

Research Article

Some New Fixed Point Theorems in Complex Valued G -Metric Spaces

Ing-Jer Lin, Wei-Shih Du, and Qiao-Feng Zheng

Department of Mathematics, National Kaohsiung Normal University, Kaohsiung 82444, Taiwan

Correspondence should be addressed to Wei-Shih Du; wsdu@nknuc.nknu.edu.tw

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Some new fixed point theorems are established in the setting of complex valued G -metric spaces. These new results improve and generalize Kang et al.'s results, the Banach contraction principle, and some well-known results in the literature.

1. Introduction and Preliminaries

It is well known that Banach contraction principle [1] plays an important role in various fields of applied mathematical analysis and scientific applications and has been generalized and improved in many various different directions; see [2–16] and references therein. In 2011, Azam et al. [2] introduced so-called complex valued metric spaces and proved the existence of fixed points under some contraction conditions. In 2006, Mustafa and Sims [3] introduced the concept of G -metric spaces to extend and generalize the notion of metric spaces. In 2013, Kang et al. [8] introduced the concept of complex valued G -metric spaces to generalize and improve the notion of G -metric spaces. In [8], the authors gave a complex valued G -metric version of Banach contraction principle.

In what follows we will give some definitions and known results which will be needed in the sequel. Throughout the present paper, the symbols \mathbb{N} , \mathbb{R} , and \mathbb{C} are used to denote the sets of positive integers, real numbers, and complex numbers, respectively.

In 2006, Mustafa and Sims [3] introduced a new class of metric spaces called generalized metric spaces or G -metric spaces as follows.

Definition 1 (see [3]). Let X be a nonempty set and let $G : X \times X \times X \rightarrow [0, \infty)$ be a function satisfying the following:

- (G1) $G(x, y, z) = 0$ if $x = y = z$,
- (G2) $0 < G(x, y, z)$ for all $x, y \in X$ with $x \neq y$,

- (G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,
- (G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$ (symmetry in all three variables),
- (G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a generalized metric or a G -metric on X and the pair (X, G) is called a G -metric space.

Example 2 (see [3]). Let (X, d) be a usual metric space. Then (X, G_1) and (X, G_2) are all G -metric spaces, where

$$\begin{aligned} G_1(x, y, z) &= d(x, y) + d(y, z) + d(z, x), \\ G_2(x, y, z) &= \max\{d(x, y), d(y, z), d(z, x)\} \end{aligned} \quad (1)$$

for all $x, y, z \in X$.

For any $z_1, z_2 \in \mathbb{C}$, we can define a partial order \leq on \mathbb{C} as follows:

$$z_1 \leq z_2 \iff \operatorname{Re}(z_1) \leq \operatorname{Re}(z_2), \quad \operatorname{Im}(z_1) \leq \operatorname{Im}(z_2). \quad (2)$$

So, it is easy to see that $z_1 \leq z_2$ holds if one of the following conditions is satisfied:

- (C1) $\operatorname{Re}(z_1) = \operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1) = \operatorname{Im}(z_2)$,
- (C2) $\operatorname{Re}(z_1) < \operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1) = \operatorname{Im}(z_2)$,

(C3) $\text{Re}(z_1) = \text{Re}(z_2)$ and $\text{Im}(z_1) < \text{Im}(z_2)$,

(C4) $\text{Re}(z_1) < \text{Re}(z_2)$ and $\text{Im}(z_1) < \text{Im}(z_2)$.

In particular, we will write $z_1 \preceq z_2$ if $z_1 \neq z_2$ and one of (C2), (C3), and (C4) is satisfied and we will write $z_1 < z_2$ if only (C4) is satisfied.

Remark 3. It is obvious that the following statements hold.

(1) If $0 \preceq z_1 \preceq z_2$, then $|z_1| < |z_2|$.

(2) If $z_1 \preceq z_2$ and $z_2 < z_3$, then $z_1 < z_3$.

The idea of complex metric space was initiated by Azam et al. [2].

Definition 4. Let X be a nonempty set. Suppose that the mapping $d : X \times X \rightarrow \mathbb{C}$ satisfies

(C1) $0 \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$,

(C2) $d(x, y) = d(y, x)$ for all $x, y \in X$,

(C3) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a complex valued metric on X and the pair (X, d) is called a complex valued metric space.

Example 5 (see [6, Example 2]). Let $X = X_1 \cup X_2$ where

$$\begin{aligned} X_1 &= \{z \in \mathbb{C} : \text{Re}(z) \geq 0, \text{Im}(z) = 0\}, \\ X_2 &= \{z \in \mathbb{C} : \text{Re}(z) = 0, \text{Im}(z) \geq 0\}. \end{aligned} \tag{3}$$

Define $d : X \times X \rightarrow \mathbb{C}$ as follows:

$$d(z_1, z_2) = \begin{cases} \frac{2}{3}|x_1 - x_2| + \frac{i}{2}|x_1 - x_2|, & \text{if } z_1, z_2 \in X_1, \\ \frac{1}{2}|y_1 - y_2| + \frac{i}{3}|y_1 - y_2|, & \text{if } z_1, z_2 \in X_2, \\ \left(\frac{2}{3}x_1 + \frac{1}{2}y_2\right) + i\left(\frac{1}{2}x_1 + \frac{1}{3}y_2\right), & \text{if } z_1 \in X_1, z_2 \in X_2, \\ \left(\frac{1}{2}y_1 + \frac{2}{3}x_2\right) + i\left(\frac{1}{3}y_1 + \frac{1}{2}x_2\right), & \text{if } z_1 \in X_2, z_2 \in X_1, \end{cases} \tag{4}$$

where $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2 \in X$. Then (X, d) is a complete complex valued metric space.

The notion of complex valued G -metric space was introduced by Kang et al. [8] to generalize the notion of complex valued metric space and G -metric space as follows.

Definition 6 (see [8]). Let X be a nonempty set and let $G_c : X \times X \times X \rightarrow \mathbb{C}$ be a function satisfying the following:

(CG1) $G_c(x, y, z) = 0$ if $x = y = z$,

(CG2) $0 \preceq G_c(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

(CG3) $G_c(x, x, y) \leq G_c(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,

(CG4) $G_c(x, y, z) = G_c(x, z, y) = G_c(y, z, x) = \dots$ (symmetry in all three variables),

(CG5) $G_c(x, y, z) \leq G_c(x, a, a) + G_c(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G_c is called a complex valued generalized metric or a complex valued G -metric on X . We call the pair (X, G_c) a complex valued G -metric space.

Remark 7. In fact, condition (CG2) defined in [8] was stated as follows:

(CG2) $0 < G_c(x, x, y)$ for all $x, y \in X$ with $x \neq y$.

In this paper, we use the weak version of (CG2) as in Definition 6.

Example 8. Let $X = \mathbb{C}$ and $G_c : X \times X \times X \rightarrow \mathbb{C}$ be defined by

$$\begin{aligned} G_c(z_1, z_2, z_3) &= (|x_1 - x_2| + |x_2 - x_3| + |x_3 - x_1|) \\ &\quad + i(|y_1 - y_2| + |y_2 - y_3| + |y_3 - y_1|), \end{aligned} \tag{5}$$

where $z_i = x_i + iy_i \in \mathbb{C}$ for any $i \in \{1, 2, 3\}$. Then (X, G_c) is a complex valued G -metric space.

Definition 9. Let (X, G_c) be a complex valued G -metric space. A point v in X is a fixed point of a mapping $T : X \rightarrow X$ if $v = Tv$. The set of fixed points of T is denoted by $\mathcal{F}(T)$.

Definition 10 (see [8]). Let (X, G_c) be a complex valued G -metric space and let $\{x_n\}$ be a sequence in X . We say that $\{x_n\}$ is *complex value G -convergent* to $x \in X$ if, for every $c \in \mathbb{C}$ with $0 < c$, there exists $k \in \mathbb{N}$ such that $G_c(x, x_n, x_m) < c$ for all $n, m \geq k$. We refer to x as the limit of the sequence $\{x_n\}$ and we write $x_n \rightarrow x$ as $n \rightarrow \infty$.

Definition 11 (see [8]). Let (X, G_c) be a complex valued G -metric space.

- (i) A sequence $\{x_n\}$ in X is said to be *complex valued G -Cauchy* if, for every $c \in \mathbb{C}$ with $0 < c$, there exists $k \in \mathbb{N}$ such that $G_c(x_n, x_m, x_l) < c$ for all $n, m, l \geq k$.
- (ii) (X, G_c) is said to be *complete* if every complex valued G -Cauchy sequence in X is complex valued G -convergent in X .

Some crucial facts in complex valued G_c -metric spaces are listed as follows. First, the following proposition follows easily due to (CG5).

Proposition 12 (see [8]). *Let (X, G_c) be a complex valued G -metric space. Then, for any $x, y, z \in X$, the following hold:*

(1) $G_c(x, y, z) \leq G_c(x, x, y) + G_c(x, x, z)$,

(2) $G_c(x, y, y) \leq 2G_c(y, x, y)$.

Proposition 13 (see [8]). *Let (X, G) be a complex valued G-metric space. Then, for a sequence $\{x_n\}$ in X and point $x \in X$, the following are equivalent.*

- (1) $\{x_n\}$ is complex valued G-convergent to x .
- (2) $|G_c(x_n, x_n, x)| \rightarrow 0$ as $n \rightarrow \infty$.
- (3) $|G_c(x_n, x, x)| \rightarrow 0$ as $n \rightarrow \infty$.
- (4) $|G_c(x_m, x_n, x)| \rightarrow 0$ as $m, n \rightarrow \infty$.

Proposition 14 (see [8]). *Let (X, G) be a complex valued G-metric space and let $\{x_n\}$ be a sequence in X . Then $\{x_n\}$ is complex valued G-Cauchy sequence if and only if $|G(x_n, x_m, x_l)| \rightarrow 0$ as $n, m, l \rightarrow \infty$.*

Proposition 15 (see [8]). *Let (X, G) be a complex valued G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

The main aim of this paper is to establish some new fixed point theorems which extend and generalize Kang et al.'s results in [8], the Banach contraction principle, and some well-known results in the literature.

2. Main Results

Recall that a function $\varphi : [0, \infty) \rightarrow [0, 1)$ is said to be an \mathcal{MT} -function (or \mathcal{R} -function) [11–16] if

$$\limsup_{s \rightarrow t^+} \varphi(s) < 1 \quad \forall t \in [0, \infty). \tag{6}$$

It is obvious that if $\varphi : [0, \infty) \rightarrow [0, 1)$ is a nondecreasing function or a nonincreasing function, then φ is an \mathcal{MT} -function. So the set of \mathcal{MT} -functions is a rich class.

Recently, Du [13] first proved the following characterizations of \mathcal{MT} -functions.

Theorem 16 (see [13]). *Let $\varphi : [0, \infty) \rightarrow [0, 1)$ be a function. Then the following statements are equivalent.*

- (a) φ is an \mathcal{MT} -function.
- (b) For each $t \in [0, \infty)$, there exist $r_t^{(1)} \in [0, 1)$ and $\varepsilon_t^{(1)} > 0$ such that $\varphi(s) \leq r_t^{(1)}$ for all $s \in (t, t + \varepsilon_t^{(1)})$.
- (c) For each $t \in [0, \infty)$, there exist $r_t^{(2)} \in [0, 1)$ and $\varepsilon_t^{(2)} > 0$ such that $\varphi(s) \leq r_t^{(2)}$ for all $s \in [t, t + \varepsilon_t^{(2)}]$.
- (d) For each $t \in [0, \infty)$, there exist $r_t^{(3)} \in [0, 1)$ and $\varepsilon_t^{(3)} > 0$ such that $\varphi(s) \leq r_t^{(3)}$ for all $s \in (t, t + \varepsilon_t^{(3)})$.
- (e) For each $t \in [0, \infty)$, there exist $r_t^{(4)} \in [0, 1)$ and $\varepsilon_t^{(4)} > 0$ such that $\varphi(s) \leq r_t^{(4)}$ for all $s \in [t, t + \varepsilon_t^{(4)}]$.
- (f) For any nonincreasing sequence $\{x_n\}_{n \in \mathbb{N}}$ in $[0, \infty)$, we have $0 \leq \sup_{n \in \mathbb{N}} \varphi(x_n) < 1$.
- (g) φ is a function of contractive factor; for any strictly decreasing sequence $\{x_n\}_{n \in \mathbb{N}}$ in $[0, \infty)$, we have $0 \leq \sup_{n \in \mathbb{N}} \varphi(x_n) < 1$.

The following new fixed point theorem is one of the main results of this paper. It can be considered as a complex valued G-metric version of Banach contraction principle and will generalize and improve [8, Theorem 2.5] and some well-known results in the literature.

Theorem 17. *Let (X, G_c) be a complete complex valued G-metric space and let $T : X \rightarrow X$ be a mapping on X . Suppose that there exists a \mathcal{MT} -function $\varphi : [0, \infty) \rightarrow [0, 1)$ such that*

$$G_c(Tx, Ty, Tz) \preceq \varphi(|G_c(x, y, z)|) G_c(x, y, z) \tag{7}$$

$\forall x, y, z \in X.$

Then T has a unique fixed point on X .

Proof. Let $x_0 \in X$ be given. Define the sequence $\{x_n\}$ by

$$x_n = T^n x_0 = T x_{n-1} \quad \text{for each } n \in \mathbb{N}. \tag{8}$$

For each $n \in \mathbb{N}$, by (7), we have

$$G_c(x_n, x_{n+1}, x_{n+1}) \preceq \varphi(|G_c(x_{n-1}, x_n, x_n)|) G_c(x_{n-1}, x_n, x_n) \tag{9}$$

which implies

$$|G_c(x_n, x_{n+1}, x_{n+1})| \leq \varphi(|G_c(x_{n-1}, x_n, x_n)|) |G_c(x_{n-1}, x_n, x_n)|. \tag{10}$$

Let $\alpha_n = |G_c(x_{n-1}, x_n, x_n)|$ for $n \in \mathbb{N}$. Then, by (10), we have

$$\alpha_{n+1} \leq \varphi(\alpha_n) \alpha_n < \alpha_n \quad \forall n \in \mathbb{N}. \tag{11}$$

So we know that $\{\alpha_n\}$ is a strictly decreasing sequence in $[0, \infty)$. Applying (g) of Theorem 16, we obtain

$$0 \leq \sup_{n \in \mathbb{N}} \varphi(\alpha_n) < 1. \tag{12}$$

That is,

$$0 \leq \sup_{n \in \mathbb{N}} \varphi(|G_c(x_{n-1}, x_n, x_n)|) < 1. \tag{13}$$

Let

$$\lambda = \sup_{n \in \mathbb{N}} \varphi(|G_c(x_{n-1}, x_n, x_n)|). \tag{14}$$

Then $\lambda \in [0, 1)$. For each $n \in \mathbb{N}$, by (10) again, we have

$$\begin{aligned} & |G_c(x_n, x_{n+1}, x_{n+1})| \\ & \leq \varphi(|G_c(x_{n-1}, x_n, x_n)|) |G_c(x_{n-1}, x_n, x_n)| \\ & \leq \lambda |G_c(x_{n-1}, x_n, x_n)| \leq \lambda^2 |G_c(x_{n-2}, x_{n-1}, x_{n-1})| \\ & \leq \dots \leq \lambda^n |G_c(x_0, x_1, x_1)|. \end{aligned} \tag{15}$$

For any $n, m \in \mathbb{N}$ with $m > n$, by the last inequality and repeated use of (CG5), we get

$$\begin{aligned} & |G_c(x_n, x_m, x_m)| \\ & \leq |G_c(x_n, x_{n+1}, x_{n+1})| + |G_c(x_{n+1}, x_{n+2}, x_{n+2})| \\ & \quad + \dots + |G_c(x_{m-1}, x_m, x_m)| \\ & \leq (\lambda^n + \lambda^{n+1} + \dots + \lambda^{m-1}) |G_c(x_0, x_1, x_1)| \\ & < \frac{\lambda^n}{1 - \lambda} |G_c(x_0, x_1, x_1)|. \end{aligned} \tag{16}$$

Since $\lambda \in [0, 1)$, $\lim_{n \rightarrow \infty} (\lambda^n / (1 - \lambda)) |G_c(x_0, x_1, x_1)| = 0$. Hence, by the last inequality, we obtain

$$|G_c(x_n, x_m, x_m)| \rightarrow 0 \text{ as } m, n \rightarrow \infty. \tag{17}$$

For any $n, m, l \in \mathbb{N}$, by Proposition 12, we obtain

$$G_c(x_n, x_m, x_l) \leq G_c(x_n, x_m, x_m) + G_c(x_l, x_m, x_m), \tag{18}$$

which implies

$$|G_c(x_n, x_m, x_l)| \leq |G_c(x_n, x_m, x_m)| + |G_c(x_l, x_m, x_m)|. \tag{19}$$

From (17) and (19), we get $|G_c(x_n, x_m, x_l)| \rightarrow 0$ as $n, m, l \rightarrow \infty$. Applying Proposition 14, $\{x_n\}$ is a complex valued G -Cauchy sequence in (X, G_c) . By the completeness of (X, G_c) , there exists $v \in X$ such that $\{x_n\}$ is complex valued G -convergent to v .

Next, we prove that $Tv = v$. Assume that $Tv \neq v$. For each $n \in \mathbb{N}$, by (7), we have

$$G_c(x_{n+1}, Tv, Tv) \leq \varphi(|G_c(x_n, v, v)|) G_c(x_n, v, v) \tag{20}$$

which deduces

$$\begin{aligned} |G_c(x_{n+1}, Tv, Tv)| & \leq \varphi(|G_c(x_n, v, v)|) |G_c(x_n, v, v)| \\ & < |G_c(x_n, v, v)|. \end{aligned} \tag{21}$$

Since $x_n \rightarrow v$ as $n \rightarrow \infty$ and G is continuous in all three of its variables, from Proposition 15 and by taking limit from both sides of (21), we get

$$|G_c(v, Tv, Tv)| \leq |G_c(v, v, v)| = 0. \tag{22}$$

Since $0 \not\leq G_c(v, Tv, Tv)$, by Remark 3, we know

$$0 < |G_c(v, Tv, Tv)|. \tag{23}$$

Hence, taking into account (22) and (23), we have

$$0 < |G_c(v, Tv, Tv)| \leq 0 \tag{24}$$

which is a contradiction. Therefore $Tv = v$ or $v \in \mathcal{F}(T)$.

Finally, we want to show the uniqueness of fixed point of T (i.e., $\mathcal{F}(T)$ is a singleton set). We have shown $v \in \mathcal{F}(T)$, so

it suffices to show that $\mathcal{F}(T) = \{v\}$. Let $w \in \mathcal{F}(T)$. Suppose $w \neq v$. By (7), we obtain

$$G_c(v, v, w) = G_c(Tv, Tv, Tw) \leq \varphi(|G_c(v, v, w)|) G_c(v, v, w) \tag{25}$$

which implies

$$|G_c(v, v, w)| \leq \varphi(|G_c(v, v, w)|) |G_c(v, v, w)|. \tag{26}$$

By (26), we have

$$(1 - \varphi(|G_c(v, v, w)|)) |G_c(v, v, w)| \leq 0. \tag{27}$$

Since $\varphi(|G_c(v, v, w)|) \in [0, 1)$, we have

$$|G_c(v, v, w)| \leq 0 \tag{28}$$

which deduces

$$|G_c(v, v, w)| = 0 \tag{29}$$

and hence $G_c(v, v, w) = 0$. This contradicts (CG2). Therefore, it must be $w = v$ and so $\mathcal{F}(T) = \{v\}$. The proof is completed. \square

Here, we give a simple example illustrating Theorem 17.

Example 18. Let $X = \mathbb{C}$ and $G_c : X \times X \times X \rightarrow \mathbb{C}$ be defined by

$$\begin{aligned} G_c(z_1, z_2, z_3) & = (|x_1 - x_2| + |x_2 - x_3| + |x_3 - x_1|) \\ & \quad + i(|y_1 - y_2| + |y_2 - y_3| + |y_3 - y_1|), \end{aligned} \tag{30}$$

where $z_i = x_i + iy_i \in \mathbb{C}$ for any $i \in \{1, 2, 3\}$. Then (X, G_c) is a complex valued G -metric space. Define $T : X \rightarrow X$ and $\varphi : [0, \infty) \rightarrow [0, 1)$ by

$$\begin{aligned} Tz & = \frac{1}{10}z \text{ for } z \in X, \\ \varphi(t) & := \begin{cases} \frac{4}{5}, & \text{if } t = 0, \\ \frac{1}{3}, & \text{if } t > 0. \end{cases} \end{aligned} \tag{31}$$

Then φ is a \mathcal{MT} -function. For any $z_1, z_2, z_3 \in \mathbb{C}$, where $z_i = x_i + iy_i$, we have

$$Tz_i = \frac{1}{10}z_i = \frac{x_i}{10} + i\frac{y_i}{10} \text{ for any } i \in \{1, 2, 3\}. \tag{32}$$

By a routine calculation, one can verify that

$$G_c(Tz_1, Tz_2, Tz_3) \leq \varphi(|G_c(z_1, z_2, z_3)|) G_c(z_1, z_2, z_3). \tag{33}$$

So all the hypotheses of Theorem 17 are fulfilled. It is therefore possible to apply Theorem 17 to get the fact that T has a unique fixed point on X (precisely speaking, 0 is the unique fixed point of T).

The following fixed point theorem established in G -metric space is immediate from Theorem 17.

Theorem 19. *Let (X, G) be a complete G -metric space and let $T : X \rightarrow X$ be a mapping on X . Suppose that there exists a \mathcal{MT} -function $\varphi : [0, \infty) \rightarrow [0, 1)$ such that*

$$G(Tx, Ty, Tz) \leq \varphi(G(x, y, z))G(x, y, z) \quad \forall x, y, z \in X. \quad (34)$$

Then T has a unique fixed point on X .

Since any nondecreasing function or any nonincreasing function $\varphi : [0, \infty) \rightarrow [0, 1)$ is an \mathcal{MT} -function, by applying Theorem 17, we have the following results.

Corollary 20. *Let (X, G_c) be a complete complex valued G -metric space and let $T : X \rightarrow X$ be a mapping on X . Suppose that there exists a nondecreasing function $\varphi : [0, \infty) \rightarrow [0, 1)$ such that*

$$G_c(Tx, Ty, Tz) \leq \varphi(|G_c(x, y, z)|)G_c(x, y, z) \quad \forall x, y, z \in X. \quad (35)$$

Then T has a unique fixed point on X .

Corollary 21. *Let (X, G) be a complete G -metric space and let $T : X \rightarrow X$ be a mapping on X . Suppose that there exists a nondecreasing function $\varphi : [0, \infty) \rightarrow [0, 1)$ such that*

$$G(Tx, Ty, Tz) \leq \varphi(G(x, y, z))G(x, y, z) \quad \forall x, y, z \in X. \quad (36)$$

Then T has a unique fixed point on X .

Corollary 22. *Let (X, G_c) be a complete complex valued G -metric space and let $T : X \rightarrow X$ be a mapping on X . Suppose that there exists a nonincreasing function $\varphi : [0, \infty) \rightarrow [0, 1)$ such that*

$$G_c(Tx, Ty, Tz) \leq \varphi(|G_c(x, y, z)|)G_c(x, y, z) \quad \forall x, y, z \in X. \quad (37)$$

Then T has a unique fixed point on X .

Corollary 23. *Let (X, G) be a complete G -metric space and let $T : X \rightarrow X$ be a mapping on X . Suppose that there exists a nonincreasing function $\varphi : [0, \infty) \rightarrow [0, 1)$ such that*

$$G(Tx, Ty, Tz) \leq \varphi(G(x, y, z))G(x, y, z) \quad \forall x, y, z \in X. \quad (38)$$

Then T has a unique fixed point on X .

Corollary 24 (see [8, Theorem 2.5]). *Let (X, G_c) be a complete complex valued G -metric space and let $T : X \rightarrow X$ be a contraction mapping on X ; that is,*

$$G_c(Tx, Ty, Tz) \leq kG_c(x, y, z) \quad (39)$$

for all $x, y, z \in X$, where $k \in [0, 1)$. Then T has a unique fixed point.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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