Decentralized and Centralized Supply Chains with Trade Credit Option

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Abstract

Decentralized and Centralized Supply Chain with Trade Credit Option

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The notion of a trade credit period is a common business practice, where a supplier allows a buyer a specified period to make a payment in full for a purchase made. The objective of this thesis is to explore the role of such a credit payment option in supply chain management. Towards this end, a two-echelon supply chain, consisting of a single supplier (e.g. manufacturer) and the cases of both a single and multiple buyers (e.g. retailers) is examined under decentralized (independent) and centralized (coordinated) decision making scenarios. The major emphasis of this research is limited to the case of a single product with price-sensitive deterministic, as well as stochastic market demand.

The conditions under which a trade credit period should be offered and its appropriate length are determined from the supplier's perspective under the decentralized case. Under the centralized decision scenario, the efficacy of a trade credit policy as a supply chain coordination mechanism is thoroughly analyzed and guidelines for pricing, production and delivery decisions are developed. The concepts developed in this study are illustrated via a number of numerical examples, in conjunction with thorough sensitivity analyses involving some selected problem parameters.

The major contribution of this thesis is that we incorporate the pricing and inventory issues in supply chains with an endogenous credit payment period. This is the first study

that examines the efficacy of trade credit option as a coordination mechanism. We propose a coordination mechanism that coordinates the supply chain, when a trade credit by itself is not sufficient to serve such a purpose, while preserving the benefits of a trade credit option. Also, this study is the first to examine the issues concerning trade credit under price sensitive stochastic demand. Another first for this work is the exploration of the implications of a trade credit policy in supply chains consisting of multiple competing retailers. The effects of the extent of competition and the market size on trade credit policy are evaluated. Our analyses lead to some important practical implications, to serve as managerial guidelines.

0.1 List of Symbols used

Deterministic Model:

Retailer:

p: Retail price charged by the buyer (\$/unit);

$D(p)$: Product's market demand rate as a function of retail price (units/year);

k, β: Constant coefficient and price elasticity in demand function;

Q: Retailer's (buyer's) order lot size (units);

A_r: Buyer's fixed ordering cost (\$/order);

h_r^c: Opportunity cost of capital for the retailer (\$/unit/year);

h_r^s: Buyer's physical inventory holding cost (\$/unit/year);

h_r: Total inventory holding cost of the buyer in \$/unit/year, where $h_r = h_r^c + h_r^s$;

Π_r: Buyer's annual gross profit (\$/year).

Suppler:

v: Unit wholesale price charged by the supplier to the buyer (\$/unit);

m: Unit manufacturing cost of the product (\$/unit);

n: Number of delivery batches per production run (batch size multiplier);

R: Supplier's production rate (units/year);

A_m: Fixed setup cost of a production batch (\$/setup);

h_m^c: Opportunity cost of capital for the supplier (\$/unit/year);

h_m^s: Supplier's physical inventory holding cost (\$/unit/year);

h_m: Total inventory holding cost for the supplier in \$/unit/year, where $h_m = h_m^c + h_m^s$;

Π_m: Supplier's annual gross profit (\$/year).

Supply chain**:**

t: Credit period length allowed for the buyer to make a purchase payment;

Π: Total gross profit for the entire supply chain ($/year);

d: Quantity discount factor as a proportion of the regular wholesale price.

Price sensitive Stochastic Model:

Retailer:

p: Retail price charged by the buyer ($/unit);

$D(p)$: Market demand rate as a function of retail price (units/year);

k, β: Constant coefficient and price elasticity in the demand function;

r: Retailer's reorder point in (r, Q) policy;

Q: Retailer's order lot size in (r, Q) policy;

S: Retailer's order up to level in (S, T) policy;

T: Retailer's replenishment order cycle time in (S, T) policy;

A_r: Retailer's fixed ordering cost ($/order);

h_r^c: Opportunity cost of capital for the retailer ($/unit/year);

h_r^s: Retailer's physical inventory holding cost ($/unit/year);

h_r: Total holding cost of the buyer in $/unit/year, where $h_r = h_r^c + h_r^s$;

Π_r: Buyer's annual gross profit ($/year).

Supplier:

v: Unit wholesale price charged by the manufacturer to the retailer ($/unit);

m: Unit manufacturing cost of the product ($/unit);

n: Number of delivery batches per production run (batch size multiplier) or the lot size multiplier for the manufacturer;

R: Supplier's production rate (units/year);

A_m: Fixed setup cost of a production batch ($/setup);

h_m^c: Opportunity cost of capital for the supplier ($/unit/year);

h_m^s: Supplier's physical inventory holding cost ($/unit/year);

h_m: Total inventory holding cost for the supplier in $/unit/year, where $h_1 = h_1^c + h_1^s$;

Π_m: Supplier's annual gross profit ($/year).

Supply chain**:**

t: Credit period length allowed for the buyer to make a purchase payment;

Π: Total gross profit for the entire supply chain ($/year);

Multiple Retailers Model:

Retailers:

p_i: Retail price charged by the buyer *i* ($/unit);

D_i: Demand for retailer *i* as a function of its retail price (units/year);

a_i, b_i, c_i: Coefficients in the demand function for retailer *i*;

q_i: Retailer *i*'s order lot size (units);

A_i: Retailer *i*'s fixed ordering cost ($/order);

h_i^c: Opportunity cost of capital for retailer *i* ($/unit/year);

h_i^s: Retailer *i*'s physical inventory holding cost ($/unit/year);

h_i: Total inventory holding cost of retailer *i* in $/unit/year, where $h_i = h_i^c + h_i^s$;

Π_i: Retailer *i*'s annual gross profit ($/year).

Suppler:

v: Unit wholesale price charged by the supplier to the buyers ($/unit);

m: Unit manufacturing cost of the product ($/unit);

θ: Production cycle length;

R: Supplier's production rate (units/year);

A_m: Fixed setup cost of a production batch ($/setup);

h_m^c: Opportunity cost of capital for the supplier ($/unit/year);

h_m^s: Supplier's physical inventory holding cost ($/unit/year);

h_m: Total inventory holding cost for the supplier ($/unit/year), where $h_m = h_m^c + h_m^s$;

Π_m: Supplier's annual gross profit ($/year).

<u>Supply chain</u>**:**

t: Credit period length allowed for the buyers to make a purchase payment;

Π: Total gross profit for the entire supply chain ($/year);

Introduction

During the past two decades, supply chain coordination has received a great deal of research attention, focusing on globally optimal supply chain decisions that can benefit all the parties (i.e. the members of a supply chain) involved, as opposed to each party making its own decisions individually. A number of mechanisms, such as price discounts, credit payment options and buy-back contracts, etc., have been used for coordination purposes.

One important means for achieving supply chain coordination is the payment credit period option, where the supplier specifies to the buyer a finite time interval (credit period) within which the payment for a purchase is to be made, in lieu of immediate payment. Trade credit option has become a common practice in business. All the members of a supply chain can benefit from a well designed and implemented trade credit policy. Furthermore, the total supply chain can also benefit from the savings of not having to borrow capital from banks or other financial institutions (which usually charge interest or a share of the equity or profit of the borrower). A trade credit option also allows the downstream supply chain entities the flexibility of increasing inventory levels, thus, increasing product availability and attracting more customers. The supplier's incentive for offering such an option is to stimulate demand by allowing credit to a downstream buyer which may not have sufficient capital to make a sizeable immediate payment, but may be induced to buy larger quantities, if some flexibility in terms of delaying purchase payments is available. Thus, from the supplier's perspective, the resulting potential increase in sales may compensate for the loss from issuing credit

(Mehta (1968)). The objective of this thesis is to explore the role of such a credit payment option in supply chain management.

Previous research has shown that, under a trade credit regime, the total supply chain profit can be improved if the cost of capital for the buyer is greater than that for the supplier (Sarmah et al. (2007)). Other benefits of the credit option are mentioned in Shinn and Hwang (2003) and Sarmah et al. (2007). For instance, a credit policy can serve as a useful tool for enhancing the supplier's competitive position and can facilitate the development of a stable, long term buyer-supplier relationship, which can yield benefits for both the parties.

Most of the related models in the existing literature assume deterministic operating conditions and explore the relevant policy implications either within the framework of a decentralized game, or assume a predetermined fixed term credit contract in a centralized game environment. Thus, several interesting questions arise. First, whether it is always beneficial for the supplier to offer a credit option, and, secondly, under what conditions should such an option be offered and the appropriate length of the credit period? Another pertinent issue is whether trade credit can be used to coordinate the supply chain and what is the range of the credit payment period, so that both parties are willing to coordinate their decisions? Consequently, we wish to analyze the effectiveness of using trade credit as a coordination mechanism and determine how the relevant parameters affect the efficacy of such a policy. Guidelines for pricing, production and delivery decisions are developed. Additionally, if market demand is stochastic, how will it affect the decision model and the trade credit policy? Finally, what is the role of market competition in a trade credit policy under a multiple retailers scenario? These issues have not received adequate attention in the existing body of work pertaining to this area of research. In summary, a significant motivating factor for undertaking this study is that, apart from addressing some important issues concerning a common business practice, it represents an important attempt to bridge some of the gaps in the existing literature, mentioned above,

In order to address these questions in a systematic manner, this study consists of three distinct parts. The first part assumes price sensitive market demand for the product in question under a deterministic environment. The analysis contained in the second part relaxes the deterministic assumptions and derives appropriate supply chain policies under stochastic conditions. Finally, the third part of this thesis extends the findings of the deterministic case to a supply chain involving multiple competing buyers with a differentiated product. In this context, the effects of the extent of competition and market size on trade credit policy are examined.

We first consider a deterministic model using the two coordination mechanisms, i.e. a credit payment option and a wholesale price discount, by analyzing five situations (with or without coordination and with or without the credit option and/or a discount) in Section 3 of this thesis. Our problem scenario, involving a single product manufactured by a single supplier for a single buyer, incorporates price-sensitive demand and the notion of the production batch size being an integer multiple of the order (or delivery) quantity, with a production rate that exceeds the retail demand rate. We determine the retail price, the buyer’s order quantity; the supplier’s manufacturing batch size and the credit period allowance, when each party attempts to derive its own individual optimal policy. Then we develop the corresponding system optimal decisions from the perspective of the entire integrated supply chain. Furthermore, we explain where the additional profit originates from, and allude to the managerial implications for utilizing a wholesale price discount and/or a credit payment option as coordination mechanisms. We also derive an appropriate policy to fully achieve supply chain coordination, and suggest ways for the two parties to equitably share the additional profit, resulting from the deployment of a price discount and/or a credit payment period.

Our analysis indicates that there are some disadvantages of using the credit option alone. First, we determine the maximum allowable credit time by setting the profit of the supplier with the credit option equal that without such an option. In this instance, there is no additional profit on the part of the supplier to be shared with the buyer. Secondly, the suitable credit time to share the excess profit can be relatively long, making it impractical under real world conditions. Also, in practice, it is not always easy to strictly enforce and follow the credit terms in buyer-supplier contracts. Finally, as our analysis indicates, more profit can be generated for the supply chain if we increase the credit time (even beyond the maximum allowable credit period), as long as the cost of capital for the supplier is lower than that for the buyer. Consequently, this work focuses on the supplier's price discount offer in conjunction with a credit payment option, which makes equitable profit sharing a tractable proposition. Generally speaking, the credit period is often negotiable and flexible, rendering the enhancement of a coordinated supply chain's profit possible.

Previous research involving the integration and coordination of supply chains has largely used either the delayed credit payment option, or a wholesale price discount, as separate mechanisms for achieving coordination. Our work differs from such endeavors in that we consider the possibility of the supplier offering to the buyer a delayed payment option, as well as a discount in the wholesale price of the product, simultaneously, in order to coordinate the supply chain and enhance its gross profit. Furthermore, in our models, we treat the credit period length as a decision variable, which allows the supply chain some flexibility, while enabling it to divide the surplus, resulting from coordination, in a fair and equitable manner, between the buyer and the vendor.

We then study the supply chain under price sensitive stochastic demand in section 4. Both continuous and periodic review policies for inventory control are examined. In the decentralized scenario, the retailer makes the replenishment quantity and frequency decisions along with the pricing decision, while the manufacturer decides the length of the credit period and its production-distribution policy. In the centralized scenario, the replenishment policies, retail prices and credit period length are determined jointly to optimize the supply chain's profit. In the independent optimization (i.e. decentralized) case, the supply chain's problem is framed as a Stackelberg game and an equilibrium solution is derived. The advantages of issuing a trade credit period are examined from the manufacturer's perspective. We focus on the major factors that determine the characteristics of the manufacturer's trade credit policy and a detailed sensitivity analysis outlines the effect of changes in some selected model parameters. In the centralized case, the optimal supply chain decisions and the effectiveness of using credit as a coordination mechanism are addressed. Some interesting managerial insights are developed via sensitivity analysis involving a set of numerical examples. In the scenario where the parameters of the decentralized supply chain results in a preferred credit option offered by the manufacturer, we find that the credit option is usually not an effective mechanism in practice to coordinate the supply chain. In contrast, under scenarios where the trade credit option can serve well as a coordination mechanism, the manufacturer would be usually unwilling to offer trade credit, without the buyer committing to an agreed upon ordering and pricing contract (for the independent optimization case).

The analysis continues with an extension of our determinist models to the case of a supply chain with multiple competing retailers in section 5. We examine correlated retailers using a Bertrand competition for a differentiated product. We determine the pricing and order quantity decisions for the retailers and the manufacturer's credit period offer applicable to all retailers. We are interested in how the extent of the competition and the changes of the whole market would affect the supply chain decisions. We develop two models, one with a fixed market and the other with a variable market (e.g. geographically dispersed retailers). How the retailers will react to the credit term offered, and the effects of the number of retailers and other relevant factors on the operational and pricing decisions of the supply chain members are explored. We provide some numerical examples, in order to demonstrate the effects of the nature of competition on the decentralized supply chain decisions and the coordination mechanisms under consideration. Finally, we provide some examples to further evaluate the effectiveness of using trade credit to coordinate the decisions of all the members of the supply chain.

2. Literature Survey

2.1 Supply Chain Coordination

The members of a supply chain typically consist of numerous parties, often separate organizational entities, e.g. suppliers, manufacturers, transporters, retailers, etc., united in the common goal of fulfilling eventual customer demand in the marketplace. Consequently, the imperative of effective management of a supply chain should be to improve and maximize the profitability of the entire supply chain, rather than focusing on the performance of any single constituent member of the chain. In order to achieve this, it is necessary to align the goals of the various disparate parties with those of the overall supply chain, through the coordination of the policies and decisions of these entities.

When a supply chain lacks coordination, many problems may arise that are likely to be detrimental from the standpoint of overall supply chain performance. One such problem is the phenomenon of double marginalization, which stems from each party in a buyer-supplier relationship attempting to maintain its profit margin towards maximizing its own profitability, at the expense of the larger chain. A market price set autonomously by a retailer, without considering the marginal costs, inventory policies and the goals of all of the other involved parties, is not likely to be the optimal decision for the supply chain as a whole, since its goals may not be aligned with those of the overall system. In order to remedy the ill effects resulting from double marginalization, coordination of the decisions made by the individual members of the supply chain is of vital importance. The surplus, or additional supply chain profit, as a consequence of reducing the double marginalization effect, can then be shared appropriately in an equitable manner to benefit all the members of the supply chain.

Another undesirable phenomenon observed in real world supply chains is the well-known “bullwhip effect”. Under market demand uncertainty, if the retail demand information is not effectively shared amongst the supply chain members, the perception of such information tends to get more and more distorted as it moves upstream along the various echelons of the chain. In effect, the perception of variability of demand increases dramatically in a cascading manner, as information moves upstream. Such increasing variability in perceived demand, due to a lack of coordination and adequate information sharing, results in non-smooth production and shipping plans, increasing related costs, such as production, set up, and inventory holding costs. Thus, in mitigating the disruptive effects of the bullwhip effect, inter-party coordination plays an important role towards improving the performance of the entire supply chain. In short, coordination can play a critical and useful role in fostering smooth physical, information and cash flows throughout a supply chain, facilitating the derivation and implementation of globally optimal operational solutions.

Thus, supply chain coordination is desirable for improving the entire chain’s efficiency and profitability. The extant literature suggests two approaches for achieving supply chain coordination. In the first of these, the optimal system policies (variables) are determined and the resulting total profit is shared via a particular contract structure (e.g. price discount, buy back, etc.). The impact of the problem parameters on the supply chain can be examined. In the second type of coordination, the parameters of a contract structure are determined, so that system optimality can be achieved in the decentralized case, where each party in the supply chain optimizes independently. In our research, we focus on the first type of contract, since system optimality is difficult to attain in a decentralized game under this scenario. In summary, this study attempts to bridge the gap in the existing literature, as mentioned earlier.

The notion of joint operation and coordination was first developed by Goyal (1976). Subsequently, Banerjee (1986) extends this work by suggesting a joint economic lot size (JELS) model for the system, while compensating the buyer through a quantity discount offer. Lee and Rosenblatt (1986) study the lot sizing issue and a quantity discount policy for increasing the supplier's profit. Chen and Chen (2005) later consider a situation where a manufacturer produces several products within the same facility and propose a joint replenishment policy using Pareto improvements, such that no party is worse off, while increasing the total supply chain profit. Along similar lines, Munson and Rosenblatt (2001) examine supply chain coordination via the quantity discount approach in three-echelon systems.

2.1.1 Supply chain coordination mechanisms

This section reviews different types of coordination mechanisms suggested in the literature. The coordination techniques in supply chain management largely focus on improving the supply chain's performance from an operational perspective, such as more accurate prediction of demand, smooth operations, shared risk, higher service level, economies of order size and inventory levels, etc., in order to enhance total supply chain profitability. Some of the most widely studied coordination procedures are outlined below.

Wholesale Quantity Discount. A wholesale quantity discount offer is perhaps the most commonly used coordination mechanism. It is easy to implement and allows the parties involved to share the resultant surplus profit effectively and fairly with considerable flexibility. There are two basic types of quantity discount offers, viz. the all units price discount (where a price reduction applies to all of the units purchased, if the purchase order quantity exceeds a specified threshold) and the incremental price discount (where a reduced price is charged for only the amount above a threshold). Other forms of quantity discount policies also exist, such as cumulative discounts, where a price discount is conditional on the cumulative demand for a product over time.

Quantity Flexibility. Tsay, Nahmias and Agrawal (1999) defines quantity flexibility as a contractual clause under which the quantity a buyer ultimately orders may deviate from a previously planned estimate. The conditions under a quantity flexibility buyer-supplier contract can include an allowable range on the order quantity, pricing rules, or both. This type of a contract tends to benefit the retailer, while shifting the bulk of the risks of overstocking and understocking to the upstream supplier. This mechanism is often used for products with seasonal demand (usually involving greater uncertainty) and in those cases where demand forecasts can be updated and improved over time.

Buyback Contracts. Under the terms of a buyback contract (applicable largely for seasonal or style goods), the supplier agrees to repurchase, albeit at a reduced price, any unsold quantity, at the end of the selling period from the retailer. Thus, both parties share the overall risk of overstocking . The proportion of the total risk distributed to each party depends on the structural details of such a contract. The buyback price is usually determined via a negotiation process and, ultimately, depends upon the relative bargaining powers of the parties concerned. Interestingly, in an earlier study Pasternack (1985) finds that the channel coordinating prices are independent of the product's market demand distribution (also see Tsay, Nahmias and Agrawal (1999)).

Trade Credit or Delayed Payment Option. The notion of a trade credit option, offered to a buyer by a supplier, allows the former to pay the latter for purchased goods or services within some specified period, instead of immediate payment. Clearly, such a contractual feature benefits the buyer in terms a more favorable cash flow position and savings in its cost of capital. Also, the possibility of delayed payment serves as an incentive for the buyer to purchase larger quantities, such that, if the product's market demand is price-sensitive, the retail can be lowered for stimulating consumer demand. In other words, the supply chain as a whole can derive significant benefits from a trade credit policy. In contrast, the supplier is burdened with the cost of capital tied up at the buyer's end and may face an unfavorable cash flow position. Nevertheless, under certain conditions, significant advantages can also accrue to it as a result of increased consumer demand and may outweigh the above mentioned drawback. Thus, it is important for the supplier to design a trade credit offer judiciously such that its overall profitability, as a consequence of implementing this type of a coordination mechanism, is enhanced. More details of trade credit policy implications are discussed in Section 2.2.

Other supply chain coordination mechanisms, such as warranty contracts, product return policies, transportation related discounts, revenue sharing contracts, etc. have also been suggested in the current literature.

2.1.2 Supply chains with stochastic Demand

Supply chains typically operate under stochastic conditions, which are characterized by uncertainty in demand, supply and a variety of other factors that have an impact on their performance. The challenges in coping with such uncertainties, particularly in the context of today's global economy, become even more important, in view of the intensely competitive nature of most business environments. In addition to stochastic demand, there are often substantial uncertainties associated with delivery lead times.

Decisions concerning inventory replenishment, production scheduling, transportation planning, etc. are closely inter-related, where safety stocks play a critical and useful role towards mitigating the unpredictable effects of environmental uncertainties and allowing decision makers some flexibility in formulating operational plans for complex supply chains. It is widely known that while safety stocks contribute towards attaining higher customer service levels, via enhanced product availability, such stocks also increase the average inventory levels throughout the supply chain and result in higher inventory carrying costs. In order to determine the level of safety stocks that is appropriate for a specific set of operating conditions, the design of an inventory replenishment policy is of utmost importance.

There are several possible types of inventory replenish policies that have been examined extensively in the literature and have found widespread implementation in practice. For a single stock location dealing with a continuously stocked inventory item with independent market demand, two major classes of replenishment policies have received much of the research attention to date. These are briefly outlined below.

Continuous review policy:

(r, Q) policy: This policy is also called a reorder point policy. The inventory position of an item is reviewed and monitored continuously. In the normal course of depletion through demand or usage, as soon as the inventory position decreases to r units, a new replenishment order of a predetermined quantity Q is placed immediately to elevate the inventory position, such that it is always above r. This replenish policy is usually used when continuous monitoring of inventory is relatively inexpensive and when a fixed replenishment quantity is desired due to reasons such as packing, shipping methods (full truck load), etc.

(r, nQ) policy: This policy is similar to the (r, Q) policy. The only difference is that in each order, an integer multiple of a base quantity Q can be placed. This policy provides more flexibility compared to the (r, Q) policy and is more suitable for scenarios with considerably high levels of uncertainty, such as non-stationary demand.

(s, S) policy: Under this replenishment policy, the inventory position is also monitored on a continuous basis. When the inventory position drops below s units, a new order of quantity *S*-*s* is placed, in order to bring the inventory position back up to *S* units. This is modification of the (*r*, *Q*) policy for handling non-unit demand or usage transactions. Note that the (*r*, *Q*) and the (*s*, *S*) replenishment policies are equivalent under unit demand transactions, where *Q* is equivalent to *S*-*s*.

Periodic review policy:

(*S*, *T*) policy: Under this replenishment policy, the inventory position is reviewed periodically (instead of continuously) at fixed intervals of T time units. There are a number of different possible policies under the periodic review category. One such policy dictates that, regardless of the inventory position, a new replenishment order is always placed at the time of each review to elevate the inventory position to *S* units. Another policy, known as the (*s*, *S*, *T*) policy, specifies that a new order is placed only when the inventory position is below *s* when a review occurs, such that the replenishment order brings back the inventory position up to the target level *S*.

One approach in determining the appropriate parameters of as chosen inventory replenishment policy utilizes the notion of customer service level. Axsäter (2000), outlines three types of service level as follows:

S1 = probability of no stockout per order cycle,

S2 = “fill rate” – fraction of demand that can be satisfied immediately and routinely from available stock,

S3 = “ready rate” – fraction of time with positive stock availability.

S1 represents the probability of satisfying demand over an order cycle without experiencing a stockout. S2 and S3, on the other hand, focus on the probability of routine demand fulfillment from the customer's perspective. It is to be noted that S2 and S3 are equivalent under continuous demand.

It is important to maintain a certain level of service, however measured, in order to meet customer expectations and reduce the costs of insufficient inventory, i.e. shortage. Nevertheless, a higher service level requires higher safety stocks, i.e. higher inventory holding costs. Thus, a balance between the costs of carrying safety stocks and the benefits of providing customer service needs to be attained for the formulation of a sound replenishment policy.

Supply chain models under stochastic demand have been extensively studied in the existing literature. Nagarajan and Rajagopalan (2008) provide a model to improve supply chain performance using holding cost subsidies for both continuous review and periodic review policies. Since it is difficult to derive closed-form solutions for all the decision variables under a periodic review policy, Eynan and Kropp (2007) adopt a Taylor's series expansion approach to approximate the cost function, resulting in an EOQ like simple solution. Cachon (1999) propose a model for a single supplier and multiple retailers supply chain using scheduled ordering policies, while leaving the ordering sequence stochastic. Viswanathan (1997), on the other hand, suggest a heuristic policy for the periodic review system with multiple items.

Maddah, Jaber and Abbound (2004) outline a procedure for the determination of stock replenishment policies, assuming a given credit period, as well as a fixed replenishment period. Robb and Silver (2006) compare four heuristics for the buyer's problem assuming gamma distributed demand. More recently, Gupta and Wang (2009) consider a discrete time model for a retailer, using a Markov decision process approach, to make decisions under both the total lost sales and full backordering scenarios. They claim that the structure of the optimal policy is not affected by the credit term. Tsao (2010) examine the notion of promotional effort on the part of the retailer for the case of multiple items. Chaharsooghi and Heydari (2010) propose a coordination model to jointly determine the retailer's reorder point and order quantity using an (r, Q) replenishment policy and sharing the additional profit via credit option offer made to the buyer. Finally, Lee and Rhee (2011) examine the newsvendor model for perishable goods with trade credit and other financing options.

2.1.3 Supply chains with price Sensitive Stochastic Demand

Price dependent stochastic demand models are mostly to be found in the literature that deal with the issues of buyback and customer rebate. Chen and Bell (2011) develop a model for price dependent stochastic demand with different buyback prices for customer returns and unsold items. Lau, Lau and Wang (2007) study the properties and related schemes for a newsvendor product supply chain. Yao, Leung and Lai (2008a) numerically examine the impact and characteristics of a return policy under price dependent stochastic demand. Arcelus, Kumar and Srinivasan (2006) study the pricing and rebate policies in a supply chain with asymmetric information and outline the conditions under which the retailer can benefit from sharing market demand information with the supplier. Zhou (2007) incorporates a price dependent stochastic demand to compare the efficiencies of four different price discount policies. Ray, Song and Verma (2010) compare two different backordering cases –time independent backordering and time dependent backordering, and find out that the former case results in longer review periods but lower retail prices. Ray, Li and Song (2005) analyze the characteristics of a supply chain with tailored decision making and show how the decisions are affected by the prevailing management paradigm.

2.1.4 Supply Chains with Multiple Retailers

Feng and Viswanathan (2007) examine a multiple buyer supply chain model using common replenishment epochs for coordinating supply chain inventories. They find that the benefits of coordination through a common replenishment period decreases as the coefficient of variation of market demand distribution increases and that such coordination may not always be desirable under very high levels of demand uncertainty. Cachon and Fisher (2000) explore the advantages of shared information in a two-echelon supply chain with a periodic review policy under stationary stochastic demand. Subsequently, Cachon (2001) formulate another model with inventory competition and propose a supply chain contract that derives the optimal policy as a Nash equilibrium. It is shown that, the smoothing of the flow of goods tends to work better than expediting information flow. Berling and Marklund (2006) introduce a new coordination mechanism with induced near optimal backorder cost and analyze its practical implications. Hsieh, Liu and Wang (2010) suggest a coordination model with price sensitive demand and short-term discounting. They point out that the distributer's profit will increase as price elasticity increases and decreases as the number of retailers increase. Similarly, Boyaci and Gallego (2002) examine the pricing and inventory polices for a multiple retailers supply chain assuming geographically dispersed retailers with uncorrelated demand.

Cases of multiple retailer models with price competition can also be found in the literature. Bernstein and Federgruen (2003) analyze a distribution system both under price competition and quantity competition. They are the first to outline the specific conditions under which a Nash equilibrium solution exists. Bernstein and Federgruen (2005) also study decentralized supply chains under demand uncertainty, and consequently suggest different types of coordination mechanisms that can enhance total supply chain profitability (Bernstein and Federgruen (2007)). Guan and Zhao (2010) consider a model with Poisson demand under an (r, Q) policy with a finite horizon. Yao, Leung and Lai (2008b) determine the structure of a manufacturer's revenue sharing contract under a Stackelberg game with two competing retailers and examine the policy implications of different competitive factors.

2.2 Trade Credit Option

The notion of a trade credit period is a widely used business practice, where a supplier of materials allows a specified period for a buyer to make a payment in full for a quantity of delivered material. This business practice is getting more and more popular. Kokemuller and Media (n.d.) lists some advantages and disadvantages of trade credit:

More Sales: From the perspective of the creditor, or supplier, trade credit should induce more sales over time by allowing customers to make purchases without immediate cash. This flexibility in purchasing methods also encourages customers to make larger purchases when prices are right than they might if they had to pay cash upfront. Along with higher sales volume, trade credit often produces interest fees and late payment fees for creditors, which increases revenue.

No Cash: From the resellers perspective, the ability to buy on credit makes it possible to buy needed inventory even when cash balances are low. Having cash to pay off long-term debt and other more urgent and immediate expenses is critical. The ability to delay cash requirements for supplies and inventory helps preserve cash for these purposes. Buyers may want to ramp up the volume of purchases at a time when demand is higher, and a trade account makes it more feasible to do so.

Immediate Replenishment: Just as consumers rely on credit cards when immediate needs come up and cash is not available, businesses have needs that include inventory and supply replenishment. If a manufacturing has a rush order of a large volume of products and low cash on hand, it needs a trade account to purchase raw materials for use in production. In essence, the trade account helps prevent delays in business activity and work performance.

Bad Debt: The potential risk to the supplier when offering trade credit is bad debt. If buyers do not pay off their debt, and in a timely manner, it has negative cash effects on the supplier. Companies eventually have to write off unpaid accounts as bad debt, which lowers their profits. Accounts that remain unpaid for a long period of time still have negative effects, though. This means the supplier has to wait to collect cash which it needs to pay its own bills.

High Costs: If buyers are not careful in the way they use trade credit, they can end up paying much higher costs for inventory. Many companies offer a 2-percent discount if you pay within 10 days, but payments received after 30 days usually include late-payment fees and interest that begins accruing. In its overview of trade credit, "Entrepreneur" notes that purchases on account can cost between 12 to 24 percent extra in interest fees if the business does not pay within the typical 30-day net payment term.

Goyal (1985) first introduced this mechanism in an EOQ model with a determined credit time from the standpoint of the buyer; a concept that has been extended and improved by Chung (1998). Kim et al. (1995) develop a model for determining the optimal credit period length from the perspective of the supplier. Also, Khouja and Mehrez (1994) compare policies with and without the credit time linked to the order quantity and show that suitable policies can lead to substantially different buyer order quantities.

Abad and Jaggi (2003) first consider the problem of delay in payments under non-cooperative and cooperative relationships with price sensitive demand. They develop their analysis by utilizing the concepts of both a Pareto efficiency solution, as well as a Nash bargaining cooperation game. Later, Jaber and Osman (2006) develop an integrated model, considering opportunity gain and loss that have a compounding rate of return. Subsequently, Yang and Wee (2006) extend their work for deteriorating items with finite replenishment rates.

More recently, Ouyang, Ho and Su (2008) have proposed an inventory model, where both trade credit terms and freight rate are determined by the order quantity. Chang et al. (2009) present a similar model where a trade credit is offered with a threshold time, without considering freight costs. Both of these papers specify fixed credit periods and consider only the coordinated situation without comparison with the un-coordinated scenario, leaving no room for profit sharing. Therefore, the profit for each party, which is determined by the relevant model parameters, is fixed. In contrast, Chen and Kang (2007) consider both the scenarios, i.e. with and without coordination, with a fixed threshold credit time, under the assumption of deterministic demand. Sarmah et al. (2007) also explore both situations and suggest a procedure that divides the supply chain surplus equitably, after both parties achieve their own profit targets. Finally, Sheen and Tsao (2007) develop a model with price sensitive demand and quantity discounts for freight cost with a fixed cost for quantities within a specified range. Their model assumes that the opportunity costs of capital are the same for both parties in determining the order quantity, price and credit time allowance to achieve maximum channel profit. This leads to the specification of the credit period range that should be offered.

Another related study considers a multiple retailers model with a credit payment option (Sarmah, Acharya and Goyal, 2008). This paper develops a procedure for determining the length of the credit period to be offered in order to induce the uncorrelated retailers to coordinate their decisions, under the assumption of a deterministic operating environment.

3. Deterministic Model

3.1 Model Preliminaries

Assumptions:

1. The operating environment is deterministic.
2. The supply chain structure considered in this study involves a single supplier (manufacturer) and a single buyer dealing with a single product.
3. For a coordinated supply chain, both parties share complete information and strictly follow the terms of the purchase/delivery contract.
4. The supplier's production rate is greater than the buyer's market demand rate
5. Shortages are not allowed.
6. The item's unit physical inventory holding costs per year are the same for both parties.
7. The item's demand is price sensitive; i.e. the demand, *D*, as a function of the unit price, p, is given by: $D(\quad) = kp^{-\beta}$ where β is the demand elasticity coefficient ($\beta > 1$) and k is a constant parameter.
8. The manufacturer's production batch size (nQ) is an integer multiple of the buyer's order quantity (Q), where *n* is a positive integer.
9. The product's regular (undiscounted) wholesale price is greater than its production cost and is exogenously determined, based on current industry practice.

3.2 Independent optimization without credit option

In this section, we consider the situation without coordination, where neither a credit option nor a wholesale discount is offered. Initially, the buyer maximizes its gross profit by determining simultaneously its order size, Q, and the retail price, p. Subsequently, based on the buyer's ordering policy, the vendor determines its manufacturing batch size via the batch size multiplier, n, in order to maximizes its own individual gross profit.

3.2.1 Buyer's problem

For a pair of (p, Q) values, the buyer's annual gross profit function consists of the sales revenue, purchasing cost, ordering cost and inventory holding cost (consisting of the physical holding cost and the cost of capital). Since there is no coordination in this scenario, total inventory carrying cost per year is computed on the basis of the average inventory level,$Q/2$, and D/Q is the number of orders per year. Thus, the annual gross profit can be expressed as

$$\Pi_r(p,Q) = (p - v)D(p) - \frac{D(p)}{Q}A_r - \frac{Q}{2}h_r. \tag{3.1}$$

The optimal price as a function of the order quantity, i.e. $p^*(Q)$, can be easily obtained by equating the first derivative of (3.1) with respect to$p$ to 0 at $p=p^*(Q)$

as follows:

$$\frac{\partial r}{\partial p} = (1-\beta)D(p) + \beta k p^{-(\beta+1)}\left(v + \frac{A_r}{Q}\right) = 0,$$

$$\text{or } p^*(Q) = \frac{\beta}{\beta-1}\left(v + \frac{A_r}{Q}\right). \tag{3.2}$$

Furthermore, the following inequality must hold for satisfying the second order optimality condition, i.e. $p^*(Q)$ in (3.2) maximizes the buyer's gross profit expressed by (3.1):

$$p < \frac{\beta+1}{\beta-1}\left(v + \frac{A_r}{Q}\right).$$

Substituting (3.2) into (3.1), the buyer's gross profit can now be expressed as a function of its lot size alone, i.e.

$$\Pi_r(Q) = \frac{k}{\beta}\left[\frac{\beta}{\beta-1}\left(v + \frac{A_r}{Q}\right)\right]^{1-\beta} - \frac{Q}{2}h_r\,. \tag{3.1′}$$

We know that $\Pi_2(Q)$ is a convex-concave function in$Q$ (see Kim et al. (1995), Abad and Jaggi (2003) and Sheen and Tsao (2007)), where it is convex over negative values of $\Pi_2(Q)$ and concave over positive values of $\Pi_2(Q)$. Restricting the buyer's gross profit to only positive values, we consider the concave part of the profit function (3.1'). Once again the first order optimality condition leads to the following expression for the buyer's optimal order lot size:

$$Q^* = \sqrt{\frac{2kA_r}{h_r}\left[\frac{\beta}{\beta-1}\left(v+\frac{A_r}{Q^*}\right)\right]^{-\beta}}\ . \tag{3.3}$$

Also, the following inequality ensures the second order optimality condition:

$$Q > \frac{A_r(\beta-2)}{2v}\,.$$

Note that in expression (3) above, $Q*$ appears on both sides of the equation. In this form, a direct solution for the optimal lot size is cumbersome to obtain. Nevertheless, the value of $Q*$ can be arrived at either via any commonly available equation solving software, or by an iterative procedure, initializing with $Q^* = A_r(\beta-2)/v,$ from the second order optimality condition above.

3.2.2 Supplier's problem

Given the buyer's lot size, the vendor's task here is to compute the number of purchase (delivery) lots comprising a single production batch, i.e. its batch size multiplier, n, in order to maximize its own gross profit. The supplier's gross profit function, shown below, consists of its wholesale revenue, production cost, ordering cost and inventory holding cost, including the physical holding cost and the cost of capital. The average inventory now is given by $\frac{Q}{2}[(n-1)(1-\rho)+\rho]$,where $\rho = D/R$ (see, for example, Pan and Yang (2002), Chang et al. (2006) and Chen and Kang (2007)). Hence, the vendor's gross profit function is

$$\Pi_m(n) = (v - m)D - \frac{DA_m}{Qn} - \frac{Q}{2}\left[(n-1) - (n-2)\frac{D}{R}\right]h_m. \tag{3.4}$$

We can easily show that the supplier's gross profit function above is strictly concave in n. Therefore, by equating the first derivative of (4), with respect to n, to zero at n=$n*$, we obtain the following expression for the optimal batch size multiplier, $n*$, which maximizesΠ_1:

$$n^* = \sqrt{\frac{2DA_m}{Q^2 h_m (1 - D/R)}}\,. \tag{3.5}$$

By definition, the lot size multiplier is restricted to be an integer. It is easy to check that profit function (3.4) is strictly concave in n. Thus, if the $n*$ value yielded by (3.5) is non-integer, we check the surrounding integers$\lfloor n^* \rfloor$and$\lceil n^* \rceil$ and, accordingly, set $n*$ to one of these integer values that maximizes the supplier's profit function (3.4).

3.3 Independent optimization with credit option

Under this scenario, with a permissible credit payment period of t, the buyer maximizes its own gross profit individually by determining its order size,Q, and retail price, p. In turn, the vendor maximizes its individual profit by choosing both the credit period length, t, and its production lot size multiplier, n, assuming it knows how the buyer will determine its individually optimal retail price and the purchase order quantity based on the credit option provided.

3.3.1 Buyer's problem

The buyer's gross profit function now results from adding one additional term, representing the gain from delaying the payment as per the credit terms, to (1). Thus, given that h_2^c is the cost of capital per unit per time unit and D is the annual demand rate, we have

$$\Pi_r(p, Q) = (p - v)D - \frac{D}{Q}A_r - \frac{Q}{2}h_r + Dt \cdot h_r^c\,. \tag{3.6}$$

As before, the first order condition yields the optimal price as a function of the order quantity, i.e.

$$p^*(Q) = \frac{\beta}{\beta - 1}\left(-th_r^c + \frac{A_r}{Q}\right). \tag{3.7}$$

Substituting (7) into (6), the buyer's gross profit as a function of Q alone is given by

$$\Pi_2(Q) = \frac{k}{\beta}\left[\frac{\beta}{\beta-1}\left(v - th_r^c + \frac{A_r}{Q}\right)\right]^{1-\beta} - \frac{Q}{2}h_r\,.$$

Hence, the first order condition obtained from the above profit function leads to the determination of the optimal order quantity, i.e.

$$Q^* = \sqrt{\frac{2kA_r}{h_r}\left[\frac{\beta}{\beta-1}\left(v - th_r^c + \frac{A_r}{Q^*}\right)\right]^{-\beta}}\,. \tag{3.8}$$

Once again, the second order optimality condition is

$$Q > \frac{A_r(\beta-2)}{2(v - th_r^c)}\,.$$

From (3.8), $Q*$ can be determined either by any standard equation solving software, or via an appropriate iterative solution technique, as mentioned earlier.

Comparing equations (3.2) with (3.7) and (3.3) with (3.8), it is clear that the availability of a credit payment option increases the buyer's order size and reduces the retail price, i.e.$Q^*_{credit} > Q^*_{nocredit}$and $p^*_{credit} < p^*_{nocredit}$. These findings appear to be logical and are consistent with real world observations. When the supplier offers a credit option, the buyer experiences less payment pressure and burden and is, thus, likely to adopt a larger purchase order quantity. In addition, the latter party now has an incentive to set a lower retail price in order to accommodate the increased transaction quantity and stimulate the item's market demand, achieving a new optimal policy with higher profit.

Proposition 3.1: In an un-coordinated supply chain with a delayed payment option, Q increases (decreases) and p decreases (increases) as the credit time, t, increases (decreases). □

Proof is shown in Appendix A.

3.3.2 Supplier's problem

The supplier's focus is now on choosing$t$ and n towards its own profit maximization. Its gross profit in this case is obtained by subtracting from the profit function (3.4) the loss resulting from offering credit over an interval of t, i.e.

$$\Pi_m(t,n) = (v-m)D - \frac{DA_m}{Qn} - \frac{Q}{2}\left[(n-1)-(n-2)\frac{D}{R}\right]h_m - Dt\cdot h_m^c, \tag{3.9}$$

$$s.t.\ \ 0 \le t \le t_0.$$

In reality, the supplier may be unwilling to offer a credit period of excessively long duration, due to a variety of practical problems. One such problem involves its own cash flow requirements. A steady and adequate cash flow stream is often necessary for the supplier's continual and smooth operation. Thus, while improving the cash flow situation of the buyer, the supplier is at a disadvantage due to payment delays. In order to capture this notion, we add a cash flow related constraint to the supplier's problem, where the credit period t should not be longer than a pre-determined threshold value of t_0, which is tolerable on the part of the supplier, in order to ensure adequate working capital levels for its own needs.

If the credit period length is fixed exogenously and specified in a purchase contract based on current industry norm, the supplier's problem is similar to the one under the scenario of non-coordination without a credit option and the optimal number of batches, n^*, can be obtained via equation (3.5).

3.4 Joint optimization with and without credit option

Under the assumptions that the supplier and the buyer share all relevant information and that both parties strictly follow the terms of a purchase contract with a delayed payment option, we compute the total supply chain profit by determining the retail price and the lot size for the buyer; the batch size multiplier for the supplier and the credit period length. Since the model and the solution algorithm for the scenario of coordination without credit will be very similar to the ones for the case with a credit option, we explore and outline only one of these cases here, viz. the one with the delayed payment option. The results for the scenario without the credit option are similar to the results obtained below, without the credit value factor $(h_r^c - h_m^c) \cdot Dt$.

With a specified credit period of *t*, the gross profit functions for the vendor and the buyer have been developed earlier, expressed by equations (3.9) and (3.6), respectively. Thus, the total supply chain gross profit, Π, is obtained by adding these, resulting in the following joint optimization problem where the optimal values of *p*, *Q*, *n* and *t* need to be determined:

Max $\Pi(p, Q, n, t) =$

$$(p - m)D - \frac{D}{Q}\left(\frac{A_m}{n} + A_r\right) - \frac{Q}{2}h_r - \frac{Q}{2}\left[(n-1) - (n-2)\frac{D}{R}\right]h_m$$

$$+(h_r^c - h_m^c) \cdot Dt \qquad (3.12)$$

Subject to: $D = k \times p^{-\beta}$,

$D \leq R$,

$p, Q, D, n \geq 0$,

$0 \leq t \leq t_0$,

and n is an positive integer.

From the objective function above, it is easy to see that if the supplier's cost of capital exceeds that of the buyer, no credit payment option should be offered, i.e. *t=0*, which maximizes (3.12).

On the other hand, if the opposite is true, i.e.$h_2^c > h_1^c$, then t can be increased for increasing the total supply chain gross profit. Needless to say that a credit period of any length (*t > 0*) increases the buyer's profit and, at the same time, it reduces the supplier's profit. From the perspective of the whole supply chain, a positive credit period would be desirable if the buyer's gain outweighs the supplier's loss. In this circumstance, *t* can increase indefinitely for maximizing overall supply chain profit. Although this observation holds true mathematically, an unbounded *t* poses some serious practical difficulties, as mentioned earlier. As alluded to above, a supplier needs working capital for day-to-day operating expenses and would prefer prompt payments from its buyers. While a relatively short delay in the receipt of payments may be tolerable for enhancing overall system performance, an inordinately long credit period is likely to be unacceptable and problematic in practice for the supplier. Nevertheless, since it may be difficult to accurately assess an appropriate penalty for an excessively long credit period from the vendor's viewpoint, we limit the credit period length to an upper bound, t_0, as was done earlier in (3.9).

Considering the objective function (3.12) above, if we take the first and second partial derivatives with respect to n, for fixed values of p and Q, we obtain the following first and second order optimality conditions:

$$\frac{\partial \Pi}{\partial n} = \frac{DA_m}{Q} n^{-2} - \frac{Q}{2}(1-\rho)h_m \text{ and } \frac{\partial^2 \Pi}{\partial n^2} = -\frac{2DA_m}{Q} n^{-3} < 0.$$

As pointed out earlier, Π is a concave function in n for fixed values of p and Q. Thus, the problem above is reduced to finding a local optimal solution. A procedure towards this end is outlined below.

3.4.1 Solution algorithm:

We suggest the following solution methodology:

Step 1. Initialize with $n = 1$,

Step 2. To find the maximum supply chain profit, $\Pi^*(n, p^*(n), Q^*(n))$,set the first derivative of (12) with respect to p to zero and obtain the corresponding optimal price as a function of Q, i.e.

$$\frac{\partial \Pi}{\partial p} = (1-\beta)kp^{-\beta} + \beta k p^{-(\beta+1)}\left[m + \frac{1}{Q}\left(\frac{A_m}{n} + A_r\right) - (h_r^c - h_m^c)\cdot t - \frac{\mathrm{Qh_m}}{2\mathrm{R}}(\mathrm{n}-2)\right]$$

$$= 0 \text{ leads to}$$

$$p^*(Q) = \frac{\beta}{\beta-1}\left[m + \frac{1}{Q}\left(\frac{\mathrm{A_m}}{n} + \mathrm{A_2}\right) - (\mathrm{h_r^c} - \mathrm{h_m^c})\cdot \mathrm{t} - \frac{\mathrm{Qh_m}}{2\mathrm{R}}(\mathrm{n}-2)\right]. \qquad (3.13)$$

Inserting equation (3.13) into (3.12), we have the total supply chain profit as a function of solely Q:

$$\Pi(Q) = \frac{k}{\beta}\left[\frac{\beta}{\beta-1}\left[m + \frac{1}{Q}\left(\frac{A_m}{n} + A_r\right) - (h_r^c - h_m^c)\cdot t - \frac{\mathrm{Qh_m}}{2\mathrm{R}}(\mathrm{n}-2)\right]\right]^{1-\beta} - \frac{Q}{2}\mathrm{h_r}$$

$$- \frac{\mathrm{Q}}{2}(\mathrm{n}-1)\mathrm{h_m}. \qquad (3.14)$$

Solving the first order optimality condition of equation (3.14) we obtain the expression involving Q as shown below:

$$k\left(\frac{\beta}{\beta-1}\right)^{-\beta}\left[m + \frac{1}{Q}\left(\frac{\mathrm{A_m}}{n} + \mathrm{A_r}\right) - (\mathrm{h_r^c} - \mathrm{h_m^c})\cdot \mathrm{t} - \frac{\mathrm{Qh_m}}{2\mathrm{R}}(\mathrm{n}-2)\right]^{-\beta}\left[\frac{1}{Q^2}\left(\frac{A_m}{n} + A_r\right)\right.$$

$$\left. + \frac{\mathrm{h_m}}{2R}(n-2)\right]$$

$$= \frac{1}{2}[\mathrm{h_r} - (n-1)\mathrm{h_m}]. \qquad (15)$$

Notice that the above equation is not in a closed form. But it can be easily solved in many software, e.g. Excel, Matlab, etc. By solving the above equation we can obtain the buyer’s jointly optimal order quantity, $Q*$.

<u>Step 3</u>. Set $n = n + 1$, and repeat step 2 to find $\Pi^*(n, p^*(n), Q^*(n))$.

Step 4. If $\Pi^*\big(n, p^*(n), Q^*(n)\big) > \Pi((n-1), p^*(n-1), Q^*(n-1))$, go to step 3, or else go to step 5.

Step 5. Set $\Pi^*(n^*, p^*(n^*), Q^*(n^*)) = \Pi^*((n-1), p^*(n-1), Q^*(n-1)$ and $n^*, p^*(n^*), Q^*(n^*)$ represents a local optimum.

Furthermore, according to Lemma 2, outlined below, the solution yielded by our algorithm is, indeed, the global optimum.

Lemma 3.1*:* $\Pi^*(n^*, p^*(n^*), Q^*(n^*))$ obtained via the algorithm outlined above is the global optimum solution. □

The proof is shown in Appendix A.

3.5 Coordination and profit sharing via wholesale discount

In this section, we discuss the notion of a wholesale price discount, as a means for encouraging the supplier to purchase larger quantities and consequently lower its retail price and increase market demand, resulting in higher supply chain profit. Clearly the overall effects of such a discount are similar to the ones resulting from a credit payment option. In addition, towards establishing an appropriate wholesale price reduction, we suggest a fair and equitable way for the two parties to share the additional total supply chain profit generated by the integration of the supply chain with (or without) the credit option, as opposed to the independent optimization scenario with (or without) credit. Two cases, outlined below, warrant some discussion.

Case 1: $\boldsymbol{h_m^c > h_r^c}$

We know that if the cost of capital for the supplier is greater than that of the buyer, it is more desirable, from a profit perspective, for the supply chain to adopt a quantity discount mechanism than to use a credit payment option (see Sarmah (2007)). Also from (3.12), it is clear that the total supply chain profit is maximized with $t=0$, i.e. when no credit option is offered.

Thus, under coordination without a credit option, we use the concept of a wholesale price discount to divide the total supply chain profit amongst the two parties in a fair and equitable manner. Let the notation of double asterix (**) denote the respective profits with a discounted wholesale price, but without a credit option, and the single asterix (*) represent the uncoordinated, individually derived optimal profits with the regular price in effect. For sharing the additional value yielded by coordination in a fair and equitable manner, in proportion to each party's original uncoordinated gross profit, we suggest a profit sharing rule that satisfies

$$\Pi_r^{**}(p^{**}, Q^{**}, n^{**}) = \Pi^{**}(p^{**}, Q^{**}, n^{**}) \frac{\Pi_r^{*}\big(p^{*}, Q^{*}, n^{*}(p^{*}, Q^{*})\big)}{\Pi^{*}\big(p^{*}, Q^{*}, n^{*}(p^{*}, Q^{*})\big)}, \quad \text{and}$$

$$\Pi_m^{**}(p^{**}, Q^{**}, n^{**}) = \Pi^{**}(p^{**}, Q^{**}, n^{**}) \frac{\Pi_m^{*}\big(p^{*}, Q^{*}, n^{*}(p^{*}, Q^{*})\big)}{\Pi^{*}\big(p^{*}, Q^{*}, n^{*}(p^{*}, Q^{*})\big)}. \tag{3.16}$$

Under this regime, the individually optimal profits, Π_i^* $(i = 1, 2)$, and the jointly optimal supply chain profit, Π^*, are obtained from (3.1), (3.4) and (3.12), respectively, with the regular wholesale price *v* and *t=0*. The Π_i^{**}andΠ^{**}profit values are based on an unknown discounted wholesale price,v*, instead of *v*, the regular wholesale price with *t=0*. Thus, the discounted unit wholesale price,v^*, is easily determined from (3.16). Consequently, the discount factor is expressed by$d = (1 - v^*/v)$. This proportional gain sharing concept appears to be fair, as well as equitable and can, at least, serve as a baseline position for negotiations between the buyer and the vendor. It is to be noted that this profit sharing scheme is only one of many possible ways to "fairly" divide the surplus resulting from coordination. The actual profit share accruing to each party in a real world scenario will, of course, depend upon the relative bargaining power enjoyed by each member of the supply chain within a negotiation framework.

Case 2: $\boldsymbol{h_r^c > h_m^c}$

As mentioned earlier, when a credit option is in effect with $h_r^c > h_m^c$, the longer the credit period, *t*, the higher the total supply chain profit, albeit with declining vendor profit (see (3.12) and (3.9), respectively). The impracticality of an excessively long credit payment period has been discussed earlier. Therefore, under this condition, a wholesale price discount in conjunction with an agreed upon credit period (based on accepted industry practice or prevailing norms) appears to be a reasonable approach in order to induce the buyer to adopt a coordinated policy, which is optimal for the entire supply chain. Once again, the discount can serve as a mechanism for sharing the additional supply chain profit fairly. A profit sharing rule similar to the one shown above by (3.16), now with a predetermined credit payment period, t, yields the discounted wholesale price and the resulting discount factor via algorithm 5.1 outlined above.

3.6 Numerical example and sensitivity analysis

In order to illustrate the concepts developed in this paper, we utilize a modified version of the example provided by Chang et al. (2009), where the relevant parameter values are

$k = 10^6,\ \beta = 2,\ R = 6000\ \text{units/year},\ m = \$8/\text{unit},\ v = \$15/\text{unit}$,

$A_m = \$400/\text{setup},\ A_r = \$50/\text{order},\ t_0 = 4/12\ year$,

$h_m^s = h_r^s = 0.1,\ h_m^c = 0.25, h_r^c = 0.4,\ h_m = 0.35,\ h_r = 0.5$ (\$/unit/year)

We solve several problem instances under a variety of operating scenarios as described below:

Case 1 - U: Uncoordinated supply chain without credit option and discount.

Case 2 - U{t}: Uncoordinated supply chain with a given credit payment option:

a. U{t=1month}, i.e. t = 1/12 year
b. U{t=3 months}, i.e. t = 3/12 year
c. U{t*} –optimal credit period by optimizing (11) subject to (7) and (10)

Case 3 – C{d}: Coordination with a price discount, but no credit option.

Case 4 – C{t}: Coordination with a credit option, but no price discount

a. C{t = 3 months}, i.e. $t = 3/12$ year

b. C{t = t'}, where t' is the credit period necessary to make the supplier's profit approximately equal to its uncoordinated optimal value, as in Case 1, without considering the cash flow related constraint on the credit period length.

Case 5 – C{d, t}: Coordination with both a discount and a credit option

a. C{d, t=1 month (1/12 year)}

b. C{d, t=3 months (3/12 year)}.

The results of solving the above mentioned ten problem instances are summarized in Table 1.

This table indicates that in an uncoordinated situation the adoption of a credit payment policy (with a reasonable credit period, such as 1 or 3 months) tends to increase the total supply chain profit, albeit slightly. The profit gained in this example primarily results from the difference in the opportunity costs of capital, $(h_r^c - h_m^c) \cdot Dt.$ Also, the profit of the buyer declines slightly while the supplier's profit, as well as the total supply chain profit increase, as the credit period is lengthened from 1 to 3 months. The optimal credit period, which maximizes the manufacturer's problem, is shown to be 0.14 years, 1.68 months. It needs to be noted that the optimal credit period for the manufacturer could also result to be zero (no credit is desired) or it could be bound by the cash flow constraint.

When the supply chain is coordinated via a credit option, or a wholesale price discount, or a combination of both, the total supply chain profit increases quite dramatically, compared to the uncoordinated case. In all cases of coordination, the product's market demand is increased with a reduced retail price, tending to result in increased profits for both the supplier and the buyer under most circumstances, as well as for the entire supply chain. Table 1 also indicates that compared to the adoption of only a wholesale price reduction, the credit payment option as a sole coordination mechanism appears to be superior in terms of total supply chain profit. When the discount approach is used in conjunction with a reasonable credit payment option, both parties' and the supply chain's profit positions improve, although not dramatically, compared to the presence of only a discount in the wholesale price.

Adopting a one or three month credit payment period, without a discount, substantially decreases the supplier's profit, but the buyer's profit increases drastically, compared to the uncoordinated case. This is to be expected, since without the discount, the supply chain surplus is not shared by the two parties. With only a credit option in effect, the supplier's profit tends to decrease, with higher buyer profit, if a multi-year credit payment period is allowable. We can see in Table 1 that a 2.25-yearcredit period is required for the supplier's profit to be approximately at par with its maximum achievable profit without any coordination. It is to be noted that the total supply chain profit increases as the credit period is increased, with or without a discount. Nevertheless, as under the uncoordinated case, a multi-year credit period is likely to be impractical and maybe highly undesirable from the supplier's standpoint.

Another observation that can be made by examining Table 1 is that supply chain coordination tends to increase the buyer's order size, with a concurrent reduction in retail price, while the supplier's production lot size multiplier tends to increase. Overall, the detailed results of our numerical example indicate that for the purpose of enhancing the total supply chain profitability, a coordinated system employing a combination of a wholesale price discount and a credit payment option with a reasonable credit period, consistent with existing industry practice appears to hold substantive merit from the perspectives of both the buyer and the supplier. In particular, if the price discount is utilized for an equitable allocation of the supply chain surplus in a manner similar to our methodology, both parties will not be averse to coordination and are likely to share the relevant information in a cooperative environment.

Finally, we examine the sensitivity of the uncoordinated and coordinated supply chain solutions to a parameter of interest, i.e. β, the demand elasticity coefficient. Table 3.2 shows the individual optimal profits of the supplier and the buyer, as well as the total supply chain profit without coordination (i.e. Π_1, Π_2 and Π, respectively) for varying values of β. This tables also contain the respective profit values, indicated by Π_1^*, Π_2^* and Π^*, that result from the implementation of a wholesale price discount mechanism for profit sharing based on (3.16), in conjunction with a credit payment period of 3 months (0.25 year). The last column in this table contains the percent increase in the entire supply chain's profit, resulting from coordination.

Table 3.2, where β varies from 1.76 to 1.80, indicates that as this parameter value increases, the benefits of supply chain coordination tend to become more pronounced, with an increasing wholesale price discount, while the producer's lot size multiplier tends to decline, with or without coordination. It is clear that as the effect of the retail price in influencing demand strengthens, the argument in favor of implementing supply chain coordination incorporating a wholesale discount and a credit option appears to gain strength.

4. Price Dependent Stochastic Demand Model

4.1 Model Preliminaries

In this model, we explore the problem scenario under price dependent stochastic demand. Uncertainty in demand gives rise to additional challenges in modeling the supply chain. More specifically, a supply chain under such operating conditions would carry safety stocks in order to reduce the risk of stockouts. Consequently, prices may need to be adjusted to accommodate higher inventory levels and the buyer and the supplier both need to balance the costs of holding additional inventories and those related to stockouts. In this context, the extent of demand uncertainty is likely to play an important role in the decision making process. In this chapter, we first analyze the effects of demand uncertainty on the credit option in the supplier's problem and how the credit option affects the buyer's inventory replenishment and pricing decisions in the decentralized case. We also examine the role of the credit option as a coordination mechanism, the ranges of trade credit period length necessary for coordination and the effectiveness of such a policy in the centralized case

In the literature, there are two basic forms of stochastic demand: the additive form and the multiplicative form. In this study, it would be reasonable to assume that the variation in demand is proportional to its magnitude or expected value. Hence, we postulate that the uncertain demand rate, δ, has a multiplicative form, such that $\delta = D(p)\varepsilon$, which has a constant coefficient of variation. $D(p)$ is the price sensitive part of demand, which is an exponential function of the retail price, i.e., $D(p) = k \cdot p^{-\beta}$, and ε is a normally distributed random variable, i.e. $\varepsilon \sim N(1, \sigma^2)$. Hence, the distribution of demand is also normally distributed with mean $\mu_d = D(p)$ and standard deviation of $\sigma_d = D(p)\sigma$. In other words, $\delta \sim N(D(p), (D(p)\sigma)^2)$. In the multiplicative case, the coefficient of variation is assumed to remain constant for different prices, i.e., $C_v = \frac{\sigma_d}{\mu_d} = \sigma$. Thus, under the ($r$, Q) policy, the demand during lead time, will have a distribution of $\delta_L \sim N(D(p)L, \left(D(p)\sqrt{L}\sigma\right)^2)$ with a coefficient of variation, $C_v(\delta_L) = \frac{\sigma}{\sqrt{L}}$. Under the periodic review (S, T) policy, the demand during the interval of cycle time plus the lead time (i.e. T+L) will also have a normal distribution, i.e. $\delta_{T+L} \sim N(D(p)(T + L), \left(D(p)\sqrt{T+L}\sigma\right)^2)$, with $C_v(\delta_{T+L}) = \frac{\sigma}{\sqrt{T+L}}$. Denote $f(\cdot)$ and $F(\cdot)$ to be the density and cumulative distribution functions, respectively, of the standard normal distribution. Also, let $f_L(\cdot)$ and $F_L(\cdot)$ denote the density and cumulative density functions of demand during lead time in the (r, Q) policy, and $f_{T+L}(\cdot)$ and $F_{T+L}(\cdot)$ denote the density and cumulative distributions functions respectively of demand T+L in the (S,T) model.

In the coordinated model, we also assume that the centralized decision maker has complete information about market demand elasticity and the relevant parameters of both parties in the supply chain. Since the manufacturer produces multiple delivery batches in each production cycle, the risk of having unsatisfied demand is much smaller than that of the buyer, especially when there are multiple buyers. For the sake of simplicity, we assume the retailer bears all the stockout related penalties.

4.2 Decentralized supply chain under (*r*, *Q*) replenishment policy

In this section, we study the underlying Stackelberg game for the supply chain. The retailer orders the products from the manufacturer using an (r, Q) policy and sells them at the retail level, where the demand is dependent upon the retail price as described earlier. Thus, the retailer chooses its reorder point r, order quantity Q and retail price p to maximize it profit. In turn, the manufacturer, upon receiving the order information from the retailer, determines the production batch size multiplier and the credit payment terms to be offered to the retailer, in order to maximize its own profit.

4.2.1 The retailer's model

In the non-cooperative scenario, the retailer maximizes its profit by deciding the replenishment policy, the reorder point of the inventory and the order quantity, and the retail price. The retailer's problem is to maximize the value of its expected profit, i.e.:

$$\text{Max:} \quad \Pi_r(r,Q,p) = (p-v)D(p) - A_r\frac{D(p)}{Q} - h_r\left(\frac{Q}{2}+r-\mu_L\right) - b\frac{D(p)}{Q}G_L(r) + D(p)th_r^c, \qquad (4.1)$$

$$\text{s.t.} \quad D(p) = k\cdot p^{-\beta},$$

$$G_L(r) = \int_r^\infty (x-r)f_L(x)dx.$$

The first term in the objective function above is the gross profit and the second represents the ordering cost. The third term in (4.1) is the expected inventory holding cost. Note that the inventory position is uniformly distributed in (r, r+Q), but the inventory level is not uniformly distributed. Thus the $(\frac{Q}{2}+r-\mu_L)$ is only a good approximation of the average inventory level (See Hadley and Whitin (1963) and Nagarajan and Rajagopalan (2008)). It can also be shown that $r = \mu_L + z\sigma_L$. The fourth term in (1) is the penalty for backorders, where the $G_L(r) = \int_r^{\infty}(x - r)f_L(x)dx$ is the expected backorder quantity when demand during lead time exceeds r. The normal loss function $G_L(r)$ can also be expressed as

$$G_L(r) = \sigma_L\left[f\left(\frac{r-\mu_L}{\sigma_L}\right) - z\left[1 - F\left(\frac{r-\mu_L}{\sigma_L}\right)\right]\right].$$

Lemma 4.1: The first order conditions of the problem can be written as:

$$b\frac{D(p)}{Q}\overline{F_L}(r) - \mathrm{h_r} = 0, \tag{4.2}$$

$$A_r\frac{D(p)}{Q^2} - \frac{h_r}{2} + b\frac{D(p)}{Q^2}G_L(r) = 0, \tag{4.3}$$

$$(1 - \beta)p + \beta\left[v - th_r^c + \frac{A_r}{Q} - h_rL + \frac{b}{Q}G_L(r)\right] = 0. \tag{4.4}$$

The sufficient condition for a local maximum is shown in Appendix A.

Proposition 4.1:

1) The optimal profit of the retailer increases in t.
2) The profit function of the retailer is submodular in (r, Q), (r, p) and (p, Q).
3) The optimal retail price of the retailer p^* decreases in t.

The proof is shown in Appendix A.

The first part of the above proposition states that a longer trade credit period always benefits the retailer, under rational decision making. The submodularity characteristic implies that for each of the pairs specified, given one of these decision variables, the other will decrease (or increase) as the former increases (or decreases). The final part of this proposition states that when a longer credit period is offered by the supplier, the retailer reacts with a lower market price, resulting in higher consumer demand, and hence, contributing towards generating greater revenue for the supply chain.

4.2.2 The manufacturer's model

The manufacturer's problem is to determine the number of delivery batches, n, for each production batch nQ, and the credit time offered to the retailer so as to maximize its profit, assuming that the retailer will react to the credit period offer as described in section 3.1. The manufacturer's problem has the following form:

Max $\Pi_m(n,t)$,

s.t. $(r, Q, p) = argmax\ \Pi_r(r, Q, p|n, t)$.

Rewriting the constraints using the optimality conditions of the retailer's problem, the manufacturer's problem can be expressed as a mixed integer non-linear problem as follows:

$$\text{Max } \Pi_m(n,t) = (v-m)D - A_m \frac{D}{nQ} - h_m\left[(n-1)-(n-2)\frac{D}{R}\right]\frac{Q}{2} - Dth_m^c \quad (4.5)$$

$$\text{s.t. } b\frac{D(p)}{Q}\overline{F_L}(r) - \mathrm{h_r} = 0 \quad (4.2)$$

$$A_r\frac{D(p)}{Q} - \frac{h_r}{2} + b\frac{D(p)}{Q^2}G_L(r) = 0 \quad (4.3)$$

$$(1-\beta)p+\beta\left[v-th_r^c+\frac{A_r}{Q}-h_rL+\frac{b}{Q}G_L(r)\right]=0 \tag{4.4}$$

$$D(p)=k\cdot p^{-\beta},$$

$$0\le t\le t_0.$$

The first term in (4.5) is the gross profit. The second term is the set up cost, and the third term represents the expected cost of holding inventory. Finally the last term in (4.5) represents the loss in terms of the cost of capital due to the credit payment option. This is a mixed integer nonlinear programming problem, which can be solved using any mixed integer nonlinear programming algorithm, such as the branch and bound or the cutting plane procedure. For a fixed n, the problem is a nonlinear programming problem, for which the KKT optimality conditions can be readily derived as indicated in the following lemma.

Lemma 4.2: The KKT conditions of the manufacturer's problem for a given n are:

$$-Dh_m^c-\lambda(-\beta h_r^c)-\mu\le 0,\ \ t\ge 0,\ \ and\ t\cdot[-Dh_m^c-\lambda(-\beta h_r^c)-\mu]=0, \tag{4.6}$$

$$b\frac{D(p)}{Q}\overline{F_L}(r)-\mathrm{h_r}=0, \tag{4.2}$$

$$A_r\frac{D(p)}{Q}-\frac{h_r}{2}+b\frac{D(p)}{Q^2}G_L(r)=0, \tag{4.3}$$

$$(1-\beta)p+\beta\left[v-th_r^c+\frac{A_r}{Q}-h_rL+\frac{b}{Q}G_L(r)\right]=0, \tag{4.4}$$

$$t\le t_0,\ \ \mu(t_0-t)=0, and\ \mu\ge 0.$$

Since the parameters associated with t appears in other parts of the problem (objective function and/or constraints), no comparative static analysis can be derived for this constrained problem directly. Therefore, we conduct sensitivity analysis in the numerical examples section.

4.2.3 Coordinated supply chain under (*r, Q*) replenishment policy

In this section, we have a centralized decision maker which acts on behalf of the entire supply chain towards optimizing its performance. The notion of a fair and negotiable credit time will be discussed so as to induce both parties to coordinate their decisions. The coordination can be proposed by either party. The manner in which the profit surplus will be shared by the buyer and the supplier will depend upon the relative bargaining powers of the two parties. The overall supply chain's problem is:

$$\text{Max } \Pi(Q,r,p,n) = (p-m)D(p) - A_r\frac{D(p)}{Q} - A_m\frac{D}{nQ} - h_r\left(\frac{Q}{2}+r-\mu_L\right)$$
$$-h_m\left[(n-1)-(n-2)\frac{D}{R}\right]\frac{Q}{2} - b\frac{D(p)}{Q}G_L(r), \quad (4.7)$$

$s.t.\;\; D(p) = k\cdot p^{-\beta},$

$p > v,$

$0 \le t \le t_0.$

The first order optimality conditions pertaining to this problem for a given multiplier of the order batch size n, are:

$$b\frac{D(p)}{Q}\overline{F_L}(r) - \mathrm{h_r} = 0, \quad (4.8)$$

$$\left(A_r+\frac{A_m}{n}\right)\frac{D(p)}{Q} - \frac{h_r}{2} - \frac{h_m}{2}\left[(n-1)-(n-2)\frac{D}{R}\right] + b\frac{D(p)}{Q^2}G_L(r) = 0, \quad (4.9)$$

$$(1-\beta)p+\beta\left[m+\frac{A_r}{Q}+\frac{A_m}{nQ}-h_rL-\frac{(n-2)h_mQ}{2R}+\frac{b}{Q}G_L(r)\right]=0. \tag{4.10}$$

Let the superscript $*$ to denote the decentralized optimal decisions for each party, and the superscript $**$ to denote the optimal supply chain decision variables. One fair way to share the extra profit is to share it proportionally based on their original profit in the decentralized model. So the retailer's profit under supply chain optimal variable values plus the saving on the proposed delayed payment should equals to the supply chain new profit times the original percentage of the retailer's profit.

$$\Pi_r{}^{**}(p^{**},Q^{**},r^{**})+Dth_r^c=\Pi^{**}(p^{**},Q^{**},r^{**},n^{**})\frac{\Pi_r^*(p^*,Q^*,r^*)}{\Pi_m^*(n^*,T^*)+\Pi_r^*(p^*,Q^*,r^*)}$$

We can, thus, have an upper and a lower bound for the credit payment period. The minimum credit period is the one that is offered such that the retailer's savings resulting from the credit option should be sufficient to compensate its loss as a consequence of adopting the supply chain optimal decisions, instead of its own optimal policy. The maximum credit payment period the manufacturer can offer is the one that will make the manufacturer's profit no different from its original profit in the decentralized case. Therefore, the entire surplus from the joint coordinated decisions of the supply chain will be transferred to the retailer. The mathematical expressions of these notions are

$$Dt_{min}h_r^c=\Pi_r^*(p^*,Q^*,r^*)-\Pi_r^{**}(p^{**},Q^{**},r^{**},n^{**}),$$

$$Dt_{max}h_r^c=\Pi_m^{**}(p^{**},Q^{**},r^{**},n^{**})-\Pi_m^*(n^*,t^*).$$

4.3 Decentralized supply chain under (S, T) replenishment policy

In this section, we study the decentralized model with the retailer adopting the *(S, T)* periodic review policy. Under this policy, inventory is reviewed at fixed intervals of T time units. During each review, an order is placed such that the inventory position reaches the order-up-to level of *S*. The retailer determines the values of *S, T* and the retail price simultaneously, based on the credit payment period offered. The manufacturer, as the Stackelberg leader, determines the credit period length and the expected number of deliveries per production batch, in order to maximize its own profit, again, subject to the first order conditions depicting the retailer's rational behavior.

4.3.1 The retailer's model

The retailer's profit can be written as:

$$\Pi_r(S,T,p) = (p-v)D(p) - \frac{A_r}{T} - h_r\left(S - \frac{DT}{2} - \mu_L\right) - \frac{b}{T}\int_S^{\infty}(x-S)f_{T+L}(x)dx + D(p)th_r^c.$$

To simplify this expression in the following analysis, we use the normal standard value z instead of S as a decision variable. The relationship between them can be expressed as $S = D(T+L) + z\sigma_d\sqrt{T+L}$. The retailer's problem now is

$$\text{Max } \Pi_r(z,T,p) = (p-v)D(p) - \frac{A_r}{T} - h_r\left(\frac{DT}{2} + zD(p)\sigma\sqrt{T+L}\right)$$

$$-\frac{b}{T}\sigma_{T+L}\big[f(z)-z[1-F(z)]\big]+D(p)th_r^c, \tag{4.11}$$

An examination of the above objective function leads to the lemma outlined below.

Lemma 4.2: The first order optimality conditions for (4.11) can be written as the following set of equalities:

$$\frac{b}{T}[F(z^*)-1]-h_r=0, \tag{4.12}$$

$$\sqrt{T+L}(hD(p)T^2-2A_r)-z\sigma_d h_r T^2+b\sigma_d\big[f(z)-z[1-F(z)]\big](T+2L)=0, \tag{4.13}$$

$$(1-\beta)p+\beta[v-th_r^c+h_r(\frac{T}{2}+z\sigma\sqrt{T+L})$$
$$+\frac{b}{T}\sigma\sqrt{T+L}\cdot[f(z)-z[1-F(z)]]]=0. \tag{4.14}$$

The sufficient condition for a local maximum is shown in Appendix A.

From (4.11) we can also obtain an expression for S in terms of $T\ and\ p$, i.e.

$$S^*(T,p)=D(p)(T+L)+D(p)\sigma\sqrt{T+L}\cdot F^{-1}\left(1-\frac{h_rT}{b}\right).$$

It is to be noted that for a given price, the value of standard normal variable z for determining the reorder point is independent of the retail price. In our game scenario, since T is a function of the price, as shown below, the value of z is related to the pricing decision indirectly.

Proposition 4.2:

1) The optimal profit of the retailer increases in t.

2) The profit function of the retailer is submodular in (z, T).

3) The optimal retail price of the retailer p^* decreases in t.

See a proof in Appendix A.

4.3.2 The manufacturer's model

Given the retailer's ordering policy, the manufacturer's task is to choose the credit payment period and the batch size multiplier in order to maximize its own profit, based on the retailer's reaction to the credit period, t, i.e. the manufacturer's problem is

$$\text{Max } \Pi_m(n,t) = (v-m)D - \frac{A_m}{nT} - h_m\left[(n-1)-(n-2)\frac{D}{R}\right]\frac{DT}{2} - Dth_m^c, \tag{4.15}$$

$$\text{s.t. } \frac{b}{T}[F(z^*)-1] - h_r = 0 \tag{4.12}$$

$$\sqrt{T+L}(hD(p)T^2 - 2A_r) - z\sigma_d h_r T^2 + b\sigma_d\big[f(z) - z[1-F(z)]\big](T+2L) = 0, \tag{4.13}$$

$$(1-\beta)p + \beta[v - th_r^c + h_r(\frac{T}{2} + z\sigma\sqrt{T+L})$$

$$+\frac{b}{T}\sigma\sqrt{T+L}\cdot[f(z) - z[1-F(z)]]] = 0, \tag{4.14}$$

$$D(p) = k\cdot p^{-\beta},$$

$$p > v,$$

$$0 \le t \le t_0.$$

Lemma 4.3: The KKT conditions for the manufacturer's problem under the (S,T) policy for a fixed n are:

$$-Dh_m^c - \lambda(-\beta h_r^c) - \mu \leq 0,\ \ t \geq 0,\ \ and\ t \cdot [-Dh_m^c - \lambda(-\beta h_r^c) - \mu] = 0, \tag{4.6}$$

$$\frac{b}{T}[F(z^*) - 1] - h_r = 0, \tag{4.12}$$

$$\sqrt{T+L}(hD(p)T^2 - 2A_r) - z\sigma_d h_r T^2 + b\sigma_d\big[f(z) - z[1 - F(z)]\big](T + 2L) = 0, \tag{4.13}$$

$$(1-\beta)p + \beta[v - th_r^c + h_r(\frac{T}{2} + z\sigma\sqrt{T+L})$$
$$+\frac{b}{T}\sigma\sqrt{T+L} \cdot [f(z) - z[1 - F(z)]]] = 0. \tag{4.14}$$

$$t \leq t_0,\ \ \mu(t_0 - t) = 0, and\ \mu \geq 0.$$

4.3.3 Coordinated supply chain under (S, T) replenishment policy

As before, we formulate the model for deriving the optimal decisions for the entire supply chain as follows:

$$\text{Max}\ \ \Pi(z,T,p,n) = (p - v)D(p) - \frac{A_r}{T} - \frac{A_m}{nT} - h_r\big(DT + zD(p)\sigma\sqrt{T+L}\big)$$
$$-h_m\left[(n-1) - (n-2)\frac{D}{R}\right]\frac{DT}{2}$$
$$-\frac{b}{T}D(p)\sigma\big[f(z) - z[1 - F(z)]\big]. \tag{4.16}$$

The first order optimality conditions for objective function (4.16) are:

$$\frac{b}{T}[F(z^*)-1]-h_r=0, \tag{4.17}$$

$$\left(\frac{A_r}{T^2}+\frac{A_m}{nT^2}\right)-\frac{h_rD(p)}{2}-zD(p)\sigma h_r\frac{1}{2\sqrt{T+L}}-h_m\left[(n-1)-(n-2)\frac{D}{R}\right]\frac{D}{2}$$

$$-bD(p)\sigma_{L+T}\big[f(z)-z[1-F(z)]\big]\frac{\frac{1}{2\sqrt{T+L}}-\sqrt{T+L}}{T^2}=0, \tag{4.18}$$

$$(1-\beta)kp^{-\beta}+\beta kp^{-(\beta+1)}[m+h_r(\frac{T}{2}+z\sigma\sqrt{T+L})$$

$$+\frac{b}{T}\sigma\sqrt{T+L}\cdot[f(z)-z[1-F(z)]]]+h_m(n-2)\frac{T}{R}D(p)\beta kp^{-(\beta+1)}=0. \tag{4.19}$$

4.4 Numerical examples and sensitivity analysis

In this section, we provide several numerical examples to explore the characteristics of our models outlined above.

Example 4.1:

Problem parameters:

$k=10^7, \beta=1.8, \sigma=0.1, v=\$50, m=\$20, L=1\ week$

$A_m=2000, A_r=300,\ h_m=h_r=15, h^c=10.5, b=20, \mathrm{t}_0=0.3,\ R=10000.$

Optimal solution for decentralized SC under (*r*, *Q*) policy:

$p^*=110.1,\ Q^*=305.0, r^*=76.8, D^*=2111.2, s=0.892, n^*=3, t^*=0.176$

$\Pi_m^*=51215.2, \Pi_i^*=125738, \Pi^*=176953.$

Optimal solution for centralized SC under (*r* ,*Q*) policy:

$p^*=48.6,\ Q^*=694.1, r^*=379.5, D^*=9208.0, t_{min}=1.57, t_{max}=2.15, \Pi^*=$ 233396.

Optimal solution for decentralized SC under (*S*, *T*) policy:

$p^* = 111.1,\ z^* = 1.31, T^* = 0.13, D^* = 2076.9, s = 0.904, n^* = 4, t^* = 0.246$

$\Pi_m^* = 48715.2, \Pi_i^* = 125891, \Pi^* = 174606.$

Optimal solutionfor centralized SC under (*S*, *T*) policy:

$p^* = 49.2,\ z^* = 1.71, T^* = 0.058, D^* = 9091.8, t_{min} = 1.59, t_{max} = 2.23, \Pi^* = 229045.$

Example 4.2:

Problem parameters:

$k = 10^6, \beta = 1.3, \sigma = 0.5, v = \$50, m = \$25, L = 1\ week$

$A_m = 4000, A_r = 100,\ h_m = h_r = 35, h^c = 30, b = 20, \mathrm{t}_0 = 0.3, R = 10000.$

Optimal solution for decentralized SC under (*r*, *Q*) policy:

$p^* = 223.6,\ Q^* = 114.8, r^* = 62.6, D^* = 882.3, s = 0.77, n^* = 4, t^* = 0$

$\Pi_m^* = 9055.4, \Pi_i^* = 147575, \Pi^* = 156630.$

Optimal solution for centralized SC under (*r*, *Q*) policy:

$p^* = 153.4,\ Q^* = 331.9, r^* = 52.2, D^* = 1140.6, t_{min} = 0.19, t_{max} = 0.29, \Pi^* = 160839.$

Optimal solution for decentralized SC under (*S*, *T*) policy:

$p^* = 259.3,\ z^* = 1.18, T^* = 0.07, D^* = 728.0, s = 0.881, n^* = 9, t^* = 0$

$\Pi_m^* = 5617.5, \Pi_i^* = 143721, \Pi^* = 149338.$

Optimal solution for centralized SC under (*S*, *T*) policy:

$p^* = 138.8,\ z^* = 1.41, T^* = 0.045, D^* = 1639.4, t_{min} = 0.31, t_{max} = 0.54, \Pi^* = 160649,$

In Fig.4.1, we depict the behaviors of the retailer's decision variables, expected annual demand and profit as a result of changes in the value of t. Notice that they all change monotonically. The market price decreases as the credit period length increases, thus increasing the expected sales and the order quantity. Due to the increase in sales, the reorder point will also need to be increased. Overall, the retailer's profit will increase, with increasing t, implying that the retailer can benefit from a longer credit period.

Figure 4.2 shows the effects of varying t on the manufacturer's profit. In this specific example, the optimal t is non zero. But in other cases, the profit function may be a decreasing function of t over its feasible domain, where the optimal value of t would be zero, indicating that no credit payment option should be offered. This phenomenon and its implicationa are discussed below. It is also possible that the optimal credit period length is bound by the cash flow constraint. In such a case, the optimal credit period length would be the maximum allowable time dictated by the cash flow requirement.

In table 4.1 and figure 4.4, we outline the influence of price elasticity on the optimal credit period offered by the manufacturer in the framework of a Stackelberg game. These show that the optimal credit period increases as the price elasticity parameter, β, increases. This is because when price elasticity is high, demand changes are more pronounced as the price varies. Thus, it would be desirable to use the advantage resulting from the increase in demand to compensate for the loss due to issuing credit to the buyer. On the other hand, when price elasticity is low, offering a longer credit period will not be attractive for the manufacturer, since the increase in its revenue would be small compared to the loss stemming from the trade credit.

The graphs shown in Figures 4.4 (b) and (c) depict the effects of the supplier's cost of capital and the coefficient of variation of consumer demand on the optimal trade credit period. When the manufacturer's cost of capital is low, the manufacturer is more reluctant to offer longer trade credit allowances. When market demand exhibits greater uncertainty, the manufacturer is more willing to offer a larger trade credit period. This is due to the fact that a trade credit option tends to increase retail inventory, which, in return, improves the customer service level and keeps the shortage at a balance level under higher demand uncertainty.

Finally, a detailed sensitivity analysis is conducted for all the parameters. Very interestingly, all the parameters have the same impact on these credit time variables. This means, in situations where the manufacturer voluntarily would like to offer credit to the retailer, it is often the case where the credit alone is not be an effective coordination mechanism. And in cases where the credit length is reasonable enough to coordinate the supply chain, the manufacturer wouldn't prefer to offer credit in the decentralized game.

5. Multiple Retailers Model

5.1 Model Preliminaries

In this section, we extend the price sensitive deterministic demand model to a supply chain with a single manufacture and multiple retailers. In order to model market competition, we adopt a linear demand function, adopted by a majority of studies found in the economics literature (e.g. Bertrand and Cournot competition scenarios). The other assumptions made in Section 2 are also adopted here. Our goal is to examine the influence of the intensity of competition (in a fixed market) and the market size (in a variable market) on the trade credit option and the supply chain inventory replenishment and pricing policies.

5.2 Supply chain with correlated retailers

The relevant models pertaining to a decentralized supply chain, where each party attempts to derive its own optimal decisions individually, are outlined here.

5.2.1 Retailer's model

First, we explore the issues starting from price competition between the retailers, via a Bertrand game with a differentiated product. The demand function can be expressed as:

$$d_i(p|p_{-i}) = a_i - b_i p_i + \sum_{j\neq i} c_{ij} p_j \ , \ \ a_i > 0, b_i > 0, c_{ij} > 0, b_i - \sum_{j\neq i} c_{ij} > 0.$$

The retailer's problem now depends on the other retailers' prices, which can be written as follows:

$$\text{Max: } \Pi_i(p_i, q_i|p_{-i}) = (p_i - v)d_i(p_i|p_{-i}) - \frac{d_i(p_i|p_{-i})}{q_i}A_i - \frac{q_i}{2}h_i + d_i(p_i|p_{-i})th_i^c, \quad (5.1)$$

$$\text{s.t.} \quad d_i(p) = a_i - b_i p_i + \sum_{j \neq i} c_{ij} p_j.$$

The first order optimality conditions of the above problem are:

$$d_i(p_i|p_{-i}) - b_i(p_i - v) + \frac{b_i}{q_i}A_i - b_i th_i^c = 0, \ \forall \ all \ i, j \quad (5.2)$$

$$\frac{d_i(p_i|p_{-i})A_i}{q_i^2} - \frac{h_i}{2} = 0, \ \forall \ all \ i, j. \quad (5.3)$$

A simultaneous solution of the above two conditions yields the optimum.

Lemma 5.1: $\Pi_i(p_i, q_i|p_{-i})$ is jointly concave on $p_i$and q_i if $4d_i(p_i|p_{-i}) - \frac{A_i b_i}{q_i} > 0$.

The proof is shown in Appendix A. This lemma assures that the solution of equations (5.2) and (5.3) represents a Nash equilibrium under this sufficiency condition. This condition is likely to be satisfied under most real world situations. Specifically, $d_i(p_i|p_{-i})$ is the annual demand, which is likely to be much larger that the ratio of the parameters, as shown.

Lemma 5.2: The equilibrium found from equation (5.2) and (5.3) is the unique equilibrium for the retailers' problem.

The proof for Lemma 5.2 is also shown in Appendix A. Lemmas 5.1 and 5.2 together guarantee a global optimum solution if the sufficiency condition stated in the former is satisfied.

5.2.2 Manufacturer's model

Given the ordering policies of the retailers, the supplier's decision is to choose its review period and the allowable delay in payments that is offered to all the retailers, such that the latter's profit is maximized, under the assumption that it knows how the retailers will react to the credit term. We further assume that there is a large number of retailers, such that its inventory depletion rate can be considered to be approximately uniform and its inventory level will be independent of the ordering sequence and the exact time between two consecutive orders. The manufacturer's problem, hence, can be expressed as

Max $\Pi_m(t,\theta)$

$$= (v-m)\sum d_i(p_i|p_{-i}) - \frac{A_m}{\theta} - \frac{1}{2}\sum d_i\,\theta\left(1-\frac{\sum d_i(p_i|p_{-i})}{R}\right)h_m$$

$$-\left(\sum d_i(p_i|p_{-i})\right)th_m^c, \tag{5.7}$$

s.t. $$d_i(p_i|p_{-i}) - b_i(p_i - v) + \frac{b_i}{q_i}A_i - b_i t h_i^c = 0,\ \forall\, all\ i,j \tag{5.2}$$

$$\frac{d_i(p_i|p_{-i})A_i}{q_i^2} - \frac{h_i}{2} = 0,\ \forall\, all\ i,j \tag{5.3}$$

$$d_i(p_i|p_{-i}) = a_i - b_i p_i + \textstyle\sum_{j\neq i} c_{ij}p_j\,, p_i > m\,,\ \forall\, all\ i,j$$

When the retailers are homogeneous, then at the equilibrium, the optimal prices and quantities are the same. In this case, the manufacturer's problem can be written as follows:

$$\text{Max} \quad \Pi_m(t,\theta)$$

$$= (v-m)nd(p) - \frac{A_m}{\theta} - \frac{1}{2}nd(p)\theta\left(1-\frac{nd(p)}{R}\right)h_m - nd(p)th_m^c, \tag{5.8}$$

$$\text{s.t.} \;\; d(p) - b(p-v) + \frac{b}{q}A_r - bth_r^c = 0, \tag{5.9}$$

$$\frac{dA_r}{q^2} - \frac{h_r}{2} = 0, \;\; \forall \; all \; i,j \tag{5.10}$$

$$d(p) = a - bp + (n-1)cp.$$

The supplier's non-linear optimization problem depicted above is solvable via one of many available software packages. We employ NEOS Solver for this purpose.

5.2.3 Supply Chain's model

We now examine the case where a single decision maker maximizes the total supply chain profit by choosing the pricing and ordering decisions for a set of heterogeneous retailers and the production cycle time for the manufacturer. The supply chain's problem, in which the total profit is the sum of equation (5.1) and (5.7), is

$$\text{Max} \;\; \Pi(p_i, q_i, \tau) = \sum_{i=1}^{n}\left[(p_i - m)d_i(p_i) - \frac{d_i(p_i|p_{-i})}{q_i}A_i - \frac{q_i}{2}h_i\right] - \frac{A_m}{\theta}$$

$$-\frac{1}{2}\sum d_i(p_i|p_{-i})\,\theta\left(1-\frac{\sum d_i(p_i|p_{-i})}{R}\right)h_m. \tag{5.11}$$

By setting the first order optimality conditions for (5.11) to zeros, we obtain the following:

$$a_i - 2b_i p_i + mb_i + \frac{A_i b_i}{q_i} + \frac{1}{2} h_m \theta \left[b_i - \frac{2b_i \sum d_i(p_i|p_{-i})}{R} \right] = 0,\ \forall\ all\ i, j \quad (5.12)$$

$$\frac{d_i(p_i|p_{-i})}{q_i^2} A_i - \frac{h_i}{2} = 0,\ \forall\ all\ i, j \quad (q_i \text{ is independent of } \theta) \quad (5.13)$$

$$\frac{A_m}{\theta^2} - \frac{1}{2} \sum d_i(p_i|p_{-i}) \left(1 - \frac{\sum d_i(p_i|p_{-i})}{R}\right) h_m = 0. \quad (5.14)$$

When the retailers are homogeneous, the problem can be simplified to:

$$\text{Max } \Pi(p, q, \theta) = n[(p - m)d(p) - \frac{d(p)}{q} A_r - \frac{q}{2} h_r] - \frac{A_m}{\theta}$$
$$- \frac{1}{2} nd(p)\theta \left(1 - \frac{nd(p)}{R}\right) h_m. \quad (5.11)$$

As before, the first order optimality conditions are:

$$a - 2bp + mb + \frac{A_r b}{q} + \frac{1}{2} h_m \theta \left[b - \frac{2bnd(p)}{R} \right] = 0, \quad (5.12)$$

$$\frac{d(p)}{q^2} A_r - \frac{h_r}{2} = 0, \quad (5.13)$$

$$\frac{A_m}{\theta^2} - \frac{1}{2} nd(p) \left(1 - \frac{nd(p)}{R}\right) h_m = 0. \quad (5.14)$$

Once again, the optimal solutions for both the heterogeneous and homogeneous retailers cases are obtained via NEOS solver.

5.3 Numerical example and sensitivity analysis

At this juncture, a set of numerical experiments, as outlined below, is conducted to explore the properties of the models developed above. A thorough sensitivity analysis is performed to examine the effects of the relevant parameters on the trade credit period length and the supply chain performance.

Example 5.1:

Parameters:

$a = 2000, b = 50, c = 1, v = \$29, m = \$10$

$A_m = 400, A_r = 50,\ h_m = h_r = 12, h^c = 9, \mathrm{t}_0 = 0.25,\ R = 5000, n = 10.$

Optimal solution for decentralized SC:

$p^* = 38.7,\ q^* = 58.3, d^* = 408.3, T^* = 0.166, t^* = 0.168$

$\Pi_m^* = 66607.3, \Pi_i^* = 3879.2, \Pi^* = 105399.$

Optimal solution for centralized SC:

$p^* = 29.3,\ q^* = 81.5, d^* = 797.3, T^* = 0.203, t_{min} = 0.53, t_{max} = 1.02, \Pi^* =$ $140431.$

Example 5.2:

Parameters:

$a = 2000, b = 30, c = 1, v = \$28, m = \$10$

$A_m = 400, A_r = 50,\ h_m = h_r = 12, h^c = 9, \mathrm{t}_0 = 0.25,\ R = 5000, n = 5.$

Optimal solution for decentralized SC:

$p^* = 45.9,\ q^* = 81.9, d^* = 805.6, T^* = 0.134, t^* = 0$

$\Pi_m^* = 66545.3, \Pi_i^* = 13467.4, \Pi^* = 133882.$

Optimal solution for centralized SC:

$p^* = 44.1,\ q^* = 84.4, d^* = 853.8, T^* = 0.131, t_{min} = 0.097, t_{max} = 0.109, \Pi^* = 134324$

In Figure 5.1, we describe the effects of the characteristics of consumer demand on the optimal credit period length of the manufacturer. It is clear from this figure that when the base market demand increases (a) increases, the optimal t declines, implying that the manufacturer is less inclined to offer a credit period option. On the other hand, as b increases, demand becomes more sensitive to price and the manufacturer is likely to increase the credit period length for obtaining the benefit of increased sales resulting from lower prices.

In Table 5.1, we depict the effects of the number of retailers on supply chain performance under a fixed market scenario, where the total demand is kept fixed for a given price. From this summary table, it is clear that a larger number of retailers tends to decrease the price and increase the total market demand, shifting the equilibrium closer to the coordinated supply chain optimum policy. Thus, less trade credit is necessary to serve such a role.

Finally, we examine the sensitivity analysis in terms of the effects of all the parameters on the range of the credit period. Table 5.2 shows the changes on the optimal credit length in our Stackelberg game and the minimum credit period length (changes in the maximum credit period length follow the same direction as the minimum period, which is offered to the retailer in the centralized supply chain.

In Table 5.2, we also show the changes on the trade credit period when the number of retailers increases in a variable market, where the individual demand function is fixed and the total market demand increases as the number of retailers increases. One example of such a case is the scenario of geographically dispersed retailers. The results indicated here show that both the trade credit variables would decrease with the number of the retailers. These changes, however, do not appear to be significant, especially in comparison with the changes in a fixed market, since an additional retailer in a variable market is unlikely to significantly change the intensity of competition.

As mentioned above, we find that the two bounds of the optimal credit period range change in the same direction. This implies that, when the upper limit of the range increases, the manufacture is willing to offer under the decentralized case, a longer credit period and can offer to coordinate the retailers. Similar analysis as the one shown in Figure 5.2 can also be done with respect to the the remaining parameters. Thus, we can conclude that in situations where the manufacturer is already offering the retailers a credit option, this alone may not be an efficient coordinating mechanism, resulting in an inordinately large credit payment period. In situations where such an option can serve well to coordinate the supply chain, the manufacturer will not offer a credit option under optimality. This implication represents an important guideline for practitioners under a centralized decision making scenario.

6. Conclusions and Future Research

This thesis has attempted to explore the role of the trade credit (i.e. delayed payment) option in supply chains. Both decentralized and centralized models are analyzed to provide some insights concerning the effectiveness of this widely used business practice as a tool for improving supply chain profitability. The conditions under which a trade credit policy is preferred as a means to stimulate market demand are outlined though detailed sensitivity analysis. The efficacy of trade credit as a coordination mechanism is also analyzed. The appropriate range of the length of the credit period necessary for proper coordination of the supply chain, benefiting all the concerned parties so that each of them has sufficient incentive to adopt system optimal decisions is derived.

In the deterministic part of this study, we compare two different coordination mechanisms, viz. a credit payment option and a wholesale price discount, for two-echelon supply chains in the presence of price sensitive consumer demand. Our analysis shows that the credit payment option is more effective, in terms of enhancing total supply chain profit via coordination, than the price discount offer. The share of the total supply chain profit, however, appears to be tilted more heavily in favor of the buyer at the expense of the supplier, especially for relatively short credit periods. As an alternative, a reasonable credit payment option, in line with existing industry practice, used in conjunction with a wholesale discount policy, focused on equitable profit sharing by the buyer and the supplier, results in greater total profit enhancement, compared to the implementation of only a trade credit period policy.

Our work makes it clear for practicing managers that supply chain coordination can yield substantial dividends in terms increased profitability. It will not be too farfetched, thus, to suggest to practitioners that a fair and equitable profit sharing policy through a wholesale price discount coupled with a reasonable credit payment option, allowing the buyer to make purchase payments with some time flexibility, can be more desirable economically for all concerned parties, compared to independent and individual decision making. The results obtained in this paper are likely to be helpful towards structuring supply chain contracts involving issues such as lot sizes, prices, credit payment and wholesale price discount terms, etc.

In the stochastic portion of this work, we develop models with uncertainty in market demand. Two inventory replenishment policies: the (r, Q) continuous review and the (S, T) periodic review policies are examined in a two-echelon supply chain environment. Under each policy, the buyer's pricing and purchasing decisions and the supplier's production, trade credit and/or wholesale price discount policies are determined. Detailed sensitivity analysis via a set of computational experiments is performed in this section, in order to examine the effects of some selected parameters on the optimal credit period length in the case of decentralized control. Under centralized decision making, we determine the appropriate range for the credit period allowance necessary for supply chain coordination without placing any of the parties in a disadvantageous position. In contrast with this approach, in the case of the deterministic coordination model presented earlier, a fixed trade credit period is offered by the supplier to the buyer. Sensitivity analysis of the lower and upper limits of the trade credit period with respect to supply chain performance is conducted for testing the practicability of our suggested approach under stochastic conditions.

Under the presence of multiple competing retailers, we examine the supply chain under two different scenarios with either a fixed market or a flexible market. Our goal is to explore the effects of changes in competition, in the fixed market case, and those due to changes in market size, in the flexible market environment, on the trade credit decision and the desirability of coordinating the supply chain.

Our attempts to address the important questions and issues raised in the Introduction Section and our foregoing explorations in this research area point to a set of key findings and conclusions. These are summarized below.

- In determining the respective inventory replenishment and pricing decisions with price sensitive demand, we find that under deterministic, as well as stochastic environments, in the presence of single or multiple retailers, the supplier may not always prefer to offer a trade credit option to the buyer. This finding holds for both centralized and decentralized decision making. Nevertheless, under certain operating conditions, a trade credit offer can serve as an effective coordination mechanism. More specifically, when market demand is relatively stable (i.e. low coefficient of variation) and when the buyer's cost of capital is relatively high, an appropriate trade credit policy can transfer the supply chain surplus from the supplier to the retailer more effectively.

- When a trade credit policy by itself is not sufficient to coordinate a supply chain effectively, a wholesale price quantity discount mechanism can be used, in conjunction with an appropriate trade credit offer, towards efficient profit sharing and achieving supply chain coordination. This result has not been reported in previous research. Furthermore, this approach is relatively easy to implement and appears to have some practical merit.

- It is only desirable for the supplier to offer the buyer a trade credit payment option in the decentralized game, under certain parametric conditions. The major factors of relevance here are the price-sensitivity of consumer demand and its own cost of capital. When market demand is highly sensitive to retail price and the supplier's cost of capital is relatively low, a judiciously formulated trade credit policy can be an attractive from the supplier's perspective. The effects of some other problem parameter's on the optimal trade credit policy are summarized in Table 4.2.

- In the multiple competing retailers case, as the number of retailers increases, the supplier is less willing to offer a trade credit, since such an increase implies an increase in competition and a decline in the gap between individual optimal decisions and supply chain optimal decisions. This phenomenon is more prominent in a fixed market scenario, in comparison with a flexible market, where a new retailer entering the market has less influence on the existing retailers.

In considering possible future research directions, it needs to be mentioned that we treat the supplier's production rate, R, as a constant parameter. In practice, this rate can be varied in many instances, albeit at additional cost. For example, the nominal production rate (i.e. nominal capacity) can be increased through overtime work, or decreased via labor underutilization. Note that the product's market demand, *D*, is price dependent, leading to the vendor's utilization factor *D/R*. Thus, a capability of varying the production rate may further enhance the supplier's, as well as the supply chain's profitability. Once again, for the sake of simplicity, we assume a constant production rate here and suggest that future studies consider the production rate (or, alternately, the utilization factor, ρ) as a decision variable.

An interesting future research idea in this area would be to examine a scenario involving multiple manufacturers competing with each other, where some of them offer trade credit, while the others do not. In such a case, a credit option may be desirable for a specific manufacturer (even though such a policy may not be preferable under the corresponding single manufacturer, single retailer scenario). Under such circumstances, a delayed payment option is expected to increase the market share of the manufacturer in question. Nevertheless, the modeling of the effect of trade credit on a supplier's market share would pose a challenging, albeit interesting, task. Here heterogeneous manufacturers should be assumed for the purpose of obtaining meaningful results. It would also be interesting to explore the interactions between the credit period and other relevant factors, such as price, quality, etc.

Other possibilities for future research may consider the incorporation of other supply chain coordination mechanisms, not examined in this study, in conjunction with a delayed payment option. It would, indeed, be interesting to ascertain as to how a trade credit policy would interact with other options offered, such as a customer return policy, a buy back option, etc.

In conclusion, it is hoped that our results have shed some light on the appropriate design and utilization of two important supply chain coordination tools: credit payment policy and wholesale price discount. We also hope that our work will be helpful for future researchers in extending the concepts developed here and our findings to more realistic and complex real world supply chains involving, for instance, multiple products, deteriorating or limited shelf life items and supply chains with more than two echelons.

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Appendix A: Proofs of Lemmas, Propositions and Mathematical Properties:

Proposition 3.1: In an un-coordinated supply chain with a delayed payment option, Q increases (or decreases) and p decreases or (increases) as the credit time, t, increases (or decreases).

Proof: In equation (3.8), shown in Section 3.3.1 of the text, let t increase and Q remain the same. The left-hand side of the equation does not change but the right-hand side increases. To achieve a new equilibrium by changing only Q, its value needs to increase, thus increasing the left-hand side and decreasing the right-hand side simultaneously. Since t and Q increase together, it is clear from equation (3.7) that, as a result, p must decrease. □

Lemma 3.1: $\Pi^*(n^*, p^*(n^*), Q^*(n^*))$ obtained via the algorithm outlined in Section 3.4.1 is the global optimum solution.

Proof:

The local optimum yielded by our algorithm above satisfies:

$\Pi^*((n^*+1), p^*(n^*+1), Q^*(n^*+1)) < \Pi^*(n^*, p^*(n^*), Q^*(n^*))$.

Assume there is another solution, $\Pi^*(\tilde{n}, p^*(\tilde{n}), Q^*(\tilde{n}))$, which is the global optimum, such that

$\Pi^*(\tilde{n}, p^*(\tilde{n}), Q^*(\tilde{n})) > \Pi^*((\tilde{n}-1), p^*(\tilde{n}-1)), Q^*(\tilde{n}-1)))$ and $\tilde{n} > n^*$.

1. Since $\Pi^*(n^*, p^*(n^*), Q^*(n^*)) > \Pi^*((n^*+1), p^*(n^*+1), Q^*(n^*+1)) > \Pi^*((n^*+1), p^*(n^*), Q^*(n^*))$ and that Π is a concave function in n for a fixed (p, Q) pair. We can conclude that $\Pi(n^*, p^*(n^*), Q^*(n^*)) > \Pi(\tilde{n}, p^*(n^*), Q^*(n^*))$.

2. Similarly, we can show that $\Pi(\tilde{n}, p^*(\tilde{n}), Q^*(\tilde{n})) > \Pi(n^*, p^*(\tilde{n}), Q^*(\tilde{n}))$.

3. From 1 and 2, we have,

   $\Pi(\tilde{n}, p^*(\tilde{n}), Q^*(\tilde{n})) - \Pi(\tilde{n}, p^*(n^*), Q^*(n^*)) > \Pi(n^*, p^*(\tilde{n}), Q^*(\tilde{n})) - \Pi(n^*, p^*(n^*), Q^*(n^*))$.

4. Since $\frac{\partial \Pi}{\partial n} = \frac{DA_1}{Q} n^{-2} - \frac{Q}{2}(1-\rho)h_1$, where $\frac{\partial \Pi}{\partial n}(n)$ is a decreasing function in n and $\frac{\partial \Pi}{\partial n}(\tilde{n}) < \frac{\partial \Pi}{\partial n}(n^*)$ for a fixed (p, Q) pair, we have

$$\Pi\big(\tilde{n}, p^*(\tilde{n}), Q^*(\tilde{n})\big) - \Pi\big(\tilde{n}, p^*(n^*), Q^*(n^*)\big) = \int_{p^*(n^*),Q^*(n^*)}^{p^*(\tilde{n}),Q^*(\tilde{n})} \frac{\partial \Pi}{\partial n}(\tilde{n})$$

$$< \Pi\big(n^*, p^*(\tilde{n}), Q^*(\tilde{n})\big) - \Pi\big(n^*, p^*(n^*), Q^*(n^*)\big)$$

$$= \int_{p^*(n^*),Q^*(n^*)}^{p^*(\tilde{n}),Q^*(\tilde{n})} \frac{\partial \Pi}{\partial n}(n^*).$$

   This contradicts statement 3 above.

5. Thus, the assumption made at the outset cannot hold and $(n^*, p^*(n^*), Q^*(n^*))$ represent the global optimum solution. □

Sufficient condition for a local maximum in the buyer's problem with (r, Q) policy:

The objective function for the buyer's problem is:

$$\Pi_r(r, Q, p) = (p - v)D(p) - A_r \frac{D(p)}{Q} - h_r\left(\frac{Q}{2} + r - \mu_L\right) - b\frac{D(p)}{Q} G_L(r)$$
$$+D(p)th_r^c. \tag{4.1}$$

For notational simplicity in the following proof, we use Π instead of Π_r, to denote the profit for the retailer, where the subscripts, respectively, represent the variables with respect to which the derivatives are obtained. The first order optimality conditions for the profit function (4.1) are

$$\Pi_r = \frac{\partial \Pi}{\partial r} = -h_r + b\frac{D(p)}{Q}\overline{F_L}(r) = 0,$$

$$\Pi_Q = \frac{\partial \Pi}{\partial Q} = A_r \frac{D(p)}{Q^2} - \frac{h_r}{2} + b\frac{D(p)}{Q^2} G_L(r) = 0,$$

$$\Pi_p = \frac{\partial \Pi}{\partial p} = (1 - \beta)kp^{-\beta} + \beta kp^{-(\beta+1)}[v - th_r^c + \frac{A_r}{Q} - h_r L + \frac{b}{Q} G_r(r) = 0,$$

Thus, the Hessian matrix of the profit function can be written as follows:

$$|H| = \begin{vmatrix} \Pi_{rr} & \Pi_{rQ} & \Pi_{rp} \\ \Pi_{Qr} & \Pi_{QQ} & \Pi_{Qp} \\ \Pi_{pr} & \Pi_{pQ} & \Pi_{pp} \end{vmatrix},$$

where the second order derivates are:

$$\Pi_{rr} = \frac{\partial^2 \Pi}{\partial r \partial r} = -b\frac{D(p)}{Q} f_l(r) < 0,$$

$$\Pi_{QQ} = \frac{\partial^2 \Pi}{\partial Q \partial Q} = -2\frac{D(p)}{Q^3}\left(A_r + bG_L(r)\right) < 0,$$

$$\Pi_{pp} = \frac{\partial^2 \Pi}{\partial p \partial p} = \beta(\beta - 1)kp^{-(\beta+1)}$$

$$-\beta(\beta + 1)kp^{-(\beta+2)}\left[v - th_r^c + \frac{A_r}{Q} - h_r L + \frac{b}{Q} G_r(r)\right] < 0,$$

$$\Pi_{Qr} = \frac{\partial^2 \Pi}{\partial Q \partial r} = -b\frac{D(p)}{Q^2}\overline{F_L}(r) < 0,$$

$$\Pi_{rp} = \frac{\partial^2 \Pi}{\partial r \partial p} = -\beta kp^{-(\beta+1)}\frac{b}{Q}\overline{F_L}(r) < 0,$$

$$\Pi_{Qp} = \frac{\partial^2 \Pi}{\partial Q \partial p} = -\beta kp^{-(\beta+1)}\frac{1}{Q^2}\left(A_r + bG_L(r)\right) < 0.$$

The second principle minor at the stationary point is:

$$\begin{vmatrix} \Pi_{rr} & \Pi_{rQ} \\ \Pi_{Qr} & \Pi_{QQ} \end{vmatrix}_{(r,Q,p)=(r^*,Q^*,p^*)}$$

$$= \left[f_L(r) - \frac{h_r}{bD(p)}\right]\frac{h_r Q^2}{D(p)}\left(\frac{bD^2(p)}{Q^2}\right) = \left[f_L(r) - \frac{\overline{F_L}(r)}{Q}\right]\frac{h_r Q^2}{D(p)}\left(\frac{bD^2(p)}{Q^2}\right)$$

$$> 0, \qquad if\ z > \frac{1}{Q} \tag{C1}$$

In order to compute the determinant, $|H|$, we first calculate the following terms:

$$\begin{vmatrix} \Pi_{QQ} & \Pi_{Qp} \\ \Pi_{pQ} & \Pi_{pp} \end{vmatrix}$$

$$= -2\frac{D(p)}{Q^3}\left(A_r + bG_L(r)\right) \cdot$$

$$\left[\beta(\beta-1)kp^{-(\beta+1)}-\beta(\beta+1)kp^{-(\beta+2)}\cdot\left[v-th_r^c+\frac{A_r}{Q}-h_rL+\frac{b}{Q}G_r(r)\right]\right]$$

$$-\left[\beta kp^{-(\beta+1)}\frac{b}{Q}\overline{F_L}(r)\right]^2,$$

$$\begin{vmatrix}\Pi_{Qr} & \Pi_{Qp}\\ \Pi_{pr} & \Pi_{pp}\end{vmatrix}$$

$$=-b\frac{D(p)}{Q^2}\overline{F_L}(r)*$$

$$\left[\beta(\beta-1)kp^{-(\beta+1)}-\beta(\beta+1)kp^{-(\beta+2)}\cdot\left[v-th_r^c+\frac{A_r}{Q}-h_rL+\frac{b}{Q}G_r(r)\right]\right]$$

$$-\beta kp^{-(\beta+1)}\frac{b}{Q}\overline{F_L}(r)\cdot\beta kp^{-(\beta+1)}\frac{1}{Q^2}\left(A_r+bG_L(r)\right).$$

Hence, we have

$$\begin{vmatrix}\Pi_{Qr} & \Pi_{QQ}\\ \Pi_{pr} & \Pi_{pQ}\end{vmatrix}$$

$$=-b\frac{D(p)}{Q^2}\overline{F_L}(r)\cdot\left[-\beta kp^{-(\beta+1)}\frac{1}{Q^2}\left(A_r+bG_L(r)\right)\right]$$

$$-2\frac{D(p)}{Q^3}\left(A_r+bG_L(r)\right)\cdot\beta kp^{-(\beta+1)}\frac{b}{Q}\overline{F_L}(r).$$

If we expand the Hessian matrix by the first row:

$$|H|_{(r,Q,p)=(r^*,Q^*,p^*)}$$

$$=\Pi_{rr}\cdot\begin{vmatrix}\Pi_{QQ} & \Pi_{Qp}\\ \Pi_{pQ} & \Pi_{pp}\end{vmatrix}_{(r,Q,p)=(r^*,Q^*,p^*)}-\Pi_{rQ}\cdot\begin{vmatrix}\Pi_{Qr} & \Pi_{Qp}\\ \Pi_{pr} & \Pi_{pp}\end{vmatrix}_{(r,Q,p)=(r^*,Q^*,p^*)}$$

$$+\Pi_{rp}\cdot\begin{vmatrix}\Pi_{Qr} & \Pi_{QQ}\\ \Pi_{pr} & \Pi_{pQ}\end{vmatrix}_{(r,Q,p)=(r^*,Q^*,p^*)}$$

$$=-b\frac{D}{Q}f_L(r)\left[\frac{h_r(\beta-1)D}{pQ}-\frac{\beta^2h_r^2}{4p^2}\right]+\frac{h_r}{Q}\left[\frac{h_r(\beta-1)D}{pQ}-\frac{\beta^2h_r^2}{2p^2}\right]-\frac{\beta h_r}{p}\left[\frac{\beta h_r^2}{2pQ}-\frac{\beta h_r^2}{pQ}\right]$$

$$= -b\frac{D}{Q}f_L(r)\left[\frac{h_r(\beta-1)D}{pQ} - \frac{\beta^2 h_r^2}{4p^2}\right] + \frac{(\beta-1)h_r^2 D}{pQ^2}$$

$$= -\frac{(\beta-1)h_r D^*(p) b f_L(r)}{p^* Q^{*2}}\left[D - \frac{\beta^2}{4(\beta-1)}\cdot\frac{h_r}{p}\cdot Q + \frac{h_r}{b}\cdot\frac{1}{f_L(r)}\right]$$

$$< 0, \qquad if\ D > \frac{\beta^2}{4(\beta-1)}\cdot\frac{h_r}{p}\cdot Q + \frac{h_r}{b}\cdot\frac{1}{f_L(r)}) \qquad \text{(C2)}$$

Thus, the second order principle minor is positive under condition C(1), and the determinant of the Hessian matrix is negative under condition C(2), which are likely to hold in practical applications. When C1 and C2 are satisfied, the Hessian matrix of the stationary point is negative definite and the stationary point is a local maximum.

It can also be proved numerically that the Hessian at the stationary point is negative definite.

Proposition 4.1:

1) The optimal profit of the retailer increases in t.
2) The profit function of the retailer is submodular in (r, Q), (r, p) and (p, Q).
3) The optimal retail price of the retailer p^* decreases in t.

Proof:

1). According to the envelop theorem,

$$\frac{d\Pi_r^*}{dt} = \frac{\partial \Pi_\mathrm{r}}{\partial t} = D(p)h_r^c > 0.$$

Thus, the maximum profit increases in t.

2). Taking the second-order partial derivative,

$$\frac{\partial^2 \Pi}{\partial r \partial Q} = -b \frac{D(p)}{Q^2} \overline{F_L}(r) < 0,$$

$$\frac{\partial^2 \Pi}{\partial r \partial p} = -\beta \cdot b \cdot \frac{D(p)}{pQ} \cdot \overline{F_L}(r) < 0,$$

$$\frac{\partial^2 \Pi}{\partial p \partial Q} = -\beta \cdot \frac{D(p)}{p} \cdot \left[\frac{A_r}{Q^2} + \frac{b}{Q^2} G_L(r)\right] < 0.$$

By the definition of submodularity, Π_r is submodular in (r, Q), (r, p) and (p, Q).

3).The variable *t* is in the objective function, associated only with *p*, i.e.

$$\frac{\partial \Pi_\mathrm{r}}{\partial p \partial t} = -\beta \frac{D(p)}{p} h_r^c < 0.$$

According to the reciprocity condition, $\frac{\partial \Pi_\mathrm{r}}{\partial p \partial t} \cdot \frac{\partial p^*}{\partial t} > 0$. Therefore, $\frac{\partial p^*}{\partial t} < 0$, i.e., the optimal price for the retailer decreases in t. □

Sufficient condition for a local maximum in the buyer's problem under the (*S, T*) policy:

The objective function for the buyer's problem is:

$$\Pi_r(z, T, p) = (p - v)D(p) - \frac{A_r}{T} - h_r\left(\frac{DT}{2} + zD(p)\sigma\sqrt{T + L}\right)$$

$$-\frac{b}{T}\sigma_{T+L}\big[f(z)-z[1-F(z)]\big]+D(p)th_r^c\,, \tag{4.11}$$

Once again, for notational simplicity in the following proof, we use Π instead of Π_r, to denote the profit for the retailer, where the subscripts, respectively, represent the variables with respect to which the derivatives are obtained. Consequently, the first order optimality conditions for (4.11) are

$$\Pi_z=\frac{\partial\Pi}{\partial z}=\frac{b}{T}[F(z^*)-1]-h_r=0,$$

$$\Pi_T=\frac{\partial\Pi}{\partial T}$$

$$=\sqrt{T+L}(hD(p)T^2-2A_r)-z\sigma_d h_r T^2+b\sigma_d\big[f(z)-z[1-F(z)]\big](T+2L)=0,$$

$$\Pi_p=\frac{\partial\Pi}{\partial p}=(1-\beta)p+\beta[v-th_r^c+h_r\left(\frac{T}{2}+z\sigma\sqrt{T+L}\right)$$

$$+\frac{b}{T}\sigma\sqrt{T+L}\cdot[f(z)-z[1-F(z)]]]=0.$$

The Hessian matrix of the profit function can be written as follows:

$$|H|=\begin{vmatrix}\Pi_{zz} & \Pi_{zp} & \Pi_{zT}\\ \Pi_{pz} & \Pi_{pp} & \Pi_{pT}\\ \Pi_{Tz} & \Pi_{Tp} & \Pi_{TT}\end{vmatrix},$$

where the second order derivates are:

$$\Pi_{zz}=\frac{\partial^2\Pi}{\partial z\partial z}=-\frac{b}{T}D(p)\sigma\sqrt{T+L}f(z)<0,$$

$$\Pi_{pp}=\frac{\partial^2\Pi}{\partial p\partial p}$$

$$= \beta(\beta-1)kp^{-(\beta+1)} - \beta(\beta+1)kp^{-(\beta+2)}[v - th_r^c + h_r\left(\frac{T}{2} + z\sigma\sqrt{T+L}\right)$$

$$+\frac{b}{T}\sigma\sqrt{T+L}\cdot[f(z) - z[1-F(z)]]],$$

$$\Pi_{TT} = \frac{\partial^2\Pi}{\partial T\partial T} = -\frac{2A_r}{T^3} + zD(p)\sigma h_r\frac{1}{4\left(\sqrt{T+L}\right)^3}$$

$$-bD(p)\sigma\left[f(z) - z[1-F(z)]\right]\cdot\frac{3T^2+12TL+4L^2}{4T^3(T+L)\sqrt{T+L}},$$

$$\Pi_{zp} = \frac{\partial^2\Pi}{\partial z\partial p} = -\beta\frac{D(p)}{p}\left(-h_r\sigma\sqrt{T+L} - \frac{b}{T}\sigma\sqrt{T+L}(F(z)-1)\right),$$

$$\Pi_{zT} = \frac{\partial^2\Pi}{\partial z\partial T} = -h_rD(p)\sigma\frac{1}{2\sqrt{T+L}} - bD(p)\sigma(F(z)-1)\frac{-(T+2L)}{2T^2\sqrt{T+L}},$$

$$\Pi_{pT} = \frac{\partial^2\Pi}{\partial p\partial T} = -\beta\frac{D(p)}{p}\left[-\frac{h_r}{2} - z\sigma h_r\frac{1}{2\sqrt{T+L}} + b\sigma\left[f(z) - z[1-F(z)]\right]\frac{(T+2L)}{2T^2\sqrt{T+L}}\right].$$

The second principle minor at the stationary point is:

$$\begin{vmatrix}\Pi_{zz} & \Pi_{zp}\\ \Pi_{pz} & \Pi_{pp}\end{vmatrix}_{(z,p,T)=(z^*,p^*,T^*)}$$

$$= \frac{bD^2(p)}{Tp}\sigma\sqrt{T+L}\cdot f(z)(\beta-1) > 0.$$

If we expand the Hessian matrix by the first row, we obtain

$$|H|_{(z,p,T)=(z^*,p^*,T^*)}$$

$$= \Pi_{zz}\cdot\begin{vmatrix}\Pi_{pp} & \Pi_{pT}\\ \Pi_{Tp} & \Pi_{TT}\end{vmatrix}_{(z,p,T)=(z^*,p^*,T^*)} - \Pi_{zp}\cdot\begin{vmatrix}\Pi_{pz} & \Pi_{pT}\\ \Pi_{Tz} & \Pi_{TT}\end{vmatrix}_{(z,p,T)=(z^*,p^*,T^*)}$$

$$+\Pi_{zT}\cdot\begin{vmatrix}\Pi_{pz} & \Pi_{pp}\\ \Pi_{Tz} & \Pi_{Tp}\end{vmatrix}_{(z,p,T)=(z^*,p^*,T^*)}$$

$$= -\frac{b}{T} D(p)\sigma\sqrt{T+L} \cdot f(z) \cdot \left[-(\beta-1)\frac{D}{p} \cdot \Pi_{TT} - \frac{\beta^2 A_r^2}{T^4 p^2}\right]$$

$$+(\beta-1)\frac{D(p)}{p}\left(h_r D(p)\sigma \frac{\sqrt{T+L}}{T}\right)^2$$

$$= -\frac{b}{T}\frac{D^2(p)}{p}\sigma\sqrt{T+L} \cdot f(z)(\beta-1) \cdot [\frac{A_r}{T^3} \cdot \frac{T^2+4L^2}{2(T+L)(T+2L)} + h_r D(p) \cdot \frac{2T^2+12TL+4L^2}{4T(T+L)(T+2L)}$$

$$+zD(p)\sigma h_r \frac{2T+L}{2T(T+L)^{\frac{3}{2}}} - \frac{\beta^2 A_r^2}{(\beta-1)D(p)pT^4} - \frac{h_r^2 D(p)\sigma\sqrt{T+L}}{bTf(z)}]. \quad \text{(C3)}$$

Thus, when C3 is negative, the Hessian matrix of the stationary point is negative definite and the stationary point is a local maximum.

It can also be proved numerically that the Hessian matrix at the stationary point is negative definite.

Proposition 4.2:

1) The optimal profit of the retailer increases in t.
2) The profit function of the retailer is submodular in (z, T).
3) The optimal retail price of the retailer p^* decreases in t.

Proof:

1). The proof is similar to proposition 4.1. Since

$$\frac{d\Pi_r^*}{dt} = \frac{\partial\Pi_\mathrm{r}}{\partial t} = D(p)h_r^c > 0,$$

the manufacturer's optimal profit increases in t.

2). Again, taking the second-order partial derivatives yield the following:

$$\frac{\partial^2 \Pi}{\partial z \partial T} = b\sigma_d[1 - F(z)]\frac{-(T + 2L)}{2T^2\left(\sqrt{T + L}\right)} < 0,$$

$$\frac{\partial^2 \Pi}{\partial z \partial p} = \beta\frac{D(p)}{p}\sigma\sqrt{T + L}\left[h_r - \frac{b}{T}\overline{F_L}(r)\right],$$

$$\frac{\partial^2 \Pi}{\partial p \partial T} = \beta h_r\frac{D(p)}{2p} - b\beta(p)\frac{D(p)}{p}\sigma[f(z) - z[1 - F(z)]]\frac{T + 2L}{2T^2(\sqrt{T + L})}\cdot$$

Thus, Π_r is only submodular in (z, T).

3). The variable t only appears in the first-order condition of p, not in Q and r, and

$$\frac{\partial \Pi_{\mathrm{r}}}{\partial p \partial t} = -\beta\frac{D(p)}{p}h_r^c < 0.$$

$\frac{\partial p^*}{\partial t}$ has the same sign as $\frac{\partial \Pi_{\mathrm{r}}}{\partial p \partial t}$, which is negative. Thus, the optimal price decreases in t. □

Lemma 5.1: $\Pi_i(p_i, q_i|p_{-i})$ is jointly concave on p_i and q_i, if $4d_i(p_i|p_{-i}) - \frac{A_i b}{q_i} > 0$.

Proof:

The objective function is given by

$$\Pi_i(p_i, q_i|p_{-i}) = (p_i - v)d_i(p_i|p_{-i}) - \frac{d_i(p_i|p_{-i})}{q_i}A_i - \frac{q_i}{2}h_i + d_i(p_i|p_{-i})th_i^c. \qquad (5.1)$$

The second order derivatives of this function with respect to p_i lead to

$$\frac{\partial^2\Pi_i}{\partial {p_i}^2} = -2b_i < 0,$$

$$\frac{\partial^2\Pi_i}{\partial q_i^2} = -2\frac{d_i(p_i|p_{-i})A_i}{q_i^2} < 0,$$

$$\frac{\partial^2\Pi_i}{\partial p_i \partial q_i} = -\frac{b_i A_i}{q_i^2} < 0.$$

Hence the determinant of the Hessian matrix is as follows:

$$|H| = \begin{vmatrix} \frac{\partial^2\Pi_i}{\partial {p_i}^2} & \frac{\partial^2\Pi_i}{\partial p_i \partial q_i} \\ \frac{\partial^2\Pi_i}{\partial p_i \partial q_i} & \frac{\partial^2\Pi_i}{\partial q_i^2} \end{vmatrix}$$

$$= \frac{b_i A_i}{q_i^3}\left[4d_i(p_i|p_{-i}) - \frac{A_i b_i}{q_i}\right]$$

Clearly, this is strictly positive if $4d_i(p_i|p_{-i}) - \frac{A_i b_i}{q_i} > 0$.

In other words if $4d_i(p_i|p_{-i}) - \frac{A_i b_i}{q_i} > 0$, the Hessian matrix is strictly negative definite, indicating that the profit function (5.1) is strictly concave on p_i and q_i. □

Lemma 5.2: The equilibrium found in equation (5.2) and (5.3) is the unique equilibrium of the retailers' problem.

Proof:

The retailer's profit function is expressed as

$$\Pi_i(p_i, q_i|p_{-i}) = (p_i - v)d_i(p_i|p_{-i}) - \frac{d_i(p_i|p_{-i})}{q_i}A_i - \frac{q_i}{2}h_i + d_i(p_i|p_{-i})th_i^c, \qquad (5.1)$$

the first order optimality conditions for which are

$$d_i(p_i|p_{-i}) - b_i(p_i - v) + \frac{b_i}{q_i}A_i - b_i t h_i^c = 0,\ \forall\ all\ i, j, \tag{5.2}$$

$$\frac{d_i(p_i|p_{-i})A_i}{q_i^2} - \frac{h_i}{2} = 0,\ \forall\ all\ i, j. \tag{5.3}$$

Rewriting condition (5.3) yields

$$q_i = \sqrt{\frac{2d_i(p_i|p_{-i})A_i}{h_i}} \tag{5.15}$$

This indicates that each q_i is a function of the price p_i only, and does not depend on the other order quantities. Inserting the result (5. 15) into the retailer's profit function (5.1), we can reduce the objective function (5.1) as a function of variable p_i only; i.e.

$$\Pi_i(p_i|p_{-i}) = (p_i - v)d_i(p_i|p_{-i}) - \sqrt{2d_iA_ih_i} + d_i(p_i|p_{-i})th_i^c,$$

leading the following second order conditions:

$$\frac{\partial\Pi_i(p_i|p_{-i})}{\partial p_i^2} = -2b_i - \sqrt{\frac{A_ih_i}{2}} \cdot b_i^2 d_i^{-\frac{3}{2}}$$

$$\frac{\partial\Pi_i(p_i|p_{-i})}{\partial p_i \partial q_i} = \sqrt{\frac{A_ih_i}{2}} \cdot b_i c_i d_i^{-\frac{3}{2}}$$

$$\frac{\partial\Pi_i(p_i|p_{-i})}{\partial p_i^2} - \sum_{j\neq i} \left|\frac{\partial\Pi_i(p_i|p_{-i})}{\partial p_i \partial q_i}\right| = -2b_i - \sqrt{\frac{A_ih_i}{2}} \cdot b_i d_i^{-\frac{3}{2}}\left(b_i - \sum_{j\neq i} c_{ij}\right) < 0$$

Thus, satisfaction of the above sufficiency condition guarantees that the equilibrium solution for the retailer's problem is unique. □

Appendix B: Tables:

Table 3.1: Summary of solutions

Cases	p	Q	D	n	t	v	d	Π_m	Π_r	Π
1 U	45.61	185.5	1032.2	3	-	-	-	9138.9	25873.8	35012.8
2a U{t=1 m}	45.13	187.2	1051.8	3	0.083	-	-	9140.7	26090.9	35231.6
2b U{t=3 m}	44.18	190.9	1092.8	3	0.250	-	-	9139.7	26537.6	35677.3
2c U{t=opt}	44.80	188.5	1065.8	3	0.14	-	-	9141.0	26244.5	35385.5
3 C{d}	22.91	319.6	3564.2	5	-	13.67	31.6%	11253.4	31909.4	43162.8
4a C{t=3 m}	23.17	320.1	3492.3	5	0.083	-	-	32571.8	10950.5	43522.3
4b C{t=2.25y}	20.70	376.7	4277.9	5	2.250	-	-	20746.0	25927.4	46673.5
5a C{d,t=1 m}	22.84	326	3582.8	5	0.083	13.81	30.9%	11268.0	31950.9	43218.9
5b C{d,t=3 m}	22.68	330	3629.5	5	0.250	14.05	29.8%	11347.1	32175.2	43522.3

Table 3.2: Effects of varying β

β	n	t	Π_1	Π_2	Π	$n*$	d	Π_1^*	Π_2^*	Π^*	Increase
1.76	3	0.00	10203.8	30209.5	40413.3	6	29.32%	12535.4	37112.6	49648.0	22.85%
1.77	3	0.00	9928.5	29059.2	38987.6	6	29.33%	12232.7	35803.4	48036.1	23.21%
1.78	3	0.04	9659.4	28057.9	37717.3	6	29.39%	11903.1	34575.3	46478.4	23.23%
1.79	3	0.09	9397.0	27135.0	36532.0	5	29.49%	11614.0	33480.3	45034.3	23.29%
1.80	3	0.18	9138.9	25873.8	35012.8	5	29.77%	11347.1	32175.2	43522.3	24.30%

Table 4.1 Effects of β_i on t^*

beta	t
1.70	0
1.72	0
1.74	0.0147
1.76	0.0721
1.78	0.1257
1.8	0.1760
1.82	0.2227
1.84	0.2665
1.86	0.3000
1.88	0.3000
1.90	0.3000

Table 4.2 Effects of parameters on t^* and t_{min}/t_{max}

(a) Parameter Changes that increase $t^*$and $t_{min}/t_{\max}$

$k\uparrow$	$\beta\uparrow$	$v\uparrow$	$L\uparrow$	$\sigma\uparrow$	$A_r\uparrow$	$b\uparrow$
$t\uparrow$	$t\uparrow$	$t\uparrow$	$t\uparrow$	$t\uparrow$	$t\uparrow$	$t\uparrow$
$t_{min}\uparrow$	$t_{min}\uparrow$	$t_{min}\uparrow$	$t_{min}\uparrow$	$t_{min}\uparrow$	$t_{min}\uparrow$	$t_{min}\uparrow$

(b) Parameter Changes that decreases $t^*$and $t_{min}/t_{\max}$

$h^c\uparrow$	$m\uparrow$	$A_m\uparrow$
$t\downarrow$	$t\downarrow$	$t\downarrow$
$t_{min}\downarrow$	$t_{min}\downarrow$	$t_{min}\downarrow$

Table 5.1 Effects of n (in variable market) on t^*

$n(FM)$	t	p	q	d	D	Π_m	Π_i	Π
7	0.3877	38.93	69.94	570.8	3995.6	57180.1	6830.8	104995
8	0.3097	38.85	64.75	503.2	4025.6	60456.8	5582.6	105118
9	0.2363	38.78	61.27	450.5	4054.5	63599.8	4628.0	105252
10	0.1676	38.71	58.33	408.3	4083.0	66607.3	3879.2	105399
11	0.1036	38.63	55.82	373.9	4112.4	69478.3	3280.1	105559
12	0.0444	38.56	53.63	345.2	4141.8	72212.4	2793.4	105733
13	0.0000	38.43	51.84	322.4	4191.7	74807.5	2419.7	106263

Table 5.2 Effects of parameters on t^* and t_{min}/t_{max}

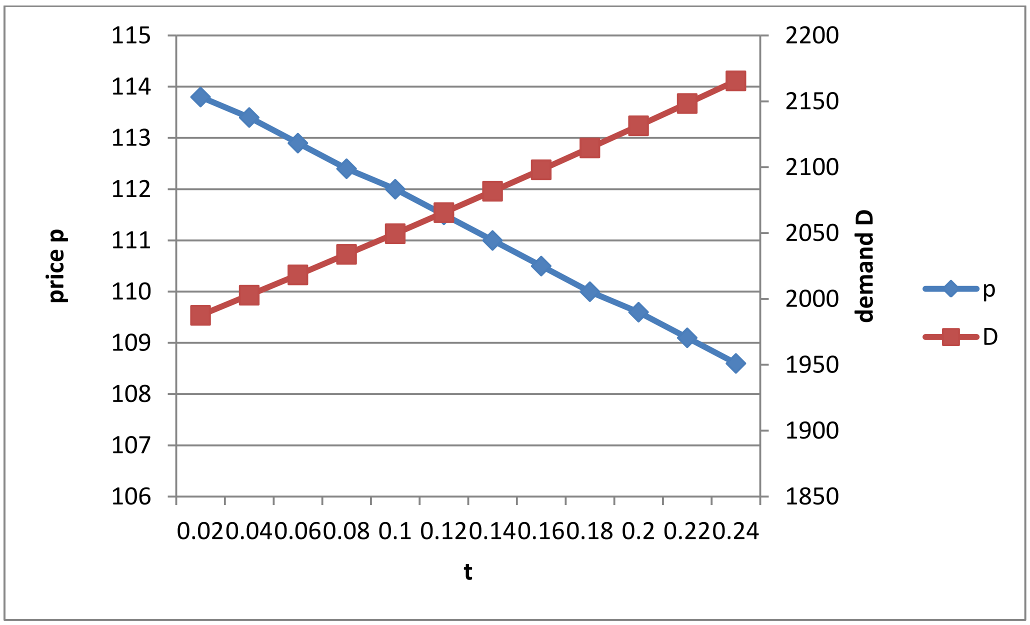
(a) Parameter Changes that increase $t^*$and $t_{min}/t_{\max}$

$b_i \uparrow$	$v \uparrow$	$A_r \uparrow$
$t \uparrow$	$t \uparrow$	$t \uparrow$
$t_{min} \uparrow$	$t_{min} \uparrow$	$t_{min} \uparrow$

(a) Parameter Changes that decreases $t^*$and $t_{min}/t_{\max}$

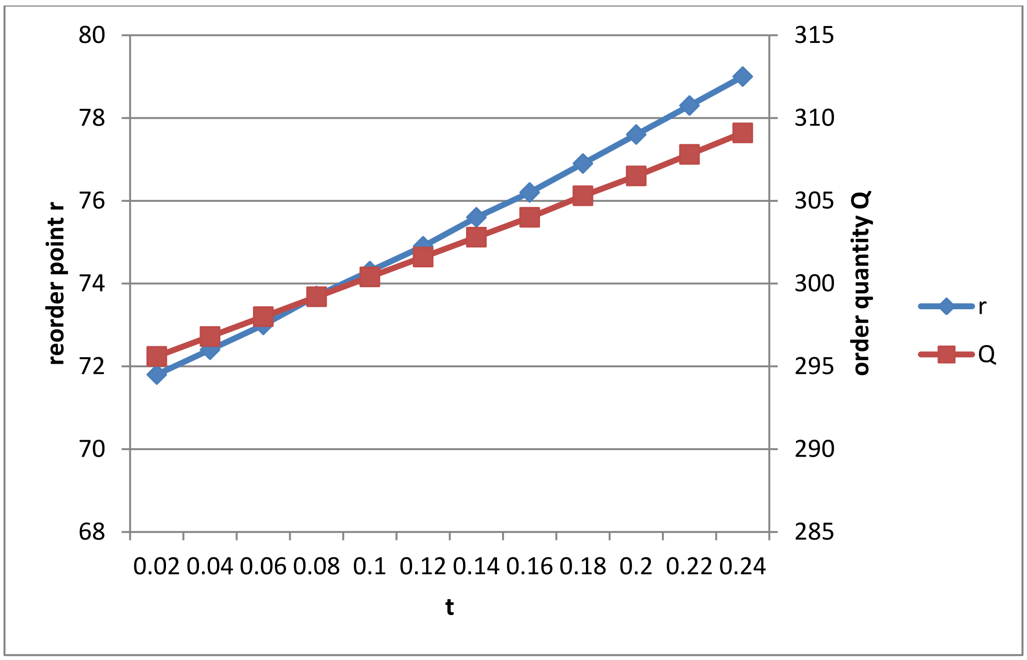
$a_i \uparrow$	$c_i \uparrow$	$h^c \uparrow$	$m \uparrow$	$A_m \uparrow$	$R \uparrow$	$n(VM) \uparrow$	$n(FM) \uparrow$
$t \downarrow$	$t \downarrow$	$t \downarrow$	$t \downarrow$	$t \downarrow$	$t \downarrow$	$t \downarrow$	$t \downarrow$
$t_{min} \downarrow$	$t_{min} \downarrow$	$t_{min} \downarrow$	$t_{min} \downarrow$	$t_{min} \downarrow$	$t_{min} \downarrow$	$t_{min} \downarrow$	$t_{min} \downarrow$

Appendix C: Figures:



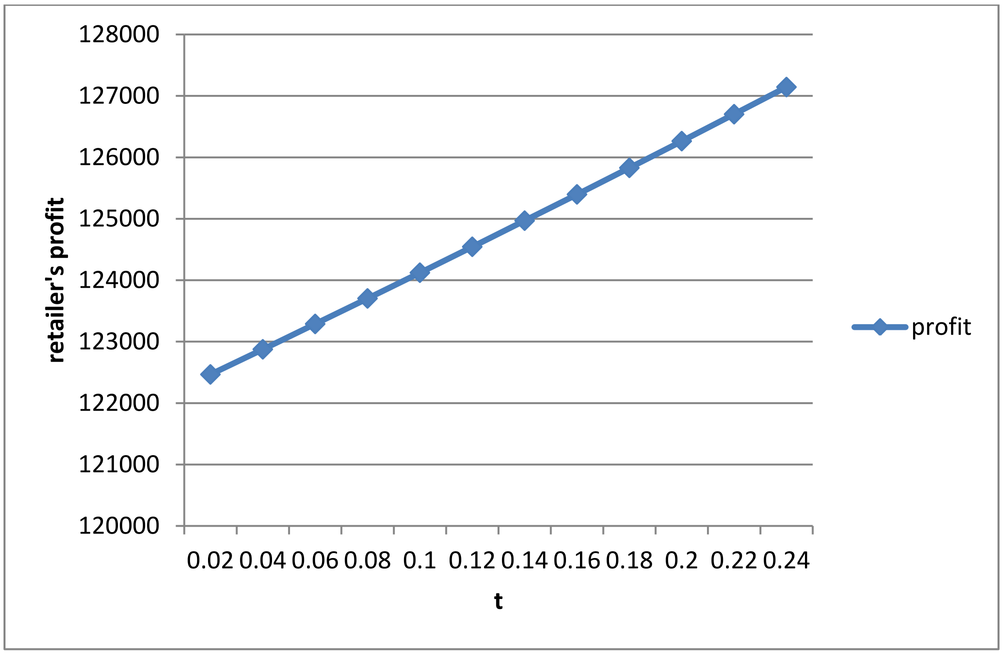
(a) Effects of t on retailer's p and D

Fig 4.1: Effects of t on retailer's variables in (r, Q) policy



(b) Effects of t on retailer's r and Q

Fig 4.1: Effects of t on retailer's variables in (r, Q) policy



(c) Effects of t on retailer's profit Π_r

Fig 4.1: Effects of t on retailer's variables in (r, Q) policy

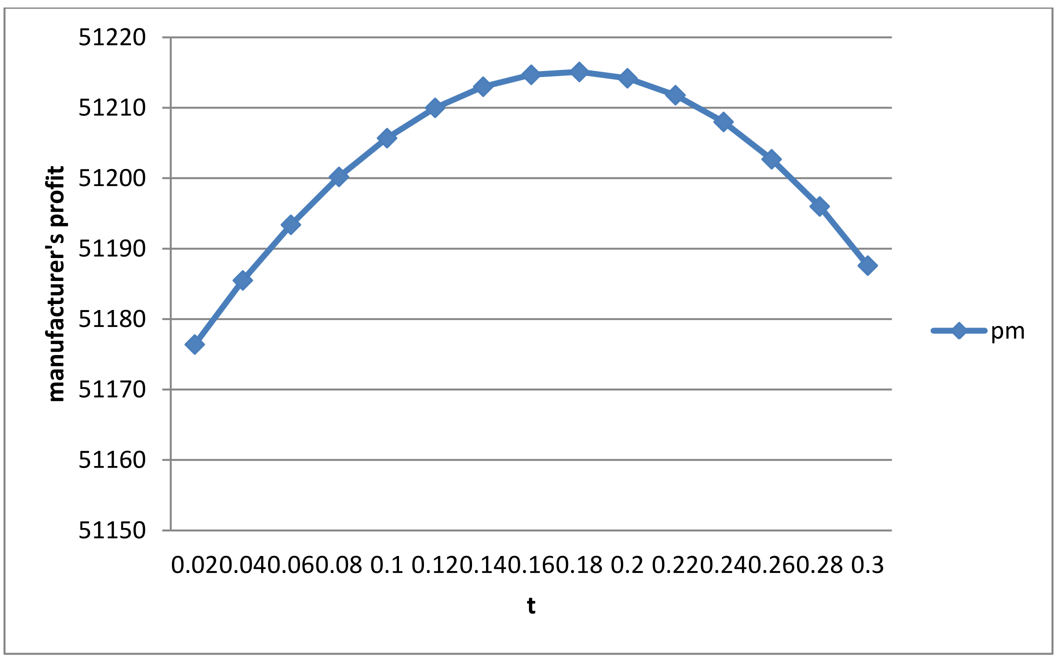
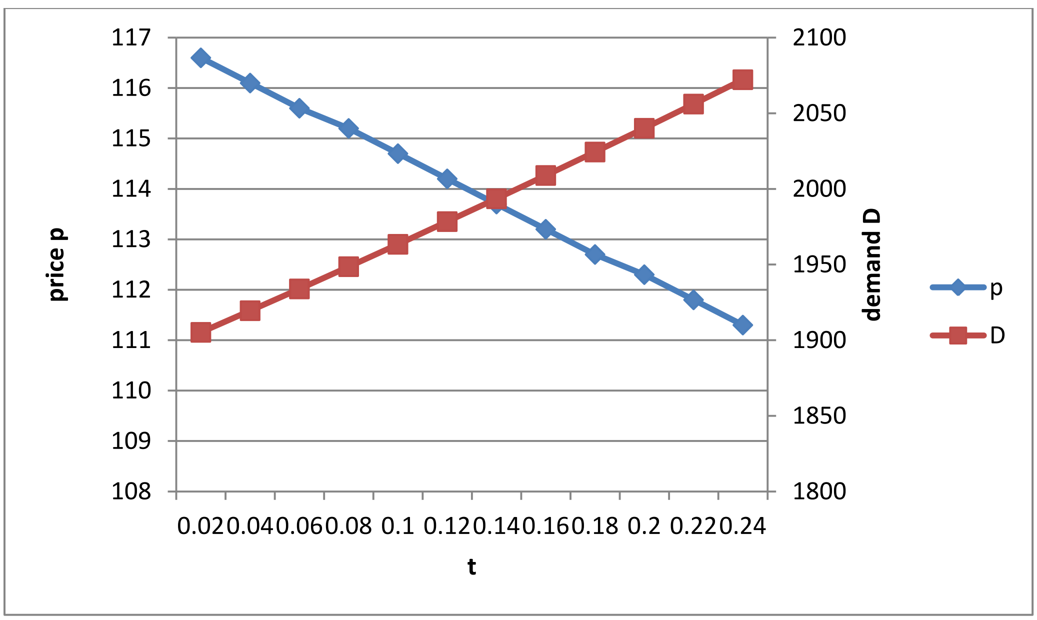
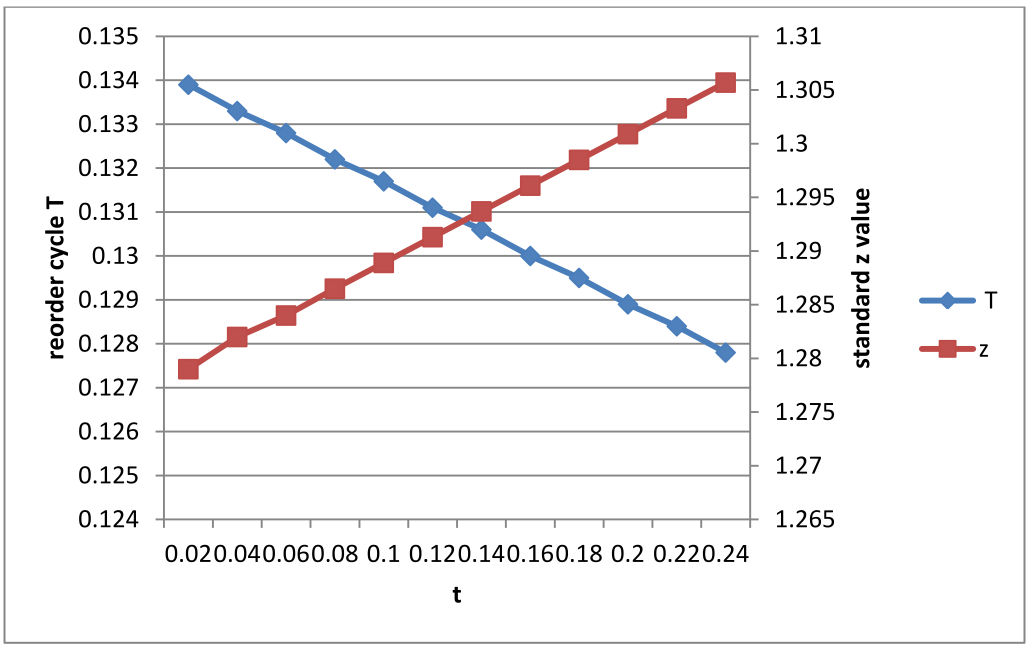


Fig.4.2 Effects of t on Π_m in (r, Q) policy



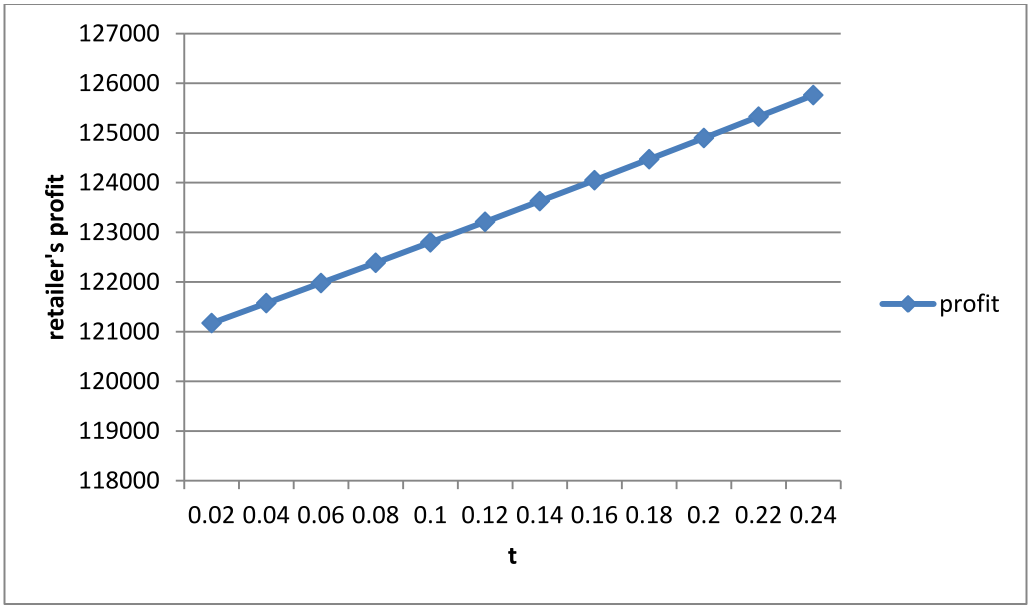
(a) Effects of t on retailer's p and D

Fig 4.3: Effects of t on retailer's variables in (S, T) policy



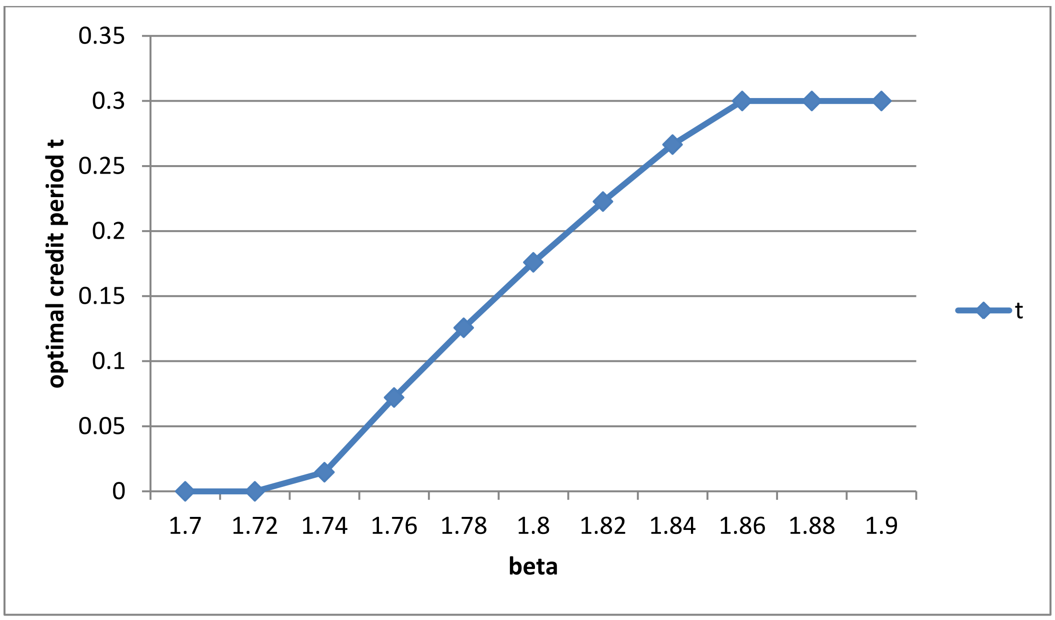
(b) Effects of t on retailer's T and z

Fig 4.3: Effects of t on retailer's variables in (S, T) policy



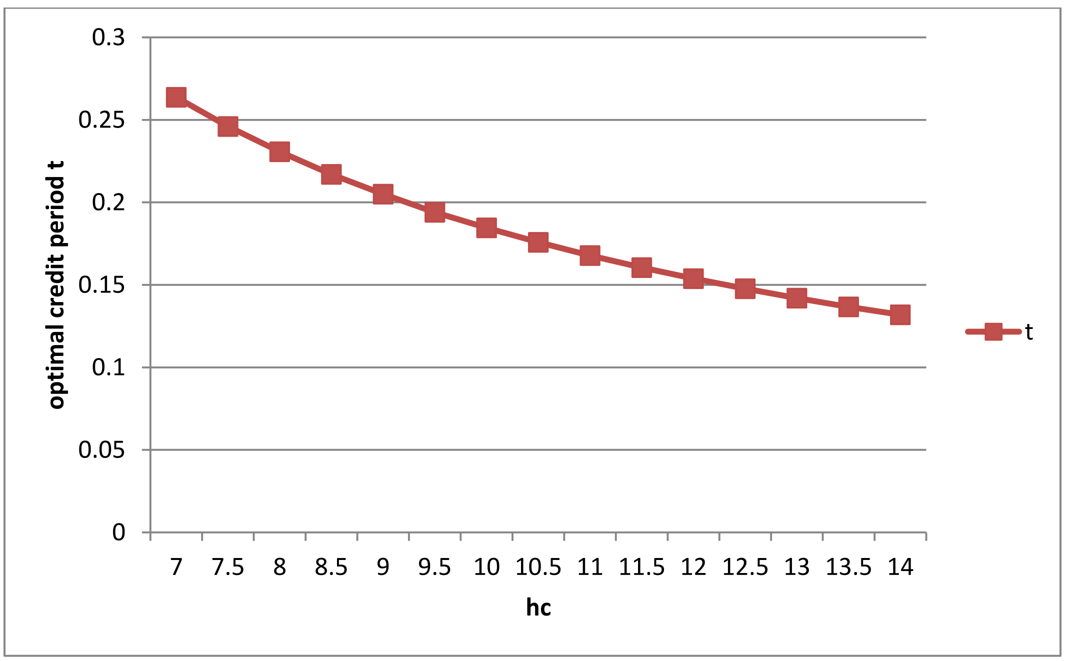
(c) Effects of t on retailer's profit Π_r

Fig 4.3: Effects of t on retailer's variables in (S, T) policy



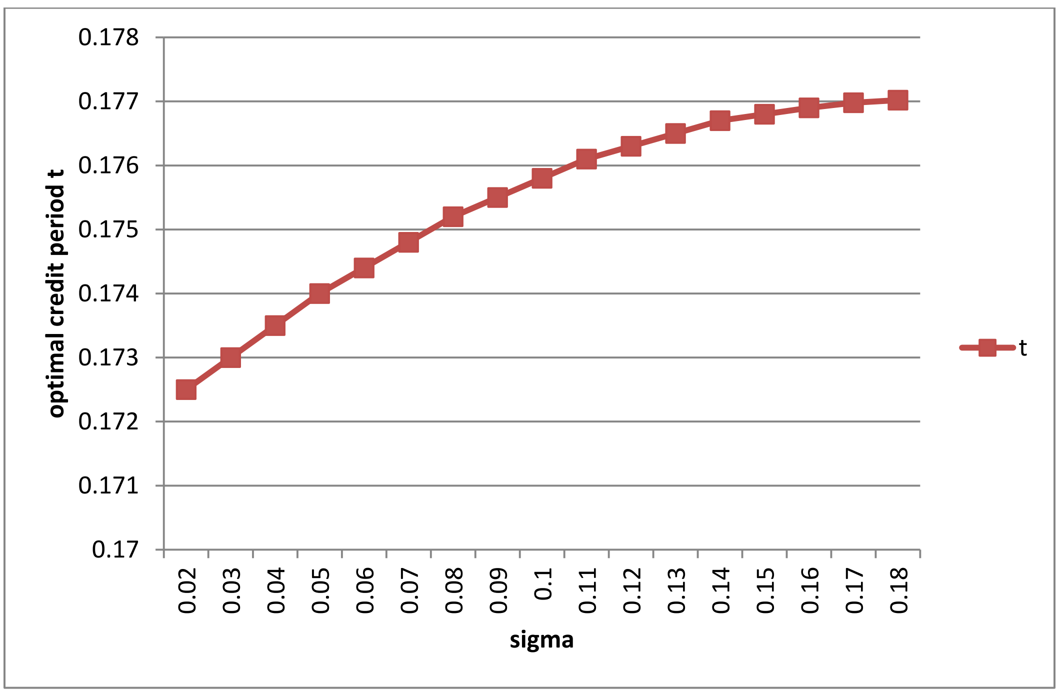
(a) Effects of β_i on t

Fig. 4.4 Effects of β_i, h_c and σ on t^*

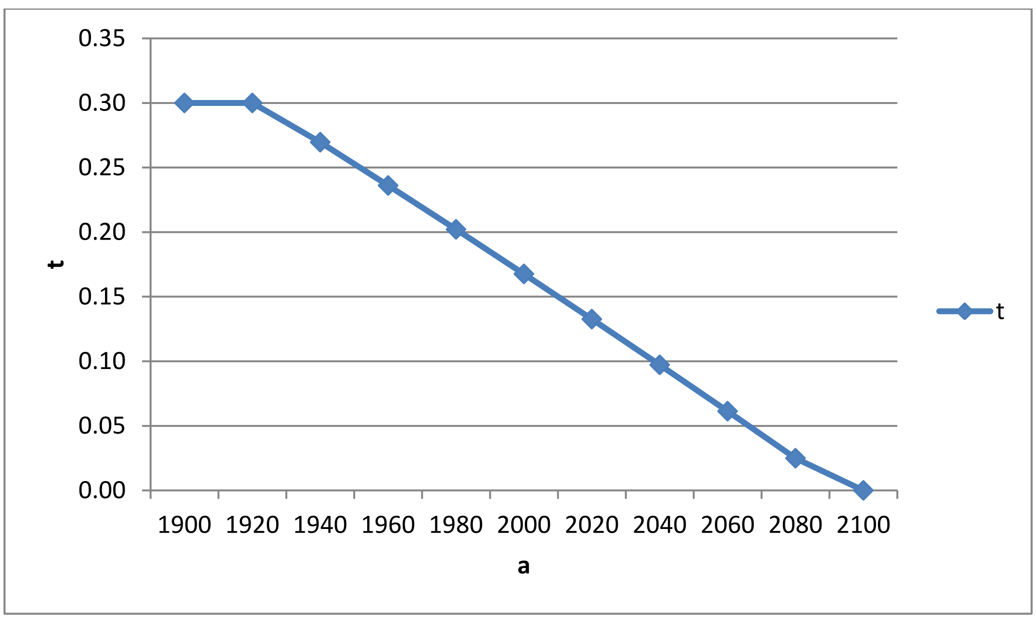


(b) Effects of h_c on t

Fig. 4.4 Effects of β_i, h_c and σ on t^*

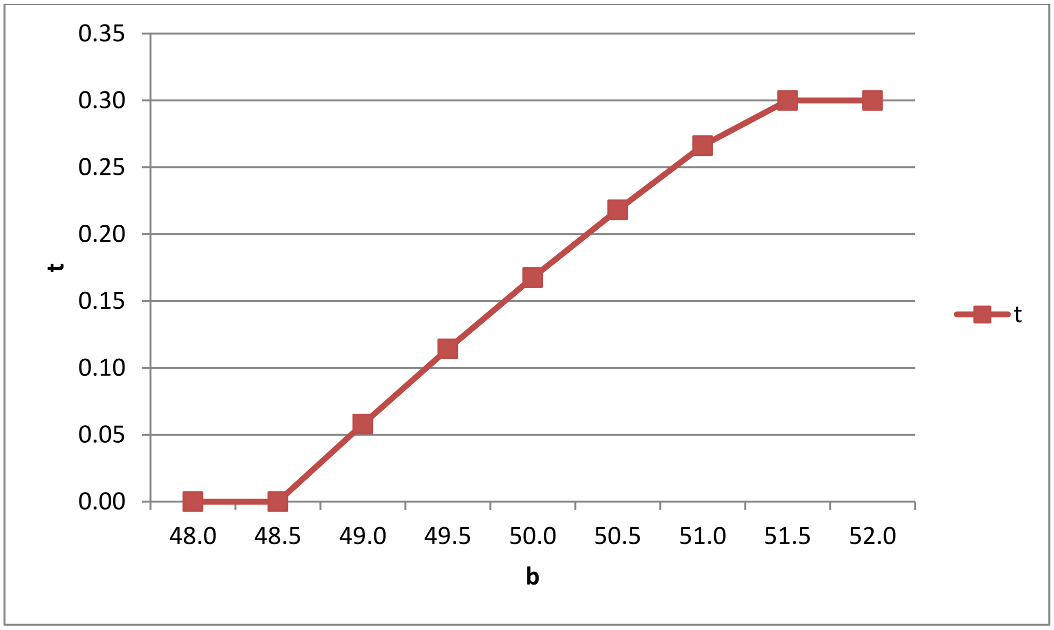


(c) Effects of σ on t
Fig. 4.4 Effects of β_i, h_c and σ on t^*



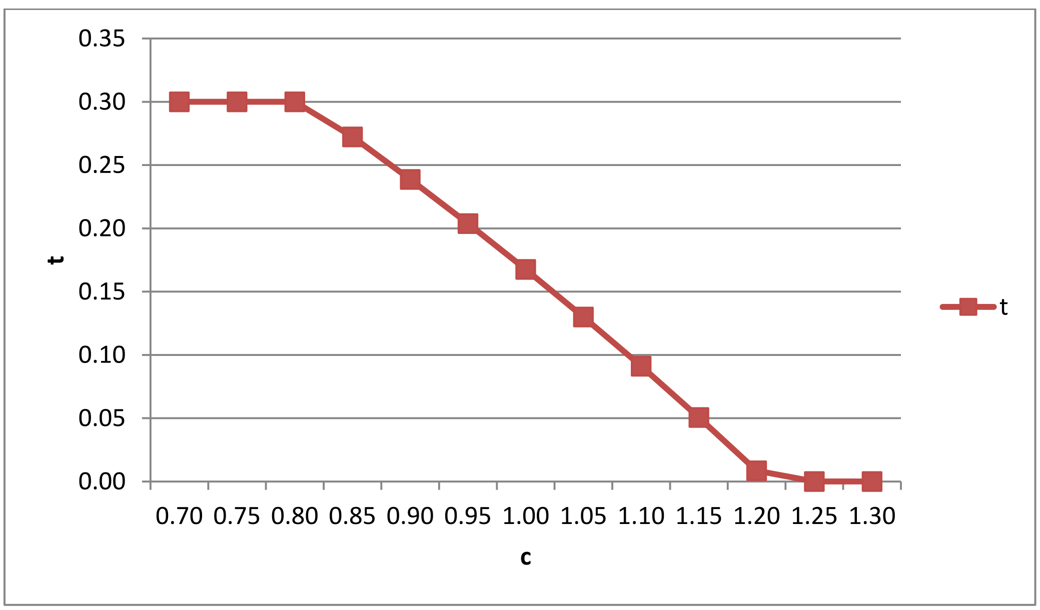
(a) Effects of a_i on t

Fig.5.1 Effects of a_i, b_i and c_i on t^*



(b) Effects of b_i on t

Fig.5.1 Effects of a_i, b_i *and* c_i on t^*



(c) Effects of c_i on t

Fig.5.1 Effects of $a_i, b_i\ ad\ c_i$ on t^*

VITA

Ruo Du was born in Mianyang, Sichuan, Peoples Republic China. She received her bachelor's degree from the University of Electronic Science and Technology of China, with a major in Electronic Information Science and Technology and a minor in Business Administration. In 2008, she was admitted to the Ph.D. program in Operations Management at Drexel university. Her doctoral dissertation, entitled "Decentralized and Centralized Supply Chains with Trade Credit Option", was co-supervised by Professors Avijit Banerjee and Seung-Lae Kim.

Her research interest is focused on supply chain management, as well as on the interface of the operations and the marketing functions of organizations. Her research publications to date include: "Coordination of two-echelon supply chains using wholesale price discount and credit option" (to appear in *International Journal of Production Economics*), "A study on the coordination of two-echelon supply chains using credit and quantity discount options," (published in the *Proceedings of the Northeast Decision Sciences Institute Conference*) and "Integrated inventory models with retail pricing and return reimbursements in a JIT environment for remanufacturing a product" (accepted for publication as a book chapter to appear in the scholarly volume: *Reverse Supply Chains: Issues and Analysis*).

While enrolled in the Ph. D program at Drexel University, Ruo has taught an advanced undergraduate course on Statistics as an independent section. In addition, she has taught, on a regular basis, recitation sections in a variety of undergraduate courses in Operations Management and Statistics, over a period of three years. As a part of her duties as a doctoral student, she has served as a teaching assistant for a number of undergraduate and master's level MBA courses such as Statistics, Operations Management, Supply Chain Management, Advanced Planning and Control of Operations, etc.