i)$ and $M(\mu)$ can be found in [64, 65]. The numerical values of LO and NLO $C_{1,2}$ with the naive dimensional regularization scheme are listed in Table 1. The values of NLO Wilson coefficients in Table 1 are consistent with those given by [64, 65] where a trick with "effective" number of active flavors $f = 4.15$ rather than formula (4) is used.

To obtain the decay amplitudes, the remaining work is how to accurately evaluate the hadronic matrix elements $\langle BM|Q_i(\mu)|B_c\rangle$ which summarize the physics contributions from scales lower than μ. Since the hadronic matrix elements involve long distance contributions, one is forced to use either nonperturbative methods such as lattice calculations and QCD sum rules or phenomenological models relying on some assumptions. Consequently, it is very unfortunate that hadronic matrix elements cannot be reliably calculated at present, and that the most intricate part and the dominant theoretical uncertainties in the decay amplitudes reside in the hadronic matrix elements.

2.2. Hadronic Matrix Elements. Phenomenologically, based on the power counting rules in the heavy quark limit, Beneke et al. proposed that the hadronic matrix elements could be written as the convolution integrals of hard scattering kernels and the light cone distribution amplitudes with the QCDF master formula [58–63]. The QCDF approach is widely applied to nonleptonic B decays and it works well [66–76],

Table 1: The numerical values of the Wilson coefficients and the effective coefficients for $B_c \to B\pi$ decay, where $m_c = 1.275 \pm 0.025$ GeV [1].

μ	LO		NLO		QCDF			
	C_1	C_2	C_1	C_2	Re (a_1)	Im (a_1)	Re (a_2)	Im (a_2)
$0.8m_c$	1.334	−0.587	1.274	−0.503	1.270	0.096	−0.450	−0.218
m_c	1.275	−0.503	1.222	−0.424	1.216	0.068	−0.361	−0.173
$1.2m_c$	1.239	−0.449	1.189	−0.373	1.184	0.054	−0.306	−0.148

which encourage us to apply the QCDF approach to the study of $B_c \to BP$, BV decays. Since the spectator is the heavy b quark who is almost always on shell, the virtuality of the gluon linked with the spectator should be $\sim \mathcal{O}(\Lambda^2_{\rm QCD})$. The dominant behavior of the $B_c \to B$ transition form factors and the contributions of hard spectator scattering interactions are governed by soft processes. According to the basic idea of the QCDF approach [69, 70], the hard and soft contributions to the form factors entangle with each other and cannot be identified reasonably, so the physical form factors are used as the inputs. The hard spectator scattering contributions are power suppressed in the heavy quark limit. Finally, the hadronic matrix elements can be written as

$$\begin{aligned}\langle BM \left|Q_{1,2}\right| B_c\rangle &= \sum_i F_i^{B_c\to B}\int dx H_i(x)\,\Phi_M(x)\\ &\propto \sum_i F_i^{B_c\to B} f_M\left\{1+\alpha_s r_1+\cdots\right\},\end{aligned} \tag{5}$$

where $F_i^{B_c\to B}$ is the transition form factor and $\Phi_M(x)$ is the light-cone distribution amplitudes of the emitted meson M with the decay constant f_M. The hard scattering kernels $H_i(x)$ are computable order by order with the perturbation theory in principle. At the leading order α_s^0, $H_i(x) = 1$, that is, the convolution integral of (5) results in the meson decay constant. The hadronic matrix elements are parameterized by the product of form factors and decay constants, which are real and renormalization scale independent. One goes back to the simple "naive factorization" (NF) scenario. At the order α_s and higher orders, the information of strong phases and the renormalization scale dependence of hadronic matrix elements could be partly recuperated. Combined the nonfactorizable contributions with the Wilson coefficients, the scale independent effective coefficients at the order α_s can be obtained [58–63] as follows:

$$\begin{aligned}a_1 &= C_1^{\rm NLO}+\frac{1}{N_c}C_2^{\rm NLO}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_2^{\rm LO}V,\\ a_2 &= C_2^{\rm NLO}+\frac{1}{N_c}C_1^{\rm NLO}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_1^{\rm LO}V,\end{aligned} \tag{6}$$

where the expressions of vertex corrections are [58–63]

$$\begin{aligned}V = {}& 6\log\left(\frac{m_c^2}{\mu^2}\right) - 18 - \left(\frac{1}{2}+i3\pi\right)a_0^M\\ &+\left(\frac{11}{2}-i3\pi\right)a_1^M - \frac{21}{20}a_2^M+\cdots,\end{aligned} \tag{7}$$

with the twist-2 quark-antiquark distribution amplitudes of pseudoscalar P and longitudinally polarized vector V meson in terms of Gegenbauer polynomials [14–16]. One has

$$\phi_M(x) = 6x\overline{x}\sum_{n=0}^{\infty}a_n^M C_n^{3/2}(x-\overline{x}), \tag{8}$$

where $\overline{x} = 1-x$; a_n^M is the Gegenbauer moment and $a_0^M \equiv 1$.

From the numbers in Table 1, it is found that (1) for the coefficient a_1 the nonfactorizable contributions accompanied by the Wilson coefficient C_2 can provide ≥10% enhancement compared with the NF's result, and a relatively small strong phase $\le 5^\circ$; (2) for the coefficient a_2, the nonfactorizable contributions assisted with the large Wilson coefficient C_1 are significant. In addition, a relatively large strong phase $\sim -155^\circ$ is obtained; (3) the QCDF's values of $a_{1,2}$ agree basically with the real coefficients $a_1 \simeq 1.20$ and $a_2 \simeq -0.317$ which are used by previous studies on the $B_c \to BP$, BV decays in [17, 18, 20–28, 34–37, 42], but with more information on the strong phases.

2.3. Decay Amplitudes. Within the QCDF framework, the amplitudes for $B_c \to BM$ decays are expressed as

$$\begin{aligned}\mathcal{A}(B_c \longrightarrow BM) &= \langle BM\left|\mathcal{H}_{\rm eff}\right|B_c\rangle\\ &= \frac{G_F}{\sqrt{2}}V_{uq_1}V_{cq_2}^{*}a_i\langle M\left|J^\mu\right|0\rangle\langle B\left|J_\mu\right|B_c\rangle.\end{aligned} \tag{9}$$

The matrix elements of current operators are defined as

$$\begin{aligned}\langle P(p)\left|\overline{q}_1\gamma^\mu(1-\gamma_5)q_2\right|0\rangle &= -if_P p^\mu,\\ \langle V(\epsilon,p)\left|\overline{q}_1\gamma^\mu(1-\gamma_5)q_2\right|0\rangle &= f_V m_V \epsilon^\mu,\end{aligned} \tag{10}$$

where f_P and f_V are the decay constants of pseudoscalar P and vector V mesons, respectively; m_V and ϵ denote the mass and polarization of vector meson, respectively.

For the mixing of physical pseudoscalar η and η' meson, we adopt the quark-flavor basis description proposed in [13] and neglect the contributions from possible gluonium and $c\overline{c}$ compositions; that is,

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix} = \begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\eta_q\\ \eta_s\end{pmatrix}, \tag{11}$$

where $\eta_q = (u\overline{u}+d\overline{d})/\sqrt{2}$ and $\eta_s = s\overline{s}$; the mixing angle $\phi = (39.3\pm1.0)^\circ$ [13]. We assume that the vector mesons are ideally mixed; that is, $\omega = (u\overline{u}+d\overline{d})/\sqrt{2}$ and $\phi = s\overline{s}$.

Table 2: The numerical values of input parameters.

Wolfenstein parameters	
$\lambda = 0.22537 \pm 0.00061$ [1]	$A = 0.814^{+0.023}_{-0.024}$ [1]
$\overline{\rho} = 0.117 \pm 0.021$ [1]	$\overline{\eta} = 0.353 \pm 0.013$ [1]
Decay constant of mesons	
$f_\pi = 130.41 \pm 0.20$ MeV [1]	$f_K = 156.2 \pm 0.7$ MeV [1]
$f_{\eta_q} = (1.07 \pm 0.02) f_\pi$ [13]	$f_{\eta_s} = (1.34 \pm 0.06) f_\pi$ [13]
$f_\rho = 216 \pm 3$ MeV [14–16]	$f_\omega = 187 \pm 5$ MeV [14–16]
$f_{K^*} = 220 \pm 5$ MeV [14–16]	
Gegenbauer moments at the scale $\mu = 1$ GeV	
$a_1^\pi = a_1^{\eta_q} = a_1^{\eta_s} = 0$ [14–16]	$a_2^\pi = a_2^{\eta_q} = a_2^{\eta_s} = 0.25 \pm 0.15$ [14–16]
$a_1^{\overline{K}} = -a_1^K = 0.06 \pm 0.03$ [14–16]	$a_2^K = a_2^{\overline{K}} = 0.25 \pm 0.15$ [14–16]
$a_{1,\rho}^\parallel = a_{1,\omega}^\parallel = 0$ [14–16]	$a_{2,\rho}^\parallel = a_{2,\omega}^\parallel = 0.15 \pm 0.07$ [14–16]
$a_{1,\overline{K}^*}^\parallel = -a_{1,K^*}^\parallel = 0.03 \pm 0.02$ [14–16]	$a_{2,K^*}^\parallel = a_{2,\overline{K}^*}^\parallel = 0.11 \pm 0.09$ [14–16]

The transition form factors are defined as [38, 39]

$$\begin{aligned} &\langle B(k) | \overline{q} \gamma^\mu (1-\gamma_5) c | B_c(p) \rangle \\ &= \left[p + k - \frac{m_{B_c}^2 - m_B^2}{q^2} q \right]^\mu F_1^{B_c \to B}(q^2) \\ &+ \frac{m_{B_c}^2 - m_B^2}{q^2} q^\mu F_0^{B_c \to B}(q^2), \end{aligned} \tag{12}$$

where $q = p - k$, and the condition of $F_0^{B_c \to B}(0) = F_1^{B_c \to B}(0)$ is required compulsorily to cancel the singularity at the pole $q^2 = 0$.

For the $B_c \to B$ transition form factors, since the velocity of the recoiled B meson is very low in the rest frame of the B_c meson, the wave functions of B and B_c mesons overlap strongly. It is believed that the form factors $F_{0,1}^{B_c \to B}$ should be close to the result using the nonrelativistic harmonic oscillator wave functions with the BSW model [17]. Consider

$$F_{0,1}^{B_c \to B} \simeq \left(\frac{2 m_{B_c} m_B}{m_{B_c}^2 + m_B^2} \right)^{1/2} \simeq 0.99. \tag{13}$$

The flavor symmetry breaking effects on the form factors are neglectable in (13). For simplification, we take $F_{0,1}^{B_c \to B_{u,d,s}} = 1.0$ in our numerical calculation to give a rough estimation.

3. Numerical Results and Discussions

The branching ratios of nonleptonic two-body B_c decays in the rest frame of the B_c meson can be written as

$$\mathscr{B}r(B_c \longrightarrow BM) = \frac{\tau_{B_c}}{8\pi} \frac{p}{m_{B_c}^2} \left| \mathscr{A}(B_c \longrightarrow BM) \right|^2, \tag{14}$$

where the lifetime of the B_c meson $\tau_{B_c} = 513.4 \pm 11.0 \pm 5.7$ fs [6] and p is the common momentum of final particles. The decay amplitudes $\mathscr{A}(B_c \to BM)$ are listed in the Appendix.

The input parameters in our calculation, including the CKM Wolfenstein parameters, decay constants, and Gegenbauer moments of distribution amplitudes in (8), are collected in Table 2. If not specified explicitly, we will take their central values as the default inputs. Our numerical results on the CP-averaged branching ratios are presented in Table 3, where theoretical uncertainties of the "QCDF" column come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.2) m_c$, decay constants, and Gegenbauer moments, respectively. For comparison, previous results calculated with the fixed coefficients $a_1 \simeq 1.22$ and $a_2 \simeq -0.4$ are also listed. There are some comments on the branching ratios.

(1) From the numbers in Table 3, it is seen that different branching ratios for the $B_c \to BP$, BV decays were obtained with different approaches in previous works, although the same coefficients $a_{1,2}$ are used. Much of the discrepancy comes from the different values of the transition form factors. If the same value of the form factor is used, then the disparities on branching ratios for the a_1-dominated B_c decays will be highly alleviated. For example, all previous predictions on $\mathscr{B}r(B_c \to B_s \pi)$ will be about 10% with the same form factor $F_0^{B_c \to B_s} \simeq 1.0$, which is generally in line with the QCDF estimation within uncertainties and also agrees with the recent LHCb measurement [12].

(2) There is a hierarchical structure between the QCDF's results on branching ratios for the $B_c \to BP$ and BV decays with the same final B_q meson, for example,

$$\begin{aligned} \mathscr{B}r(B_c \longrightarrow B_q \pi) &> \mathscr{B}r(B_c \longrightarrow B_q \rho), \\ \mathscr{B}r(B_c \longrightarrow B_q K) &\gtrsim 5 \mathscr{B}r(B_c \longrightarrow B_q K^*), \end{aligned} \tag{15}$$

which differs from the previous results. There are two decisive factors. One is the kinematic factor. The phase space for the $B_c \to BV$ decays is more compressed than that for the $B_c \to BP$ decays, because the mass of the light pseudoscalar meson (except for the exotic η' meson) is generally less than the mass of the corresponding vector meson with the same valence quark components. The other is the dynamical factor. The

TABLE 3: The CP-averaged branching ratios for the $B_c \to BP, BV$ decays.

Decay mode	Case	Reference [17][a]	Reference [18][b]	Reference [19][c]	Reference [20][d]	Reference [21, 22][e]	Reference [23][f]	Reference [24][g]	Reference [25][h]	Reference [26–28][i]	Reference [29, 30][j]	QCDF
$B_c \to B_s^0\pi^+$	1-a	1.0×10^{-1}	6.8×10^{-2}	1.8×10^{-2}	2.9×10^{-2}	4.0×10^{-2}	4.3×10^{-2} (4.3×10^{-2})	1.3×10^{-1}	4.6×10^{-2}	1.9×10^{-1}	8.8×10^{-2}	$(1.13^{+0.00+0.11+0.01}_{-0.00-0.06-0.01})\times10^{-1}$
$B_c \to B_s^0K^+$	1-b	7.6×10^{-3}	4.9×10^{-3}	2.0×10^{-3}	2.4×10^{-3}	3.3×10^{-3}	3.3×10^{-3} (3.3×10^{-3})	8.5×10^{-3}	3.4×10^{-3}	1.2×10^{-2}	5.2×10^{-3}	$(7.41^{+0.04+0.70+0.09}_{-0.04-0.39-0.09})\times10^{-3}$
$B_c \to B_s^0\rho^+$	1-a	6.3×10^{-2}	5.2×10^{-2}	4.6×10^{-2}	1.6×10^{-2}	2.7×10^{-2}	3.0×10^{-2} (2.7×10^{-2})	1.1×10^{-1}	2.7×10^{-2}	8.4×10^{-2}	3.2×10^{-2}	$(4.44^{+0.00+0.41+0.13}_{-0.00-0.23-0.13})\times10^{-2}$
$B_c \to B_s^0K^{*+}$	1-b	3.2×10^{-4}		1.2×10^{-3}	3.5×10^{-5}	1.5×10^{-4}	8.0×10^{-5} (7.1×10^{-5})	4.0×10^{-4}	1.3×10^{-4}		9.7×10^{-5}	$(1.25^{+0.01+0.12+0.06}_{-0.01-0.07-0.06})\times10^{-4}$
$B_c \to B_d^0\pi^0$	1-b	7.4×10^{-3}	3.8×10^{-3}	1.2×10^{-3}	1.2×10^{-3}	1.3×10^{-3}	1.8×10^{-3} (1.5×10^{-3})	8.4×10^{-3}	2.4×10^{-3}	1.2×10^{-2}	6.9×10^{-3}	$(7.83^{+0.04+0.73+0.04}_{-0.04-0.41-0.04})\times10^{-3}$
$B_c \to B_d^0K^+$	1-c		3.0×10^{-4}	1.2×10^{-4}	1.0×10^{-4}	1.1×10^{-4}	1.5×10^{-4} (1.2×10^{-4})	5.9×10^{-4}	1.8×10^{-4}	8.1×10^{-4}	4.4×10^{-4}	$(5.29^{+0.06+0.50+0.07}_{-0.06-0.28-0.07})\times10^{-4}$
$B_c \to B_d^0\rho^+$	1-b	8.3×10^{-3}	6.9×10^{-3}	3.3×10^{-3}	1.5×10^{-3}	1.6×10^{-3}	2.2×10^{-3} (1.7×10^{-3})	1.4×10^{-2}	2.4×10^{-3}	1.1×10^{-2}	4.3×10^{-3}	$(5.32^{+0.03+0.49+0.15}_{-0.03-0.28-0.15})\times10^{-3}$
$B_c \to B_d^0K^{*+}$	1-c		2.1×10^{-4}	1.5×10^{-4}	4.6×10^{-5}	4.4×10^{-5}	4.9×10^{-5} (3.7×10^{-5})	3.5×10^{-4}	5.7×10^{-5}	1.7×10^{-4}	8.3×10^{-5}	$(1.06^{+0.01+0.10+0.05}_{-0.01-0.06-0.05})\times10^{-4}$
$B_c \to B_u^+\overline{K}^0$	2-a	2.1×10^{-2}	1.2×10^{-2}	4.9×10^{-3}	4.2×10^{-3}	4.4×10^{-3}	6.0×10^{-3} (4.9×10^{-3})	2.3×10^{-2}	6.8×10^{-3}	3.6×10^{-2}	2.2×10^{-3}	$(1.97^{+0.00+1.11+0.05}_{-0.00-0.54-0.05})\times10^{-2}$
$B_c \to B_u^+K^0$	2-c						1.6×10^{-5} (1.3×10^{-5})				6.3×10^{-6}	$(5.71^{+0.06+3.20+0.15}_{-0.06-1.58-0.14})\times10^{-5}$
$B_c \to B_u^+\overline{K}^{*0}$	2-a	7.8×10^{-3}	8.5×10^{-3}	5.8×10^{-3}	1.6×10^{-3}	1.6×10^{-3}	1.9×10^{-3} (1.4×10^{-3})	1.3×10^{-2}	2.1×10^{-3}	8.0×10^{-3}	2.0×10^{-4}	$(3.72^{+0.00+2.09+0.21}_{-0.00-1.03-0.20})\times10^{-3}$
$B_c \to B_u^+K^{*0}$	2-c						5.0×10^{-6} (3.7×10^{-6})				5.6×10^{-7}	$(1.07^{+0.01+0.60+0.06}_{-0.01-0.30-0.06})\times10^{-5}$
$B_c \to B_u^+\pi^0$	2-b	4.0×10^{-4}	2.1×10^{-4}	6.4×10^{-5}	6.2×10^{-5}	6.7×10^{-5}	9.7×10^{-5} (8.1×10^{-5})	4.5×10^{-4}	1.3×10^{-4}	6.6×10^{-4}	5.2×10^{-5}	$(4.23^{+0.02+2.37+0.07}_{-0.02-1.17-0.07})\times10^{-4}$
$B_c \to B_u^+\rho^0$	2-b	4.4×10^{-4}	3.7×10^{-4}	1.7×10^{-4}	8.7×10^{-5}	8.9×10^{-5}	1.2×10^{-4} (9.4×10^{-5})	7.4×10^{-4}	1.3×10^{-4}	5.5×10^{-4}	1.8×10^{-5}	$(2.86^{+0.01+1.60+0.10}_{-0.01-0.79-0.10})\times10^{-4}$
$B_c \to B_u^+\omega$	2-b	4.1×10^{-4}					9.0×10^{-5} (7.0×10^{-5})				1.3×10^{-5}	$(2.05^{+0.01+1.15+0.12}_{-0.01-0.57-0.12})\times10^{-4}$

Table 3: Continued.

Decay mode	Case	Reference [17][a]	Reference [18][b]	Reference [19][c]	Reference [20][d]	Reference [21, 22][e]	Reference [23][f]	Reference [24][g]	Reference [25][h]	Reference [26–28][i]	Reference [29, 30][j]	QCDF
$B_c \to B_u^+\eta$							5.0×10^{-4} (4.1×10^{-4})				1.4×10^{-4}	$(1.46^{+0.01+0.82+0.13}_{-0.01-0.40-0.12})\times10^{-3}$
$B_c \to B_u^+\eta'$							6.7×10^{-6} (5.6×10^{-6})				4.2×10^{-6}	$(7.28^{+0.04+4.09+1.66}_{-0.04-2.01-1.47})\times10^{-5}$

[a]It is estimated with the form factors $F_0^{B_c\to B_s}=0.925$, $F_0^{B_c\to B}=0.91$ and parameter $\omega=1$ GeV [17] based on the BSW model.

[b]It is estimated with the instantaneous nonrelativistic approximation and the potential model based on the BS equation.

[c]It is estimated in a relativistic model with a one-gluon interaction plus a scalar confinement potential based on the BS equation.

[d]It is estimated with the form factors $F_0^{B_c\to B_s}=0.5$, $F_0^{B_c\to B}=0.39$ using a quasipotential in the relativistic quark model [20].

[e]It is estimated with the form factors $F_0^{B_c\to B_s}=0.58$, $F_0^{B_c\to B}=0.39$ in the nonrelativistic constituent quark model [21, 22].

[f]It is estimated with the form factors $F_0^{B_c\to B_s}=0.573$ (0.571), $F_0^{B_c\to B}=0.467$ (0.426) in the light-front quark model based on the Coulomb plus linear (harmonic oscillator) potential, together with the hyperfine interaction [23].

[g]It is estimated with the form factors $F_0^{B_c\to B_s}=1.03$, $F_0^{B_c\to B}=1.01$ in the relativistic independent quark model [24].

[h]It is estimated within a relativistic constituent quark model [25].

[i]It is estimated with the form factors $F_0^{B_c\to B_s}=1.3$, $F_0^{B_c\to B}=1.27$ in the QCD sum rules [26–28].

[j]It is estimated with the perturbative QCD approach based on the k_T factorization scheme [29, 30].

Table 4: Hierarchy of amplitudes among the QCDF's branching ratios for B_c decay.

Case	Coefficient	CKM factor	Branching ratio	Decay modes
1 a	a_1	$\lvert V_{ud}V_{cs}^*\rvert \sim 1$	$\gtrsim 10^{-2}$	$B_s\pi$, $B_s\rho$
1 b	a_1	$\lvert V_{ud}V_{cd}^*\rvert, \lvert V_{us}V_{cs}^*\rvert \sim \lambda$	$\gtrsim 10^{-3}$	B_sK, $B_d\pi$, $B_d\rho$
1 c	a_1	$\lvert V_{us}V_{cd}^*\rvert \sim \lambda^2$	$\gtrsim 10^{-5}$	B_dK, B_dK^*
2 a	a_2	$\lvert V_{ud}V_{cs}^*\rvert \sim 1$	$\gtrsim 10^{-3}$	$B_u^+\overline{K}^0$, $B_u^+\overline{K}^{*0}$
2 b	a_2	$\lvert V_{ud}V_{cd}^*\rvert, \lvert V_{us}V_{cs}^*\rvert \sim \lambda$	$\gtrsim 10^{-4}$	$B_u\pi$, $B_u\rho$, $B_u\omega$
2 c	a_2	$\lvert V_{us}V_{cd}^*\rvert \sim \lambda^2$	$\gtrsim 10^{-6}$	$B_u^+K^0$, $B_u^+K^{*0}$

orbital angular momentum for the BP final states is $\ell_{BP} = 0$, while the orbital angular momentum is $\ell_{BV} = 1$ for the BV final states.

(3) According to the CKM factors and the coefficients $a_{1,2}$, there is another hierarchy of amplitudes among the QCDF's branching ratios for the B_c decays, which could be subdivided into different cases (see Table 4). The CKM-favored a_1-dominated $B_c \rightarrow B_s\pi$ decays are expected to have the largest branching ratio, ~10%, within the QCDF framework. In addition, the branching ratios for the Cabibbo favored $B_c^+ \rightarrow B_s^0\rho^+$, $B_u^+\overline{K}^0$ decays are also larger than 1%, which might be promisingly detected at experiments.

(4) There are many uncertainties on the QCDF's results. The first uncertainty from the CKM factors is small due to the high precision on Wolfenstein parameter λ with only 0.3% relative errors [1]. Large uncertainty comes from the renormalization scale, especially for the a_2 dominated $B_c \rightarrow B_uP$, B_uV decays. In principle, the second uncertainty could be reduced by the inclusion of higher order α_s corrections to hadronic matrix elements. It has been showed [77, 78] that tree amplitudes incorporating with the NNLO corrections are relatively less sensitive to the choice of scale than the NLO amplitudes. As aforementioned, large uncertainty mainly comes from hadron parameters, such as the transition form factors, which is expected to be cancelled from the rate of branching ratios. For example,

$$\frac{\mathscr{B}r(B_c \rightarrow B_sK)}{\mathscr{B}r(B_c \rightarrow B_s\pi)} \approx \left|V_{us}\right|^2 \frac{f_K^2}{f_\pi^2} \approx \frac{\mathscr{B}r(B_c \rightarrow B_dK)}{\mathscr{B}r(B_c \rightarrow B_d\pi)}, \quad (16)$$

$$\frac{\mathscr{B}r(B_c \rightarrow B_sK^*)}{\mathscr{B}r(B_c \rightarrow B_s\rho)} \approx \left|V_{us}\right|^2 \frac{f_{K^*}^2}{f_\rho^2} \approx \frac{\mathscr{B}r(B_c \rightarrow B_dK^*)}{\mathscr{B}r(B_c \rightarrow B_d\rho)}, \quad (17)$$

$$\frac{\mathscr{B}r(B_c \rightarrow B_u\pi)}{\mathscr{B}r(B_c \rightarrow B_d\pi)} \approx \frac{1}{2}\frac{\left|a_2\right|^2}{\left|a_1\right|^2} \approx \frac{\mathscr{B}r(B_c \rightarrow B_u\rho)}{\mathscr{B}r(B_c \rightarrow B_d\rho)}. \quad (18)$$

Particularly, the relation of (18) might be used to give some information on the coefficients $a_{1,2}$ and to provide an interesting feasibility research on the validity of the QCDF approach for the charm quark decay. Finally, we would like to point out that many uncertainties from other factors, such as the final state interactions, which deserve the dedicated study, are not considered here. So one should not be too serious about the numbers in Table 3. Despite this, our results will still provide some useful information to experimental physicists; that is, the Cabibbo favored $B_c \rightarrow B_s\pi$, $B_s\rho$, $B_u\overline{K}$ decays have large branching ratios $\gtrsim$ 1%, which could be detected earlier.

4. Summary

In prospects of the potential B_c meson at the LHCb experiments, accurate and thorough studies of the B_c decays will be accessible very soon. The carefully theoretical study on the B_c decays is urgently desiderated. In this paper, we concentrated on the nonfactorizable contributions to hadronic matrix elements within the QCDF framework, while the transition form factors are taken as nonperturbative inputs, which is different from previous studies. It is found that the branching ratios for the Cabibbo favored $B_c \rightarrow B_s\pi$, $B_s\rho$, $B_u\overline{K}$ decays are very large and could be measured earlier by the running LHCb experiment in the forthcoming years.

Appendix

Decay Amplitudes

$$\mathscr{A}\left(B_c^+ \longrightarrow B_s^0\pi^+\right) = -i\frac{G_F}{\sqrt{2}}F_0^{B_c\rightarrow B_s} f_\pi\left(m_{B_c}^2 - m_{B_s}^2\right)V_{ud}V_{cs}^*a_1,$$

$$\mathscr{A}\left(B_c^+ \longrightarrow B_s^0K^+\right) = -i\frac{G_F}{\sqrt{2}}F_0^{B_c\rightarrow B_s} f_K\left(m_{B_c}^2 - m_{B_s}^2\right)V_{us}V_{cs}^*a_1,$$

$$\mathscr{A}\left(B_c^+ \longrightarrow B_s^0\rho^+\right) = \sqrt{2}G_F F_1^{B_c\rightarrow B_s} f_\rho m_\rho\left(\epsilon_\rho\cdot p_{B_c}\right)V_{ud}V_{cs}^*a_1,$$

$$\mathscr{A}\left(B_c^+ \longrightarrow B_s^0K^{*+}\right) = \sqrt{2}G_F F_1^{B_c\rightarrow B_s} f_{K^*} m_{K^*}\left(\epsilon_{K^*}\cdot p_{B_c}\right)V_{us}V_{cs}^*a_1,$$

$$\mathscr{A}\left(B_c^+ \longrightarrow B_d^0\pi^+\right) = -i\frac{G_F}{\sqrt{2}}F_0^{B_c\rightarrow B_d} f_\pi\left(m_{B_c}^2 - m_{B_d}^2\right)V_{ud}V_{cd}^*a_1,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_d^0 K^+\right) = -i\frac{G_F}{\sqrt{2}} F_0^{B_c\to B_d} f_K \left(m_{B_c}^2 - m_{B_d}^2\right) V_{us} V_{cd}^* a_1,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_d^0 \rho^+\right) = \sqrt{2} G_F F_1^{B_c\to B_d} f_\rho m_\rho \left(\epsilon_\rho \cdot p_{B_c}\right) V_{ud} V_{cd}^* a_1,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_d^0 K^{*+}\right) = \sqrt{2} G_F F_1^{B_c\to B_d} f_{K^*} m_{K^*} \left(\epsilon_{K^*} \cdot p_{B_c}\right) V_{us} V_{cd}^* a_1,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \overline{K}^0\right) = -i\frac{G_F}{\sqrt{2}} F_0^{B_c\to B_u} f_K \left(m_{B_c}^2 - m_{B_u}^2\right) V_{ud} V_{cs}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ K^0\right) = -i\frac{G_F}{\sqrt{2}} F_0^{B_c\to B_u} f_K \left(m_{B_c}^2 - m_{B_u}^2\right) V_{us} V_{cd}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \overline{K}^{*0}\right) = \sqrt{2} G_F F_1^{B_c\to B_u} f_{K^*} m_{K^*} \left(\epsilon_{K^*} \cdot p_{B_c}\right) V_{ud} V_{cs}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ K^{*0}\right) = \sqrt{2} G_F F_1^{B_c\to B_u} f_{K^*} m_{K^*} \left(\epsilon_{K^*} \cdot p_{B_c}\right) V_{us} V_{cd}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \pi^0\right) = +i\frac{G_F}{2} F_0^{B_c\to B_u} f_\pi \left(m_{B_c}^2 - m_{B_u}^2\right) V_{ud} V_{cd}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \rho^0\right) = -G_F F_1^{B_c\to B_u} f_\rho m_\rho \left(\epsilon_\rho \cdot p_{B_c}\right) V_{ud} V_{cd}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \omega\right) = +G_F F_1^{B_c\to B_u} f_\omega m_\omega \left(\epsilon_\omega \cdot p_{B_c}\right) V_{ud} V_{cd}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta_q\right) = -i\frac{G_F}{2} F_0^{B_c\to B_u} f_{\eta_q} \left(m_{B_c}^2 - m_{B_u}^2\right) V_{ud} V_{cd}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta_s\right) = -i\frac{G_F}{\sqrt{2}} F_0^{B_c\to B_u} f_{\eta_s} \left(m_{B_c}^2 - m_{B_u}^2\right) V_{us} V_{cs}^* a_2,$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta\right) = \cos\phi \mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta_q\right) - \sin\phi \mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta_s\right),$$

$$\mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta'\right) = \sin\phi \mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta_q\right) + \cos\phi \mathcal{A}\left(B_c^+ \longrightarrow B_u^+ \eta_s\right). \quad \text{(A.1)}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The work is supported by the National Natural Science Foundation of China (Grant nos. 11475055, 11275057, and U1232101). N. Wang thanks the support from CCNU-QLPL Innovation Fund (QLPL201411). Q. Chang is also supported by the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant no. 201317) and the Program for Science and Technology Innovation Talents in Universities of Henan Province (Grant no. 14HASTIT036).

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