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RECENT DEVELOPMENTS FOR THE CALCULATION OF ELASTIC AND NON-ELASTIC BREAKUP OF WEAKLY-BOUND NUCLEI*

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In this contribution, we review some recent theoretical advances for the calculation of breakup cross sections in reactions induced by weakly-bound nuclei.

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1. Introduction

It is well-known that breakup is a major reaction channel in reactions induced by weakly-bound nuclei. The analysis of breakup experiments has provided important structure information on these nuclei, such as spectroscopic factors, separation energies, positions and widths of resonances, and electric responses to the continuum, among others (see, *e.g.* [1]).

For a two-body projectile, the process can be schematically represented as $a+A \to b+x+A$, where a is the projectile nucleus, b and x its constituents, and A the target. When the final state of the three outgoing fragments is fully determined, the reaction is said to be *exclusive*. If, in addition, the three particles are emitted in their ground state, the corresponding cross section is referred to as *elastic breakup* (EBU).

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A qualitatively different scenario occurs when the final state of one or more particles is not determined (*i.e.*, measured). Using the notation introduced above, this is the case of reactions of the form of $A(a,b)X$, in which only one of the two projectile constituents (the fragment b in this case) is observed. In this case, the reaction is said to be *inclusive* with respect to the unobserved particle(s). The angular/energy distribution of these b fragments will contain contributions from all possible final states of the $x + A$ system, namely:

(i) The elastic breakup process introduced above, *i.e.*, $a+A \to b+x+A_{\rm gs}$.

(ii) Inelastic breakup (INBU), in which the breakup is accompanied by the excitation of some of the fragments. This includes target excitations ($a+A \to b+x+A^*$) as well as core excitations ($a+A \to b^*+x+A_{\rm gs}$).

(iii) Particle transfer, leading to bound states of the $A + x \equiv B$ system, *i.e.* $a + A \to b + B$.

(iv) Incomplete fusion (ICF), in which the fragment x is absorbed by the target, forming a compound nucleus C, which will eventually decay by particle or gamma-ray emission: $a + A \to b + C$.

(v) Complete fusion (CF) followed by evaporation. If b is among the evaporation products, it will also contribute to the inclusive b yield.

As an example, we illustrate in Fig. 1 these different breakup modes for a ^{11}Be+A process, assuming that ^{11}Be breaks into ^{10}Be+n.

Following some authors [2], we will refer generically to the processes *(ii)–(iv)* as *non-elastic breakup* (NEB). Note that the processes *(i)*, *(ii)* and *(iii)* correspond to direct reaction modes, whereas *(v)* is a purely compound nucleus mode. Process *(iv)* can be considered as a hybrid mode comprising a direct breakup mechanism (the dissociation of the projectile) followed by the formation of the compound nucleus (*i.e.*, fusion) $C = A + x$.

From the theoretical point of view, breakup reactions are difficult to describe because they involve three or more particles in the final state. In fact, theoretical models are usually tailored to specific types of breakup. In the following, we describe some recent advances in reaction theory for the description of the breakup modes listed above. We focus the discussion on the direct (or hybrid) modes *(i)–(iv)*. Although the calculation of total CF cross sections can be performed with the direct reaction formalisms discussed here, the detailed calculation of these observables (for example, energy distribution of evaporation products) requires the use of statistical models, which will not be discussed here.

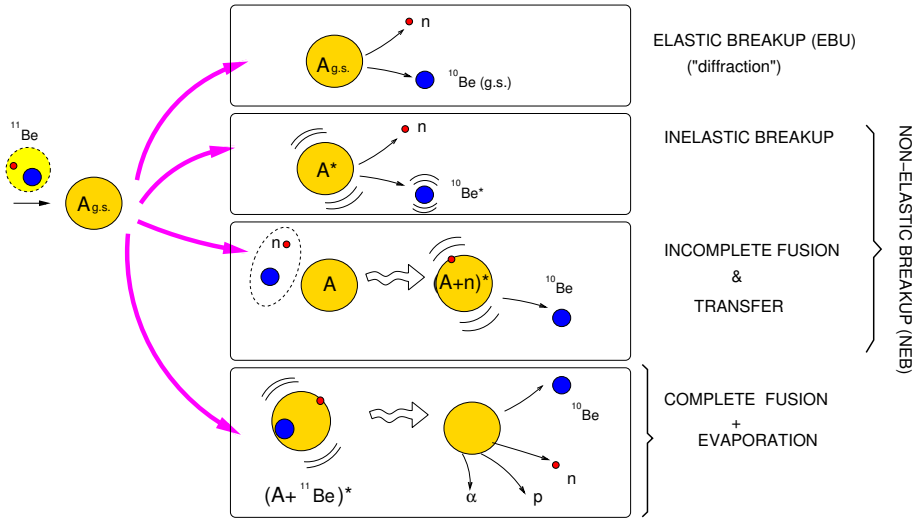

Fig. 1. Two-body breakup modes for the $^{11}\mathrm{Be}+A$ reaction.

2. Exclusive breakup

Most of the existing theoretical models are designed to describe exclusive breakup reactions. For two-body projectiles, these models usually assume an effective three-body scattering problem with some effective pairwise interactions adjusted to reproduce the properties of the corresponding two-body systems. These reactions have been usually described using the distorted-wave Born approximation (DWBA) [3] or the continuum-discretized coupled-channels (CDCC) method [4]. At high energies, semiclassical methods provide a simpler and popular alternative [5–8]. More recently, it has been also possible to obtain the *exact* solution of this problem by solving the so-called Faddeev equations [9].

2.1. Elastic breakup

Assuming a two-body projectile, the elastic breakup process can be denoted as $a+A \to b+x+A_{\mathrm{gs}}$. Theoretical models for elastic breakup assume a Hamiltonian of the form

$$H = H_{\mathrm{proj}} + \hat{T}_{\boldsymbol{R}} + U_{bA}(\boldsymbol{r}_{bA}) + U_{xA}(\boldsymbol{r}_{xA})\,, \tag{1}$$

with $H_{\mathrm{proj}} = \hat{T}_{\boldsymbol{r}} + V_{bx}$ the projectile internal Hamiltonian, $\hat{T}_{\boldsymbol{r}}$ and $\hat{T}_{\boldsymbol{R}}$ are kinetic energy operators, V_{bx} the intercluster interaction and U_{bA} and U_{xA} are the fragment-target optical potentials evaluated at the corresponding incident energy per nucleon.

The procedure to solve the scattering problem varies from one method to another. For instance, in CDCC, the three-body wave function of the system is expanded in terms of the eigenstates of $H_{\rm proj}$, including both bound and unbound states. To make the expansion finite, the continuum spectrum is first approximated by a discrete representation in terms of square-integrable functions (continuum *discretization*).

The CDCC method was originally proposed by Rawitscher [10] to describe the effect of breakup in deuteron scattering. The method was refined and applied to other weakly-bound projectiles (such as ^{6,7}Li) by the Pittsburgh–Kyushu Collaboration [4, 11] and, more recently, it has been extensively applied to halo nuclei, composed by a *core* (b) and a weakly bound *valence* nucleon (x) (such as ^{8}B and ^{11}Be).

Despite its success, several limitations are apparent in this formulation. First, the projectile fragments b and x are considered as structureless bodies. This is a good approximation for deuteron scattering, but not necessarily for more complex systems, like ^{11}Be (treated as ^{10}Be+n). Second, target excitations, which may occur simultaneously with projectile breakup, are also ignored. Third, for some nuclei, the two-body structure model will not be accurate and may require more sophisticated descriptions. This is the case of the Borromean systems, genuine three-body systems in which the binary sub-systems are unbound, for which a three-body structure model will be required.

In the remainder, we discuss some recent works and developments aimed at understanding and overcoming several of these limitations.

2.2. Inclusion of fragments collective excitations

Excitations of the projectile constituents (b and x in our case) may take place concomitant with the projectile breakup. This mechanism is neglected in the standard formulation of the CDCC method. For example, for the scattering of halo nuclei, collective excitations of the core b may be important. These core excitations will affect both the structure of the projectile as well as the reaction dynamics. In the inert core picture, the projectile states will correspond to pure single-particle or cluster states but, if the core is allowed to excite, these states will contain, in general, admixtures of core-excited components. Additionally, the interaction of the core with the target will produce excitations and de-excitations of the former during the collision, and this will modify the reaction observables to some extent. These two effects (structure and dynamic effects) have been recently investigated within extended versions of the DWBA and CDCC methods [12–15]. To this end, the following effective three-body Hamiltonian has been used

$$H = H_{\rm proj}(\boldsymbol{r}, \xi_b) + \hat{T}_{\boldsymbol{R}} + U_{bA}(\boldsymbol{r}_{bA}, \xi_b) + U_{xA}(\boldsymbol{r}_{xA})\,. \tag{2}$$

Note that the core degrees of freedom (ξ_b) appear in the projectile Hamiltonian (structure effect) as well as in the core-target interaction (dynamic effect). In the weak coupling limit, the projectile Hamiltonian can be written more explicitly as

$$H_{\rm proj} = \hat{T}_{\boldsymbol{r}} + V_{bx}(\boldsymbol{r}, \xi_b) + h_{\rm core}(\xi_b)\,, \tag{3}$$

where $h_{\rm core}(\xi_b)$ is the internal Hamiltonian of the core. The eigenstates of this Hamiltonian are of the form

$$\Phi_{i,JM}(\xi) \equiv \sum_{\alpha} \left[\varphi_\alpha(\boldsymbol{r}) \otimes \Phi_I(\xi_b)\right]_{JM}\,, \tag{4}$$

where i is an index labeling the states with angular momentum J, M, $\xi \equiv \{\xi_b, \boldsymbol{r}\}$, $\alpha \equiv \{\ell, s, j, I\}$, with I the core intrinsic spin, $\vec{j} = \vec{\ell} + \vec{s}$ and $\vec{J} = \vec{j} + \vec{I}$. The functions $\Phi_I(\xi_b)$ and $\varphi_\alpha(\boldsymbol{r})$ describe, respectively, the core states and the valence–core relative motion. For continuum states, a procedure of continuum discretization is used.

Once the projectile states (4) have been calculated, the three-body wave function is written as an expansion in terms of these states, as in the standard CDCC method. Calculations using this extended CDCC method (XCDCC) were first performed by Summers *et al.* [14, 16] for ^{11}Be and ^{17}C on ^{9}Be and ^{11}Be+p. These calculations predicted very little core excitation effect in all the cases analyzed by the authors.

Later on, an extended DWBA (XDWBA) method with core excitation was proposed in Refs. [12, 13]. Contrary to the XCDCC results of [14, 16], the application of this XDWBA method to the ^{11}Be+p reaction suggested that the dynamic core excitation mechanism enhances significantly the breakup cross sections. The discrepancy between the XCDCC and XDWBA calculations was found to be due to an inconsistency in the numerical implementation of the XCDCC formalism presented in Ref. [14], as clarified in [17].

The XDWBA results were ratified by a new implementation of the XCDCC method developed by De Diego *et al.* [15]. Moreover, this work provided calculations for ^{11}Be on ^{64}Zn and ^{208}Pb showing that, for these heavier targets, the dynamic core excitation mechanism is small but the effect of core excitation on the projectile structure is still important.

As an example of these XCDCC calculations, we show in Fig. 2 the differential breakup cross section, as a function of the n–^{10}Be relative energy, for the reaction ^{11}Be+p at 63.7 MeV/nucleon. The calculations use the same structure model and potentials as those employed in Ref. [15]. Continuum states with angular momentum/parity $J = 1/2^{\pm}$, $3/2^{\pm}$ and $5/2^{+}$ were included using a pseudostate representation, that is, diagonalizing the projectile Hamiltonian ($H_{\rm proj}$) in a basis of square-integrable functions. For

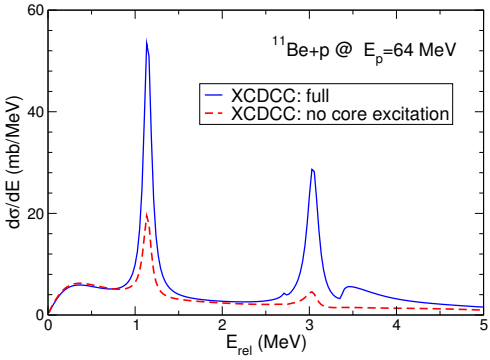

Fig. 2. (Color on-line) Differential breakup cross sections, with respect to the n-^{10}Be relative energy, for the breakup of ^{11}Be on protons at 63.7 MeV/nucleon.

that, a transformed harmonic oscillator (THO) basis was employed [18]. To get a smooth function of the energy, the calculated differential cross sections were then convoluted with the *true* scattering states of $H_{\rm proj}$. The two peaks correspond to the $5/2^+$ and $3/2^+$ resonances which, in the assumed structure model, appear at $E_{\rm rel} = 1.2$ and 3.2 MeV, respectively. The solid line is the full XCDCC calculation, including the ^{10}Be deformation in the structure of the projectile as well as dynamic excitations. The dashed line is the XCDCC calculation omitting the effect of the dynamic core excitation mechanism. It is clearly seen that the inclusion of this mechanism increases significantly the breakup cross sections, particularly in the region of the $3/2^+$ resonance, owing to the dominant $^{10}\mathrm{Be}(2^+)\otimes 2s_{1/2}$ configuration of this resonance [12, 13, 18].

In addition to the enhancement of the breakup cross sections, XDWBA calculations performed for the ^{11}Be+^{12}C reaction evidenced that the interplay between the single-particle and dynamic core excitation mechanisms produces a distinctive effect on the interference pattern of the resonant breakup angular distributions [19].

Similarly to the case of the projectile constituents, collective excitations of the target may also take place, and compete with the projectile breakup mechanism. The explicit inclusion of target excitation was first done by the Kyushu group in the 1980s [11], and applied to deuteron scattering. The motivation was to compare the roles of target-excitation process with those of the deuteron breakup process in elastic and inelastic scattering of deuterons. In this way, they could study the relative importance and mutual influence of these two mechanisms. They applied the formalism to the d+^{58}Ni reaction at $E_d = 22$ and 80 MeV, including the ground state and the first excited state of $^{58}\mathrm{Ni}(2^+)$, finding that, in this case, the deuteron breakup process is more important than the target-excitation. Except for a few exceptions [20], the problem seems to have received little attention since then.

2.3. Application to (p, pN) reactions

Breakup experiments of the form $a(p, pn)b$ and $a(p, 2p)b$ were used extensively in the 1970s as a tool to extract spectroscopic information on proton-hole and neutron-hole states in nuclei, such as separation energies, spin-parity assignments, and occupation probabilities. In these reactions, an energetic proton beam ($E > 100$ MeV) collides with a stable target nucleus, removing one or more nucleons, and leaving a residual nucleus (b), either in its ground state, or in an excited state. Recently, the technique has been extended to the study of unstable nuclei, using inverse kinematics, *i.e.*, bombarding a hydrogen target with an energetic radioactive beam. This technique is analogous to the knockout experiments with composite targets used extensively in the past years [21–24].

Theoretical analyses of the $(p,\ pN)$ reactions with stable nuclei have been traditionally performed using the distorted-wave impulse approximation (DWIA) [25, 26], in which one assumes that the binding potential of the removed particle can be neglected in comparison with the projectile–target kinetic energy. At sufficiently high energies (several hundreds of MeV per nucleon) this approximation is expected to be well-justified but, for not so high energies (tens of MeV per nucleon), the impulse approximation may be questionable.

In recent years, there have been some attempts to apply the CDCC formalism to these reactions [27, 28], treating the process as an inelastic-like mechanism of the form $a + p \to (b + N) + p$ and using an expansion of the three-body final wave function in terms of $b + N$ states. However, due to the relatively large angular momentum and energy transfer involved in the process, this expansion converges very slowly with the size of the model space. A recently proposed alternative [29] is to expand the three-body scattering wave function in terms of $p + N$ states. In this case, the reaction can be viewed as a transfer-like process of the form $p + a \to b + (p + N)$, leading to unbound $p + N$ states, thus receiving the name of *transfer to the continuum* method.

In inverse-kinematics experiments, measured observables usually correspond to parallel or longitudinal momentum distributions of the residual nucleus b. The shapes of these momentum distributions are very sensitive to the orbital angular momentum of the struck nucleon. Moreover, their magnitude is proportional to the occupation probability of the orbital from which this nucleon has been removed (spectroscopic factor). Therefore, the comparison of the measured distributions with a suitable reaction framework provide useful spectroscopic information.

As an example, we show in Fig. 3 the momentum distributions of the ^{22}O core, resulting from the ^{23}O(p,pn) reaction at 445 MeV/nucleon (quoted from Ref. [29]), calculated with the transfer to the continuum method, for different single-particle configurations of the removed nucleon. The dependence on this configuration is clearly evidenced.

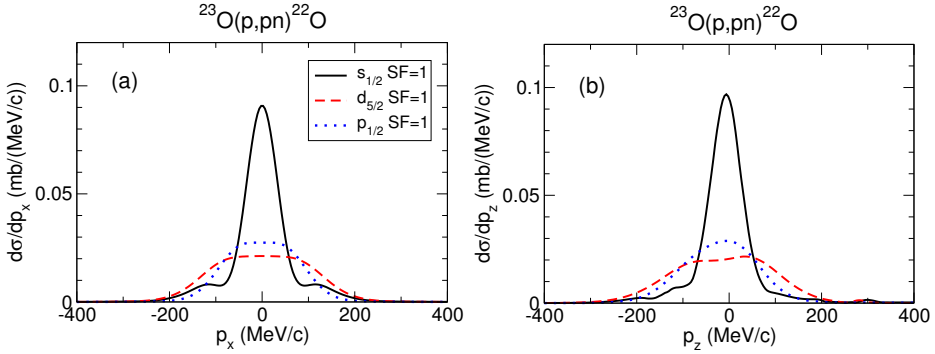

Fig. 3. (Color online) Transverse (a) and longitudinal (b) momentum distributions for the ^{22}O residual nucleus from the one-neutron removal of ^{23}O at 445 MeV/nucleon. Solid, dashed and dotted lines correspond to the removal from $2s_{1/2}$, $1d_{5/2}$, and $1p_{1/2}$ configurations in ^{23}O, respectively, assuming in all cases unit spectroscopic factor.

3. Evaluation of inclusive breakup cross sections

Whereas the calculation of EBU can be accurately done within the CDCC method and other approaches, the calculation of NEB is more difficult because it involves the sum over all possible states of the $x + A$ system. In the 1980s, several groups found that the explicit sum over these final states could be avoided making use of formal techniques based on the completeness of the $x+A$ final states and the Feshbach projection formalism. For instance, in the pioneering works by Baur and co-workes [30–32], the sum was performed making use of unitarity and a surface approximation of the form factors of the residual nucleus excited states. Later on, Udagawa and Tamura [33, 34] proposed a prior-form formula, whereas Austern and Vincent [35] gave a formally similar post-form expression. The latter model was refined by Kasano and Ichimura [36], who found a formal separation between the EBU and NEB contributions. These results were carefully reviewed by Ichimura, Austern and Vincent [2] and the model was subsequently referred to as the IAV formalism. Later on, Austern *et al.* reformulated this theory within a more complete three-body model [4]. Although the final expression differs from one theory to another, it is interesting that in all these works the non-elastic breakup cross section is calculated as some expectation value of the imaginary part of the $x + A$ optical potential.

We consider here the expression obtained by IAV, in which the double differential cross section for the NEB with respect to the angle and energy of the b fragments is given by

$$\left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|_{\rm NEB} = -\frac{2}{\hbar v_i}\rho_b(E_b)\left\langle \psi_x^{(0)}(\boldsymbol{k}_b)\right| W_x \left|\psi_x^{(0)}(\boldsymbol{k}_b)\right\rangle , \tag{5}$$

where $\rho_b(E_b) = k_b\mu_b/((2\pi)^3\hbar^2)$ is the density of states for the particle b, W_x is the imaginary part of the optical potential describing $x + A$ elastic scattering and $\psi_x^{(0)}(\boldsymbol{k}_b, \boldsymbol{r}_x)$ is a wave function describing the evolution of x after the projectile dissociation when b scatters with momentum $\boldsymbol{k}_b$ and the target remains in the ground state. This function satisfies the following inhomogeneous differential equation

$$\left(E_x^+ - K_x - U_{xA}\right)\psi_x^{(0)}(\boldsymbol{k}_b, \boldsymbol{r}_x) = \left(\chi_b^{(-)}(\boldsymbol{k}_b)\left|V_{\rm post}\right|\Psi^{3b}\right\rangle , \tag{6}$$

where the round bracket denotes integration over $\boldsymbol{r}_b$ only, $E_x = E - E_b$, $\chi_b^{(-)}$ is the distorted-wave describing the scattering of b in the final channel with respect to the $x + A$ system, and $V_{\rm post} \equiv V_{bx} + U_{bA} - U_b$ (with U_b the optical potential in the final channel) is the post-form transition operator. This equation is to be solved with outgoing boundary conditions.

IAV suggest approximating the three-body wave function appearing in the source term of Eq. (6), Ψ^{3b}, by the CDCC one. Since the CDCC wave function is also a complicated object by itself, a simpler choice is to use the DWBA approximation, *i.e.*, $\psi_x^{3b} \approx \chi_a^{(+)}(\boldsymbol{R})\phi_{bx}(\boldsymbol{r})$, where $\chi_a^{(+)}$ is a distorted wave describing $a+A$ elastic scattering and ϕ_{bx} is the projectile ground state wave function.

The IAV model has been recently revisited by several groups [37–39]. The calculations performed so far by these groups make use of the DWBA approximation for the incoming wave function. In Refs. [38, 39], the theory was applied to deuteron induced reactions of the form of $A(d,p)X$, whereas in Ref. [37] the calculations were extended to ^{6}Li induced reactions of the form of $A(^6\mathrm{Li},\alpha)X$. In general, the agreement with the data has been found to be very promising, and several extensions and improvements are under way.

As an application of the IAV model, we show in Fig. 4 (left panel) the angular distribution of α particles produced in the ^{6}Li+^{208}Pb breakup reaction. The dashed and dot-dashed lines are the EBU and NEB contributions computed, respectively, with the CDCC and IAV DWBA models. The solid line is the sum of both contributions which, except for some slight overestimation, reproduces rather well the data from Signorini *et al.* [40]. In this case, the NEB is seen to dominate the inclusive breakup cross sections. It is

worth noting, nevertheless, that the relative importance of EBU and NEB may largely depend on the system and on the energy. For example, in reactions induced by halo nuclei on heavy targets at near-barrier energies the inclusive breakup cross section is, in general, dominated by the EBU mechanism [41, 42]. This is a consequence of the long-range Coulomb couplings arising from the strong polarizability of the projectile, which favor distant breakup and suppress the absorption of the unobserved particle (the halo neutron(s) in this case). As an example, we show in the right panel of Fig. 4 the angular distribution of ^{9}Li fragments produced in the reaction ^{11}Li+^{208}Pb, measured in Ref. [43]. The solid line is the EBU contribution, calculated with CDCC. It is seen that, unlike the ^{6}Li case, the EBU mechanism dominates the inclusive breakup cross section, whereas other channels are expected to play a minor role. For example, the dashed line is the two-neutron transfer contribution, which is found to contribute only at the largest angles (adapted from Ref. [42]).

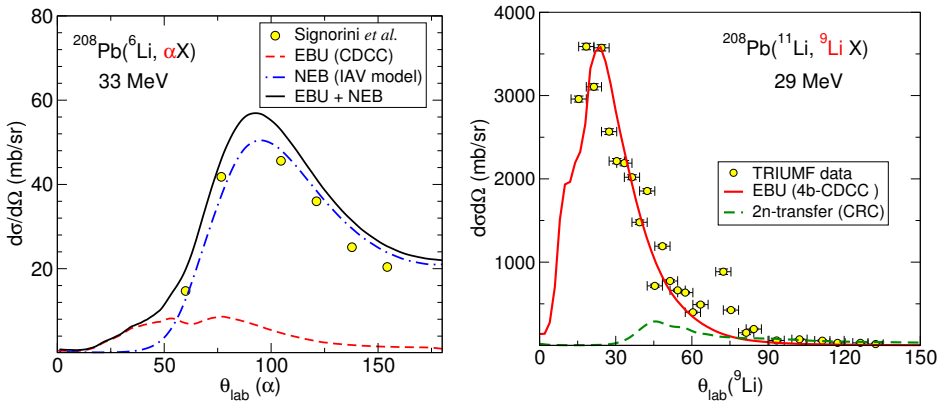

Fig. 4. Comparison of ^{6}Li+^{208}Pb and ^{11}Li+^{208}Pb breakup cross sections.

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