Adaptive Feed Rate Policies for Spiral Drilling Using Markov Decision Process

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***Abstract*-In this study, the feed rate optimization model based on a Markov Decision Process (MDP) was introduced for spiral drilling process. Firstly, the experimental data on spiral drilling was taken from literature for different axial force parameters and with various feed rate decisions made, having the length of a hole being drilled as a reward. Proposed optimization model was computed using value iteration method. Secondly, the results of computations were displayed for optimal decision to be made on each state. Proposed decisions for an optimal feed rate could be utilized in order to improve the efficiency of spiral drilling process in terms of cost and time.**

I. Introduction

Spiral drilling is a process that produces straight holes with the ratio of hole depth to hole diameter less than ten. Nowadays spiral drilling has lots of applications in automotive, aeronautics, hydraulic, petrochemical and oil & gas industries. As an illustration, automotive industries are performing drilling process in order to manufacture various types of engine parts. Perforating a number of holes in an engine parts requires precise choice of the feed rates in order to maintain stability of the drill bit. Depending upon different feed rates the axial force is subjected to changes, which itself tends to the variation of the cutting speed. It is said that the cutting speed has an influence on the length of the hole being drilled [1]. In other words, even slight changes in feed rates cause significant changes on the length of the drilled hole.

This paper aims to present an optimal feed rate choice policy for spiral drilling system using MDP [2, 3]. The data required for the MDP model include axial force values and number of feed rates corresponding to each this value, length of the hole drilled for each of the feed rate value, corresponding transition probabilities [1]. Depending upon certain axial force value a feed rate is to be chosen for drilling hole with maximum possible length.

II. Case Study on Spiral Drilling

Spiral Drilling of holes in an automotive engine parts require getting sufficient quality of holes in terms of surface finish and straightness. The spiral drilling system is required to be time consumable to increase productivity of the work. To meet these requirements for quality and productivity the system is proposed to have adaptive feed rate choice policy using MDP to identify any significant change in the tool parameters and proceed with the most optimal parameter. At the time when changes in axial force identified, the system is set to make immediate and optimal action for the feed rate. The data required for the MDP model of transformers was taken from experimental results for spiral drilling using a number of different tryouts [1]. For each set of the axial force values there are five sets of different feed rates along with final length of the hole being drilled for each of these chosen parameters. Axial force values are set as conditions for given problem, so there are ten conditions for description of the axial force in total: $F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}$. For each of the axial force parameter there are five different feed rates, which are considered as set of actions: $S_1^1, S_2^1, S_3^1, S_4^1, S_5^1, \ldots, S_1^{10}, S_2^{10}, S_3^{10}, S_4^{10}, S_5^{10}$. For each of the parameter chosen for feed rate in particular axial force value there is a result which expressed as a length of the drilled hole. This result is considered as a reward for given problem: $L_1^1, L_2^1, L_3^1, L_4^1, L_5^1, \ldots, L_1^{10}, L_2^{10}, L_3^{10}, L_4^{10}, L_5^{10}$. The data for MDP model is presented in Table 1.

TABLE I

DATA FOR MDP MODEL OF DRILLING DEEP HOLES WITH A SPIRAL DRILL

State I, Axial force rates per drilled hole - F, Newton		Decision k, Feed rate - S, mm/rev		Transition probabilities										Reward q - Length of hole=L, x10^{-2} mm
				P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	
1	50.92	1	0.0522	0.75	0.15	0.1	0	0	0	0	0	0	0	7161.82
		2	0.0582	0.65	0.2	0.15	0	0	0	0	0	0	0	7430.09
		3	0.0658	0.5	0.5	0	0	0	0	0	0	0	0	5009.24
		4	0.0742	0.2	0.65	0.15	0	0	0	0	0	0	0	2056.31
		5	0.0831	0.15	0.75	0.1	0	0	0	0	0	0	0	498.15
2	83.75	1	0.0832	0	0.75	0.15	0.1	0	0	0	0	0	0	8278.23
		2	0.0931	0	0.65	0.2	0.15	0	0	0	0	0	0	8500.39
		3	0.1053	0	0.5	0.5	0	0	0	0	0	0	0	6036.86
		4	0.1185	0	0.2	0.65	0.15	0	0	0	0	0	0	2827.92
		5	0.1324	0	0.15	0.75	0.1	0	0	0	0	0	0	854.43
3	112.12	1	0.1029	0	0	0.75	0.15	0.1	0	0	0	0	0	7831.99
		2	0.1162	0	0	0.65	0.2	0.15	0	0	0	0	0	9450.55
		3	0.1341	0	0	0.5	0.5	0	0	0	0	0	0	7471.69
		4	0.154	0	0	0.2	0.65	0.15	0	0	0	0	0	3568.89
		5	0.1749	0	0	0.15	0.75	0.1	0	0	0	0	0	995.89
4	147.36	1	0.1254	0	0	0	0.75	0.15	0.1	0	0	0	0	9228.49
		2	0.1481	0	0	0	0.65	0.2	0.15	0	0	0	0	9141.06
		3	0.1753	0	0	0	0.5	0.5	0	0	0	0	0	4731.06
		4	0.2043	0	0	0	0.2	0.65	0.15	0	0	0	0	1192.48
		5	0.2341	0	0	0	0.15	0.75	0.1	0	0	0	0	142.72
5	159.54	1	0.1316	0	0	0	0	0.75	0.15	0.1	0	0	0	8061.25
		2	0.1513	0	0	0	0	0.65	0.2	0.15	0	0	0	10198.16
		3	0.1781	0	0	0	0	0.5	0.5	0	0	0	0	7858.75
		4	0.2077	0	0	0	0	0.2	0.65	0.15	0	0	0	3344.22
		5	0.2387	0	0	0	0	0.15	0.75	0.1	0	0	0	758.47
6	187.06	1	0.1461	0	0	0	0	0	0.75	0.15	0.1	0	0	8909.61
		2	0.1729	0	0	0	0	0	0.65	0.2	0.15	0	0	10270.01
		3	0.2067	0	0	0	0	0	0.5	0.5	0	0	0	6290.49
		4	0.2431	0	0	0	0	0	0.2	0.65	0.15	0	0	1871.15

		5	0.2807	0	0	0	0	0	0.15	0.75	0.1	0	0	262.34
		1	0.1602	0	0	0	0	0	0	0.75	0.15	0.1	0	9622.59
		2	0.1863	0	0	0	0	0	0	0.65	0.2	0.15	0	10449.93
7	203.92	3	0.2177	0	0	0	0	0	0	0.5	0.5	0	0	7090.61
		4	0.2512	0	0	0	0	0	0	0.2	0.65	0.15	0	2841.58
		5	0.2858	0	0	0	0	0	0	0.15	0.75	0.1	0	658.41
		1	0.1738	0	0	0	0	0	0	0	0.75	0.15	0.1	10206.75
		2	0.2053	0	0	0	0	0	0	0	0.65	0.2	0.15	10166.77
8	228.71	3	0.2418	0	0	0	0	0	0	0	0.5	0.5	0	5848.13
		4	0.2804	0	0	0	0	0	0	0	0.2	0.65	0.15	1845.82
		5	0.3199	0	0	0	0	0	0	0	0.15	0.75	0.1	313.76
		1	0.173	0	0	0	0	0	0	0	0	0.75	0.25	9206.44
		2	0.2046	0	0	0	0	0	0	0	0	0.65	0.35	10927.64
9	240.35	3	0.2442	0	0	0	0	0	0	0	0	0.5	0.5	7327.75
		4	0.2869	0	0	0	0	0	0	0	0	0.35	0.65	2556.97
		5	0.3308	0	0	0	0	0	0	0	0	0.25	0.75	451.9
		1	0.1835	0	0	0	0	0	0	0	0	0.2	0.8	9701.99
		2	0.2203	0	0	0	0	0	0	0	0	0.15	0.85	10842.65
10	262.94	3	0.2648	0	0	0	0	0	0	0	0	0.1	0.9	6355.54
		4	0.3122	0	0	0	0	0	0	0	0	0.05	0.95	1810.01
		5	0.3608	0	0	0	0	0	0	0	0	0	1	244.43

The MDP model based on data shown in Table 1 is solved using backward induction algorithm. This method is applied in order to determine which decision to make in every state in each drilled hole of a ten holes to be drilled planning horizon so that the vector of expected total rewards is maximized. Optimal policy for a finite horizon is defined as which maximizes the vector of expected total rewards received until the end of the horizon. An optimal policy can be found by utilizing a value iteration method.

For the present case study the value iteration equations are:

$$v_i(10) = 0, \text{ for } i = 1, 2, \dots, 10$$

$$v_i(n) = \max_k \left[q_i^k + \sum_{j=1}^{10} p_{ij}^k v_j(n+1) \right]$$

$$= \max_k \left[q_i^k + p_{i1}^k v_1(n+1) + p_{i2}^k v_2(n+1) + p_{i3}^k v_3(n+1) + p_{i4}^k v_4(n+1) + p_{i5}^k v_5(n+1) + p_{i6}^k v_6(n+1) + p_{i7}^k v_7(n+1) + p_{i8}^k v_8(n+1) + p_{i9}^k v_9(n+1) + p_{i10}^k v_{10}(n+1) \right]$$

for $n = 0, 1, \dots, 9$, and $i = 1, 2, \dots, 10$. Firstly, following values are specified for all states at the end of the hole number 10:

$$v_1(10) = v_2(10) = v_3(10) = v_4(10) = v_5(10) = v_6(10) = v_7(10) = v_8(10) = v_9(10) = v_{10}(10) = 0$$

Since the value iteration is a form of dynamic programming, the calculations for each epoch is displayed in a tabular format.

The calculations for 9th hole are shown in Table 2.

$$\mathrm{v_i}(9) = \max_{k}\left[\mathrm{q_i^k} + \mathrm{p_{i1}^k v_1}(10) + \mathrm{p_{i2}^k v_2}(10) + \mathrm{p_{i3}^k v_3}(10) + \mathrm{p_{i4}^k v_4}(10) + \mathrm{p_{i5}^k v_5}(10) + \mathrm{p_{i6}^k v_6}(10) + \mathrm{p_{i7}^k v_7}(10) + \mathrm{p_{i8}^k v_8}(10) + \mathrm{p_{i9}^k v_9}(10) + \mathrm{p_{i10}^k v_{10}}(10)\right] = \max_{k}\left[\mathrm{q_i^k}\right]$$

for $\mathrm{i} = 1, 2, \ldots, 10$. At the end of hole 9, where n=9, the optimal decision is to select the maximum reward in each state. Consequently, the decision vector is computed as d(9)=[2 2 2 1 2 2 2 1 2 2]^T.

TABLE II

DATA VALUE ITERATION FOR N=9

State	q_i^1	q_i^2	q_i^3	q_i^4	q_i^5	Expected total reward, x10^{-2} mm	Decision
i	k=1	k=2	k=3	k=4	k=5	$v_i(9) = \max_k[q_i^1, q_i^2, q_i^3, q_i^4, q_i^5]$, for $i = 1, 2, \ldots, 10$	k
1	7161.82	7430.09	5009.24	2056.31	498.15	$v_1(9) = \max_k[7161.82, 7430.09, 5009.24, 2056.31, 498.15] = 7430.09$	2
2	8278.23	8500.39	6036.86	2827.92	854.43	$v_2(9) = \max_k[8278.23, 8500.39, 6036.86, 2827.92, 854.43] = 8500.39$	2
3	7831.99	9450.55	7471.69	3568.89	995.89	$v_3(9) = \max_k[7831.99, 9450.55, 7471.69, 3568.89, 995.89] = 9450.55$	2
4	9228.49	9141.06	4731.06	1192.48	142.72	$v_4(9) = \max_k[9228.49, 9141.06, 4731.06, 1192.48, 142.72] = 9228.49$	1
5	8061.25	10198.16	7858.75	3344.22	758.47	$v_5(9) = \max_k[8061.25, 10198.16, 7858.75, 3344.22, 758.47] = 10198.16$	2
6	8909.61	10270.01	6290.49	1871.15	262.34	$v_6(9) = \max_k[8909.61, 10270.01, 6290.49, 1871.15, 262.34] = 10270.01$	2
7	9622.59	10449.93	7090.61	2841.58	658.41	$v_7(9) = \max_k[9622.59, 10449.93, 7090.61, 2841.58, 658.41] = 10449.93$	2
8	10206.75	10166.77	5848.13	1845.82	313.76	$v_8(9) = \max_k[10206.75, 10166.77, 5848.13, 1845.82, 313.76] = 10206.75$	1
9	9206.44	10927.64	7327.75	2556.97	451.9	$v_9(9) = \max_k[9206.44, 10927.64, 7327.75, 2556.97, 451.9] = 10927.64$	2
10	9701.99	10842.65	6355.54	1810.01	244.43	$v_{10}(9) = \max_k[9701.99, 10842.65, 6255.54, 244.43] = 10842.65$	2

The calculations for 8th hole are shown in Table 3.

$$v_i(8) = \max_{k}\left[q_i^k + p_{i1}^k v_1(9) + p_{i2}^k v_2(9) + p_{i3}^k v_3(9) + p_{i4}^k v_4(9) + p_{i5}^k v_5(9) + p_{i6}^k v_6(9) + p_{i7}^k v_7(9) + p_{i8}^k v_8(9) + p_{i9}^k v_9(9) + p_{i10}^k v_{10}(9)\right]$$

$$v_i(8) = \max_k [q_i^k + p_{i1}^k(7430.09) + p_{i2}^k(5800.39) + p_{i3}^k(9450.55) + p_{i4}^k(9228.49) + p_{i5}^k(10198.16) + p_{i6}^k(10270.01) + p_{i7}^k(10449.93) + p_{i8}^k(10206.75) + p_{i9}^k v_9(10927.64) + p_{i10}^k(10842.65)]$$

At the end of hole 8, where n=8, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(8)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE III

DATA VALUE ITERATION FOR N=8

State i		Decision k		$v_i(8)$	Expected total reward, x10^{-2}mm	Decision
1	50.92	1	0.0522	7161.82+0.75(7430.09)+0.15(8500.39)+0.1(9450.55)=14954.5	15377.3	2
		2	0.0582	7430.09+0.65(7430.09)+0.2(8500.39)+0.15(9450.55)=15377.3		
		3	0.0658	5009.24+0.5(7430.09)+0.5(8500.39)=12974.5		
		4	0.0742	2056.31+0.2(7430.09)+0.65(8500.39)+0.15(9450.55)=10485.2		
		5	0.0831	498.15+0.15(7430.09)+0.75(8500.39)+0.1(9450.55)=8933.01		
2	83.75	1	0.0832	8278.23+0.75(8500.39)+0.15(9450.55)+0.1(9228.49)=16994	17300	2
		2	0.0931	8500.39+0.65(8500.39)+0.2(9450.55)+0.15(9228.49)=17300		
		3	0.1053	6036.86+0.5(8500.39)+0.5(9450.55)=15012.3		
		4	0.1185	2827.92+0.2(8500.39)+0.65(9450.55)+0.15(9228.49)=12055.1		
		5	0.1324	854.43+0.15(8500.39)+0.75(9450.55)+0.1(9228.49)=10140.3		
3	112.12	1	0.1029	7831.99+0.75(9450.55)+0.15(9228.49)+0.1(10198.2)=17324	18968.8	2
		2	0.1162	9450.55+0.65(9450.55)+0.2(9228.49)+0.15(10198.2)=18968.8		
		3	0.1341	7471.69+0.5(9450.55)+0.5(9228.49)=16811.2		
		4	0.154	3568.89+0.2(9450.55)+0.65(9228.49)+0.15(10198.2)=12987.2		
		5	0.1749	995.89+0.15(9450.55)+0.75(9228.49)+0.1(10198.2)=10354.7		
4	147.36	1	0.1254	9228.49+0.75(9228.49)+0.15(10198.2)+0.1(10270.01)=18706.6	18719.7	2
		2	0.1481	9141.06+0.65(9228.49)+0.2(10198.2)+0.15(10270.01)=18719.7		
		3	0.1753	4731.06+0.5(9228.49)+0.5(10198.2)=14444.4		
		4	0.2043	1192.48+0.2(9228.49)+0.65(10198.2)+0.15(10270.01)=11207.5		
		5	0.2341	142.72+0.15(9228.49)+0.75(10198.2)+0.1(10270.01)=10202.6		
5	159.54	1	0.1316	8061.25+0.75(10198.2)+0.15(10270.01)+0.1(10449.9)=18295.4	20448.5	2
		2	0.1513	10198.16+0.65(10198.2)+0.2(10270.01)+0.15(10449.9)=20448.5		
		3	0.1781	7858.75+0.5(10198.2)+0.5(10270.01)=18092.8		
		4	0.2077	3344.22+0.2(10198.2)+0.65(10270.01)+0.15(10449.9)=13626.8		
		5	0.2387	758.47+0.15(10198.2)+0.75(10270.01)+0.1(10449.9)=11035.7		
6	187.06	1	0.1461	8909.61+0.75(10270.01)+0.15(10449.9)+0.1(10206.8)=19200.3	20566.5	2
		2	0.1729	10270.01+0.65(10270.01)+0.2(10449.9)+0.15(10206.8)=20566.5		
		3	0.2067	6290.49+0.5(10270.01)+0.5(10449.9)=16650.5		
		4	0.2431	1871.15+0.2(10270.01)+0.65(10449.9)+0.15(10206.8)=12248.6		
		5	0.2807	262.34+0.15(10270.01)+0.75(10449.9)+0.1(10206.8)=10661		
7	203.92	1	0.1602	9622.59+0.75(10449.9)+0.15(10206.8)+0.1(10927.6)=20083.8	20922.9	2

		2	0.1863	10449.93+0.65(10449.9)+0.2(10206.8)+0.15(10927.6)=20922.9		
		3	0.2177	7090.61+0.5(10449.9)+0.5(10206.8)=17419		
		4	0.2512	2841.58+0.2(10449.9)+0.65(10206.8)+0.15(10927.6)=13205.1		
		5	0.2858	658.41+0.15(10449.9)+0.75(10206.8)+0.1(10927.6)=10973.7		
8	228.71	1	0.1738	10206.75+0.75(10206.8)+0.15(10927.6)+0.1(10842.7)=20585.2	20613.1	2
		2	0.2053	10166.77+0.65(10206.8)+0.2(10927.6)+0.15(10842.7)=20613.1		
		3	0.2418	5848.13+0.5(10206.8)+0.5(10927.6)=16415.3		
		4	0.2804	1845.82+0.2(10206.8)+0.65(10927.6)+0.15(10842.7)=12616.5		
		5	0.3199	313.76+0.15(10206.8)+0.75(10927.6)+0.1(10842.7)=11124.8		
9	240.35	1	0.173	9206.44+0.75(10927.6)+0.25(10842.7)=20112.8	21825.5	2
		2	0.2046	10927.64+0.65(10927.6)+0.35(10842.7)=21825.5		
		3	0.2442	7327.75+0.5(10927.6)+0.5(10842.7)=18212.9		
		4	0.2869	2556.97+0.35(10927.6)+0.65(10842.7)=13429.4		
		5	0.3308	451.9+0.25(10927.6)+0.75(10842.7)=11315.8		
10	262.94	1	0.1835	9701.99+0.2(10927.6)+0.8(10842.7)=20561.6	21698	2
		2	0.2203	10842.65+0.15(10927.6)+0.85(10842.7) =21698		
		3	0.2648	6355.54+0.1(10927.6)+0.9(10842.7)=17206.7		
		4	0.3122	1810.01+0.05(10927.6)+0.95(10842.7)=12656.9		
		5	0.3608	244.43+(10842.7)=11087.1		

The calculations for 7th hole are shown in Table 4.

$$v_i(7) = \max_k\left[q_i^k + p_{i1}^k v_1(8) + p_{i2}^k v_2(8) + p_{i3}^k v_3(8) + p_{i4}^k v_4(8) + p_{i5}^k v_5(8) + p_{i6}^k v_6(8) + p_{i7}^k v_7(8) + p_{i8}^k v_8(8) + p_{i9}^k v_9(8) + p_{i10}^k v_{10}(8)\right]$$
$$= \max_k\left[q_i^k + p_{i1}^k(15377.3) + p_{i2}^k(17300) + p_{i3}^k(18968.8) + p_{i4}^k(18719.7) + p_{i5}^k(20448.5) + p_{i6}^k(20566.5) + p_{i7}^k(20922.9) + p_{i8}^k(20613.1) + p_{i9}^k v_9(21825.5) + p_{i10}^k(21698)\right]$$

At the end of hole 7, where n=7, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(7)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE IV

DATA VALUE ITERATION FOR n=7

State i		Decision k		$v_i(7)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	23186.7	23730.7	2
		2	0.0582	23730.7		
		3	0.0658	21347.9		
		4	0.0742	19222.1		
		5	0.0831	17676.6		
2	83.75	1	0.0832	25970.5	26347.1	2
		2	0.0931	26347.1		
		3	0.1053	24171.3		

		4	0.1185	21425.6		
		5	0.1324	19548		
3	112.12	1	0.1029	26911.4	28591.5	2
		2	0.1162	28591.5		
		3	0.1341	26316		
		4	0.154	22597.7		
		5	0.1749	19925.8		
4	147.36	1	0.1254	28392.2	28483.5	2
		2	0.1481	28483.5		
		3	0.1753	24315.1		
		4	0.2043	21312.9		
		5	0.2341	20343.7		
5	159.54	1	0.1316	28574.9	30741.4	2
		2	0.1513	30741.4		
		3	0.1781	28366.2		
		4	0.2077	23940.6		
		5	0.2387	21342.9		
6	187.06	1	0.1461	29534.2	30914.8	2
		2	0.1729	30914.8		
		3	0.2067	27035.2		
		4	0.2431	22676.3		
		5	0.2807	21100.8		
7	203.92	1	0.1602	30589.3	31446.2	2
		2	0.1863	31446.2		
		3	0.2177	27858.6		
		4	0.2512	23698.5		
		5	0.2858	21439.2		
8	228.71	1	0.1738	31110.2	31185.1	2
		2	0.2053	31185.1		
		3	0.2418	27067.4		
		4	0.2804	23409.7		
		5	0.3199	21944.7		
9	240.35	1	0.173	31000.1	32708.6	2
		2	0.2046	32708.6		
		3	0.2442	29089.5		
		4	0.2869	24299.6		
		5	0.3308	22181.8		
10	262.94	1	0.1835	31425.5	32559.8	2
		2	0.2203	32559.8		
		3	0.2648	28066.3		
		4	0.3122	23514.4		

		5	0.3608	21942.5		

The calculations for 6th hole are shown in Table 5.

$$v_i(6) = \max_k[q_i^k + p_{i1}^k v_1(7) + p_{i2}^k v_2(7) + p_{i3}^k v_3(7) + p_{i4}^k v_4(7) + p_{i5}^k v_5(7) + p_{i6}^k v_6(7) + p_{i7}^k v_7(7) + p_{i8}^k v_8(7) + p_{i9}^k v_9(7) + p_{i10}^k v_{10}(7)]$$

$$= \max_k[q_i^k + p_{i1}^k(23730.7) + p_{i2}^k(26347.1) + p_{i3}^k(28591.5) + p_{i4}^k(28483.5) + p_{i5}^k(30741.4) + p_{i6}^k(30914.8) + p_{i7}^k(31446.2) + p_{i8}^k(31185.1) + p_{i9}^k v_9(32708.6) + p_{i10}^k(32559.8)]$$

At the end of hole 6, where n=6, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(6)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE V

DATA VALUE ITERATION FOR N=6

State i		Decision k		$v_i(6)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	31771	32413.2	2
		2	0.0582	32413.2		
		3	0.0658	30048.1		
		4	0.0742	28216.8		
		5	0.0831	26677.2		
2	83.75	1	0.0832	35175.7	35616.9	2
		2	0.0931	35616.9		
		3	0.1053	33506.2		
		4	0.1185	30954.4		
		5	0.1324	29098.5		
3	112.12	1	0.1029	36622.3	38342.9	2
		2	0.1162	38342.9		
		3	0.1341	36009.2		
		4	0.154	32412.7		
		5	0.1749	29721.4		
4	147.36	1	0.1254	38293.8	38440.9	2
		2	0.1481	38440.9		
		3	0.1753	34343.5		
		4	0.2043	31508.3		
		5	0.2341	30562.8		
5	159.54	1	0.1316	38899.1	41080	2
		2	0.1513	41080		
		3	0.1781	38686.8		
		4	0.2077	34304		
		5	0.2387	31700.4		
6	187.06	1	0.1461	39931.1	41331.6	2

		2	0.1729	41331.6		
		3	0.2067	37471		
		4	0.2431	33171.9		
		5	0.2807	31602.8		
7	203.92	1	0.1602	41155.9	42033.3	2
		2	0.1863	42033.3		
		3	0.2177	38406.3		
		4	0.2512	34307.4		
		5	0.2858	32035		
8	228.71	1	0.1738	41757.8	41862.8	2
		2	0.2053	41862.8		
		3	0.2418	37795		
		4	0.2804	34227.4		
		5	0.3199	32778.9		
9	240.35	1	0.173	41877.8	43584.1	2
		2	0.2046	43584.1		
		3	0.2442	39961.9		
		4	0.2869	35168.8		
		5	0.3308	33048.9		
10	262.94	1	0.1835	42291.6	43424.8	2
		2	0.2203	43424.8		
		3	0.2648	38930.2		
		4	0.3122	34377.3		
		5	0.3608	32804.3		

The calculations for 5th hole are shown in Table 6.

$$v_i(5) = \max_k\left[q_i^k + p_{i1}^k v_1(6) + p_{i2}^k v_2(6) + p_{i3}^k v_3(6) + p_{i4}^k v_4(6) + p_{i5}^k v_5(6) + p_{i6}^k v_6(6) + p_{i7}^k v_7(6) + p_{i8}^k v_8(6) + p_{i9}^k v_9(6) + p_{i10}^k v_{10}(6)\right]$$

$$= \max_k\left[q_i^k + p_{i1}^k(32413.2) + p_{i2}^k(35616.9) + p_{i3}^k(38342.9) + p_{i4}^k(38440.9) + p_{i5}^k(41080) + p_{i6}^k(41331.6) + p_{i7}^k(42033.3) + p_{i8}^k(41862.8) + p_{i9}^k v_9(43584.1) + p_{i10}^k(43424.8)\right]$$

At the end of hole 5, where n=5, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(5)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE VI

DATA VALUE ITERATION FOR N=5

State i		Decision k		$v_i(6)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	40648.5	41373.5	2
		2	0.0582	41373.5		
		3	0.0658	39024.3		

		4	0.0742	37441.3		
		5	0.0831	35907.1		
2	83.75	1	0.0832	44586.4	45086.1	2
		2	0.0931	45086.1		
		3	0.1053	43016.8		
		4	0.1185	40640.3		
		5	0.1324	38798.3		
3	112.12	1	0.1029	46463.3	48223.6	2
		2	0.1162	48223.6		
		3	0.1341	45863.6		
		4	0.154	42386		
		5	0.1749	39686		
4	147.36	1	0.1254	48354.3	48543.4	2
		2	0.1481	48543.4		
		3	0.1753	44491.5		
		4	0.2043	41782.4		
		5	0.2341	40852		
5	159.54	1	0.1316	49274.3	51471.5	2
		2	0.1513	51471.5		
		3	0.1781	49064.5		
		4	0.2077	44730.8		
		5	0.2387	42122.5		
6	187.06	1	0.1461	50399.6	51821.6	2
		2	0.1729	51821.6		
		3	0.2067	47973		
		4	0.2431	43738.5		
		5	0.2807	42173.3		
7	203.92	1	0.1602	51785.4	52681.7	2
		2	0.1863	52681.7		
		3	0.2177	49038.6		
		4	0.2512	44996.7		
		5	0.2858	42718.9		
8	228.71	1	0.1738	52483.9	52608.1	2
		2	0.2053	52608.1		
		3	0.2418	48571.6		
		4	0.2804	45061.8		
		5	0.3199	43623.8		
9	240.35	1	0.173	52750.7	54456	2
		2	0.2046	54456		
		3	0.2442	50832.2		
		4	0.2869	46037.5		

		5	0.3308	43916.5		
10	262.94	1	0.1835	53158.6	54291.3	2
		2	0.2203	54291.3		
		3	0.2648	49796.3		
		4	0.3122	45242.8		
		5	0.3608	43669.2		

The calculations for 4th hole are shown in Table 7.

$$v_i(4) = \max_k \big[q_i^k + p_{i1}^k v_1(5) + p_{i2}^k v_2(5) + p_{i3}^k v_3(5) + p_{i4}^k v_4(5) + p_{i5}^k v_5(5) + p_{i6}^k v_6(5) + p_{i7}^k v_7(5)$$
$$+ p_{i8}^k v_8(5) + p_{i9}^k v_9(5) + p_{i10}^k v_{10}(5)\big]$$
$$= \max_k \big[q_i^k + p_{i1}^k (41373.5) + p_{i2}^k (45086.1) + p_{i3}^k (48223.6) + p_{i4}^k (48543.4)$$
$$+ p_{i5}^k (51471.5) + p_{i6}^k (51821.6) + p_{i7}^k (52681.7) + p_{i8}^k (52608.1) + p_{i9}^k v_9(54456)$$
$$+ p_{i10}^k (54291.3)\big]$$

At the end of hole 4, where n=4, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(4)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE VII

DATA VALUE ITERATION FOR N=4

State i		Decision k		$v_i(6)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	49777.2	50573.6	2
		2	0.0582	50573.6		
		3	0.0658	48239		
		4	0.0742	46870.5		
		5	0.0831	45341.1		
2	83.75	1	0.0832	54180.7	54732.6	2
		2	0.0931	54732.6		
		3	0.1053	52691.7		
		4	0.1185	50472		
		5	0.1324	48639.4		
3	112.12	1	0.1029	56428.4	58225.3	2
		2	0.1162	58225.3		
		3	0.1341	55855.2		
		4	0.154	52487.5		
		5	0.1749	49784.1		
4	147.36	1	0.1254	58538.9	58761.8	2
		2	0.1481	58761.8		
		3	0.1753	54738.5		
		4	0.2043	52130.8		
		5	0.2341	51210		
5	159.54	1	0.1316	59706.3	61921.2	2

		2	0.1513	61921.2		
		3	0.1781	59505.3		
		4	0.2077	55224.8		
		5	0.2387	52613.6		
6	187.06	1	0.1461	60938.9	62381.6	2
		2	0.1729	62381.6		
		3	0.2067	58542.2		
		4	0.2431	54369.8		
		5	0.2807	52807.7		
7	203.92	1	0.1602	62470.7	63383.1	2
		2	0.1863	63383.1		
		3	0.2177	59735.5		
		4	0.2512	55741.6		
		5	0.2858	53462.4		
8	228.71	1	0.1738	63260.4	63396.9	2
		2	0.2053	63396.9		
		3	0.2418	59380.2		
		4	0.2804	55907.5		
		5	0.3199	54476.1		
9	240.35	1	0.173	63621.3	65326	2
		2	0.2046	65326		
		3	0.2442	61701.4		
		4	0.2869	56905.9		
		5	0.3308	54784.4		
10	262.94	1	0.1835	64026.3	65158.7	2
		2	0.2203	65158.7		
		3	0.2648	60663.3		
		4	0.3122	56109.6		
		5	0.3608	54535.8		

The calculations for 3rd hole are shown in Table 8.

$$\begin{aligned} v_i(3) = \max_k\big[q_i^k &+ p_{i1}^k v_1(4) + p_{i2}^k v_2(4) + p_{i3}^k v_3(4) + p_{i4}^k v_4(4) + p_{i5}^k v_5(4) + p_{i6}^k v_6(4) + p_{i7}^k v_7(4) \\ &+ p_{i8}^k v_8(4) + p_{i9}^k v_9(4) + p_{i10}^k v_{10}(4)\big] \\ &= \max_k\big[q_i^k + p_{i1}^k(50573.6) + p_{i2}^k(54732.6) + p_{i3}^k(58225.3) + p_{i4}^k(58761.8) \\ &+ p_{i5}^k(61921.2) + p_{i6}^k(62381.6) + p_{i7}^k(63383.1) + p_{i8}^k(63396.9) + p_{i9}^k v_9(65326) \\ &+ p_{i10}^k(65158.7)\big] \end{aligned}$$

At the end of hole 3, where n=3, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(3)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE VIII

DATA VALUE ITERATION FOR n=3

State i		Decision k		$v_i(6)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	59124.4	59983.2	2
		2	0.0582	59983.2		
		3	0.0658	57662.3		
		4	0.0742	56481		
		5	0.0831	54956.1		
2	83.75	1	0.0832	63937.6	64535.9	2
		2	0.0931	64535.9		
		3	0.1053	62515.8		
		4	0.1185	60435.1		
		5	0.1324	58609.5		
3	112.12	1	0.1029	66507.3	68337.5	2
		2	0.1162	68337.5		
		3	0.1341	65965.2		
		4	0.154	62697.3		
		5	0.1749	59993.1		
4	147.36	1	0.1254	68826.2	69077.7	2
		2	0.1481	69077.7		
		3	0.1753	65072.5		
		4	0.2043	62550.9		
		5	0.2341	61636		
5	159.54	1	0.1316	70197.7	72430.7	2
		2	0.1513	72430.7		
		3	0.1781	70010.2		
		4	0.2077	65784		
		5	0.2387	63171.2		
6	187.06	1	0.1461	71543	73004.2	2
		2	0.1729	73004.2		
		3	0.2067	69172.9		
		4	0.2431	65056		
		5	0.2807	63496.6		
7	203.92	1	0.1602	73202	74127.2	2
		2	0.1863	74127.2		
		3	0.2177	70480.6		
		4	0.2512	66525.1		
		5	0.2858	64246.2		
8	228.71	1	0.1738	74069.2	74213.8	2
		2	0.2053	74213.8		
		3	0.2418	70209.6		
		4	0.2804	66760.9		
		5	0.3199	65333.7		

9	240.35	1	0.173	74490.6	76195.1	2
		2	0.2046	76195.1		
		3	0.2442	72570.1		
		4	0.2869	67774.2		
		5	0.3308	65652.4		
10	262.94	1	0.1835	74894.1	76026.4	2
		2	0.2203	76026.4		
		3	0.2648	71531		
		4	0.3122	66977.1		
		5	0.3608	65403.1		

The calculations for 2nd hole are shown in Table 9.

$$v_i(2) = \max_k \big[q_i^k + p_{i1}^k v_1(3) + p_{i2}^k v_2(3) + p_{i3}^k v_3(3) + p_{i4}^k v_4(3) + p_{i5}^k v_5(3) + p_{i6}^k v_6(3) + p_{i7}^k v_7(3)$$

$$+ p_{i8}^k v_8(3) + p_{i9}^k v_9(3) + p_{i10}^k v_{10}(3)\big]$$

$$= \max_k \big[q_i^k + p_{i1}^k(59983.2) + p_{i2}^k(64535.9) + p_{i3}^k(68337.5) + p_{i4}^k(69077.7)$$

$$+ p_{i5}^k(72430.7) + p_{i6}^k(73004.2) + p_{i7}^k(74127.2) + p_{i8}^k(74213.8)$$

$$+ p_{i9}^k v_9(76195.1) + p_{i10}^k(76026.4)\big]$$

At the end of hole 2, where n=2, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(2)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE IX

DATA VALUE ITERATION FOR N=2

State i		Decision k		$v_i(6)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	68663.4	69577	2
		2	0.0582	69577		
		3	0.0658	67268.8		
		4	0.0742	66251.9		
		5	0.0831	64731.3		
2	83.75	1	0.0832	73838.5	74477.9	2
		2	0.0931	74477.9		
		3	0.1053	72473.6		
		4	0.1185	70516.1		
		5	0.1324	68695.7		
3	112.12	1	0.1029	76689.9	78550.1	2
		2	0.1162	78550.1		
		3	0.1341	76179.3		
		4	0.154	73001.5		
		5	0.1749	70297.9		
4	147.36	1	0.1254	79201.8	79478.3	2
		2	0.1481	79478.3		

		3	0.1753	75485.3		
		4	0.2043	73038.6		
		5	0.2341	72127.8		
		1	0.1316	80747.7		
		2	0.1513	82998.1		
5	159.54	3	0.1781	80576.2	82998.1	2
		4	0.2077	76402.2		
		5	0.2387	73789		
		1	0.1461	82203.2		
		2	0.1729	83680.3		
6	187.06	3	0.2067	79856.2	83680.3	2
		4	0.2431	75786.8		
		5	0.2807	74229.8		
		1	0.1602	83969.6		
		2	0.1863	84904.6		
7	203.92	3	0.2177	81261.1	84904.6	2
		4	0.2512	77335.2		
		5	0.2858	75057.3		
		1	0.1738	84899		
		2	0.2053	85048.7		
8	228.71	3	0.2418	81052.6	85048.7	2
		4	0.2804	77619.3		
		5	0.3199	76194.8		
		1	0.173	85359.4		
		2	0.2046	87063.7		
9	240.35	3	0.2442	83438.5	87063.7	2
		4	0.2869	78642.4		
		5	0.3308	76520.5		
		1	0.1835	85762.2		
		2	0.2203	86894.4		
10	262.94	3	0.2648	82398.8	86894.4	2
		4	0.3122	77844.9		
		5	0.3608	76270.9		

The calculations for 1st hole are shown in Table 10.

$$v_i(1) = \max_k \big[q_i^k + p_{i1}^k v_1(2) + p_{i2}^k v_2(2) + p_{i3}^k v_3(2) + p_{i4}^k v_4(2) + p_{i5}^k v_5(2) + p_{i6}^k v_6(2) + p_{i7}^k v_7(2)$$

$$+ p_{i8}^k v_8(2) + p_{i9}^k v_9(2) + p_{i10}^k v_{10}(2)\big]$$

$$= \max_k \big[q_i^k + p_{i1}^k(69577) + p_{i2}^k(74477.9) + p_{i3}^k(78550.1) + p_{i4}^k(79478.3)$$

$$+ p_{i5}^k(82998.1) + p_{i6}^k(83680.3) + p_{i7}^k(84904.6) + p_{i8}^k(85048.7)$$

$$+ p_{i9}^k v_9(87063.7) + p_{i10}^k(86894.4)\big]$$

At the end of hole 1, where n=1, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(1)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE X

DATA VALUE ITERATION FOR n=1

State i		Decision k		$v_i(6)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	78371.3	79333.2	2
		2	0.0582	79333.2		
		3	0.0658	77036.7		
		4	0.0742	76164.8		
		5	0.0831	74648.1		
2	83.75	1	0.0832	83867	84542.8	2
		2	0.0931	84542.8		
		3	0.1053	82550.8		
		4	0.1185	80702.8		
		5	0.1324	78886.5		
3	112.12	1	0.1029	86966.1	88853.5	2
		2	0.1162	88853.5		
		3	0.1341	86485.9		
		4	0.154	83389.5		
		5	0.1749	80687		
4	147.36	1	0.1254	89655	89953.6	2
		2	0.1481	89953.6		
		3	0.1753	85969.3		
		4	0.2043	83588.9		
		5	0.2341	82681		
5	159.54	1	0.1316	91352.3	93618.7	2
		2	0.1513	93618.7		
		3	0.1781	91197.9		
		4	0.2077	87071.7		
		5	0.2387	84458.9		
6	187.06	1	0.1461	92910.4	94400.4	2
		2	0.1729	94400.4		
		3	0.2067	90583		
		4	0.2431	86552.5		
		5	0.2807	84997.7		
7	203.92	1	0.1602	94764.8	95707.2	2
		2	0.1863	95707.2		
		3	0.2177	92067.3		
		4	0.2512	88163.7		
		5	0.2858	85887		
8	228.71	1	0.1738	95742.3	95895.3	2

		2	0.2053	95895.3		
		3	0.2418	91904.3		
		4	0.2804	88481.1		
		5	0.3199	87058.3		
9	240.35	1	0.173	96227.8	97932.1	2
		2	0.2046	97932.1		
		3	0.2442	94306.8		
		4	0.2869	89510.6		
		5	0.3308	87388.6		
10	262.94	1	0.1835	96630.2	97762.4	2
		2	0.2203	97762.4		
		3	0.2648	93266.9		
		4	0.3122	88712.9		
		5	0.3608	87138.8		

Finally, the calculations for hole 0 are indicated in Table 11.

$$v_i(0) = \max_k\left[q_i^k + p_{i1}^k v_1(1) + p_{i2}^k v_2(1) + p_{i3}^k v_3(1) + p_{i4}^k v_4(1) + p_{i5}^k v_5(1) + p_{i6}^k v_6(1) + p_{i7}^k v_7(1) + p_{i8}^k v_8(1) + p_{i9}^k v_9(1) + p_{i10}^k v_{10}(1)\right]$$

$$= \max_k\left[q_i^k + p_{i1}^k(79333.2) + p_{i2}^k(84542.8) + p_{i3}^k(88853.5) + p_{i4}^k(89953.6) + p_{i5}^k(93618.7) + p_{i6}^k(94400.4) + p_{i7}^k(95707.2) + p_{i8}^k(95895.3) + p_{i9}^k v_9(97932.1) + p_{i10}^k(97762.4)\right]$$

At the end of hole 0, which is the beginning of hole 1, the optimal decision is to select the second alternative in each state. Consequently, the decision vector is computed as d(0)=[2 2 2 2 2 2 2 2 2 2]^T.

TABLE XI

DATA VALUE ITERATION FOR N=0

State i		Decision k		$v_i(6)$	Expected total reward, x10^{-2} mm	Decision
1	50.92	1	0.0522	88228.5	89233.3	2
		2	0.0582	89233.3		
		3	0.0658	86203.8		
		4	0.0742	86203.8		
		5	0.0831	84690.6		
2	83.75	1	0.0832	94008.7	94716.9	2
		2	0.0931	94716.9		
		3	0.1053	92735		
		4	0.1185	90984.3		
		5	0.1324	89171.3		
3	112.12	1	0.1029	97327	99238.8	2
		2	0.1162	99238.8		
		3	0.1341	96875.3		

		4	0.154	93852.3		
		5	0.1749	91151		
4	147.36	1	0.1254	100177	100495	2
		2	0.1481	100495		
		3	0.1753	96517.2		
		4	0.2043	94195.4		
		5	0.2341	93289.8		
5	159.54	1	0.1316	102006	104286	2
		2	0.1513	104286		
		3	0.1781	101868		
		4	0.2077	97784.3		
		5	0.2387	95172.3		
6	187.06	1	0.1461	103656	105156	2
		2	0.1729	105156		
		3	0.2067	101344		
		4	0.2431	97345.2		
		5	0.2807	95792.4		
7	203.92	1	0.1602	105581	106529	2
		2	0.1863	106529		
		3	0.2177	102892		
		4	0.2512	99004.8		
		5	0.2858	96729.2		
8	228.71	1	0.1738	106594	106750	2
		2	0.2053	106750		
		3	0.2418	102762		
		4	0.2804	99345.1		
		5	0.3199	97923.4		
9	240.35	1	0.173	107096	108800	2
		2	0.2046	108800		
		3	0.2442	105175		
		4	0.2869	100379		
		5	0.3308	98256.7		
10	262.94	1	0.1835	107498	108631	2
		2	0.2203	108631		
		3	0.2648	104135		
		4	0.3122	99580.9		
		5	0.3608	98006.9		

III. RESULTS AND DISCUSSION

The results of presented above calculations for the expected total rewards and the optimal decisions at the end of each hole of the 10-hole planning horizon are displayed in Table 12, 13. As it could be clearly seen from results that if the process starts at state 10, the expected total reward would be 108630.525 x10-2 mm, which is the highest for any other state. However, if the process starts at state 1, the expected total reward is 89233.2667 x10-2 mm, which would be the lowest among other states. The decision matrixes show slight change in states 4 and 8 at the hole number 9, but apart from that, decision 2 is showing dominance throughout the process.

TABLE XII

EXPECTED TOTAL REWARDS FOR F PLANNING HORIZON OF 10 HOLES

End of hole	Expected total reward, x10^{-2} mm										
	0	1	2	3	4	5	6	7	8	9	10
$v_1(n)$	89233.2	79333.2	69576.9	59983.2	50573.6	41373.4	32413.1	23730.6	15377.3	7430.09	0
$v_2(n)$	94716.9	84542.7	74477.8	64535.8	54732.5	45086.0	35616.8	26347.1	17300.0	8500.39	0
$v_3(n)$	99238.8	88853.4	78550.0	68337.5	58225.2	48223.6	38342.9	28591.4	18968.8	9450.55	0
$v_4(n)$	100494.	89953.6	79478.3	69077.7	58761.7	48543.3	38440.8	28483.5	18719.7	9228.49	0
$v_5(n)$	104286.	93618.6	82998.0	72430.7	61921.1	51471.4	41079.9	30741.3	20448.4	10198.1	0
$v_6(n)$	105156.	94400.4	83680.2	73004.2	62381.6	51821.6	41331.6	30914.7	20566.5	10270.0	0
$v_7(n)$	106528.	95707.2	84904.6	74127.2	63383.0	52681.7	42033.2	31446.2	20922.8	10449.9	0
$v_8(n)$	106749.	95895.3	85048.7	74213.7	63396.9	52608.1	41862.7	31185.0	20613.0	10206.7	0
$v_9(n)$	108800.	97932.0	87063.6	76195.0	65326.0	54456.0	43584.1	32708.5	21825.5	10927.6	0
$v_{10}(n)$	108630.	97762.4	86894.3	76026.4	65158.6	54291.3	43424.7	32559.8	21698.0	10842.6	0

TABLE XIII

EXPECTED OPTIMAL DECISIONS FOR F PLANNING HORIZON OF 10 HOLES

Decisions for each state	Decision at each state, n										
	0	1	2	3	4	5	6	7	8	9	10
$d_1(n)$	2	2	2	2	2	2	2	2	2	2	-
$d_2(n)$	2	2	2	2	2	2	2	2	2	2	-
$d_3(n)$	2	2	2	2	2	2	2	2	2	2	-
$d_4(n)$	2	2	2	2	2	2	2	2	2	1	-
$d_5(n)$	2	2	2	2	2	2	2	2	2	2	-
$d_6(n)$	2	2	2	2	2	2	2	2	2	2	-
$d_7(n)$	2	2	2	2	2	2	2	2	2	2	-
$d_8(n)$	2	2	2	2	2	2	2	2	2	1	-
$d_9(n)$	2	2	2	2	2	2	2	2	2	2	-
$d_{10}(n)$	2	2	2	2	2	2	2	2	2	2	-

Based on axial force value conditions the optimal decisions for feed rate could be chosen from Table 13. For each state of axial force there exists a specific optimal feed rate value, which would lead to higher reward than other feed rate values. General trend seem that the feed rate increases with the increase of axial force value. However, if all possible decisions for one state could be compared and examined, it can be seen that the optimal ones will not be the highest or the lowest values, but rather feed rate values close to the middle range. This is due to the fact that the lower feeds provides lower cutting speed, which seem to increase the total life of the tool, but appear to decrease the length of the hole being drilled for particular amount of time. In the other hand, higher feeds are likely to be more inefficient in terms of hole length because the drills operating on high speeds are subjected to excessive wear. The worn out tools could not have the same cutting speed, even when operating at higher speeds. To summarize, the optimal feed rate decisions for each axial force value during drilling deep holes are computed and presented in the given case study.

IV. CONCLUSION

Engineers working in the manufacturing area could adopt more effective ways for feed rate policies in spiral drilling. For instance, spiral drilling requires most optimal choice of the feed rate depending upon the hardness of the material and geometry of the tool. In other words, even slight deviations in the feed rate can cause deviation of the hole straightness or even breakage of the tool inside of a hole, which will lead to the wastage of the workpiece. In addition, slight deviations in the feed rate could cause the failure of the tool, which will waste the time spent for drilling. In this paper, the feed rate optimization model based on a MDP was introduced for spiral drilling process. In particular, the experimental data on drilling was implemented for 10 states of axial force parameters with 5 feed rate decisions made in each of the states, having the length of a hole being drilled as a reward. Proposed optimization model was computed using value iteration method for 10 holes planning horizon. Furthermore, the results of computations were displayed as tables for optimal decision as well as the expected total reward in each state. In conclusion, adaptive choice of the feed rates based on MDP model is claimed to improve the efficiency of the spiral drilling in terms of cost and time.

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