A Beyond Mean Field study of Bose gases in a double-well potential with a Feshbach resonance

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Abstract. The Bose-Hubbard model coupled to a Feshbach resonance is studied. Quantum phase transitions are analyzed within a beyond mean field framework in order to get finite size corrections to the simple mean field approach. Analytical results for the ground state energy and the first few energy gaps are presented.

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The system considered consists of atoms trapped in a double-well potential. The atoms can occupy just two single-particle levels, one in the left and the other in the right-well [1]. Besides the interacting atoms, these can interact with a molecule via a Feshbach resonance [2]. Thus, two left or right atoms can be combined into a single molecule. The model Hamiltonian is

$$H = -J(a_L^\dagger a_R + a_R^\dagger a_L) + \frac{U}{N}(a_L^\dagger a_L^\dagger a_L a_L + a_R^\dagger a_R^\dagger a_R a_R) + \omega b^\dagger b - \frac{g}{\sqrt{2N}}\left[b^\dagger(a_L a_L + a_R a_R) + (a_L^\dagger a_L^\dagger + a_R^\dagger a_R^\dagger)b\right], \quad (1)$$

where ω, U, J, and g are control parameters. In an obvious notation, $a_R^\dagger$, $a_L^\dagger$ and $b^\dagger$ are creation operators of an atom in the right well, an atom in the left well and a molecule, respectively. $\hat{N}$ is the total number of particles operator, $\hat{N} = 2b^\dagger b + \left(a_L^\dagger a_L + a_R^\dagger a_R\right)$. This quantity is conserved. This model can be exactly solved numerically. However, an analytic study is required if scale laws at the critical points are to be studied. Here, we report on analytical mean field and beyond [3] studies of this problem and validate the method by comparison with exact numerical calculations. The results presented were obtained for N=200 as a function of the control parameter U. The rest of the control parameters in (1) were fixed to $J = 1$, $\omega = 5$ and $g = 5$ (in a longer publication other cases will be presented [4]). In Fig. 1, the energy per particle in the ground state is plotted versus U. In the left panel, exact (symbols) and two analytical calculations, mean field (full line) and beyond (dot-dashed line), are shown. Both analytical calculation reproduce the exact results at this scale. In the right panel, the difference between the exact and the two analytical calculations is presented to show how the beyond mean field match better the exact results. The only differences are found close to the critical point, which is at $U \approx -1.14$. The beyond mean field scheme allows to compute also energy

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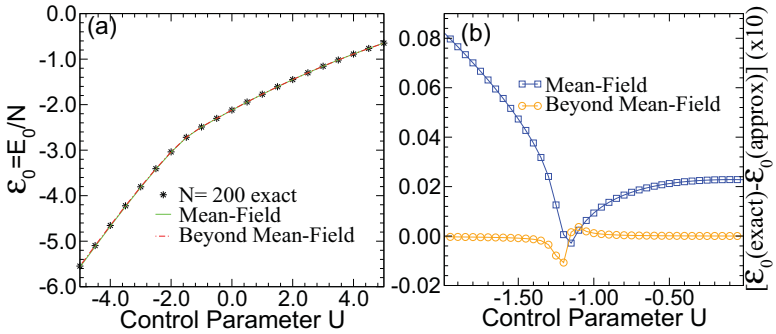

FIGURE 1. (Color online) Ground state energy per particle in the Bose-Hubbard Hamiltonian with a Feshbach resonance as a function of the control parameter U is plotted for N=200. In (a) the calculated g.s. energy (symbols) is compared with two analytical calculations (lines). In (b) The energy difference between numerical exact and mean field and beyond mean field analytical calculations is given.

gaps of the system excited states. This is presented in Fig. 2. The pairing gap collapse at the critical point and the first few gaps at both phases are well reproduced. It is worth noting that states are degenerated in pairs in the phase at the left of the critical point.

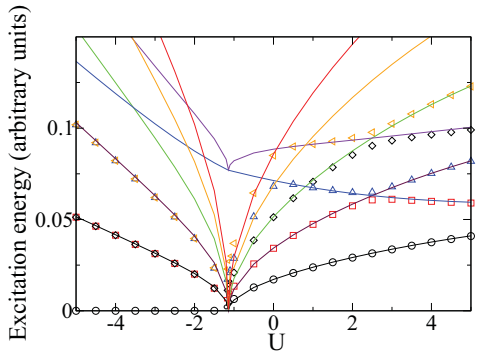

FIGURE 2. (Color online) Several excited state energy gaps for the Bose-Hubbard Hamiltonian with a Feshbach resonance as a function of the control parameter U. Symbols are the exact numerical calculations and lines are the the beyond mean field results.

In conclusion, a second-order QPT in a Bose-Hubbard model with a Feshbach resonance is studied within a beyond mean field approach. This reproduces the behavior of g.s. energy and, in addition, gives correctly the first few energy gaps of the system.

REFERENCES

1. See for a recent review, D. Jaksch and P. Zoller, *Ann. Phys.* **315**, 52 (2005).
2. S. Pethick, *Bose-Einstein Condensation in Dilute Gases*, Cambridge, 2002.
3. J. P. Blaizot and G. Ripka, *Quantum Theory of Finite Systems*, The MIT Press, 1986.
4. A. Relaño, J. Dukelsky, P. Pérez-Fernández, and J.M. Arias, *to be published.*