

Updating spin-dependent Regge intercepts

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(Received 29 March 2018; published 29 August 2018)

We use new high statistics data from CLAS and COMPASS on the nucleon's spin structure function at low Bjorken x and low virtuality, $Q^2 < 0.5 \text{ GeV}^2$, together with earlier measurements from the SLAC E-143, HERMES, and GDH experiments to estimate the effective intercept(s) for spin dependent Regge theory. We find $\alpha_{a_1} = 0.31 \pm 0.04$ for the intercept describing the high-energy behavior of spin dependent photoabsorption together with a new estimate for the high-energy part of the Gerasimov-Drell-Hearn sum rule: $-15 \pm 2 \mu\text{b}$ from photon-proton center-of-mass energy greater than 2.5 GeV . Our value of α_{a_1} suggests QCD physics beyond a simple straight-line a_1 trajectory.

DOI: [10.1103/PhysRevC.98.025209](https://doi.org/10.1103/PhysRevC.98.025209)

I. INTRODUCTION

The high-energy behavior of the spin dependent part of the photon-proton total cross section is important for determining the Gerasimov-Drell-Hearn sum rule for polarized photoabsorption with real photons [1,2], as well as studies of the transition from polarized photoproduction to deep inelastic scattering [3].

Here we investigate this behavior using the new high statistics measurements from CLAS at Jefferson Laboratory [4] and COMPASS at CERN [5] of the spin asymmetry for polarized photon-proton collisions at low photon virtuality $Q^2 < 0.5 \text{ GeV}^2$ and center-of-mass energy $\sqrt{s} \geq 2.5 \text{ GeV}$, together with earlier measurements from the E-143 experiment at SLAC [6], HERMES at DESY [7], and the GDH Collaboration in Bonn [8].

The large s dependence of hadronic total cross sections is usually described in terms of Regge exchanges [9,10], e.g., summing the exchanges of hadrons with given quantum numbers that occur along Regge trajectories with slope (often taken as a straight line) related to the confinement potential. Regge phenomenology has had considerable success in describing unpolarized high-energy scattering processes [11].

II. SPIN DEPENDENT REGGE THEORY

Let σ_A and σ_P denote the two cross sections for the absorption of a transversely polarized photon with spin antiparallel

σ_A or parallel σ_P to the spin of the target nucleon. The Regge prediction for the isovector and isoscalar parts of $(\sigma_A - \sigma_P)$ for a real photon, $Q^2 = 0$, with $s \rightarrow \infty$ is [12–14]

$$\begin{aligned} (\sigma_A - \sigma_P)^{(p-n)} &\sim \sum_i N_i^{(3)} s^{\alpha_{a_1}-1}, \\ (\sigma_A - \sigma_P)^{(p+n)} &\sim \sum_i N_i^{(0)} s^{\alpha_{f_i}-1} + N_g \frac{\ln s / \mu^2}{s}. \end{aligned} \quad (1)$$

Here, the α_i denote the Regge intercepts for isovector $a_1(1260)$ Regge exchange and the a_1 -pomeron cuts [12]. The α_{f_i} denote the intercepts for the isoscalar $f_1(1285)$ and $f_1(1420)$ Regge trajectories and their f_1 -pomeron cuts. The logarithm $\ln s/s$ term comes from two non-perturbative gluon exchanges in the t channel [13] with a vector short-range exchange potential [14], and the mass parameter μ is taken as a typical hadronic scale. The coefficients $N_i^{(3)}$, $N_i^{(0)}$, and N_g are to be determined from experiment.

If one makes the usual assumption that the a_1 Regge trajectories are straight lines parallel to the (ρ, ω) trajectories, then one finds $\alpha_{a_1} \simeq -0.4$ for the leading trajectory, within the range of possible α_{a_1} values between -0.5 and zero discussed in Ref. [15]. Fitting straight line trajectories through the $a_1(1260)$ and $a_3(2030)$ states, the $a_1(1640)$ and $a_3(2310)$ states, and the $f_1(1285)$ and $f_3(2050)$ states yields near parallel trajectories with slopes 0.79 , 0.76 , and 0.78 GeV^{-2} respectively. The two leading trajectories then have slightly lower intercepts, $\alpha_{a_1} = -0.25$ and $\alpha_{f_1} = -0.29$. With this value of α_{a_1} the effective intercepts corresponding to the a_1 soft-pomeron cut and the a_1 hard-pomeron cut are -0.17 and $+0.15$ respectively if one takes the soft pomeron with intercept 1.0808 and the hard pomeron proposed in Ref. [16] with intercept 1.4 as two distinct exchanges. Values of α_{a_1} close to zero could be achieved with curved Regge trajectories;

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$\alpha_{a_1} = -0.03 \pm 0.07$ is found in the model of Ref. [17]. For this value the intercepts of the a_1 soft-pomeron cut and the a_1 hard-pomeron cut are $\sim +0.05$ and $\sim +0.37$.

Before presenting our new results, we first recall the challenge of understanding the proton's internal spin structure in high- Q^2 deep inelastic scattering and Q^2 dependence of the intercepts α_i describing the asymptotic high-energy behavior.

In deep inelastic kinematics the nucleon's g_1 spin structure function is related to $(\sigma_A - \sigma_P)$ by

$$(\sigma_A - \sigma_P) \simeq \frac{4\pi^2 \alpha_{\text{QED}}}{pq} g_1, \quad (2)$$

where p and q are the proton and photon four-momenta respectively and α_{QED} is the electromagnetic coupling. The Regge prediction for the isovector $g_1^{p-n} = g_1^p - g_1^n$ at small Bjorken x ($=Q^2/2pq$) is

$$g_1^{p-n} \sim \sum_i N_i^{(3)} \left(\frac{1}{x}\right)^{\alpha_i} \quad (3)$$

with all data taken at the same Q^2 . Equation (3) follows from $s = (p+q)^2 = Q^2 \frac{(1-x)}{x} + M^2$, where M is the proton mass and $s \simeq Q^2/x$ in the small- x limit. There is possible Q^2 dependence in the α_i and $N_i^{(3)}$. The COMPASS experiment found

$$g_1^{p-n} \sim x^{-0.22 \pm 0.07}, \quad (4)$$

corresponding to an effective intercept $\alpha_{a_1}(Q^2) = 0.22 \pm 0.07$ at $Q^2 = 3 \text{ GeV}^2$, with small- x data down to $x_{\text{min}} \sim 0.004$ [18].

The isoscalar spin structure function $g_1^{p+n} \sim 0$ for $x < 0.03$ at deep inelastic Q^2 [19], in sharp contrast to the unpolarized structure function F_2 where the isosinglet part dominates through gluonic exchanges. The proton spin puzzle, why the quark spin content of the proton is so small ~ 0.3 , concerns the collapse of the isoscalar spin sum structure function to near zero at this small x . The spin puzzle is now understood in terms of pion cloud effects with transfer of quark spin to orbital angular momentum in the pion cloud [20], a modest polarized gluon correction $-3 \frac{\alpha_s}{2\pi} \Delta g$ with Δg less than about 0.5 at the scale of the experiments [19], and a possible topological effect at $x = 0$ [21].

The observed rise in g_1^{p-n} at deep inelastic values of Q^2 is required to reproduce the area under the fundamental Bjorken sum rule,

$$\int_0^1 dx g_1^{(p-n)}(x, Q^2) = \frac{g_A^{(3)}}{6} C_{\text{NS}}(Q^2). \quad (5)$$

Here $g_A^{(3)} = 1.270 \pm 0.003$ is the isovector axial charge measured in neutron β decays and $C_{\text{NS}}(Q^2)$ is the perturbative QCD Wilson coefficient, $\simeq 0.85$ with QCD coupling $\alpha_s = 0.3$ [19]. The Bjorken sum rule is connected to pion physics and chiral symmetry through the Goldberger-Treiman relation $2M g_A^{(3)} = f_\pi g_{\pi NN}$, where f_π is the pion decay constant and $g_{\pi NN}$ is the pion-nucleon coupling constant. The sum rule has been confirmed in polarized deep inelastic scattering experiments at the level of 5% [18]. About 50% of the sum rule comes from x values less than about 0.15. The g_1^{p-n} data are consistent with quark model and perturbative QCD predictions

in the valence region $x > 0.2$ [22]. The size of $g_A^{(3)}$ forces us to accept a large contribution from small x , and the observed rise in g_1^{p-n} is required to fulfill this nonperturbative constraint.

Perturbative QCD evolution acts to push the weight of the distribution to smaller Bjorken x with increasing Q^2 , with perturbative calculations predicting rising g_1^{p-n} at small x and deep inelastic Q^2 [23,24]. Regge phenomenology should describe the high-energy part of g_1 close to photoproduction and provide the input for perturbative QCD evolution at deep inelastic values of Q^2 . One then applies perturbative QCD, typically above $Q^2 > 1 \text{ GeV}^2$. These perturbative QCD calculations involve Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution and double logarithm, $\alpha_s^m \ln^n \frac{1}{x}$, resummation at small x [25], in possible combination with vector meson dominance terms at low Q^2 [26]. For g_1^{p-n} with DGLAP evolution this approach has the challenging feature that the input and output (at soft and hard scales) are governed by nonperturbative constraints with perturbative QCD evolution in the middle, unless the a_1 Regge input has information about $g_A^{(3)}$ and chiral symmetry built into it. One possibility is a separate hard-exchange contribution (perhaps an a_1 hard-pomeron cut) in addition to the soft a_1 term [27].

III. FITTING THE HIGH ENERGY SPIN ASYMMETRY

We next estimate the spin-dependent Regge intercepts. Good statistics measurements of the spin asymmetry for photon-proton collisions, $A_1^p = (\sigma_A - \sigma_P)/(\sigma_A + \sigma_P)$, at large \sqrt{s} and low Q^2 have recently become available from the CLAS and COMPASS experiments, complementing earlier measurements from SLAC, HERMES, and the GDH Collaboration. We make a Regge motivated fit to this data on $\Delta\sigma = \sigma_A - \sigma_P = A_1^p (\sigma_A + \sigma_P)$ with the constraints $\sqrt{s} \geq 2.5 \text{ GeV}$ where Regge theory is expected to apply [11] and $Q^2 < 0.5 \text{ GeV}^2$. Keeping $Q^2 < 0.5 \text{ GeV}^2$ is a compromise between keeping Q^2 as low as possible and including the maximum amount of data. This input data involves 18 points from COMPASS with \sqrt{s} between 11 and 15 GeV [5], 2 data points from HERMES with \sqrt{s} at 6.6 and 6.8 GeV [7], 7 points from SLAC E-143 with \sqrt{s} between 2.5 and 3.1 GeV [6], and 102 points from CLAS between 2.5 and 2.9 GeV [4]. These data are consistent with A_1^p being Q^2 independent in each experiment within the chosen kinematics. We also consider the highest energy single data point from the GDH photoproduction experiment with $\sqrt{s} = 2.5 \text{ GeV}$ and $Q^2 = 0$ [8]. Data at higher Q^2 values between 0.5 and 1 GeV^2 are in principle sensitive to the extra effects of turning on DGLAP evolution and decay of higher-twist terms with increasing Q^2 .

The unpolarized total cross section, $\sigma_{\text{tot}} = \sigma_A + \sigma_P$, measurements from HERA were found to be well described by a combined Regge and generalized vector meson dominance (GVMD) motivated fit in the kinematics $Q^2 < 0.65 \text{ GeV}^2$ and $s \geq 3 \text{ GeV}^2$ [28–30]. The ZEUS Collaboration used the four-parameter fit [28]

$$\sigma_{\text{tot}}^{*p}(s, Q^2) = \left(\frac{M_0^2}{M_0^2 + Q^2} \right) (A_R s^{\alpha_R - 1} + A_P s^{\alpha_P - 1}) \quad (6)$$

to describe the low- Q^2 region, also including fixed target data from the E665 Collaboration [31], with $A_R = 147.8 \pm 4.6 \mu\text{b}$, $\alpha_R = 0.5$ (fixed), $A_P = 62.0 \pm 2.3 \mu\text{b}$, $\alpha_P = 1.102 \pm 0.007$, and $M_0^2 = 0.52 \pm 0.04 \text{ GeV}^2$.

In the HERA kinematical region the total γ^*p cross-section is related to $F_2(x, Q^2)$ by

$$\sigma_{\text{tot}}^{\gamma^*p}(s, Q^2) \simeq \frac{4\pi^2\alpha_{\text{QED}}}{Q^2} F_2(x, Q^2), \quad (7)$$

where $s \simeq Q^2/x$. For Q^2 larger than 1 GeV^2 the HERA data on F_2 seem to be well described by DGLAP evolution. Parametrizing $F_2 \sim Ax^{-\lambda}$ at small x , the effective intercept λ is observed to grow from 0.11 ± 0.02 at $Q^2 = 0.3 \text{ GeV}^2$ to 0.18 ± 0.03 at $Q^2 = 3.5 \text{ GeV}^2$ and to 0.31 ± 0.02 at 35 GeV^2 [29,30,32]. The value 0.4 was found at the highest Q^2 , motivating suggestions of a new hard pomeron [16,33].

Here, we first assume A_1^p to be Q^2 independent in our chosen kinematics with $Q^2 < 0.5 \text{ GeV}^2$. That is, we conjecture

$$(\sigma_A - \sigma_P)^{\gamma^*p}(s, Q^2) = \left(\frac{M_0^2}{M_0^2 + Q^2} \right) (\sigma_A - \sigma_P)^{\gamma p}(s, 0) \quad (8)$$

at large s and small Q^2 with the same value of M_0^2 in both Eqs. (6) and (8) and Q^2 -independent values of the spin Regge intercepts α_i at this low Q^2 .

Second, we assume that the isoscalar deuteron asymmetry A_1^d can be taken as zero in first approximation. The deuteron data on A_1^d are consistent with zero in each experiment in our chosen kinematics [6,7,34,35] (as well as in g_1^d measurements at deep inelastic Q^2 and low $x < 0.03$ [19]). This means that we set the normalization factors $N_i^{(0)} = N_g = 0$ in Eq. (1).

Third, we take σ_{tot} from a fit to unpolarized data. We assume that the errors on σ_{tot} can be neglected compared to the errors on A_1^p . For the total photoproduction cross section we take

$$(\sigma_A + \sigma_P) = 67.7 s^{+0.0808} + 129 s^{-0.4545} \quad (9)$$

(in units of μb), which provides a good Regge fit for \sqrt{s} between 2.5 and 250 GeV [11]. The $s^{+0.0808}$ contribution is associated with gluonic pomeron exchange and the $s^{-0.4545}$ contribution is associated with the isoscalar ω and isovector ρ trajectories.

Our best fit of form $(\sigma_A - \sigma_P) \sim Ns^\alpha$ including all data is

$$(\sigma_A - \sigma_P) = (35.3 \pm 3.6) s^{-0.69 \pm 0.04} \mu\text{b} \quad (10)$$

for $\sqrt{s} \geq 2.5 \text{ GeV}$, corresponding to an effective Regge intercept

$$\alpha_{a_1} = +0.31 \pm 0.04; \quad (11)$$

see Fig. 1. The χ^2/ndf for the fit is 0.98. Statistical and systematic errors for each data point have been added in quadrature.

To convert the fit results in Eqs. (9)–(11) into a prediction for the asymmetry A_1^p as a function of x , it is important to note that $s \simeq Q^2/x$ at large center-of-mass energy and to take into account that experimental measurements in different x bins are typically taken at different Q^2 values. For example, the COMPASS measurements using a 160 GeV muon beam at $\langle x \rangle = 0.000052$ were taken at $\langle Q^2 \rangle = 0.0062 \text{ GeV}^2$, whereas

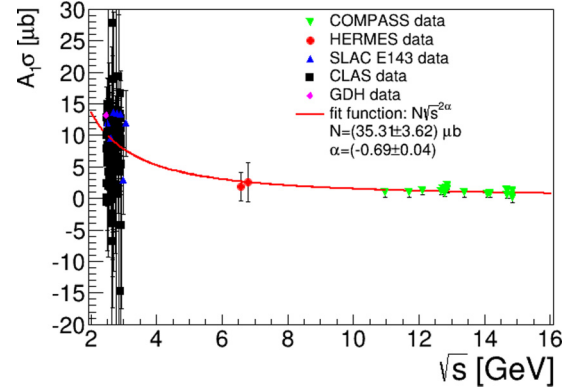


FIG. 1. Regge fit to $(\sigma_A - \sigma_P) = A_1^p (\sigma_A + \sigma_P)$ with spin data from the CLAS [4], COMPASS [5], GDH [8], HERMES [7], and SLAC E-143 [6] experiments with $Q^2 < 0.5 \text{ GeV}^2$.

their measurements at $\langle x \rangle = 0.0020$ were taken at $\langle Q^2 \rangle = 0.33 \text{ GeV}^2$ [5], a factor of 53 greater in Q^2 . Within each x bin taken separately, Q^2 was varied over a more limited factor of about 5 and the experimental uncertainties were too large to make definite conclusions about possible Q^2 dependence within individual x bins. All of our COMPASS points with $Q^2 < 0.5 \text{ GeV}^2$ are in the range \sqrt{s} between 11 and 15 GeV. One expects A_1^p to vanish in the small- x limit, which follows in these data when all points are shifted to the same Q^2 by dividing out the factor $(Q^2)^{\alpha_{a_1} - 1.0808} \sim (Q^2)^{-0.77}$ from Eqs. (9)–(11).

IV. DISCUSSION

It is very interesting that the intercept in Eq. (11) is close to the value found in deep inelastic scattering, viz., $\alpha_{a_1}(Q^2) = 0.22 \pm 0.07$ at $Q^2 = 3 \text{ GeV}^2$ in Eq. (4). Our new low- Q^2 value signifies either the presence of a hard exchange, perhaps involving an a_1 hard pomeron cut, or a curved Regge trajectory instead of just a simple straight-line a_1 Regge trajectory.

More valuable experimental input could come from the proposed future electron-ion-collider, which could extend the experimental data up to \sqrt{s} values between 40 and 140 GeV [36,37]; that is, up to an order of magnitude higher in \sqrt{s} than the present highest center-of-mass energy COMPASS data. Estimates for the expected asymmetries are given in [3]. The fit values in Eqs. (10) and (11) suggest low- Q^2 asymmetries $A_1^p = (1.7 \pm 0.5) \times 10^{-3}$ at $\sqrt{s} = 40 \text{ GeV}$ and $A_1^p = (2.5 \pm 1.0) \times 10^{-4}$ at $\sqrt{s} = 140 \text{ GeV}$.

Taking the fit values in Eq. (10), we estimate the high-energy contribution to the Gerasimov-Drell-Hearn sum-rule from $\sqrt{s} \geq 2.5 \text{ GeV}$ to be

$$\int_{s_0}^{\infty} \frac{ds}{s - M^2} (\sigma_P - \sigma_A) = -15 \pm 2 \mu\text{b}. \quad (12)$$

This determination compares with previous estimates: $-15 \pm 10 \mu\text{b}$ for $\sqrt{s} \geq 2.5 \text{ GeV}$ based on an extrapolation of lower energy photoproduction data which also gave $\alpha_{a_1} = 0.42 \pm 0.23$ [38], $-25 \pm 10 \mu\text{b}$ from an early estimate using lower statistics low- Q^2 data (prior to CLAS and COMPASS) for $\sqrt{s} \geq 2.5 \text{ GeV}$ [39], and $-26 \pm 7 \mu\text{b}$ for $\sqrt{s} \geq 2 \text{ GeV}$ [40]

from early Regge fits to low- Q^2 data. The new result in Eq. (12) is a factor of 3.5 times more accurate than the previous most accurate determination. The corresponding integral from threshold up to $\sqrt{s} = 2.5$ GeV has been extracted from proton fixed target experiments with photon energy up to 2.9 GeV. One finds $226 \pm 12 \mu\text{b}$ [8,38]. Combining this number and the result in Eq. (12) gives

$$\int_{M^2}^{\infty} \frac{ds}{s - M^2} (\sigma_P - \sigma_A) = 211 \pm 13 \mu\text{b} \quad (13)$$

for the Gerasimov-Drell-Hearn sum rule. This value compares with the sum-rule prediction $2\pi^2\alpha_{\text{QED}}\kappa^2/M^2 = 205 \mu\text{b}$, with $\kappa = 1.79$ being the proton's anomalous magnetic moment [1,2].

ACKNOWLEDGMENTS

We thank R. Fersch for helpful communications. This work was supported by the Foundation for Polish Science through the TEAM/2017-4/39 project, and by the National Science Centre of Poland through Grant No. 2016/23/B/ST2/00784.

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