

# Quantum phase transitions in odd- $A$ nuclei: The effect of the odd particle from spherical to oblate shapes

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## Abstract.

Quantum shape-phase transitions in odd-nuclei are investigated within the framework of the interacting boson-fermion model (IBFM). We consider the case of a single- $j$  fermion coupled to an even-even boson core that performs a transition from spherical to oblate shapes varying a control parameter in the boson Hamiltonian. The aim of this work is to see the effect of the coupling of the unpaired fermion on the transition, to understand how the coupled single particle modifies the geometric shape of the system and how each of the odd states behaves when the boson core shifts along the transitional path.

## 1. Introduction

The study of phase transitions in finite nuclear quantal systems has been the subject of many investigations in Nuclear Physics. These studies include the Quantum Phase Transitions (QPT) in nuclei within both the Geometric Collective Model (GCM) [1] and also the Interacting Boson Model (IBM) [2], mostly on even-even systems [3, 4]. Moreover the corresponding phase transitions in Bose-Fermi systems [5-16] have been studied within the IBFM [16].

In our previous works [11, 15], we focused on quantum phase transitions for Bose-Fermi systems: the effect of the coupling of a single  $j = 9/2$  fermion to an even-even boson core that performs the transition from spherical to  $\gamma$ -unstable shapes [11] and from spherical to axially prolate shapes [15]. These situations are described within the framework of the intrinsic frame formalism for the IBFM [17, 18, 19]. In Ref [15], results of both studies are also compared to see the differences of the odd particle effect along both transitional paths.

In this work, we intend to see the effect of the coupling of the unpaired fermion in a orbit of definite angular momentum  $j = 9/2$  on the transition from spherical to oblate shapes, to understand how the coupled single particle modifies the geometric shape of the system and how each of the odd states behaves when the boson core shifts along the transitional region. Moreover, the overall results of the odd particle effect along the spherical to oblate transition are presented for different angular momenta, with  $j$  ranging from  $3/2$  to  $13/2$ .



The structure of this proceeding is organized as follows: we first give the general IBFM Hamiltonian in section 2, then the intrinsic frame formalism for Bose-Fermi systems is described in section 3. Finally, results are presented and conclusions are remarked in the last section.

## 2. The IBFM Hamiltonian

The interacting boson-fermion (IBFM) Hamiltonian is generally written as

$$H = H_B + H_F + V_{BF} , \quad (1)$$

where  $H_B$  describes the bosonic part, the  $H_F$  term is the fermionic part, and the  $V_{BF}$  is the interaction between bosons and fermion.

The boson Hamiltonian in equation (1) is given by

$$H_B = (1 - c)n_d - \frac{c}{4N_B} Q_B^X \cdot Q_B^X , \quad (2)$$

where  $N_B$  is the total boson number,  $c$  is the control parameter,  $n_d$  is the the d-boson number operator defined as

$$n_d = \sum_{\mu} d_{\mu}^{\dagger} d_{\mu} , \quad (3)$$

and  $Q_B^X$  is the boson quadrupole operator given by

$$Q_B^X = (s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s})^{(2)} + \chi(d^{\dagger} \times \tilde{d})^{(2)} . \quad (4)$$

Taking into account that the pure fermion part is a constant for the single-j shell case as the one presented here, the boson-fermion Hamiltonian (1) can be written as follows:

$$H = (1 - c)n_d - \frac{c}{4N_B} Q_{BF}^X \cdot Q_{BF}^X , \quad (5)$$

where  $Q_{BF}^X$  is the quadrupole operator for the odd-even system

$$Q_{BF}^X = Q_B^X + q_F , \quad (6)$$

and  $q_F$  is the fermion quadrupole operator given by

$$q_F = (a_j^{\dagger} \times \tilde{a}_j)^{(2)} . \quad (7)$$

In this case, the boson-fermion interaction  $V_{BF}$  included in equation (1) is

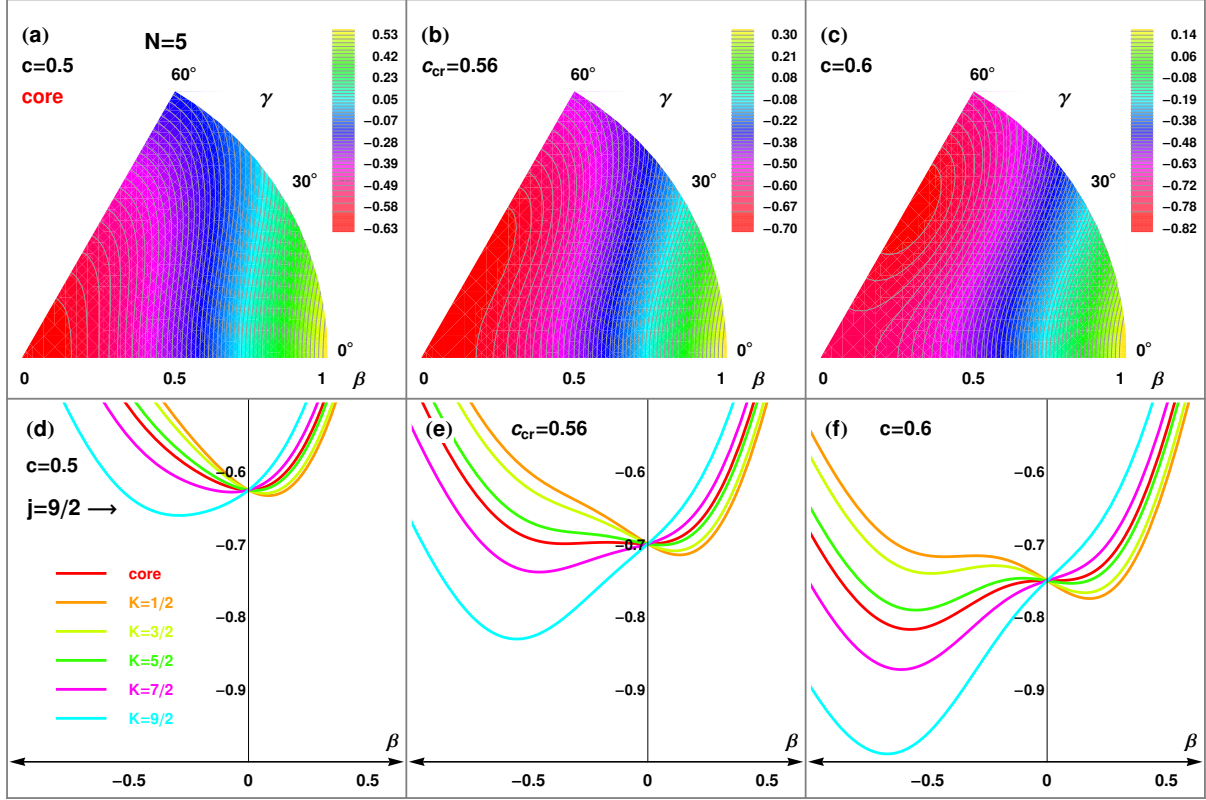
$$V_{BF} = -\frac{c}{2N_B} Q_B^X \cdot q_F . \quad (8)$$

## 3. The Intrinsic Frame Formalism

The concept of intrinsic frame formalism is one of the useful ways to look into quantum shape-phase transitions in atomic nuclei. This concept associates a potential energy surface to a Hamiltonian as (2) in terms of shape variables. Within the IBM, the intrinsic state for the ground state band of an even-even nucleus is written as

$$|\Phi_{gs}(\beta, \gamma)\rangle = \frac{1}{\sqrt{N_B!}} [b_{gs}^{\dagger}(\beta, \gamma)]^{N_B} |0\rangle , \quad (9)$$

where  $|0\rangle$  is the boson vacuum,  $b_{gs}^{\dagger}$  is the ground state boson creation operator and the variational parameters, also called deformation parameters  $\beta$  and  $\gamma$ , play a similar role to the one played



**Figure 1.** Even-even and odd-even energy surfaces. In panels (a), (b) and (c) energy surfaces for the even-even system in the  $\beta$ - $\gamma$  plane are shown for different values of  $c$ . The selected values for the control parameter  $c$  are chosen around the critical value,  $c_{cr}$ , in the Hamiltonian describing the transition from spherical to oblate shapes ( $\chi = +\sqrt{7}/2$  is fixed). In panels (d), (e) and (f), the five energy surfaces for the different  $K$  states coming from  $j = 9/2$  in the odd-even system are plotted as a function of  $\beta$  for the same control parameters. In these panels, the energy surface for the even-even case (red color) is also plotted for reference.

by the intrinsic collective shape variables in the Bohr Hamiltonian. The  $\beta$  variable measures the axial deviation from sphericity and the angle variable  $\gamma$  controls the departure from axial deformation.

The ground state boson creation operator is given by

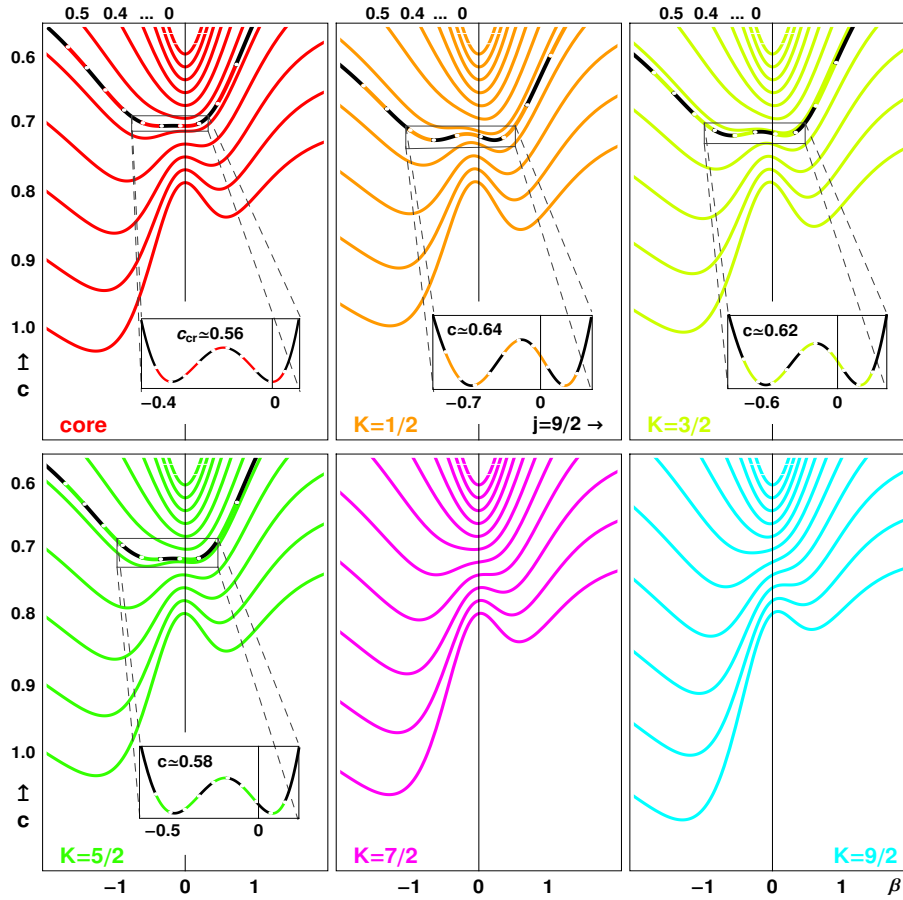
$$b_{gs}^\dagger(\beta, \gamma) = \frac{1}{\sqrt{1 + \beta^2}} \left[ s^\dagger + \beta \cos \gamma d_0^\dagger + \frac{\beta}{\sqrt{2}} \sin \gamma (d_2^\dagger + d_{-2}^\dagger) \right]. \quad (10)$$

The ground state energy surface is obtained by calculating the expectation value of the boson Hamiltonian (2) in the intrinsic state (9)

$$E_{gs}(\beta, \gamma) = \langle \Phi_{gs}(\beta, \gamma) | H_B | \Phi_{gs}(\beta, \gamma) \rangle. \quad (11)$$

Intrinsic frame states for the mixed boson-fermion system can be constructed by coupling the odd single-particle states to the intrinsic states of the even core. The lowest intrinsic states of the odd nucleus originate from this coupling to the intrinsic ground-state  $\Phi_{gs}(\beta, \gamma)$ . To obtain them, we first construct the coupled states

$$|\Psi_{jK}(\beta, \gamma)\rangle = |\Phi_{gs}(\beta, \gamma)\rangle \otimes |jK\rangle, \quad (12)$$



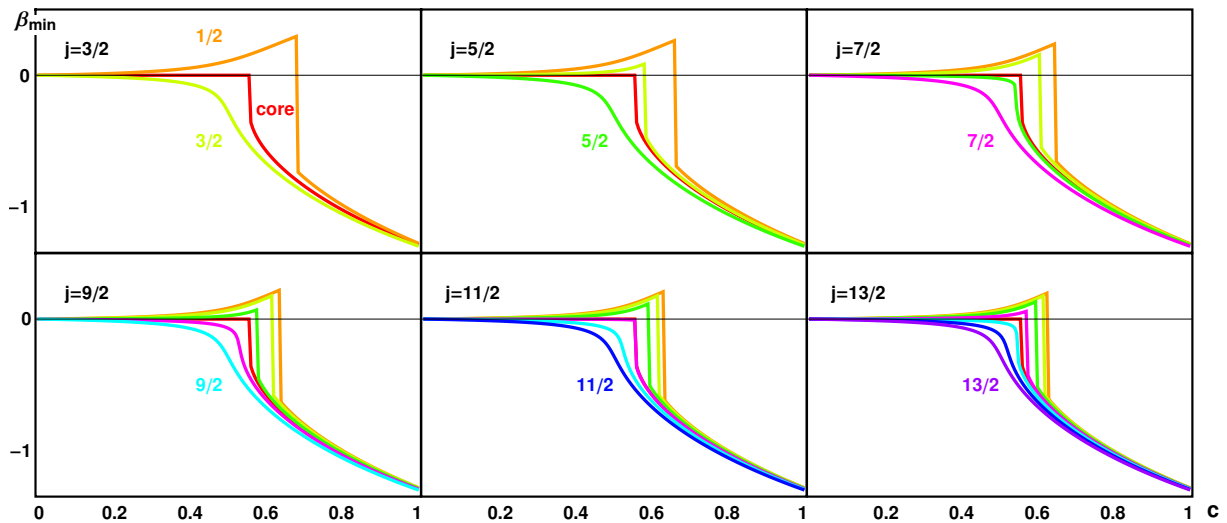
**Figure 2.** Evolution of the energy surfaces for the bosonic core (red color) and for the different  $K$  states (different colors) in the odd-even system with  $j = 9/2$  as a function of  $\beta$ . Here, the control parameter  $c$  is changed from 0 to 1 for the transition from spherical to oblate shape. Calculations are performed for  $N_B = 5$ .

and then diagonalize the total boson-fermion Hamiltonian in this basis, yielding a set of energy eigenvalues  $E_n(\beta, \gamma)$ , where  $n$  is an index to count solutions in the odd-even system and  $K$  is the projection of the total angular momentum  $j$  on the symmetry axis.

#### 4. Results and Conclusions

A single fermion with  $j = 9/2$  is coupled to an even-even boson core that undergoes the shape phase transition from spherical to deformed oblate shapes to observe the corresponding shape phase transition in a mixed Bose-Fermi system. The spherical shape is obtained for  $c = 0$ , while  $c = 1$  gives pure oblate shape, fixing  $\chi = +\sqrt{7}/2$  in equation (2) and equation (4), unlike Ref. [15] where  $\chi = -\sqrt{7}/2$  to obtain pure prolate shape. If it were only for the boson part, the two cases would be completely symmetric with respect to the change of sign in  $\chi$ , as expected. However, in the case of coupling to a fermion, the behaviours of  $K$ -states are quite different as described in the following.

The expectation value of the boson Hamiltonian in the intrinsic state (9),  $E_{gs}(\beta, \gamma)$ , is calculated to obtain the ground state energy surface. For the bosonic system, a first-order shape phase transition is observed at  $c_{cr} = 16N_B/(34N_B - 27)$  in between  $U(5)$ - $SU(3)$ , same as in Ref. [10] for spherical to prolate shapes. Therefore, the critical point of the first-order transition is  $c_{cr} = 0.56$  in the case of *five* bosons, as in Ref. [15]. The energy surfaces for the



**Figure 3.** Evolution of the equilibrium deformation parameter in odd-even system as a function of the control parameter  $c$  along the  $U(5)$  to  $SU(3)$  transition. Calculation have been done for  $N_B = 5$ . Each panel represents a different single- $j$  case. Within a panel, different lines correspond to the different possible  $K$ -states. The red line, which is the same in all panels, gives the even-even value.

boson core as a function of the deformation parameters  $(\beta, \gamma)$  are plotted around the critical point as shown in panels (a), (b), (c) of figure 1. In the case of  $c_{cr} = 0.56$  (middle panels in figure 1) the even-even surface has two degenerate minima (one spherical and one oblate). The even-even system is spherical before this point then it jumps to oblate deformation.

Then the odd  $j = 9/2$  particle is coupled. The behaviour of the odd-even energy surfaces as a function of  $\beta$  is illustrated around the critical point in panels (d), (e) and (f) of figure 1. In these panels, the boson core is denoted with red color as a reference. For these cases, around the even-even critical point, we see that independently of the core shape (spherical in figure 1(a), critical figure 1(b), or oblate in figure 1(c)) the intrinsic states of the odd-even system prefer either prolate or oblate shapes depending on their  $K$  value (see figures 1(d), 1(e) and 1(f), respectively).

In the case of angular momentum  $j = 9/2$ , the possible  $K$ -projections are  $K = -9/2, \dots, 9/2$ . Therefore there are 10 different states that are restricted to 5 because of the symmetry  $K \leftrightarrow -K$ . The evolution of the energy surfaces along the shape phase transition for the boson core and the odd-even system with five magnetic components  $K$  are shown in figure 2. Different lines in a panel correspond to different control parameter values  $c$ . The bosonic core (red color) is given in the first panel. For  $c = 0$ , uppermost line, the core is spherical and it changes to well defined oblate shape for  $c = 1$  (lowest line). At the critical point,  $c_{cr} = 0.56$ , the energy surface presents two degenerate minima as seen in the inset of this panel. The rest of panels correspond to the evolution of the energy surface as a function of  $\beta$  for the different  $K$ -values allowed for  $j = 9/2$  in the odd-even system. States with  $K = 7/2, 9/2$  always favor oblate shapes, while states with  $K = 1/2, 3/2, 5/2$  are prolate up to  $c_{1/2} \simeq 0.64$ ,  $c_{3/2} \simeq 0.62$  and  $c_{5/2} \simeq 0.58$ , respectively. After these points, they suddenly change to oblate shapes. As seen in the insets for  $K = 1/2, 3/2$  and  $5/2$ , they present for the mentioned values of the control parameter two degenerate minima. Therefore, we can suggest that components with  $K = 1/2, 3/2, 5/2$  show first order transitions. This is similar to the case of transitional nuclei along the  $U(5) \rightarrow SU(3)$  line but the role and behaviour of high- $K$  and low- $K$  states are interchanged in a non-trivial way: states with  $K = 1/2, 3/2, 5/2$  always favor prolate shapes, while states with  $K = 7/2, 9/2$

are oblate up to  $c_{7/2} \simeq 0.6$  and  $c_{9/2} \simeq 0.69$ . This is evidently seen when comparing figure 2 of Ref. [15].

Now we study the same transition for a single- $j$  case but considering different angular momenta  $j$ : from  $3/2$  to  $13/2$ . The behaviour of the order parameter  $\beta$  that minimizes the energy surface as a function of the control parameter  $c$  for the different  $j$  cases is depicted in figure 3. Here, the number of  $K$  states changes according to the angular momentum  $j$  considered. The vertical scale displays the  $\beta$  deformation, with positive values indicating prolate deformation and negative ones for oblate deformed shapes. The bosonic case is plotted as a reference in red. Panels of figure 3 refer to the angular momenta;  $j = 3/2, 5/2, 7/2, 9/2, 11/2$  and  $13/2$ . It is evident from figure 3 that along the transitional path, states with lower  $K$  values prefer to be prolate up to a critical point, and then jump to oblate shape at specific  $c$  values. On the contrary, the states with higher  $K$  values always prefer oblate shapes for all different angular momenta. In the pure oblate deformed region,  $c = 1$ , the dominant action of the strongly oblate core drives all odd states into the oblate side and they have approximately the same  $\beta_{min}$  as the oblate core. Moreover, notice that on left side of figure 3 of Ref.[15] high- $K$  states jump from oblate to prolate, while in figure 3 of this work, they behave smoothly for  $j = 9/2$ . Their tendency to the oblate side is not hampered by the core's deformation. The reverse is true for low- $K$  states.

Summarizing, in this work the coupling of a single  $j = 9/2$  particle to a boson core that undergoes a transition from spherical to oblate shapes has been investigated and results are compared with our previous work [15]. The overall results of the odd particle effect are also presented for different angular momenta,  $j$  from  $3/2$  to  $13/2$ , along the spherical to oblate transition.

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