Reconfigurable Control of Aircraft undergoing Sensor and Actuator Failures

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Abstract

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Significant number of fatal aircraft accidents in recent years have been linked to component failures. With the predicted increase in air traffic these numbers are likely to increase. With reduction of fatal accidents as motivation, this dissertation investigates design of fault tolerant control systems for aircrafts undergoing sensor and/or actuator failures. Given that the nominal controller may perform inadequately in the event of sensors and/or actuator failure, the feasible approach for such a control scheme is to predesign various controllers anticipating these failures and then switching to an appropriate controller when the failure occurs. This is enabled by the available redundancy in sensing and actuation and allows the system to perform adequately even when these failures occur. The predesign of controllers for sensor and actuator failures is considered. Sensor failures are easily accommodated if certain detectability conditions are met. However, the predesign for actuator failures is not trivial as the position at which the actuators fail is not known *a priori*. It is shown that this problem can be tackled by reducing it to the classical control problem of disturbance decoupling, in which, the functional control enables the steady state output of dynamical system to reject any disturbance due to the failed actuators. For linear systems, conditions for existence of a controller capable of accommodating these failures can be understood in geometric terms and calculations are linked to solvability of coupled matrix equations. Although control design for aircrafts is done using linear techniques, failures can cause excursions into nonlinear regimes due to ensuing changes in the flight conditions. This dissertation also uses the recent results in the nonlinear regulator theory to address actuator failures in nonlinear systems. The utility of design techniques is illustrated using flight control examples with failures. The symbolic computational tools are developed and are available in the appended disk. A section on the use of variable structure servomechanisms to perform the regulation needed in case of actuator failures is also included.

Chapter 1. Introduction

Fatal crashes, when they occur, cause tragic loss of life and are accompanied by colossal loss of money associated with destruction of property, cost of investigation, and reduced public confidence in air travel. In the coming years, commercial air traffic is likely to increase significantly. Several regulatory and investigative agencies are pushing for creating technologies that will significantly reduce the fatal accident rate, which is presently little under 2 per million departures (not including sabotage, terrorism and military action) making air travel the safest mode of transport. In many cases, inadequate airport facilities and human errors are cited as causes by the investigative agencies for these accidents. These present challenges that are not easily overcome. However, numerous aircraft accidents in the recent years have been caused when components in the control loop have malfunctioned. Failed sensors or actuators, such as dysfunctional gyros, stuck horizontal stabilizers or other control surfaces have led to catastrophic consequences. Therefore, there is need to design reconfigurable control schemes, which actively address such failures in the control loop. The nominal controller tends to perform inadequately because of the changes in the system dynamics following a failure. Since online computation of the appropriate control algorithm makes tremendous demands on the available resources, a feasible approach for such a control scheme is to predesign various controllers anticipating component failures and switching to an appropriate controller when failure occurs. The key in designing such a bank of controllers is the available redundancy in the aircraft sensing and actuation. Modern day aircraft are instrumented with redundant sensors and have many control surfaces that may be used even if one or few failures occur. Once these controllers are designed, they can be stored in the on-board memory. A supervisory switching scheme can be designed based on the failure detection and identification mechanism. In this dissertation, the predesign of controllers for sensor and actuator failures is considered. The ability to design controllers for sensor failures is linked to the property of the system, which allows the design of stable observers for the purpose of reconstructing the states for using state feedback. However, the predesign for actuator failures is not just related to the stabilizability or controllability of the system. When actuators fail they not only reduce control authority, but also may present persistent disturbances, which the functional actuators have to compensate for. The most common failure of the actuators is when the control surface gets stuck due to mechanical, hydraulic or electrical failure at any position. It is shown that the problem of predesigning a controller for stuck actuators can be reduced to the classic problem of designing a regulator with internal stability. The regulator has the capacity to reject persistent disturbance caused by the stuck actuators in the steady state. The transient response can be shaped using standard control techniques. For linear systems, the necessary and sufficient conditions for accommodating such a failure can be understood in geometric terms i.e. finding an invariant subspace in the kernel of the output and the calculations are linked to solvability of coupled matrix equations called “Sylvester’s equations”. The existence of the solution to these equations along with the stabilizability and detectability conditions together provide *necessary and sufficient* conditions for the system to reach the desired steady state. The reconfigurable controller provides a supervisor, which switches to the controller designed for the impaired system, when the failures are detected. Hence, it usually takes a period of time before the actuator failure is diagnosed by the fault detection and identification mechanism. During this time the impaired system continues to operate with the original, but now inappropriate controller. If the failures are detected immediately, the linear control techniques used in the control design may suffice since the system states remain close to the linear region. However, delays in detection may cause significant excursion of the system states. In many cases, nonlinearities in the aircraft dynamics may cause the linear design to be rendered inappropriate. Nonlinear regulators may provide larger domains of stability, providing a larger window of safety in face of delays in detection and identification. Therefore, this dissertation also investigates various approaches to design state feedback regulators for nonlinear systems to address actuator failures. The utility of design techniques is illustrated using flight control examples. The design techniques are implemented by developing symbolic computational tools. These tools are included in the disk attached with the thesis. In the next section, the organization of the rest of the thesis is explained.

1.1. Organization of the thesis

The rest of the thesis is organized as follows. In the remaining sections of this chapter the motivation for this research is presented followed by a section on the background of fault tolerant control and a survey of the existing methodologies for achieving fault tolerance. The contribution of this thesis to the available techniques is summarized in the final section. In chapter 2, the predesign of controllers in case of sensor failures is considered. It is shown that the sensor failure can be adequately addressed by designing appropriate observers and a switching mechanism based on range and spectral checks of the sensory data. The design strategy is demonstrated using a model of the F/A-18A Automatic Carrier Landing System taken from literature subjected to sensor failures. In chapter 3, we consider the predesign of controller for stuck actuators in case of linear systems. It is shown that the problem can be reduced to design of a regulator with internal stability by the addition of a dynamic equation for the stuck actuators. The problem can now be solved subject to certain conditions, which are explained. The design procedure is demonstrated by using the earlier carrier-landing example subjected to actuator failures. The details of design are presented. The necessary and sufficient conditions presented in chapter 3 leads to a novel way to assess the redundancy in systems with regards to actuator failures and this forms the topic of chapter 4. Chapter 5 presents the design of nonlinear regulators using series approximation for the regulating functions. The improvement in steady state regulation in face of actuator failures is demonstrated by using the longitudinal dynamics of a jet transport aircraft when the primary control surface is stuck. It is seen that the longitudinal axis can be controlled solely by modulating the thrust input. Chapter 6 discusses the use of variable structure control theory to design controllers to address actuator failures. Chapter 7 summarizes the work and presents pointers to further research.

All through the thesis computations are emphasized, definitions, theorems and their proofs are relegated to the appendices to enhance the readability and the utility of the thesis.

1.2. Motivation

In the year 1959, which was probably the first full year of commercial jet operations, the world's air carriers averaged 100,000 jet-flying hour per hull loss*; today they average nearly 800,000 flying hours per hull loss [69]. The record varies greatly globally; even so, air transportation is the safest of all major modes of transportation. However, the current accident rate (which is little under 2/million departures) will be soon become unacceptable because of the predicted increase in the commercial air traffic, which is expected to triple in the next 20 years [37]. Several agencies including the regulatory authority Federal Aviation Authority (FAA), National Aeronautics and Space Administration (NASA), International Civil Aviation Organization (ICAO), non-profit organizations like Flight Safety Foundation and Aviation Safety Network are pushing for a significant reduction in airplane accidents in the next few years [34, 36-38]. e.g. the NASA Aviation Safety Program Goal is to "*develop and demonstrate technologies that contribute to a reduction in the aviation fatal accident rate by a factor of 5 by year 2007 and by a factor of 10 by year 2022* [35]."

Accidents have been attributed to mistakes by the flight crew, the airworthiness of the airplane, harsh environmental conditions, lapses in maintenance or a combination of these factors [69]. This varied set of reasons present difficult challenges that technology can't easily overcome. However, in a large number of accidents the sequence of events

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* National Transportation Safety Board (NTSB) and International Civil Aviation Organization (ICAO) definition of Hull loss: Airplane damage that is substantial and is beyond economic repair. Hull loss also includes events in which: Airplane is missing or Search for wreckage has been terminated without it being located or the Airplane is substantially damaged and inaccessible.

leading to the fatality, rather than being pilot error happens to be a component failure. A modern day aircraft has a few million parts, e.g. A Boeing 767 has approximately 3,140,000 parts. Although, each of these parts are tested the likelihood of some part malfunctioning is finitely large. Many accidents can be directly linked to the failures of the control system components and a few real accidents are presented in the sequel. Figure 1.1 shows the reduction in the fatal accident rate over the years from the late 1950s. The reduction in fatal accident rate has made flying the safest mode of all forms of transport.

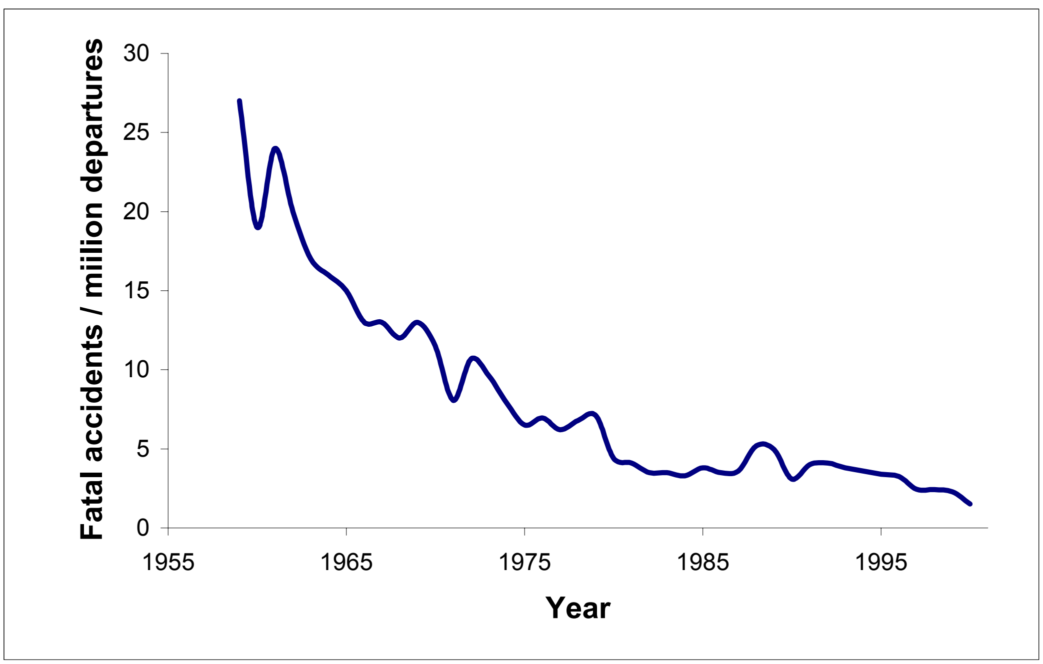


Figure 1.1. Fatal accident rate (i.e. those involving hull loss), not included are acts of sabotage, terrorism and military action

Fatal accidents when they occur cause violent loss of life and property and are the subject of newspaper headlines all over the world. This is followed by the immediate reduced public confidence and this makes fatal crashes an expensive affair for the airline industry. It takes a significant time, effort and money on the part of the investigating agencies to determine the exact cause of the accident [38].

With the lowering of accident rates the number of commercial airline operations worldwide has continued to grow steadily as shown in Figure 1.2 [69].

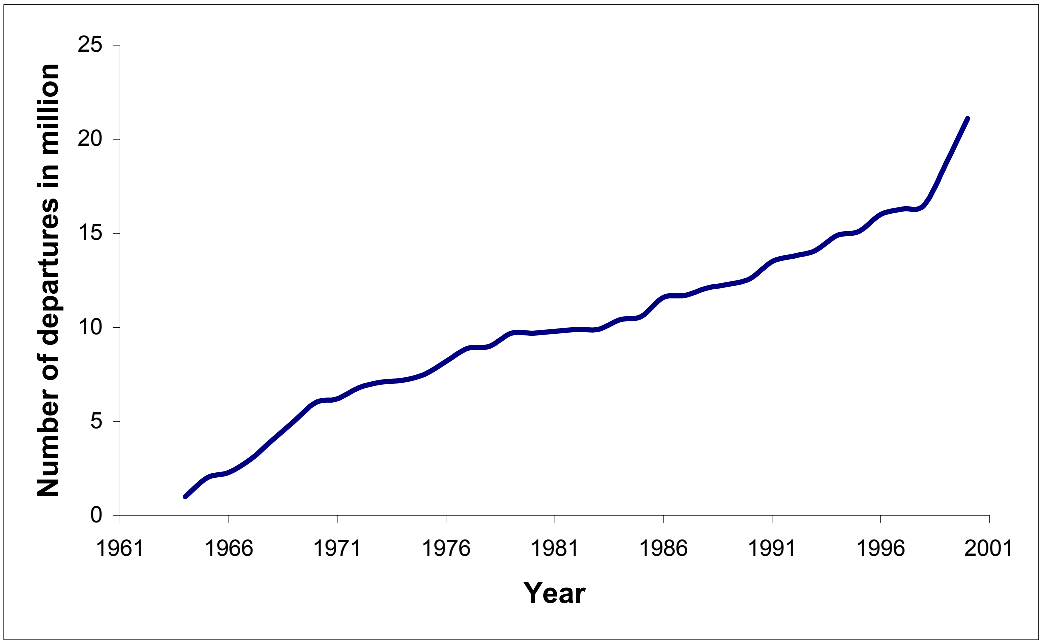


Figure 1.2. Commercial airline traffic growth - worldwide

Indeed if the accident rate of the early years of commercial jet accidents were maintained with the increase in traffic we would see a few fatal crashes everyday. Thankfully, the accident rate has reduced dramatically. However, if the air traffic continues to grow even the present low accident rate will become unacceptable [35]. Figure 1.3 shows the extrapolation of data from Figure 1.1 and Figure 1.2 for the last few years using smoothed polynomial functions. This figure shows a dramatic increase in the number of accidents in the coming years. The technological challenge in making an already safe mode of transport safer are significant.

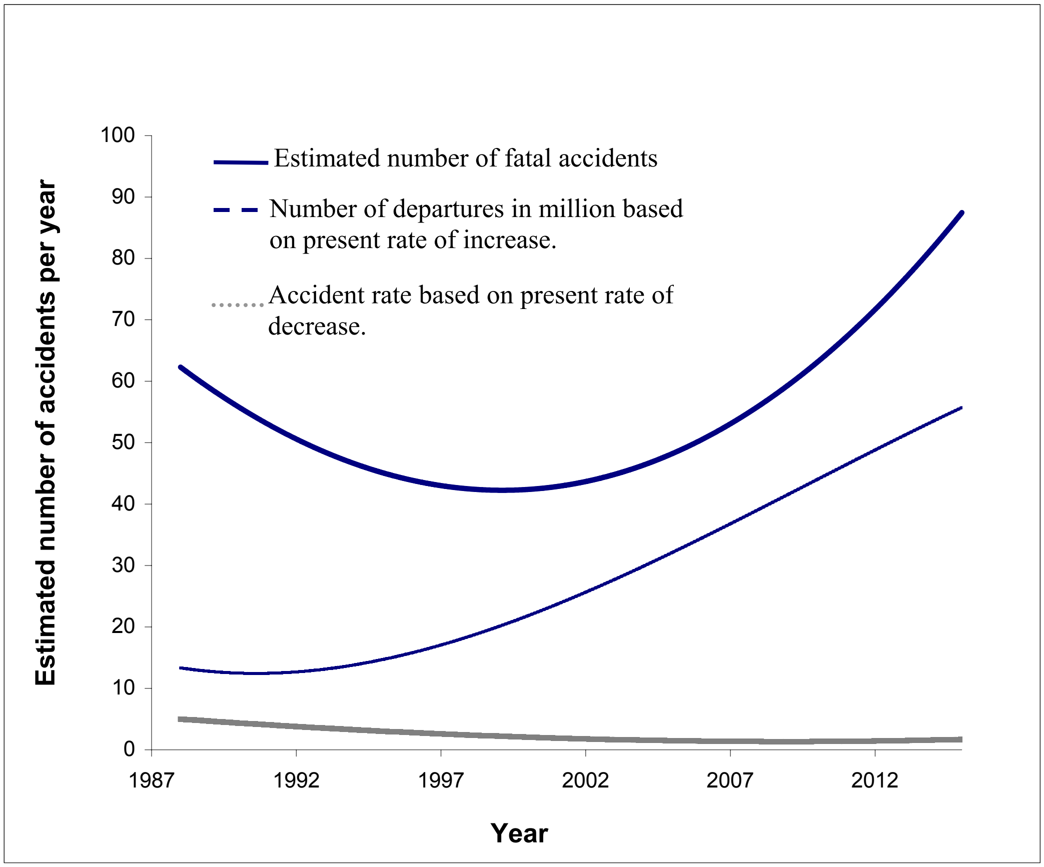


Figure 1.3. Predicted increase in air traffic and resultant increase in fatal accidents in the coming years

It takes months of investigation on the part of the authorities to determine the exact sequence of events that led to a fatal air crash. Sometimes these investigations prove inconclusive and it is left to imagination as to what may have gone wrong with the airplane. Here are some recent examples of fatal accidents that may have been caused by failures in the control loop components.

1. The Alaska airline flight 261 – Jan. 2000: On January 31, the McDonnell Douglas MD-83 of the Alaska Airlines on flight 261 crashed off the coast of California about 4:20 p.m. (PST) en route from Puerto Vallarta, Mexico, to San Francisco killing all of its 83 passengers and 5 members of the crew. Although, the investigation in still under progress it is clear from the flight data recorder that the crew was unable to maintain vertical control due to dysfunctional stabilizer. The FDR shows that the stabilizer trim changed to a full-nose down trim and remained jammed there until the crash. During the last 12 minutes before the crash the crew attempted to diagnose and troubleshoot their stabilizer trim problems in vain and the MD-83 finished just off Point Mugu, CA, 650 ft deep in water. The failure of the actuation assembly during the final minutes of flight was confirmed when the navy recovered the parts from the sea. It can be hypothesized that mechanical failure of the actuation assembly of the horizontal stabilizer is the primary cause of the tragic accident [39].
2. On 12 August, 1985 Japan Airlines (JAL) flight took off from Tokyo-Haneda at 18.12h for a flight to Osaka. At 18.24h, while climbing through 23900ft at a speed of 300kts, an unusual vibration occurred. An impact force raised the nose of the aircraft and control problems were experienced. Two minutes later hydraulic pressure had dropped and ailerons, elevators and yaw damper became inoperative, followed by dutch roll and plughoid oscillations (unusual movement in which altitude and speed change significantly in a 20-100sec. cycle without change of angle of attack). The aircraft started to descend to 6600ft while the crew tried to control the aircraft by using engine thrust. Upon reaching 6600ft

the airspeed had dropped to 108kts. The aircraft then climbed with a 39deg. angle of attack to a maximum of approx. 13400ft and started to descend again. JAL123 finally brushed against a tree covered ridge, continued and struck another ridge, bursting into flames. Probable cause was cited as: "Deterioration of flight characteristics and loss of primary flight controls due to rupture of the aft pressure bulkhead with subsequent ruptures of the tail, vertical fin and hydraulic flight control systems. The reason for the aft pressure bulkhead rupture was that its strength was reduced by the fatigue cracks propagating in the spliced portion of the bulkhead's webs. The initiation and propagation of the fatigue cracks are attributable to the improper repairs of the bulkhead, conducted in 1978, and since the fatigue cracks were not found in the later maintenance inspections, this contributed to the accident." 520 people lost their lives in what lives as the among the worst aircraft disasters of all time [36].

3. On January 10, 2000, a Saab 340B on a scheduled passenger flight (number 498) took off normally from Zurich-Kloten to Dresden. Quickly after take off resulting from improper right aileron input caused increase in roll rate. Meanwhile, the pitch decreased rapidly, accompanied by a marked increase in speed and the airplane entered an irrecoverable high speed high rate spiral descent crashing in a open field killing all the 7 passengers and 3 crew members on board [38].

This list is by no means exhaustive but only representative of the set of fatal, commercial jet accidents. It is clearly evident from the list these problems are not restricted to any particular plane manufacturer, carrier, make, or region of the world.

They also demonstrate a need for designing control schemes that actively address failures. These schemes are important for safety of the passengers and crew when such failures occur.

The next section presents the various functions that the flight control computer performs in a modern fly-by-wire aircraft and a survey of control design techniques available in the literature.

1.3 Background and literature review

The subject of fault-tolerant control has attracted many researchers and is available in widely scattered publications. This research is spread under the varied nomenclature of "Fault-tolerant Control" [8, 12, 27, 31, 62, 68] "Reconfigurable Control Systems" [13, 14, 68, 86], "Intelligent Control Systems" [71], "Self-repairing Control Systems" [42], "Restructurable Control Systems" [17, 27], "Fault Accommodation" [26, 66], Failure Compensation" [75], etc. Closely related work has also been done in "Supervisory Control" [12, 45], "Fault detection and isolation" [29, 59, 65, 70], "Fault Diagnosis" [28, 32, 45], "Hierarchical Control Systems" [5]. The motivation for all the research has been to overcome unsatisfactory response of a conventional feedback control design in the event of a malfunction of the components. Some survey papers [12, 67, 71] exist and provide an overview of the various techniques developed by researchers and the various approaches taken before 1997. In this section, we begin by listing the functions of the flight control computer and then examine the relevant principles of fault tolerant design. We also present a survey of some of the recent approaches to sensor and actuator failures developed by researchers more recently. In this thesis, we shall not consider avoidance of faults by shielding of the flight computer, use of filters and surge suppressing devices or use of optical cables for data transmission. We shall adopt the view that some faults are inevitable and an active approach to fault-tolerance is needed. The problem definition is left at a pertinent fuzzy level. Later on we define the problem more clearly when we start considering the specific failures and predesigning controllers for those failures. Right now the idea is to present a generic architecture which can provide fault tolerance. Subsequently, this generic control architecture can be tailored to fit a specific aircraft by imposing the design requirements.

1.3.1 Functions of the flight control computer.

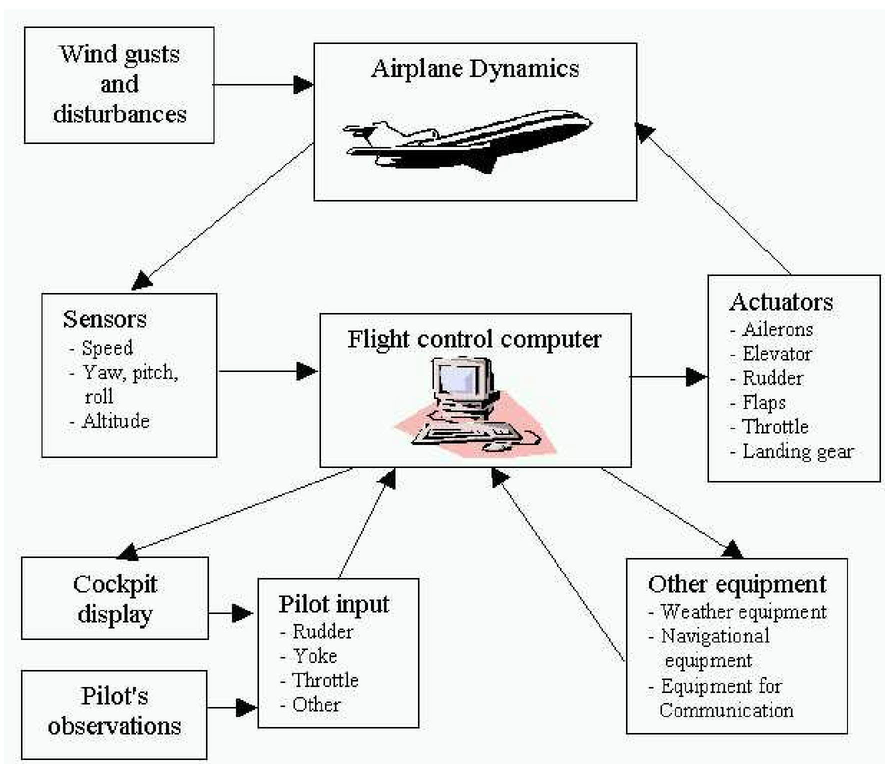


Figure 1.4. The schematics of role of flight control computer in a modern fly-by-wire aircraft

The various functions of a generic fly-by-wire flight control computer are shown in Figure 1.4, the flight control computer performs a multitude of functions. It is responsible to translate the pilot commands from the cockpit (inputs to the pedals, yoke and throttle) to the movement of the actuators (ailerons, elevator, flaps, rudder and the actuators for engine speed). The control computer also performs the function to augment the stability of the aircraft, which involves feedback from the sensors and augmenting the pilot input to the controls. At the same time, the control computer is responsible to present information to the pilot on the various CRTs and display panels. Displayed information includes not only present flight data but also weather and navigational information. The flight computer is also involved in controlling the communications equipment. It is useful to characterize these functions that the flight control computer is involved into three major categories**:** performance, comfort and safety. These categories can then help determine the necessary levels of redundancy in functionality needed to make the system fault-tolerant.

In the first category of functions, we ideally want all the control systems to function at top performance and without glitches. This is the level for performance, because either all the systems are operating without fault, or minor faults can be compensated so as not to cast any apparent sacrifice of performance of the aircraft. The second category of functions that the flight computer performs is to maintain the comfort of the passengers and crew. We can let the fault-tolerant control system tradeoff performance if it is more desirable to ensure the comfort of the passengers in the presence of non-critical faults. The third category of functions can be termed flight-critical functions. To maintain the flight safety, the fault tolerant flight control computer must continue to perform these flight-critical functions without error at all times even in an emergency. Creating this hierarchy of control objectives of the control computer helps to develop modular fault tolerant architecture of the control system.

1.3.2 Principles of fault tolerant control

In case of failure the fault must be determined, identified, contained and compensated. The important feature of fault tolerant designs is fault identification and isolation. The control system must at all times be aware of the health of the system. Fault identification in fault tolerant designs entails the need for supervision or health monitoring sub-systems. This supervision may include data range checks, spectral nature checks, parity checks, logic checks, checksums, etc. to detect the nature and severity of faults. A modular approach to design will enable the capability to contain the fault in a very small region of the entire system. In case faults occur the system can be reconfigured to either perform without the faulty module, or other functional modules must compensate the services of the faulty module. The supervisor for reconfiguration must also be able to identify the recovery of the module in the case of transient failures. This reconfiguration of the system is only possible if there is some form of redundancy in the system. Redundancy can be understood as additional resources beyond those required for nominal operation. In the following subsection we examine these principles in some detail.

1.3.2.1 Failure detection and isolation

According to the generally accepted terminology the fault detection and isolation involves the determination and localization of the faulty elements of the system. Some researchers also include identification as one of the tasks. So the fault detection and isolation may involve the following functions

1. Failure detection: the indication that something is gone or going wrong in the system.
2. Failure isolation: the determination of the exact location or quality of the failure.
3. Failure identification: the determination of the size or quantity of the failure.

The relative importance of the above functions is usually subjective to the specific application and also related to the cost associated with performing any of the above functions. All failure detection and isolation involve filtering of the sensory data available for supervision and control. A large number of schemes merely involve data range checks, compatibility checks and spectral checks on the sensory data. However, there are schemes in which the failure detection and isolation method makes explicit use of some mathematical model of the system. These ideas based on "analytical redundancy" employ techniques using thresholds or spectral analysis on residuals or by identifying patterns sometimes called signatures of the residuals [29]. More recently researchers have employed deterministic nonlinear observer-based approach to fault diagnosis [28]. The above techniques based on direct sensory measurements and on model-based methods have been extensively studied. The key indicators of a failure detection and identification scheme are the time needed to identify the failure(speed) and the absence of false alarms(robustness).

1.3.2.2 Key role of redundancy

Not all kinds of redundancy lead to fault tolerance. Even if similar modules are available for replacement of faulty module or if many similar modules are working in parallel using some form of voting, it does not preclude the common modes of failure. To be truly fault tolerant the system must be redundant but also disallow common modes of failure. One can identify the following forms of redundancy in most systems.

1. Hardware redundancy: This kind of redundancy involves processors with redundant registers and buses, extra processors, alternate sensors and actuators and multiple data lines. Rapid advances in VLSI technology allow for more powerful, cheaper and power efficient processors. Even today aircraft are equipped with redundant processors or a multi-processor to perform the calculations for the flight control. As pointed out in the last paragraph, attention needs to be paid to guard against common modes of failure.

2. Software redundancy: The algorithms responsible for the smooth functioning of the aircraft must have error checks for the calculations. There must be parity checks for all the digital signals and some form of checksum for the calculation. Conducting the same calculation independently in two different ways enables the system to perform consistency checks.

3. Data redundancy: There must be at all times a backup clean copy of the flight critical data so that all the important calculations can be restarted using clean data when faults occur.

Besides the above redundancies there may be need in the system for manual overrides that the pilot can invoke in case of emergencies. In addition to all these we have some inherent redundancy in sensing and actuation. Design of control systems for aircraft and submarines are usually done using generalized inputs. The number of physical effectors usually exceeds the number generalized controls. The control authority is usually apportioned among the physical effectors. The controller produces the command signals for the servo loops, which are in turn responsible for positioning the control surfaces or the throttle at the commanded values. Usually the dynamics of the servo loops are fast enough so that the control design can be carried out without considering the actuator states. Failure of the servo loop or the positioning mechanism may cause the control surface to get stuck at constant value. It is possible that when one or some of the actuators fail the remaining functioning actuators are sufficient for safe control of the vehicle. This notion of redundancy in terms of the ability to design and implement an effective feedback controller to meet the performance objective is developed further in Chapter 4.

1.3.2.3 Supervision and coordination

Since the aircraft is a complex assembly of sub-systems operating closely together it would not be effective for one supervisor to monitor the health of all the modules. Therefore, we propose the hierarchical architecture shown in Figure 1.5, with local supervisors monitoring the health of a group of modules and sharing information with each other and a central coordinator.

An effective model for the supervisor is that of the 'directed graphs' structure where the 'nodes' of the graph represent modules and the 'edges' the communication between the modules. Data range checks can easily be included in the models as the capacity of the edges. We can maintain a running buffer of previous values transmitted through these edges to perform spectral checks. These error checks can be programmed as interrupts or flags into the control software for supervision. The supervisor inference mechanism can be an automaton allowing it to take requisite action when faults occur. We can assign states to the nodes of the graph e.g. 'operational' and 'faulty'. The act of reconfiguration will involve changing the edge structure to isolate all the 'faulty' nodes and using the 'operational' nodes to carry out the required tasks. When the need for reconfiguration is identified at the design stage mechanisms that allow reconfiguration may be designed and built or programmed. The directed graph structure allows the supervisors to capture the topology of the connections of the system. The representation allows for a more knowledge-rich representation than just the dynamical equations. In this development, the description of the model is left sufficiently abstract to enable modeling all kinds of modules of the aircraft control system and their interfaces.

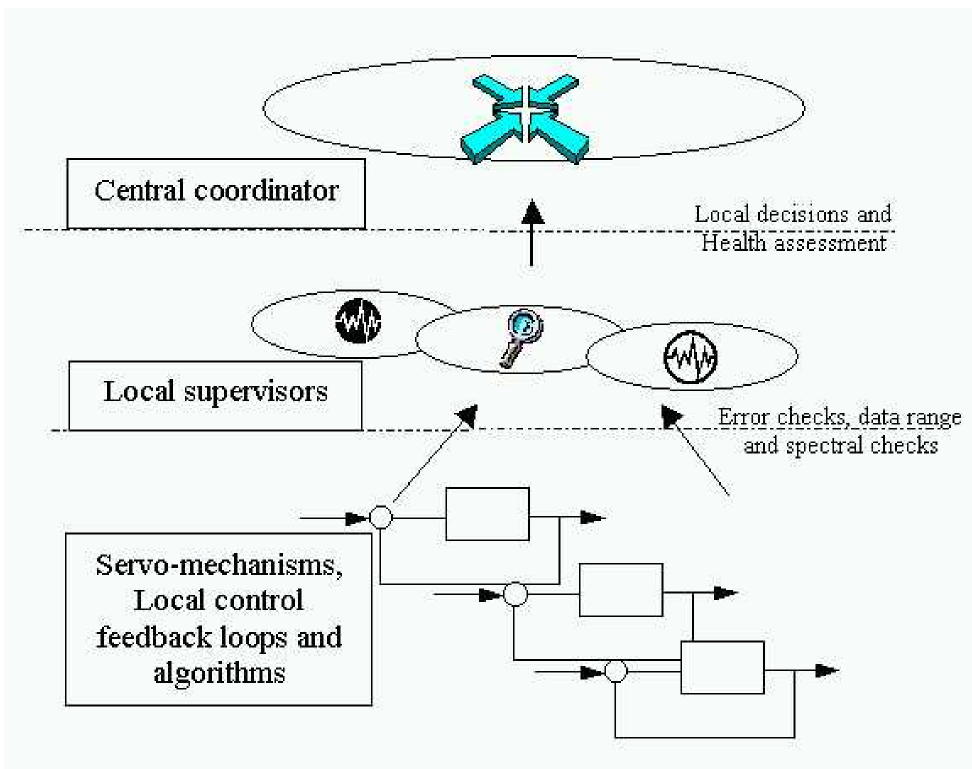


Figure 1.5. Hierarchical architecture for fault-tolerant control systems

The local supervisors allow quick fault identification and rapid fault isolation. They also share information with the central coordinator, which is responsible for global decisions and ensuring that the local supervisors perform with consistency. There are other approaches that have been developed using bond-graphs and other directed graph techniques [45].

1.3.3 Recent work to address sensor and actuator failures

The “Pseudo-inverse method” or the “Control Mixer” approach was described as the key approach to reconfigurable control [17, 26, 27, 42, 66, 68, 85, 86]. The main design objective is to maintain as much similarity as possible to the original closed loop with the aim of providing graceful degradation. This is achieved by reassignment of the feedback gains and the approach can be explained as follows.

Let the open-loop plant dynamics be described by the equations

$$\dot{x} = Ax + Bu \tag{1.1}$$

$$y = Cx \tag{1.2}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control vector and $y \in R^p$ is the output vector. A, B and C are matrices of appropriate dimensions. The nominal closed-loop strategy is to design a feedback gain matrix

$$u = Kx \tag{1.3}$$

where K is a matrix of dimension $m \times n$, generating a closed-loop dynamics given by

$$\dot{x} = (A + BK)x \tag{1.4}$$

Let the dynamics after failure be given by

$$\dot{x} = A_f x + B_f u \tag{1.5}$$

$$y = C_f x \tag{1.6}$$

where the new matrices A_f, B_f and C_f model the changes in the system after failure. The new closed-loop after feedback be given by

$$\dot{x} = (A_f + B_f K_f)x \tag{1.7}$$

where K_f is the new feedback matrix to be determined. Since the idea is to maintain the original closed-loop dynamics we have

$$K_f = B_f^{\perp}(A - A_f + BK) \tag{1.8}$$

where $B_f^{\perp}$ denotes the pesudo-inverse of $B^{\perp}$. These feedback matrices can be calculated for many anticipated failures. The main drawback of the method is that the stability of the reconfigured system is not guaranteed. Hence if the above approach is applied with no appropriate safeguard can lead to instability. The authors in [27] proposed a modified pseudo-inverse method. Authors using the frequency domain mixers and other embellishment have developed this method further [85, 86]. However, this method is *ad hoc* and doesn't address the additional persistent disturbances that the failed actuators may cause.

Other authors have proposed multiple models, switching and tuning schemes [13, 31]. Assuming that the controlled plant belongs to a set of plant models, a multiple model, switching and tuning design has several forms: one based all-fixed plant models, one based on all adaptive plant models, one based on fixed models and one based on adaptive model, and one based on fixed models with one free-running and one reinitialized adaptive model [63]. Among other things an all-fixed model design needs sufficient density of fixed models in the set of plant models. More recently, the authors in [77] proposed an adaptive state feedback control approach to achieve plant-model state matching in the presence of actuator failures. Some actuators were assumed to fail and jam at any fixed position during the operation. The problem formulated in [77] was to adaptively adjust the remaining effective controls using the measured state information to achieve the plant-model state tracking without the knowledge of the system parameters and without knowing which and how many actuators have failed and at what fixed positions. The condition for the problem to be solvable is that the associated matching equations need to be satisfied. A drawback of the approach is that the online adaptive algorithms may require too much time and computation to react to the problem created by actuator failures.

Researchers have also used soft computing techniques to the problem of sensor and actuator failures. In [62] Neural networks are employed for sensor and actuator failure detection, identification and accommodation. The scheme for sensor accommodation consists of a main neural network and a set of many decentralized neural networks, one for each sensor in the flight control system without physical redundancy. The outputs of the main neural network are the estimates of the same parameters measured by the real sensors. Using thresholds on residuals the sensor failure can be detected. Similarly, actuator failure detection is based on spotting substantial changes in the aircraft angular velocities following any type of failure. The accommodation scheme involves the use of pretrained neural networks in the feedforward sense. The drawbacks for using such schemes on a commercial aircraft are the computational burden and the cost of verification of the scheme for certification.

The key to practical reconfigurable schemes is to balance the many factors of computational complexity, verifiability, maintainability, reliability and cost for each one of the aforementioned. It is worthwhile to work with deterministic verifiable schemes for which stability and performance of the system is guaranteed before the control scheme is put on an aircraft.

1.4. Contribution of the thesis

The subject of this thesis is to investigate and develop techniques of fault accommodation for fault-tolerant control. The thesis presents approaches to predesign of anticipated component failures of the control loop, which have been responsible for many a fatal crashes in the recent years. It uses verifiable well-understood techniques for designing controllers for anticipated failures. This greatly enhances the verifiability of the control scheme consequently reducing the cost of certification for commercial purposes. The connection of the actuator failure problem with the classical problem of regulator design with internal stability helps establish measures of redundancy that take into account hard constraints of magnitude saturations into account. The thesis also examines the role of nonlinearities in system dynamics when failures occur and the benefits of the nonlinear design strategy over that of a linear one in face of delays in failure detection and identification. It is shown that the nonlinear strategy may provide a larger window of safety by increasing the domain of attraction of the regulator. The symbolic computational tools for all the control calculations are developed and greatly enhance the applicability of the techniques to real systems.

Chapter 2. Predesign of controllers for sensor failures in aircraft

This chapter deals with the predesign of controllers for sensor failures. Most sensory data has some amount of noise in the measurement of the quantity. Robust filtering techniques have been developed and are used to generate reliable data from measurements[31]. When the sensory data becomes unreliable i.e. when the measurable quantities unrecoverable from the readings the controller must either find a way to estimate the quantity or work without the information. In this chapter we present ways to analyze the sensory redundancy using detectability as the governing system property, we also design a bank of observers with hierarchical switching logic to actively address sensor failures.

Sensors are used not only for feedback information but also for fault detection. In this chapter, we consider only the case where the sensors whose information is used by feedback controllers to make a control decision. The sensors for fault detection are addressed in available literature. The accepted definitions for fault detection are given by authors in [19], faults to be (strongly) detectable if it is possible to construct a residual generator that is sensitive to the (constant) fault while decoupling all disturbances. The calculations for linear systems rely on the well-established theory on polynomial matrices and rational vector spaces [48].

For the case in which the sensory information is used for feedback the supervisory system must be capable of performing range checks and spectral checks on the data to detect the faulty sensors. Other methods for detecting failures can be encoded by studying real precedents of sensor failures. The design procedure for sensor failures also gives a measure for redundancy in the sensory system for feedback. If a part of the system becomes undetectable for a sensor failure then it calls for hardware redundancy to accommodate the failure. This idea is made precise in the next section.

2.1 Determining detectability for sensor failures.

Feasible observers can be designed only if adequate measurements are available to satisfy detectability conditions even after failures. Those sensor failures which the system becomes undetectable the designer can choose to include hardware redundancy to harden the system against such failures or a dummy set of expected values or a virtual model of the physical system [73]. Observability is the property of a system that enables complete choice in the dynamics of the state estimators. Observability can be defined in terms of determination of the initial state vector given the output measurements. Those modes that remain undetermined are called unobservable. Unobservable states have no effects on the outputs, and maybe viewed, as outside the system boundary, and they would be of no interest to control design except for system instability. Observability, is important for measurement selection and designing observers. Detectability is a property that can be understood as asymptotic observability. A system is detectable if all the unobservable modes of the system are stable. It follows that a linear dynamical system $\dot{x} = Ax + Bu$, $y = Cx + Du$, where $x \in R^n$, $u \in R^m$ and $y \in R^p$, also, A, B, C and D are constant matrices of appropriate dimensions, (or the pair (A, C)) is state detectable if and only if there exists a matrix L of appropriate dimensions such that $(A + LC)$ has a spectral list belonging to the open left half of the complex plane, i.e. is stable (Hurwitz). Computations for computing the observable modes and unobservable modes cab be accomplished using the well-known decomposition techniques. Finally detectability can be established by checking the stability of the modes associated with the unobservable part of the system.

If the system remains detectable when one or more sensors fail, one can design another controller with the reduced number of measurements using the same state feedback design. The design for observers is considered in the next section.

2.2 Problem formulation and sensor failure models

The nominal plant $G(s)$ is described by the following linear dynamics equations,

$$\dot{x}(t) = Ax(t) + B_1 w_1(t) + B_2 u(t) \tag{2.1a}$$

$$G(s) \quad : \qquad z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} C_{1u} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ D_{12d} \end{bmatrix} u(t) \tag{2.1b}$$

$$y(t) = C_2 x(t) + v(t) + D_{22} u(t) \tag{2.1c}$$

where $x \in R^n$ is the state vector, $w_1 \in R^{mw1}$ the exogenous input vector to the system, $u \in R^{mu}$ the control input vector, $z \in R^{pz}$ the regulated signal vector, $y \in R^{py}$ the measured output, and $v \in R^{py}$ the measurement noise. A, B_1, B_2, C_{1u}, D_{12d}, C_2, and D_{22} are constant matrices of appropriate dimensions. The regulated vector z consists of z_1 and z_2 where $z_1 \in R^{pz1}$ is the error to be minimized and $z_2 \in R^{pz2}$ represents the control-input constraints. Without loss of generality, w_1 and v are assumed white noises with the following covariances,

$$E(w_1 w_1^T) = I_{mw1}, \quad E(vv^T) = V_1, \quad E(w_1 v^T) = N_1 \tag{2.1d}$$

where I_{mw1} is an identity matrix with dimension $mw1$, V_1 a diagonal matrix of size py, and N_1 an $mw1 \times py$ constant matrix.

When sensors fail the number of measurements $y(t)$ reduces and the relevant rows in the matrices C_2, D_{22} and the vector $v(t)$ can be disregarded. The system is now given by

$$\dot{x}(t) = Ax(t) + B_1 w_1(t) + B_2 u(t) \tag{2.2a}$$

$$G(s) \quad : \qquad z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} C_{1u} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ D_{12d} \end{bmatrix} u(t) \tag{2.2b}$$

$$\overline{y}(t) = \overline{C}_2 x(t) + \overline{v}(t) + \overline{D}_{22} u(t) \tag{2.2c}$$

The quantities with the bar represent the change in the system due to sensor failures.

Also, the relevant covariances are given by

$$E(w_1 w_1^T) = I_{mw1}, \quad E(\overline{v}\overline{v}^T) = \overline{V}_1, \quad E(w_1 \overline{v}^T) = \overline{N}_1 \tag{2.2d}$$

Now an optimal estimator can easily be designed by solving the relevant Riccati equation associated with the system given by equations (2.2) [22, 30].

2.3 Design of the observer using linear quadratic estimator design

To construct an observer for the states assuming that detectability conditions are met. The dynamics of the state estimates, $\hat{x}$ are given by the following equation

$$\dot{\hat{x}}(t) = (A - L\overline{C}_2)\hat{x}(t) + (B_2 - L\overline{D}_{22})u(t) + Ly(t) \tag{2.3a}$$

where the observer gain or the output injection matrix L is given by

$$L = Y\overline{C}_2 \overline{V}_1^{-1} \tag{2.3b}$$

and Y is the positive semi-definite stabilizing solution of the following algebraic Ricatti equation

$$A - B_1\overline{N}_1\overline{V}_1^{-1}\overline{C}_2)Y + Y(A - B_1\overline{N}_1\overline{V}_1^{-1}\overline{C}_2)^T - Y\overline{C}_2^T\overline{V}_1^{-1}\overline{C}_2Y + B_1(I - \overline{N}_1\overline{V}_1^{-1}\overline{N}_1)B_1^T = 0 \tag{2.3c}$$

The eigenvalues of $(A - L\overline{C}_2)$, the poles of the observer, are identical to the stable eigenvalues of following Hamiltonian matrix.

$$H_{obs} = \begin{bmatrix} (A - B_1\overline{N}_1\overline{V}_1^{-1}C_2)^T & -\overline{C}_2^T\overline{V}_1^{-1}\overline{C}_2 \\ B_1(-I + \overline{N}_1\overline{V}_1^{-1}\overline{N}_1^T)B_1^T & -A + B_1\overline{N}_1\overline{V}_1^{-1}\overline{C}_2 \end{bmatrix} \tag{2.3d}$$

2.4 A note on supervisory switching logic

A supervisory logic then needs to be designed based on a fault detection system. The fault detection system can rely on simple consistency checks for average value, limiting values, spectral nature of the data from the sensors. When a sensor fails the logic switches to the relevant observer starting the equations from the best available estimate i.e. the last credible value of the state. Additionally, the supervisor also should keep the pilot informed of all the changes in the system and alert him as to the unreliable measurements, if any. For cases a stable observer cannot be designed, changes in the hardware or *ad hoc* remedies using a dummy set of values; open-loop control may be resorted to. Alternatively, changes in the regulated variable or the definition of the system can also help.

2.5 Automatic carrier landing example

Consider the F/A-18A Automatic Carrier Landing System [72]. The longitudinal small perturbation equations of the F/A-18 A at 136 kts and an altitude of 50 ft with full flaps, i.e., those of the unimpaired system are given by

$$\dot{x} = Ax + B_1 u + B_2 w \qquad (2.4a)$$

$$y = C_2 x + D_{21} u + D_{22} w \qquad (2.5b)$$

where the system vectors are given by $x = [\overline{u}/V \;\; \alpha \;\; \theta \;\; q \;\; h/V]^T$, $u = [\delta_H \;\; \delta_{LEF} \;\; \delta_{RT} \;\; \delta_{PL}]^T$, and $w = [\alpha_g \;\; w_1 \;\; w_2 \;\; w_3 \;\; w_4]^{\mathrm{T}}$. The terms are explained in Table 2.1.

Table 2.1. Explanation of the term in the F/18 A carrier landing model

Term	**Physical quantity**
$\overline{u}/V$	Normalized Velocity
α	Perturbed angle of attack (rad.)
θ	Perturbed pitch angle (rad.)
q	Perturbed pitch rate (rad/s)
h/V	Perturbed normalized altitude
δ_H	Perturbed horizontal tail deflection
δ_{LEF}	Perturbed leading edge flap deflection
δ_{RT}	Perturbed rudder toe-in deflection
δ_{PL}	Engine power lever control angle
α_g	Incremental angle of attack
w_{1-4}	Sensor noises

The system matrices are given below

$$A=\begin{bmatrix} -0.0705 & 0.0475 & -0.1403 & 0.0000 & -0.000058 \\ -0.3110 & -0.3430 & 0.0000 & 0.99133 & 0.00102 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0218 & -1.1660 & 0.0000 & -0.2544 & 0.0000 \\ 0.0000 & -1.0000 & 1.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

$$B_1=\begin{bmatrix} 0.0121 & 0.00248 & 0.1690 & 0.2316 \\ -0.0721 & 0.0140 & 0.0128 & -0.0338 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -1.8150 & -0.0790 & 0.1681 & 0.0023 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

$$B_2=\begin{bmatrix} 0.0475 & 0 & 0 & 0 & 0 \\ -0.343 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1.166 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_2=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0.311 & 0.343 & 0 & 0.0087 & -0.001 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{21}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0721 & -0.014 & -0.0218 & 0.0338 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{22}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.343 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The general aim is to make the system sufficiently stable to keep the steady-state perturbed angle of attack near zero under a step vertical rate command.

Analysis for detectability for various cases of sensor failures reveals that the first measurement, perturbed normalized altitude is crucial for the design of stable observers. The case in which this sensor fails is therefore not considered for simulation. Observers for all other cases of failures can be designed. The calculations for the output injection matrices are conducted according to design strategy given in Section 2.3. Additionally, we use the same state feedback matrix based on linear regulator design to complete the controller design.

For the sake of simulation of the system, the carrier is given a step command in the vertical rate. In the cases, Figure 2.1 is the response of the angle of attack of system when all the four sensors are working fine. Figure 2.2 shows disastrous result of using the controller designed for the all the four sensors when the readings from 2^{nd} and 4^{th} sensors is corrupted by noise. In the third and fourth cases, shown in figures 2.3 and 2.4, the 2^{nd} and 4^{th} sensors and 2^{nd} and 3^{rd} sensors respectively, fail at time = 0. The hierarchical switch logic immediately switches it to a different controller and although the transient response of the closed-loop system is not as fast as the first case, it is acceptable. It is also seen for the cases shown in the figures 2.1, 2.3 and 2.4 the command signal is tracked fairly well too. It is seen here that multiple sensor failures can also be accommodated by the above approach.

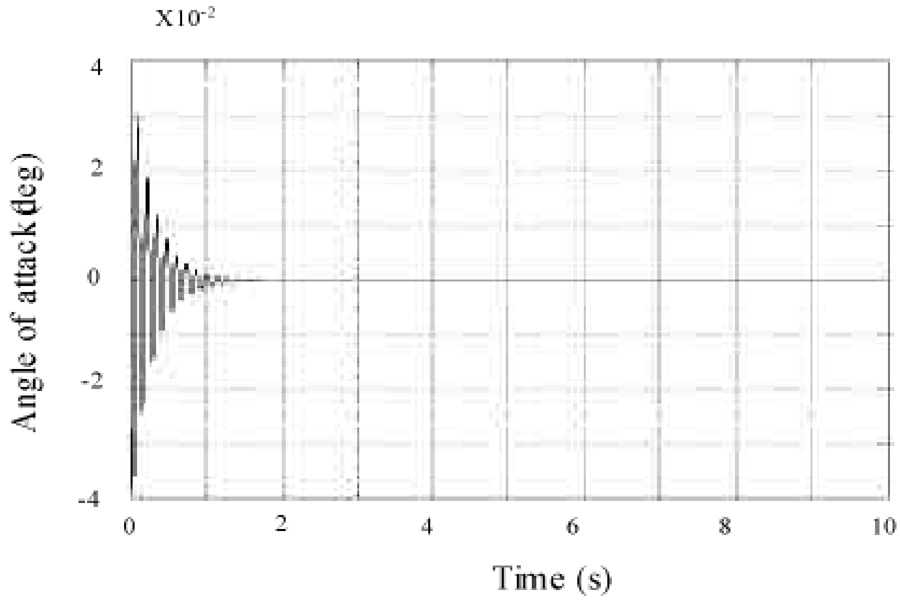


Figure 2.1. Response of the system with all measurements

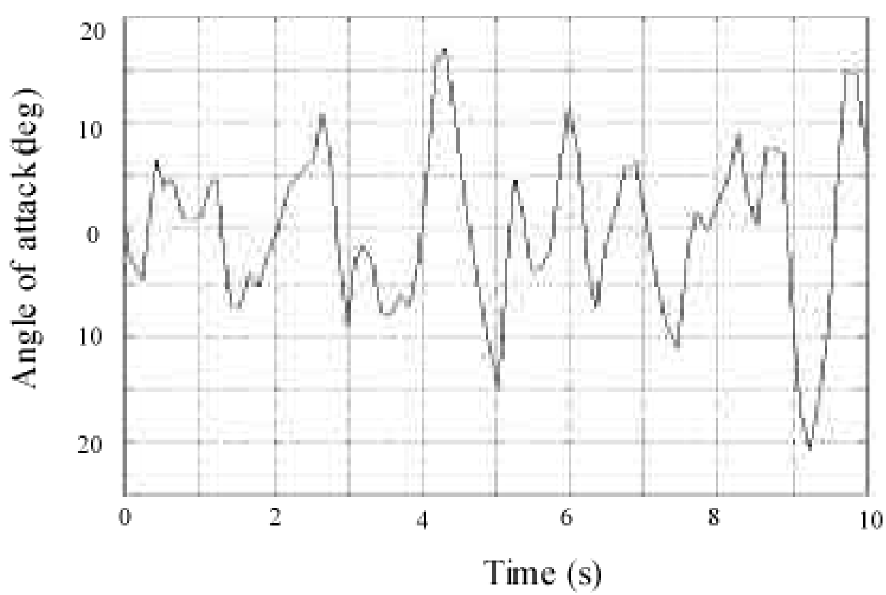


Figure 2.2. Response of the system with failed 2nd and 4th measurements but no hierarchical controller

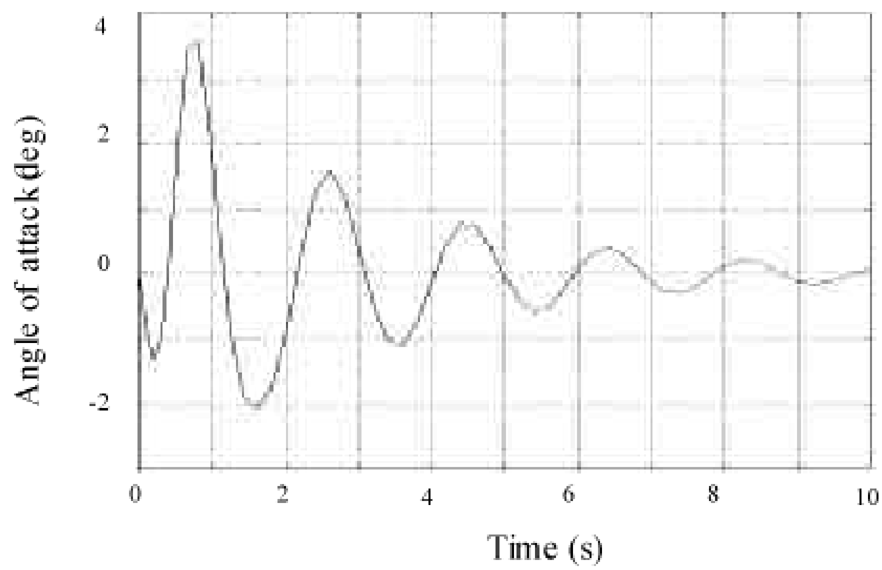


Figure 2.3. Response of the system with failed 2nd and 4th measurements with proposed hierarchical scheme

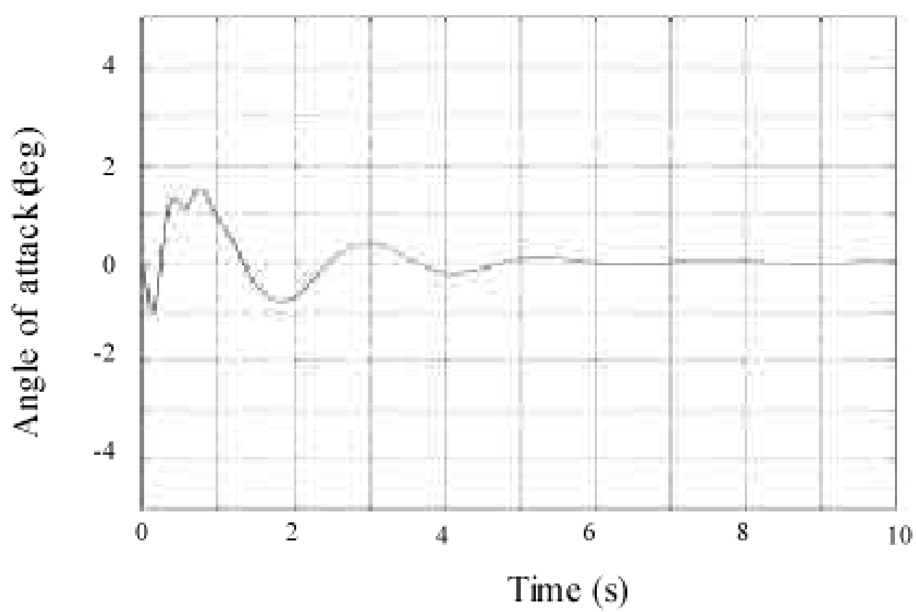


Figure 2.4. Response of the system with failed 2nd and 3rd measurements with proposed hierarchical scheme

2.6 Conclusion

In this chapter, the predesign of controllers for sensor failures was accomplished. Using detectability as the governing property of the system observers can be designed ignoring the measurements for anticipated sensor failures and utilized when the sensor failure occur. The more severe problem is that of actuator failure. For when an actuator fails it cannot be ignored. Also, in modern aircraft the sensory system has a greater hardware redundancy that the actuators. Sensor's are also easier to maintain and are cheaper. Also, the failures are easier to diagnose and switching to a new observer doesn't present great problems regards system stability, as the dynamics of state estimators are much faster than the dynamical system to be controlled. For these reasons, the rest of the thesis concentrates on the important problem of designing controllers to accommodate anticipated actuator failures.

Chapter 3. Predesign of controllers for actuator failures in linear systems

In this chapter, we consider the predesign of actuators for failed actuators. Other researchers as mentioned in section 1.3.3 of Chapter 1 have addressed the problem of control design for actuator failures. The most commonly used methods are *ad hoc* involving a control mixer. The mixer is a device responsible for distributing the control command signal to the actual physical effectors. When sensors fail they can always be ignored but actuator failures are more complicated than that.

Various actuator failure scenarios include partial failures, floating surfaces and jammed actuators. Partial failures of actuators can be addressed by rescaling the control law on the actuators. However, accurate failure identification is a must for rescaling and is generally not an easily accomplished if there are no direct measurements of the servo loop of the actuator. Floating surfaces present less of a problem determining the model of the failed system as the entire column with associated control variable disappears from the equations. This may severely restrict the redesign of the control law if adequate control authority is not available in the remaining effective actuators. However, the most common failure and mathematically most severe failures of an actuator is when it gets jammed due to hydraulic/mechanical/electrical failure at a possibly unknown position.

3.1 Problem formulation for jammed actuators

In our proposed remedy to the problem caused by jammed actuators, we assume the information of which and how many actuators have failed can be detected although the jammed positions need not to be known. We also assume that the plant models for all possible failure scenarios are given a priori. With these assumptions a set of controllers specifically tailored for their corresponding actuator failure scenarios are ready to work with the controller switching mechanism to accommodate every possible actuator failure. As mentioned before, the occurrence of actuator failures changes the structure of the plant model. The jammed actuators not only reduce the number of control inputs but also present persistent disturbances to the system. We employ the regulator theory [21, 25, 57, 80, 81] to address the persistent disturbance problem so that the steady-state response due to the persistence disturbance is zero. We will also use LQR (linear quadratic regulator) [22, 30] and LQE (linear quadratic estimator or Kalman estimator) [22, 30, 31] to respectively compute the state feedback gain and construct the observer that gives an estimate of the plant states and the persistent dynamics states. The state feedback gain and the observer determine the transient behavior of the system.

Consider the feedback control system shown in Figure 3.1 in which $G(s)$ is the nominal plant and $K(s)$ is the nominal controller that gives an optimal H_2 closed-loop performance [22, 30].

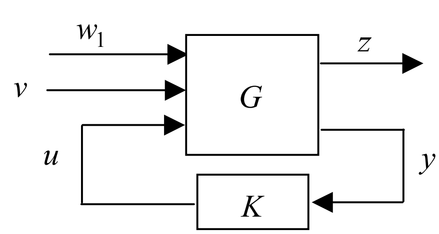


Figure 3.1. The nominal closed-loop system

The nominal plant $G(s)$ is described by the following linear dynamics equations,

$$\dot{x}(t) = Ax(t) + B_1 w_1(t) + B_2 u(t) \tag{3.1a}$$

$$G(s) \quad : \qquad z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} C_{1u} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ D_{12d} \end{bmatrix} u(t) \tag{3.1b}$$

$$y(t) = C_2 x(t) + v(t) + D_{22} u(t) \tag{3.1c}$$

where $x \in R^n$ is the state vector, $w_1 \in R^{mw1}$ the exogenous input vector to the system, $u \in R^{mu}$ the control input vector, $z \in R^{pz}$ the regulated signal vector, $y \in R^{py}$ the measured output, and $v \in R^{py}$ the measurement noise. A, B_1, B_2, C_{1u}, D_{12d}, C_2, and D_{22} are constant matrices of appropriate dimensions. The regulated vector z consists of z_1 and z_2 where $z_1 \in R^{pz1}$ is the error to be minimized and $z_2 \in R^{pz2}$ represents the control-input constraints. Without loss of generality, w_1 and v are assumed white noises with the following covariances,

$$E(w_1 w_1^T) = I_{mw1}, \quad E(vv^T) = V_1, \quad E(w_1 v^T) = N_1 \tag{3.1d}$$

where I_{mw1} is an identity matrix with dimension $mw1$, V_1 a diagonal matrix of size py, and N_1 an $mw1 \times py$ constant matrix. The nominal controller $K(s)$ is assumed to be an optimal H_2 controller, i.e., it is the one that minimizes the following quadratic performance index

$$J = \lim_{T \to \infty} \frac{1}{T} E\left[\int_0^T z^T(t) z(t) dt \right] \tag{3.1e}$$

The nominal optimal H_2 control solution can be found in [22,30].

3.1.1. Change in the system due to actuator failure

In the following, we will investigate the structure change in the system caused by jammed actuators. This type of actuator failure is very common in practice; for example in the flight control of aircraft, some of the control surfaces may get stuck due to hydraulic, mechanical or electrical failure. The control input u is partitioned into two parts and the controller is also divided into two parts accordingly as

$$u = \begin{bmatrix} u_f \\ u_e \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} K_f \\ K_e \end{bmatrix} \tag{3.2}$$

where $u_f \in R^{muf}$ represents the failed actuators and $u_e \in R^{mue}$ the remaining effective controls.

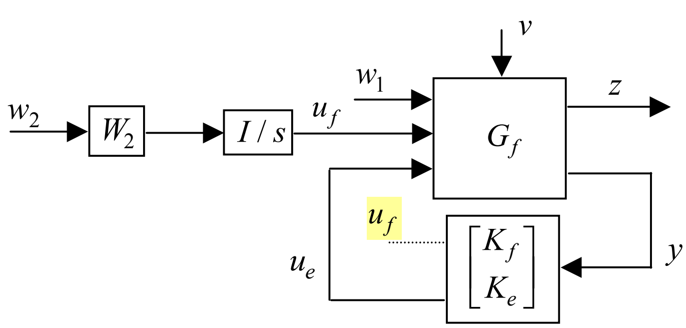


Figure 3.2. The system with actuator failure

When the actuator failure occurs, the system becomes that described in Figure 3.2. The input u_f, which originally was connected to the controller output, now is a persistent disturbance vector generated by the integrators I_{muf} / s, the weighting matrix W_2, and the white noise vector w_2 with covariance $E(w_2 w_2^T) = I_{muf}$. The plant model now is changed to $G_f(s)$:

$$\dot{x}(t) = Ax(t) + B_1 w_1(t) + B_{2f} u_f + B_{2e} u_e(t) \tag{3.3a}$$

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} C_{1u} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ D_{12de} \end{bmatrix} u_e(t) \tag{3.3b}$$

$$y(t) = C_2 x(t) + v(t) + D_{22f} u_f(t) + D_{22e} u_e(t) \tag{3.3c}$$

and the persistent disturbance model is described by

$$\dot{u}_f(t) = W_2 w_2(t) \tag{3.3d}$$

In the above, B_{2f}, B_{2e}, D_{22f}, D_{22e}, and D_{12de} are obtained from the following partitions,

$$B_2 = \begin{bmatrix} B_{2f} & B_{2e} \end{bmatrix}, \quad D_{22} = \begin{bmatrix} D_{22f} & D_{22e} \end{bmatrix}, \quad D_{12d} = \begin{bmatrix} D_{12df} & D_{12de} \end{bmatrix} \tag{3.3e}$$

Note that the system dynamics have been altered significantly because of the actuator failure. Continuing to use the nominal controller usually will lead to unacceptable performance or even system instability. A new controller is needed to accommodate the altered system dynamics.

3.2. Regulator controller design to accommodate the actuator failure

The failed actuators not only reduce the number of effective controls but also introduce persistent disturbances to the system. A new controller using the remaining effective controls needs to be designed so that the close-loop system is internally stable, steady-state regulation takes place, and the quadratic performance index is minimized.

The steady-state regulation means that the error $z_1(t)$ due to the persistent disturbance will approach to zero as $t \to \infty$.

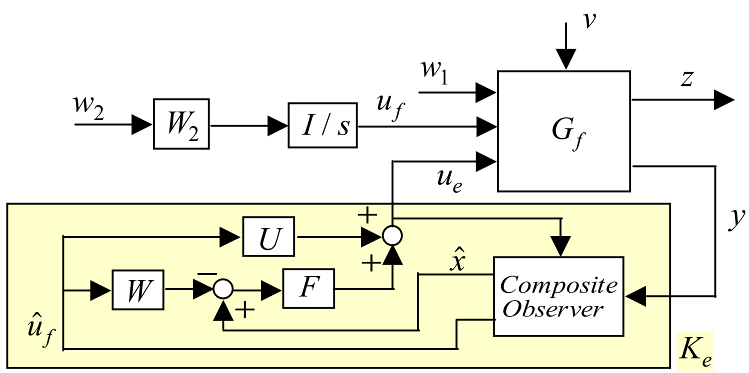


Figure 3.3. Regulator controller design to accommodate the actuator failure

3.2.1 Steady-state regulation

The block diagram of the proposed regulator controller to accommodate the actuator failure is shown in Figure 3.3. The condition for the existence of stabilizing controllers is that the system (A, B_{2e}, C_2) is stabilizable and detectable. As long as the closed-loop system is internally stable, the steady-state regulation will take place if W and U are chosen so that the following equations are satisfied [21, 76],

$$AW + B_{2f} + B_{2e}U = 0 \tag{3.4a}$$

$$C_{1u}W = 0 \tag{3.4b}$$

It is worthwhile to mention here that if the above regulator equations are underdetermined and further embellishment in order to introduce weighting or additional constraints in terms of the minimization of control for regulation can be conducted to make the system of equations exactly determined. If however, these equations are overdetermined and no solutions exist then one cannot meet the zero error criterion and further tradeoff by of modification of the error variables is necessary. The nature of the regulator equations provides a measure for assessing the redundancy available for reconfiguration. This is elaborated in the next chapter.

3.2.2. Composite observer construction using linear quadratic estimator design

A composite observer will be constructed to generate $\hat{x}(t)$, the estimated plant state, and $\hat{u}_f(t)$, the estimated disturbance dynamics state. For this purpose, a composite system is formed as follows,

$$\dot{x}_c(t) = A_c x_c(t) + B_{c1} w(t) + B_{c2} u_e(t) \tag{3.5a}$$

$$z(t) = C_{c1} x_c(t) + D_{c12} u_e(t) \tag{3.5b}$$

$$y(t) = C_{c2} x_c(t) + v(t) + D_{22e} u_e(t) \tag{3.5c}$$

where

$$x_c(t) = \begin{bmatrix} x(t) \\ u_f(t) \end{bmatrix}, \quad w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$$

$$A_c = \begin{bmatrix} A & B_{2f} \\ 0 & 0 \end{bmatrix}, \ B_{c1} = \begin{bmatrix} B_1 & 0 \\ 0 & W_2 \end{bmatrix}, \ B_{c2} = \begin{bmatrix} B_{2e} \\ 0 \end{bmatrix} \tag{3.5d}$$

$$C_{c1} = \begin{bmatrix} C_{1u} & 0 \\ 0 & 0 \end{bmatrix}, \ D_{c12} = \begin{bmatrix} 0 \\ D_{12de} \end{bmatrix}, \ C_{c2} = \begin{bmatrix} C_2 & D_{22f} \end{bmatrix}$$

and w and v are white noises with the following covariances,

$$E(ww^T) = I_{mw}, \quad E(vv^T) = V, \quad E(wv^T) = N \tag{3.5e}$$

where I_{mw} is an identity matrix with dimension $mw = mw1 + muf$, V a diagonal matrix of size py, and N an $mw \times py$ constant matrix.

For the ease of presentation, we assume that (A_c, C_{c2}) is detectable. With this assumption, a stable composite observer can be constructed as follows,

$$\dot{\hat{x}}_c(t) = \left(A_c - LC_{c2}\right)\hat{x}_c(t) + \left(B_{c2} - LD_{22e}\right)u_e(t) + Ly(t) \tag{3.6a}$$

where the composite observer gain L is

$$L = YC_{c2}^T V^{-1} \tag{3.6b}$$

and Y is the positive semi-definite stabilizing solution of the following algebraic Riccati equation

$$(A_c - B_{c1}NV^{-1}C_{c2})Y + Y(A_c - B_{c1}NV^{-1}C_{c2})^T - YC_{c2}^T V^{-1}C_{c2}Y + B_{c1}(I - NV^{-1}N^T)B_{c1}^T = 0 \tag{3.6c}$$

Note that the eigenvalues of $A_c - LC_{c2}$, the poles of the composite observer, are identical to the stable eigenvalues of the following Hamiltonian matrix

$$H_{obs} = \begin{bmatrix} (A_c - B_{c1}NV^{-1}C_{c2})^T & -C_{c2}^T V^{-1}C_{c2} \\ B_{c1}(-I + NV^{-1}N^T)B_{c1}^T & -A_c + B_{c1}NV^{-1}C_{c2} \end{bmatrix} \tag{3.6d}$$

3.2.3. Computation of the state feedback gain using linear quadratic regulator design

Define

$$Q = C_{1u}^T C_{1u}, \quad R = D_{12de}^T D_{12de} \tag{3.7a}$$

Then the state feedback gain matrix F can be computed as follows

$$F = -R^{-1} B_{2e}^T X \tag{3.7b}$$

where X is the positive semi-definite stabilizing solution of the following algebraic Riccati equation

$$A^T X + XA - XB_{2e} R^{-1} B_{2e}^T X + Q = 0 \tag{3.7c}$$

Note that the eigenvalues of $A + B_{2e} F$, the regulator poles, are identical to the stable eigenvalues of the following Hamiltonian matrix

$$H_{reg} = \begin{bmatrix} A & -B_{2e} R^{-1} B_{2e}^T \\ -Q & -A^T \end{bmatrix} \tag{3.7d}$$

3.2.4. State-space realization of the regulator

From the interconnection of Figure 3.3 and the composite observer dynamics given by Eq. (3.6a), we have the following state space realization for the regulator controller $K_e(s)$:

$$\dot{x}_K(t) = A_K x_K(t) + B_K y(t) \tag{3.8a}$$

$$u_e(t) = C_K x_K(t) \tag{3.8b}$$

where

$$B_K = L \tag{3.8c}$$

$$C_K = \begin{bmatrix} F & U - FW \end{bmatrix} \tag{3.8d}$$

$$A_K = A_c - LC_{c2} + (B_{c2} - LD_{22e}) C_K \tag{3.8e}$$

The state vector of the controller, $x_K(t)$, is the same as that of the composite observer, $\hat{x}_c(t)$. It is easy to show that the closed-loop systems poles are the eigenvalues of $A+B_{2e}F$, the regulator poles, and those of $A_c - LC_{c2}$, the composite observer poles. Note that there are n regulator poles and composite observer $n+muf$ poles, where n is the order of the plant and muf is the number of failed actuators.

In the next section, an F/18-A automatic carrier landing system undergoing actuator failures is employed to illustrate the proposed design procedure.

3.3 An F/A 18-A automatic carrier landing system with actuator failures

In the aircraft flight control system the number of control surfaces or thrust vectoring actuators usually exceeds the number of control inputs that is necessary for the system to be controllable. This redundancy in control actuation may be utilized for controller reconfiguration allowing the aircraft to deliver acceptable performance even when some of the control surfaces get stuck at any fixed position during operation.

3.3.1 Nominal plant

In this section, the proposed regulator controller design is applied to an aircraft landing system with an actuator failure. The nominal plant is again taken from [72] and is a F/A 18-A aircraft in a landing maneuver onto a carrier. The system matrices for the nominal plant, which is the longitudinal small perturbation equations of F/A-18A at 136 kts and an altitude of 50 ft with full flaps, are given as

$$\dot{x}(t) = Ax(t) + B_1 w_1(t) + B_2 u(t) \tag{3.9a}$$

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} C_{1u} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ D_{12d} \end{bmatrix} u(t) \tag{3.9b}$$

$$y(t) = C_2 x(t) + v(t) + D_{22} u(t) \tag{3.9c}$$

where

$$A = \begin{bmatrix} -0.0705 & 0.0475 & -0.1403 & 0 & -0.000058 \\ -0.3110 & -0.3430 & 0 & 0.99133 & 0.00102 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0218 & -1.1660 & 0 & -0.2544 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.0475 \\ -0.343 \\ 0 \\ -1.166 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0121 & 0.00248 & 0.1690 & 0.2316 \\ -0.0721 & 0.0140 & 0.0128 & -0.0338 \\ 0 & 0 & 0 & 0 \\ -1.8150 & -0.0790 & 0.1681 & 0.0023 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_{12u} = \begin{bmatrix} -5 & 17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 35 \end{bmatrix}, \quad D_{12d} = \begin{bmatrix} 0.075 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0.311 & 0.343 & 0 & 0.0087 & -0.001 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{22} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0721 & -0.014 & -0.0218 & 0.0338 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and w_1 and v are assumed white noises with the following covariances,

$$E(w_1 w_1^T) = 1, \quad E(vv^T) = V_1, \quad E(w_1 v^T) = N_1 \tag{3.9d}$$

where $V_1 = diag\{1, \quad 1, \quad 1+0.343^2, \quad 1\}$ and $N_1 = [0 \quad 0 \quad 0.343 \quad 0]$.

The F/A 18-A has five pairs of aerodynamic control surfaces: stabilators, rudders, ailerons, leading-edge flaps, and trailing edge flaps. The twin vertical stabilizers, with trailing edge rudders, are canted outboard at approximately 20° from the vertical. Being a modern day aircraft the flight control system features measurements from a redundant production sensor sets and air data. The actuation of the aircraft surfaces is provided by redundant hydraulic systems and so it is highly unlikely that all the control surfaces fail at the same time. The magnitude and rate saturation for the relevant control surfaces is given in [44] and the relevant ones are reproduced here in Table 3.1.

Table 3.1 Rate and magnitude saturations of the aircraft actuators

Surface	Magnitude Saturation (deg.)	Rate Saturation(deg/sec)
Stabilator:		
Trailing edge up (+)	24	40
Trailing edge down (-)	10.5	40
Leading edge flap:		
Up (+)	3	15
Down (-)	33	15
Rudder:		
Trailing edge left (+)	30	82
Trailing edge right (-)	30	82

The reconfigurable controller has to be designed to accommodate the complete failure of the leading-edge flaps and the rudder of the aircraft.

3.3.2. Change in the system due to actuator failure

Now we assume that the 2nd and 3rd controls of the control input vector, $u(t)$, fail and jam at an unknown position while the 1st and 4th controls are still effective. The failed controls are represented by u_f, which now act like persistent disturbances, and the remaining effective ones by u_e. The plant now has been changed to that described as in Eq. (3.3 a, b, c) where

$$B_{2f} = \begin{bmatrix} 0.00248 & 0.1690 \\ 0.0140 & 0.0128 \\ 0 & 0 \\ -0.0790 & 0.1681 \\ 0 & 0 \end{bmatrix}, \quad B_{2e} = \begin{bmatrix} 0.0121 & 0.2316 \\ -0.0721 & -0.0338 \\ 0 & 0 \\ -1.8150 & 0.0023 \\ 0 & 0 \end{bmatrix} \qquad (3.10a)$$

$$D_{12de} = \begin{bmatrix} 0.075 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad D_{22e} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0721 & 0.0338 \\ 0 & 0 \end{bmatrix}, \quad D_{22f} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.014 & -0.0218 \\ 0 & 0 \end{bmatrix}$$

(3.10b)

The persistent disturbance model is described in Eq. (3.2-3.3d) where w_2 is assumed a white noise with covariance $E(w_2 w_2^T) = I_2$ and W_2 is a weighting matrix chosen to be $W_2 = 3I_2$.

3.3.3. Regulator controller design to accommodate the actuator failure

The block diagram of the controller is shown in Figure 3.3. The first step is to check if the failed-actuator system (A, B_{2e}, C_2) is stabilizable and detectable, which is the necessary and sufficient condition of the existence of stabilizing controllers for the system. Using PBH rank test [48], it is easy to find that (A, B_{2e}, C_2) is controllable and observable.

3.3.3.1 Steady-state regulation

The steady-state regulation takes place if W and U are chosen so that the equations of (3.4) are satisfied. The steady-state regulation means that the error response $z_1(t)$ due to the persistent disturbance at u_f will approach to zero as $t \to \infty$. A solution of (3.4) can be found as follows,

$$W = \begin{bmatrix} 0.04211270800091091 & 0.07496200503388845 \\ 0.012386090588503209 & 0.022047648539378954 \\ 0.012386090588503209 & 0.022047648539378954 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.11a}$$

$$U = \begin{bmatrix} -0.050965140412176015 & 0.07846372577233009 \\ 0.009736845076272217 & -0.70215271132341 \end{bmatrix} \tag{3.11b}$$

3.3.3.2 Composite observer construction using linear quadratic estimator design

To construct a composite observer for $\hat{x}(t)$ and $\hat{u}_f(t)$, the composite system is formed as in Eq. (3.5) with which w and v are white noises with the following covariances,

$$E(ww^T) = I_3, \quad E(vv^T) = V, \quad E(wv^T) = N \tag{3.12a}$$

where $V = diag\{1, \ 1, \ 1+0.343^2, \ 1\}$ and

$$N = \begin{bmatrix} 0 & 0 & 0.343 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.12b}$$

It is easy to see that the composite system is observable. A stable composite observer can be constructed as

$$\dot{\hat{x}}_c(t) = \left(A_c - LC_{c2}\right)\hat{x}_c(t) + \left(B_{c2} - LD_{22e}\right)u_e(t) + Ly(t) \tag{3.13a}$$

where the composite observer gain L is

$$L = YC_{c2}^T V^{-1} \tag{3.14b}$$

and Y is the positive semi-definite stabilizing solution of the algebraic Riccati equation in Eq. (3.2-3.6c). With this composite observer gain L, the composite observer poles or the eigenvalues of $A_c - LC_{c2}$ are

{ -0.6622 ± 1.1368i, -0.3057 ± 0.5960i, -0.5423 ± 0.2010i, -0.1309 } (3.15)

3.3.3.3 Computation of the state feedback gain F using linear quadratic regulator design

The state feedback gain F can be found using

$$F = -R^{-1}B_{2e}^T X \tag{3.16}$$

where X is the positive semi-definite stabilizing solution of the algebraic Riccati equation in Eq. (3.7c). With this state feedback gain F, the regulator poles or the eigenvalues of $A+B_{2e}F$ are

$$\{ -16.8703 \pm 11.3696i, \ -1.2612 \pm 1.7704i, \ -1.7641\} \tag{3.16a}$$

3.3.3.4 State-space realization of the regulator controller

From the interconnection of Figure 3.3 and the composite observer dynamics given by Eq. (3.6a), we have the following state space realization for the regulator controller $K_e(s)$:

$$\begin{aligned} \dot{x}_K(t) &= A_K x_K(t) + B_K y(t) \\ &= \left\{A_c - LC_{c2} + (B_{c2} - LD_{22e})\begin{bmatrix} F & U - FW \end{bmatrix}\right\} x_K(t) + Ly(t) \end{aligned} \tag{3.17a}$$

$$u_e(t) = \begin{bmatrix} F & U - FW \end{bmatrix} x_K(t) \tag{3.17b}$$

where the composite observer gain is

$$L = \begin{bmatrix} 0.5098 & 0.9203 & 0.7197 & 0.1257 \\ 0.0167 & 0.0673 & 0.2026 & 0.8371 \\ 0.4880 & 0.6275 & 0.5385 & 0.9044 \\ 0.1674 & 0.2151 & 0.1109 & 0.6718 \\ 0.8384 & 0.4713 & 0.1244 & 0.0167 \\ 0.5103 & 0.8887 & 0.6321 & -2.7392 \\ 0.8598 & 1.8569 & 1.7314 & 1.2092 \end{bmatrix} \tag{3.17c}$$

and the regulator gain is

$$F = \begin{bmatrix} -57.9344 & 127.7988 & 99.5565 & 12.5596 & 102.4137 \\ -10.0780 & 68.3006 & -65.9201 & -2.4374 & -85.3418 \end{bmatrix} \tag{3.17d}$$

With this controller realization, it is easy to verify that the set of the closed-loop system poles,

$$\begin{Bmatrix} -16.8703 \pm 11.3696i, & -1.2612 \pm 1.7704i, & -1.7641, & -0.1309 \\ -0.6622 \pm 1.1368i, & -0.3057 \pm 0.5960i, & -0.5423 \pm 0.2010i & \end{Bmatrix} \tag{3.18}$$

includes the composite observer poles

$\{ -0.6622 \pm 1.1368i, \ -0.3057 \pm 0.5960i, -0.5423 \pm 0.2010i, -0.1309 \}$

and the regulator poles

$\{ -16.8703 \pm 11.3696i, -1.2612 \pm 1.7704i, \ -1.7641 \}$

With this controller, the error response $z_1(t)$ due to the persistent disturbance $u_f(t)$ is shown in Figure 3.4.

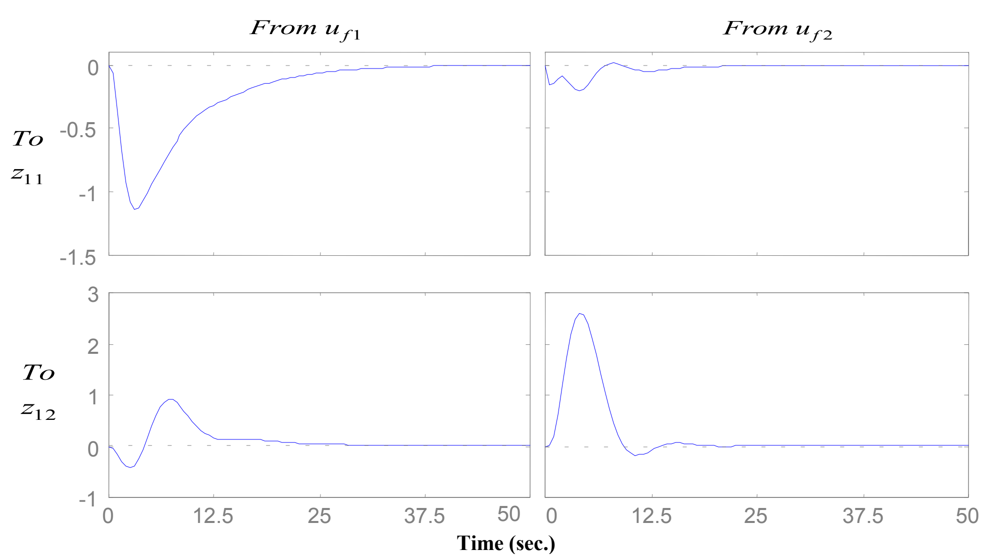


Figure 3.4. Error response $z_1(t)$ due to step input at u_f

Note that the error response $z_1(t)$ due to the persistent disturbance input at u_f will approach to zero as $t \to \infty$. The transient behavior is determined by the composite observer gain L and the state feedback gain F, which in turn depend on the choice of weighting matrices. The transient behavior of the error response due to u_f can be traded off with the control-input constraint or the system performance concerning other exogenous inputs.

3.4 Conclusion

In this chapter, the predesign for controllers for actuator failure was considered. The design was accomplished using the well-established regulator theory with internal stability. It is worthwhile to note that the example presented multiple actuator failures were overcome using the design procedure. In comparison the most widely used procedure, that of control mixer or pseudo-inverse method could not overcome such failures completely. The theory allows the controller to utilize the system dynamics in addition to the redundancy in the control actuation for the purpose of output regulation. The natural extension of this design procedure would be to analyze and synthesize fault-tolerant systems. The reduction in control authority can also be addressed by using the computed matrices from the regulator equations. This forms the subject of next chapter.

4. Characterizing redundancy in systems with actuator failures.

Redundancy can be understood as the addition of resources beyond those needed for nominal operation. In modern day aircrafts redundancy is provided in terms of same or similar hardware/software/data components. This notion of duplex/triplex/quadruplex redundancy albeit expensive does help in providing the increase in safety by allowing the system to perform even when some of the components fail [67]. The notion of analytical redundancy like that used in model based methods for failure detection and identification have also proved useful [29]. These notions are however insufficient when it comes to describing the redundancy with respect to actuator failures. Authors in [82] have made an attempt to define "Control reconfigurability" trying to capture the transparency needed to design an effective feedback controller. They measure the redundancy in connection with feedback control by assuming that the foreseeable faults are parameterized in the model of the process. The change in the smallest second order mode, which is taken to be a measure of combined controllability and observability, this is the potential of the system to maintain or meet a certain performance criterion through control reconfiguration at the occurrence of the worst fault in the fault parameter space. Since the definition requires the parameterization of faults this definition works only for systems with partial failures in actuation. However, when an actuator gets stuck in possibly non-zero position there may be drastic changes in the input output structure as seen in the last chapter. This chapter develops a way to characterize redundancy for systems undergoing complete failures of actuators.

It has known that the existence of solution to the coupled matrix equations(regulator equations) along with the conditions for stabilizability and detectability give not only the sufficient, but also the necessary conditions to design a control to meet the zero error objective. This idea can then be developed to define what redundancy the system might have with regards to stuck actuators. In the following development the systems are considered so as not to include the zero mean variables associated with white noise and the feedforward measurements. Addtionally, the failures are restricted to single failures however the discussion could easily be generalized to multiples failures.

4.1. Using regulator conditions to establish reconfigurability with respect to actuator failures.

Consider then the systems for which the failure models are given by

$$\dot{x}(t) = Ax(t) + B_{2f}u_f + B_{2e}u_e(t) \qquad (4.1a)$$

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} C_{1u} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ D_{12de} \end{bmatrix} u_e(t) \qquad (4.1b)$$

$$y(t) = C_2 x(t) \qquad (4.1c)$$

Additionally, the dynamics of the persistent dynamics caused by the failed actuator is given by

$$\dot{u}_f = 0 \qquad (4.1d)$$

Since, in the above equations we restrict u_f to be a scalar, the column associated with it, B_{2f} is a vector.

The necessary and sufficient conditions for output regulation for output regulation are given by

1. (A, B_{2e}) is stabilizable.

2. (A, C_2) is detectable.

3. There exist two matrices W and U of appropriate dimensions that are solutions to the following matrix equations

$$AW + B_{2e}U = -B_{2f} \tag{4.2a}$$

$$C_{1u}W = 0 \quad (4.2b)$$

or

$$\begin{bmatrix} A & B_{2e} \\ C_{1u} & 0 \end{bmatrix}\begin{bmatrix} W \\ U \end{bmatrix} = \begin{bmatrix} -B_{2f} \\ 0 \end{bmatrix} \tag{4.2}$$

If any of the conditions are not met then the goal of $z_1(t) \to 0$ as $t \to \infty$ cannot be met [80]. It is useful then to examine how these conditions might not be satisfied.

The first condition fails when the column B_{2f} is instrumental in stabilizing an unstable mode and the span of B_{2e} doesn't include B_{2f}. If the matrix associated with the remaining functional controls contains the column associated with the failed control in it's span then the effect of failed control on stabilizability can be overcome.

The second condition doesn't depend upon the failed controls in any way hence if the original system is detectable then the failed system remains detectable and this condition is satisfied automatically.

Finally, if the system remains stabilizable and detectable but no W and U exist that satisfy the two given equations, the outputs $z_1(t)$ cannot be regulated to zero if u_f is not zero, i.e., the actuator fails at a non-zero position. The solvability of the matrix equations can also be understood in geometric terms as the requirement of finding an (A, B_{2e}) invariant mapping in the kernel of C_{1u}. The first matrix equation is associated finding the linear map $x = Wu_f$ that is rendered locally invariant by the feedback law

$u = Uu_f$. The second equations represents the kernel calculations associated with the output matrix. Together the conditions imply that the variable u_f can be made not to affect the output variables z_1 in the steady state. The existence of solution implies that the set of linear equations given by 4.2 are undetermined or exactly determined. If the system is underdetermined they can be made exactly determined by imposing additional constraints, like the reduction of the weighted norms associated with the matrices W and U. This may be done to get a unique solution for these matrices.

These important conditions are considered by other authors as changes in the zero structure and the relative degree of the system with and without the persistent disturbance dynamics [80]. These conditions presented here are also equivalent to the matching conditions presented in geometric terms in [77] for the design of adaptive controllers. However, when the conditions are presented as here they lead to two interesting extensions, one to address the required control authority in the effective controls to overcome failure and the generalization to nonlinear systems, which form the subject of the next chapter.

4.2. Addressing loss of control authority

All physical actuators have magnitude and rate restrictions. The discussion here is restricted to magnitude saturations. In this section, we seek to determine how much control authority is needed in the functioning actuators in order to overcome the failed controls assuming that the three conditions for output regulation are met if the magnitude saturations for the failed control are known. Consider that for nominal operation $u_f \in [u_{f\min}, u_{f\max}]$, i.e., $u_{f\min} \le u_f \le u_{f\max}$. The control then can fail at any value between the two extreme values. This implies the additional need for control authority in the functional control is the difference between the extreme values $(KW-U)u_{f\min}$ and $(KW-U)u_{f\max}$.

4.3. Examples

Consider then the example taken from [82]. The unimpaired model has a state space description given as

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$z = y = Cx(t)$$

where $A = \begin{bmatrix} -0.0226 & -36.6 & -18.9 & -32.1 \\ 0 & -1.9 & 0.983 & 0 \\ 0.0123 & -11.7 & -2.63 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -0.414 & 0 \\ -77.8 & 22.4 \\ 0 & 0 \end{bmatrix}$ and

$$C = \begin{bmatrix} 0 & 5.73 & 0 & 0 \\ 0 & 0 & 0 & 5.73 \end{bmatrix}.$$

The above plant has fours states: physically they represent forward velocity, angle of attack, pitch rate and pitch angle. The plant model has two inputs: elevon command and canard command. The two outputs to be regulated are also the measurements: angle of attack and pitch angle. It can be shown that using one column at a time the above system remains stabilizable and detectable but the regulator equations wind up being overdetermined and hence no solutions exist. This implies we cannot meet the objective of regulating both the variables of the output if the failures of the actuators occur at non-zero positions. The only resort then is to change the system design by adding additional controls or by relaxing the control objectives. One way to relax the control objective would be to combine the angle of attack and the pitch angle into a single variable. In this case it makes physical sense to take the difference between the two variables and define the flight path angle and make that the regulated variable. It is then seen that the following matrices for the regulation can be calculated for the failure associated with the second control, i.e. the canard command.

$$W = \begin{bmatrix} -221.104 \\ 0.07274 \\ 0 \\ 0.07274 \end{bmatrix} \text{ and } U = \begin{bmatrix} -0.33381 \end{bmatrix}$$

This means that we can regulate the flight path angle to zero but each of the variables composing the flight path angle cannot be each regulated. If we want to continue to regulate both variable additional actuators are needed in the system. Additionally, the extra requirement in control authority to regulate the variable can be easily calculated.

In the example of the F/A-18 A aircraft in a carrier landing maneuver given in the last chapter it can be shown that failure of any two actuators not involving the stabilator can be overcome by the system. However, for the failure of the stabilator although one can solve the regulator equations the control authority required in the remaining actuators is prohibitive if the failure occurs at a value away from the zero position. This can be shown by comparing the extreme values with available authority given earlier in Table 3.1.

4.4. Conclusion

In this chapter, the necessary and sufficient conditions for regulation are developed to determine the amount of redundancy required in systems to completely over failed actuators. This analysis can be very useful in synthesizing fault-tolerant systems and assessing the amount of control authority required to overcome failures in which the surface get stuck. The analysis is demonstrated on one example taken from literature and the analyzing control authority issues for the example given in the previous chapter. These ideas can be further developed to arrive at measures of redundancy in the dynamical sense for actuator failures. The advantage that the regulator design gives over the previous methods of control design for actuator failures is clearly demonstrated using these ideas for redundancy management. Regulator design the entire system with the input-output structure is used to overcome failures. All the calculations are easily verifiable and allow the engineer to make smart choices when designing the system.

Chapter 5. Predesign of controllers for actuator failures in nonlinear systems

Chapter 3 provides a regulator approach to predesigning control for linear systems with stuck actuators. In presented regulator design provides necessary and sufficient conditions for the design of controllers for linear systems but also addresses transient performance when the system is trying to reconfigure.

As mentioned earlier an important component of a fault tolerant system is the fault detection and identification mechanism. Approaches to reconfigurable control provide hierarchical switching mechanisms that switch to a controller designed for the impaired system when the failures are detected. It usually takes some time before the actuator failure is correctly diagnosed by the fault detection and identification mechanism. During this time the impaired system continues to operate with the original but now inappropriate controller.

If the faults are detected immediately the linear control techniques may suffice since the system states are close to the linear region. However, delays in the detection may cause significant excursion of the states. In many cases, nonlinearities in the aircraft dynamics may cause the linear design to be rendered inappropriate. Nonlinear regulators may provide larger domains of stability providing a larger window of safety in face of delays in detection and identification.

5.1. Formulation of the nonlinear actuator failure problem

In this section, we consider the actuator failures of nonlinear systems. We consider systems of the form

$$\dot{x} = f(x,u) \tag{5.1}$$

with state x defined in the neighborhood X of the origin in R^n and the input $u \in R^m$, We also assume that $f(0,0) = 0$. We also have a set of output variables that we are trying to regulate given as

$$e = h(x) \tag{5.2}$$

where $e \in R^p$.

we consider only the case in which some of the controls get stuck at a particular position. We consider the value at which the controls get stuck is available for feedback by measurement.

Let $u_f \in U_f$ represent the set of failed controls and $u_e \in U_e$ the set of effective controls after failure. The dynamics of the plant and the output equation can now be rewritten as

$$\dot{x} = f(x, u_f, u_e) \tag{5.3}$$

$$e = h(x) \tag{5.4}$$

Since the actuators represented by the control inputs u_f are stuck they can be thought of as outputs of a dynamical system given as

$$\dot{u}_f = 0 \tag{5.5}$$

Note: $U_e \times U_f = U$

Equations (5.3), (5.4) and (5.5) then form the basis of control design for the impaired system. We are interested in designing a control strategy for the effective controls as a function of the states and the position of the failed controls i.e.

$$u_e = \alpha(x, u_f) \tag{5.6}$$

Problem Statement: Given a nonlinear system of the form (5.3) and the disturbance dynamics of failed actuators given by (5.5), find, if possible, a mapping of the form (5.6) such that

the equilibrium $x = 0$ of

$$\dot{x} = f(x, 0, \alpha(x, 0)) \tag{5.7}$$

is asymptotically stable in the first approximation,

and there exists a neighborhood $V \subset X \times U_f$ of *(0,0)* such that, for each initial condition $(x_o, u_{fo}) \in V$, the solution of $\dot{x} = f(x, u_f, \alpha(x, u_f))$ satisfies

$$\lim_{t \to \infty} h(x(t), w(t)) = 0 . \tag{5.8}$$

Note: Since the stability of the equilibrium in the first approximation of the closed loop system is demanded, the stabilizability of the linearized system is a requirement.

In the following section the design of nonlinear regulator for the systems are described. The linearization of the system described by equation (5.3) about the origin results in equations of the form.

$$\dot{x} = Ax + B_f u_f + B_e u_e \tag{5.9}$$

where $A = \left.\frac{\partial f}{\partial x}\right|_{x=0, uf=0, ue=0}$, $B_f = \left.\frac{\partial f}{\partial u_f}\right|_{x=0, uf=0, ue=0}$, $B_e = \left.\frac{\partial f}{\partial u_e}\right|_{x=0, uf=0, ue=0}$. In addition the output equation also needs to be linearized to get

$$y = Cx \tag{5.10}$$

where $C = \left.\frac{\partial h}{\partial x}\right|_{x=0}$.

As is seen earlier the necessary and sufficient conditions for the design of a linear state feedback regulator are

1. The pair (A, B_e) is stabilizable

2. There exist two matrices W and U such that the following equations are satisfied.

$$AW + B_e U = -B_f \tag{5.11}$$

$$CW = 0 \tag{5.12}$$

or $\begin{bmatrix} A & B_e \\ W & 0 \end{bmatrix} = \begin{bmatrix} -B_f \\ 0 \end{bmatrix}$

The design is then completed using the linear quadratic theory to design a state feedback gain matrix K, that minimizes a weighted quadratic norm formed using the states and the functional controls. For details of the linear design the reader is referred to earlier chapter.

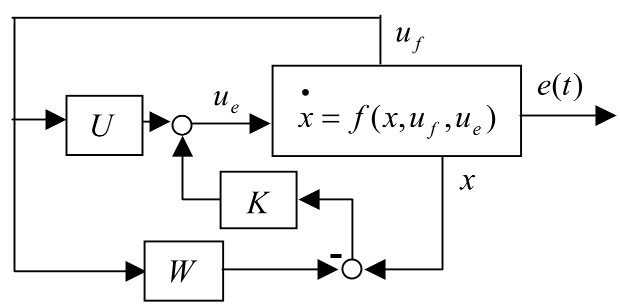


Figure 5.1. Schematics for the full information linear regulator for actuator failure

The matrices are then used to put together the feedback controller as shown in figure 5.1.

5.2 Nonlinear regulator design to address actuator failures

Theorem: The full information feedback problem is solvable if and only if the pair (A, B_e) is stabilizable and there exist mappings $x = \pi(u_f)$ and $u = c(u_f)$, with $\pi(0) = 0$ and $c(0) = 0$, both defined in the neighborhood of the origin, satisfying the conditions

$$f(\pi(u_f), u_f, c(u_f)) = 0 \tag{5.13}$$

$$h(\pi(u_f)) = 0 \tag{5.14}$$

for all u_f.

For the proof of the above theorem the reader is referred to [46]. The above conditions can be understood as a special case of the output regulation in the case of full information.

Note that the conditions (5.13) represent the fact that there exists a submanifold contained in $V \subset X \times U_f$ namely a graph of the mapping $x = \pi(u_f)$, which is rendered locally invariant by means of a suitable feedback law, namely $u = c(u_f)$. The equations (5.14) express the fact that the error map, i.e. the output of the system, is zero for every initial condition $x(0) = x_0$ and $u_f(0) = u_{f0}$ in the neighborhood of the origin. Again it can seen that the search involves finding a mapping that is locally invariant and lies in the kernel of the output.

Equations (5.13) and (5.14) are in general nonlinear algebraic equations that must be solved for the mappings $\pi(u_f)$ and $c(u_f)$. It may not always be possible to solve for these mappings explicitly. In order to calculated we can assume the required mappings to be power series in the variable u_f as follows

$$\pi(u_f) = \pi_0 + \pi_1 u_f + \pi_2 u_f^2 + \pi_3 u_f^3 + ... \tag{5.15}$$

$$c(u_f) = c_0 + c_1 u_f + c_2 u_f^2 + c_3 u_f^3 + ... \tag{5.16}$$

The conditions $\pi(0) = 0$ and $c(0) = 0$ give us the first coefficients of the series

i.e. $\pi_0 = 0$ and $c_0 = 0$ (5.17)

Using the expressions from (5.15) and (5.16) along (5.17) in the equations (5.13) and (5.14) we obtain power series expressions that are needed to be solved for the coefficients π_1, π_2, ... and c_1, c_2, ...It turns out that the coefficients of the power series obtained by the above substitution can solved.

The full information nonlinear regulator can then be put together as shown in Figure 5.2.

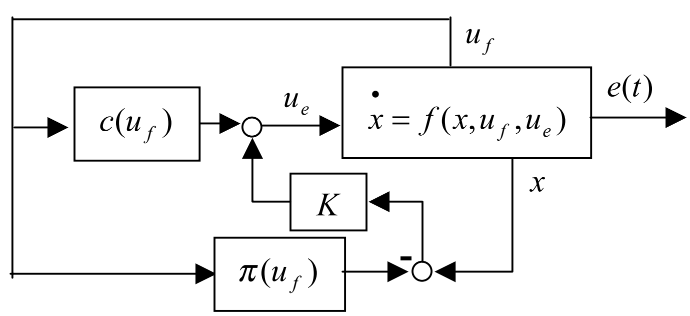


Figure 5.2: Schematics of the full information nonlinear regulator for actuator failure.

The above design procedure is made clear by an example of the longitudinal dynamics of an airplane in the next section. The solutions to the regulating functions can be obtained to any order desired.

5.3. The example of longitudinal dynamics

In this section we design linear and nonlinear regulators for the longitudinal dynamics of an aircraft. The purpose of the controller is to maintain a zero flight path angle in face of complete elevator failure.

The nondimensional equation of motion for aircraft can be written by normalizing the three equations: linear momemtum balance in the body X axes, linear momemtum balance in the Z direction and angular momemtum balance in the body X-Z plane, by choosing a appropriate nominal velocity. The development of the model follows the same by authors in [52].

The nondimensional equations can be written as follows

$$\begin{bmatrix} Cos(\alpha) & -\upsilon Sin(\alpha) & \upsilon Sin(\alpha) & 0 \\ Sin(\alpha) & \upsilon Cos(\alpha) & -\upsilon Cos(\alpha) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{d}{d\tau} \begin{bmatrix} \upsilon \\ \alpha \\ \theta \\ q \end{bmatrix} = \begin{bmatrix} -Sin(\theta) + \Lambda_w Sin(\alpha) + \Lambda_t Sin(\alpha_t) + \Pi - \Delta Cos(\alpha) \\ Cos(\theta) - \Lambda_w Cos(\alpha) - \Lambda_t Sin(\alpha_t) - \Delta Sin(\alpha) \\ q \\ \frac{v_0^2 l}{gr^2}\left[\Sigma_w + \kappa\Lambda_w Cos(\alpha) - (1-\kappa)\Lambda_t Cos(\alpha_t)\right] - \frac{cv_0}{mgr^2} q \end{bmatrix}$$

Where, υ is normalized velocity, α the angle of attack, θ is the pitch angle, q the nondimensional pitch rate, α_t is the tail angle of attack and is related to the angle of attack α, tail angle downwash angle i_t and elevator deflection δ_e via a relation

$$\alpha_t = \alpha + i_t - \varepsilon + \delta_e$$

The equations are written in nondimensional time, $\tau := \frac{g}{v_0} t$ where g is the gravitational acceleration, v_0 the nominal velocity and t is time. Also, κ is a nondimensional parameter for the center of gravity location, Λ_w, Λ_t, Δ and Σ_w are nondimensional lift forces, drag force and moment which take the following form.

$$\Lambda_w = f_w(\alpha)\rho\upsilon^2$$

$$\Lambda_t = f_t(\alpha)\rho\upsilon^2$$

$$\Delta = (a + b[f_w(\alpha)]^2)\rho\upsilon^2$$

$$\Sigma_w = \sigma_w(\alpha)\rho\upsilon^2$$

In order to use the above model for illustrative purposes a complete set of aerodynamic properties must be defined. We choose characteristic values that correspond to hypothetical, subsonic, jet transport flying at high altitude. For more details see sections 9.1-9.4 of [23]. We choose

$$\rho = 1, \quad \varepsilon = 0, \quad \sigma_w(\alpha) = 0, \quad a = 0.05, \quad b = 0.05, \quad f_w(\alpha) = \frac{\alpha - 2.08(\alpha - \alpha_0)^3}{\alpha_0},$$

$$f_w(\alpha) = 0.1\frac{(\alpha - \alpha_0 + \delta_e) - 3(\alpha - \alpha_0 + \delta_e)^3}{\alpha_0} \quad \text{with} \quad \alpha_0 = 0.05, \quad \frac{v_0^2 l}{gr^2} = 300, \quad \frac{cv_0}{mgr^2} = 8,$$

$\kappa = 0$.

Under nominal operating conditions control inputs are the elevator deflection δ_e and thrust input Π. We are to design state feedback controllers that regulate the flight path angle given by $\gamma = \alpha - \theta$ and $\gamma = 0$ corresponds level flight.

For control design we assume the linearisation of the of the above model about the equilibrium given by $\delta_e = 0.0005$, $\Pi = 0.1$, $\upsilon = 1.0$, $\alpha = 0.0495$, $\theta = -0.0495$, $q = 0.0$.

The control before the switch to the regulator is made can be formed by designing a linear quadratic regulator using the equations formed by linearizing the equation for the dynamics about the above equilibrium.

Using identity matrices of appropriate dimensions for the weighting matrices we can calculate the feedback matrix for nominal operation and is given by

$$K = \begin{bmatrix} -0.779181 & -0.030292 & 0.361829 & 0.566283e^{-4} \\ -0.325643 & -0.923677 & 1.26007 & 0.987161 \end{bmatrix}$$

Now consider the predesign for a stuck elevator. The Taylor linearized model for the impaired system is given by equations of the form (5.9) and (5.10) with

$$A = \begin{bmatrix} -0.10024 & 0.981453 & -0.995104 & 0 \\ -1.98016 & -22.1987 & 0.0988384 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & -599.25 & 0 & -8 \end{bmatrix}$$

$$B_e = \begin{bmatrix} 0.998775 \\ -0.0494798 \\ 0 \\ 0 \end{bmatrix} \text{ and } B_f = \begin{bmatrix} 0.0010 \\ -2.0 \\ 0 \\ -599.25 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix}$$

The solution to linear regulator equations (5.11) and (5.12) for the above linearized model is given by

$$W = \begin{bmatrix} 10.1256 \\ -1.0 \\ -1.0 \\ 0.0 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1.00157 \end{bmatrix}.$$

The feedback matrix that minimizes the norm again with weighting matrices chosen to be identity matrices is given by

$$K_f = \begin{bmatrix} -1.8109 & 0.183034 & -1.83687 & 0.0146075 \end{bmatrix}$$

For nonlinear design it is almost impossible to do all the calculations by hand. Symbolic computation packages like Mathematica® have to be employed to make the design feasible.

```
eqns = Simplify@Inverse@88Cos@αD, −ν Sin@αD, ν Sin@αD, 0<, 8Sin@
        αD, ν Cos@αD, −ν Cos@αD, 0<, 80, 0, 1, 0<, 80, 0, 0, 1<<D D.
   88−Sin@θD + LambdaW Sin@αD + LambdaT Sin@αtD + Th − δ Cos@αD<,
    8Cos@θD − LambdaW Cos@αD − LambdaT Cos@αtD − δ Sin@αD<, 8q<,
    8V0 HσW + κ LambdaW Cos@αD − H1 − κL LambdaT Cos@αtDL − Cv0 q<<;

LambdaW = fw ρ ν^2; LambdaT = ft ρ ν^2; fw = (α − 2.08 Hα − α0L^3) / α0;
ft = 0.1 (Hα − α0 + δeL − 3 Hα − α0 + δeL^3) / α0;

δ = Ha + b Hfw αL^2L ρ ν^2;

σw = 0;

eqns1 = eqns ê. 8ρ → 1, a → .05, b → .05, α0 → .05, V0 → 300, Cv0 → 8,
   κ → 0, αt → α + δe<

:9Sin@αD I−20. H−2.08 H−0.05 + αL^3 + αL ν^2 Cos@αD −
      2. H−0.05 + α + δe − 3 H−0.05 + α + δeL^3L ν^2 Cos@α + δeD +
      Cos@θD − I0.05 + 20. α^2 H−2.08 H−0.05 + αL^3 + αL^2M ν^2 Sin@αDM +
   Cos@αD ITh − I0.05 + 20. α^2 H−2.08 H−0.05 + αL^3 + αL^2M ν^2 Cos@αD +
      20. H−2.08 H−0.05 + αL^3 + αL ν^2 Sin@αD +
      2. H−0.05 + α + δe − 3 H−0.05 + α + δeL^3L ν^2 Sin@α + δeD − Sin@θDM=,
 :q + (1/ν) ICos@αD I−20. H−2.08 H−0.05 + αL^3 + αL ν^2 Cos@αD −
       2. H−0.05 + α + δe − 3 H−0.05 + α + δeL^3L ν^2 Cos@α + δeD +
       Cos@θD − I0.05 + 20. α^2 H−2.08 H−0.05 + αL^3 + αL^2M ν^2 Sin@αDMM −
   (1/ν) ISin@αD ITh − I0.05 + 20. α^2 H−2.08 H−0.05 + αL^3 + αL^2M ν^2 Cos@αD +
       20. H−2.08 H−0.05 + αL^3 + αL ν^2 Sin@αD +
       2. H−0.05 + α + δe − 3 H−0.05 + α + δeL^3L ν^2 Sin@α + δeD − Sin@θDMM>,
 8q<, 8−8 q − 600. H−0.05 + α + δe − 3 H−0.05 + α + δeL^3L ν^2 Cos@α + δeD<>
```

Shifting the equilibrium.

```
eqns2 = eqns1 ê. 8Th → Th + .1, δe −> δe + .0005, ν → ν + 1., α → α + 0.0495,
   θ → θ − 0.0495, q → q<
```

```
:9Sin@0.0495 + αD
    I-20. H0.0495 - 2.08 H-0.0005 + αL^3 + αL H1. + νL^2 Cos@0.0495 + αD -
      2. H0. + α + δe - 3 H0. + α + δeL^3L H1. + νL^2 Cos@0.05 + α + δeD + Cos@0.0495 - θD -
      I0.05 + 20. H0.0495 + αL^2 H0.0495 - 2.08 H-0.0005 + αL^3 + αL^2M
       H1. + νL^2 Sin@0.0495 + αDM + Cos@0.0495 + αD
    I0.1 + Th - I0.05 + 20. H0.0495 + αL^2 H0.0495 - 2.08 H-0.0005 + αL^3 + αL^2M
       H1. + νL^2 Cos@0.0495 + αD + 20. H0.0495 - 2.08 H-0.0005 + αL^3 + αL
       H1. + νL^2 Sin@0.0495 + αD + 2. H0. + α + δe - 3 H0. + α + δeL^3L
       H1. + νL^2 Sin@0.05 + α + δeD + Sin@0.0495 - θDM=,
 :q +    1
       ------  ICos@0.0495 + αD I-20. H0.0495 - 2.08 H-0.0005 + αL^3 + αL
       1. + ν
         H1. + νL^2 Cos@0.0495 + αD - 2. H0. + α + δe - 3 H0. + α + δeL^3L
         H1. + νL^2 Cos@0.05 + α + δeD + Cos@0.0495 - θD -
        I0.05 + 20. H0.0495 + αL^2 H0.0495 - 2.08 H-0.0005 + αL^3 + αL^2M
                                            1
         H1. + νL^2 Sin@0.0495 + αDMM -  ------  ISin@0.0495 + αD
                                         1. + ν
      I0.1 + Th - I0.05 + 20. H0.0495 + αL^2 H0.0495 - 2.08 H-0.0005 + αL^3 + αL^2M
         H1. + νL^2 Cos@0.0495 + αD + 20. H0.0495 - 2.08 H-0.0005 + αL^3 + αL
         H1. + νL^2 Sin@0.0495 + αD + 2. H0. + α + δe - 3 H0. + α + δeL^3L
         H1. + νL^2 Sin@0.05 + α + δeD + Sin@0.0495 - θDMM>,
 8q<, 8-8 q - 600. H0. + α + δe - 3 H0. + α + δeL^3L H1. + νL^2
    Cos@0.05 + α + δeD<>
```

Setting up regulator equations

```
regeqs = Flatten@Join@eqns2, 8α - θ<DD
```

```
:Sin@0.0495 + αD I−20. H0.0495 − 2.08 H−0.0005 + αL^3 + αL H1. + νL^2 Cos@0.0495 + αD −
    2. H0. + α + δe − 3 H0. + α + δeL^3L H1. + νL^2 Cos@0.05 + α + δeD + Cos@0.0495 − θD −
    I0.05 + 20. H0.0495 + αL^2 H0.0495 − 2.08 H−0.0005 + αL^3 + αL^2M
     H1. + νL^2 Sin@0.0495 + αDM + Cos@0.0495 + αD
  I0.1 + Th − I0.05 + 20. H0.0495 + αL^2 H0.0495 − 2.08 H−0.0005 + αL^3 + αL^2M
     H1. + νL^2 Cos@0.0495 + αD +
    20. H0.0495 − 2.08 H−0.0005 + αL^3 + αL H1. + νL^2 Sin@0.0495 + αD +
    2. H0. + α + δe − 3 H0. + α + δeL^3L H1. + νL^2 Sin@0.05 + α + δeD + Sin@0.0495 − θDM,
 q +    1
     ------  ICos@0.0495 + αD I−20. H0.0495 − 2.08 H−0.0005 + αL^3 + αL
     1. + ν
      H1. + νL^2 Cos@0.0495 + αD − 2. H0. + α + δe − 3 H0. + α + δeL^3L
      H1. + νL^2 Cos@0.05 + α + δeD + Cos@0.0495 − θD −
     I0.05 + 20. H0.0495 + αL^2 H0.0495 − 2.08 H−0.0005 + αL^3 + αL^2M
                                             1
      H1. + νL^2 Sin@0.0495 + αDMM −   ------  ISin@0.0495 + αD
                                           1. + ν
    I0.1 + Th − I0.05 + 20. H0.0495 + αL^2 H0.0495 − 2.08 H−0.0005 + αL^3 + αL^2M
      H1. + νL^2 Cos@0.0495 + αD + 20. H0.0495 − 2.08 H−0.0005 + αL^3 + αL
      H1. + νL^2 Sin@0.0495 + αD + 2. H0. + α + δe − 3 H0. + α + δeL^3L
      H1. + νL^2 Sin@0.05 + α + δeD + Sin@0.0495 − θDMM,
 q, −8 q − 600. H0. + α + δe − 3 H0. + α + δeL^3L H1. + νL^2
   Cos@0.05 + α + δeD, α − θ>
```

Declaring the order of approximation and creating the series

```
n = 3
```

```
3
```

```
rules = 8ν → Series@ν@δeD, 8δe, 0, n<D, α → Series@α@δeD, 8δe, 0, n<D,
   θ → Series@θ@δeD, 8δe, 0, n<D, q → Series@q@δeD, 8δe, 0, n<D,
   Th → Series@Th@δeD, 8δe, 0, n<D< ê.
  8ν@0D → 0, α@0D → 0, θ@0D → 0, Th@0D → 0, q@0D → 0<
```

```
:ν → ν'@0D δe + 1/2 ν''@0D δe^2 + 1/6 ν^H3L@0D δe^3 + O@δeD^4,
 α → α'@0D δe + 1/2 α''@0D δe^2 + 1/6 α^H3L@0D δe^3 + O@δeD^4,
 θ → θ'@0D δe + 1/2 θ''@0D δe^2 + 1/6 θ^H3L@0D δe^3 + O@δeD^4,
 q → q'@0D δe + 1/2 q''@0D δe^2 + 1/6 q^H3L@0D δe^3 + O@δeD^4,
 Th → Th'@0D δe + 1/2 Th''@0D δe^2 + 1/6 Th^H3L@0D δe^3 + O@δeD^4>
```

Solving for the first set of coefficients

```
first =
 Flatten@Solve@Transpose@regeqns1D@@2DD « 80, 0, 0, 0, 0<,
   8Th'@0D, α'@0D, θ'@0D, ν'@0D, q'@0D<DD
8ThÃ@0D → 1.00157, αÃ@0D → -1., θÃ@0D → -1., νÃ@0D → 10.1256, qÃ@0D → 0.<
```

Solving for the second set of coefficients using the solutions for the first

```
second =
 Flatten@
  Solve@Flatten@Transpose@regeqns1D@@3DD ê. firstD « 80, 0, 0, 0, 0<,
   8Th''@0D, α''@0D, θ''@0D, ν''@0D, q''@0D<DD
8ThÃÃ@0D → 20.6228, αÃÃ@0D → 0., θÃÃ@0D, νÃÃ@0D → 153.241, qÃÃ@0D → 0.<
```

Similarly solving for the third set of coefficients using the above solutions

```
third =
 Flatten@
  Solve@Flatten@Transpose@regeqns1D@@4DD ê. Join@first, secondDD «
    80, 0, 0, 0, 0<, 8Th'''@0D, α'''@0D, θ'''@0D, ν'''@0D, q'''@0D<DD
8ThH3L@0D → 409.977, αH3L@0D → 0.,
 θH3L@0D, νH3L@0D → 25558.1, qH3L@0D → 0.<<
```

and so on we can find all the coefficients and construct the solution.

As a result of the procedure we obtain the following regulating functions, four for each of the states and one for the functional control. These can then be used to construct the nonlinear regulator as shown in Figure 5.2.

$$\pi_1 = 10.1256\delta_e + 153.241\delta_e^2 + 2558.06\delta_e^3 + 44939.9\delta_e^4 + 812408.\delta_e^5 + O[\delta_e^6]$$

$$\pi_2 = -\delta_e + O[\delta_e^6]$$

$$\pi_3 = -\delta_e + O[\delta_e^6]$$

$$\pi_4 = 0 + O[\delta_e^6]$$

$$c = 1.00157\delta_e + 20.6228\delta_e^2 + 409.977\delta_e^3 + 8258.2\delta_e^4 + 165947\delta_e^5 + O[\delta_e^6]$$

Note: The first terms of the above series are exactly the same as the linear regulator.

5.4 Simulation results

In these simulations it is assumed that the elevator fails at $\tau = 0$ and continues to operate with the inappropriate controller till a switch is made the magnitude of elevator failure and the switching time are indicated at the bottom of each figure. It is seen that the mechanism of regulation in this case involves change in the velocity of the aircraft which is also plotted with the flight path angle. It is also noteworthy to mention that each second on the x-axis corresponds to roughly 22.8 sec. in real time.

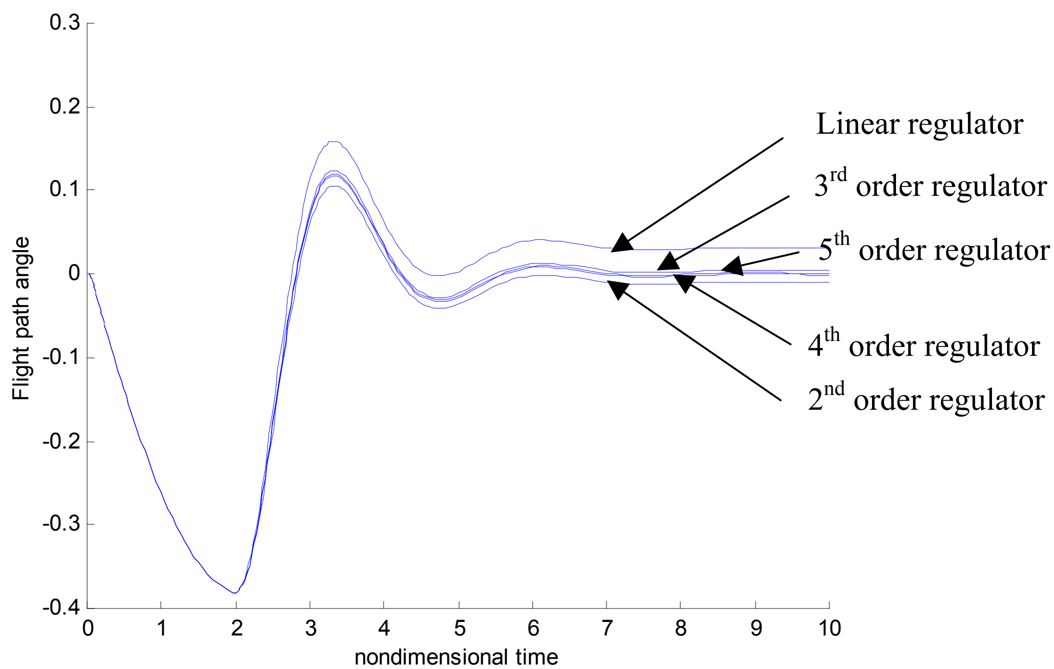


Figure 5.1. Flight path angle for elevator failure $\delta_e = -.02$ and switching time $\tau = 2$

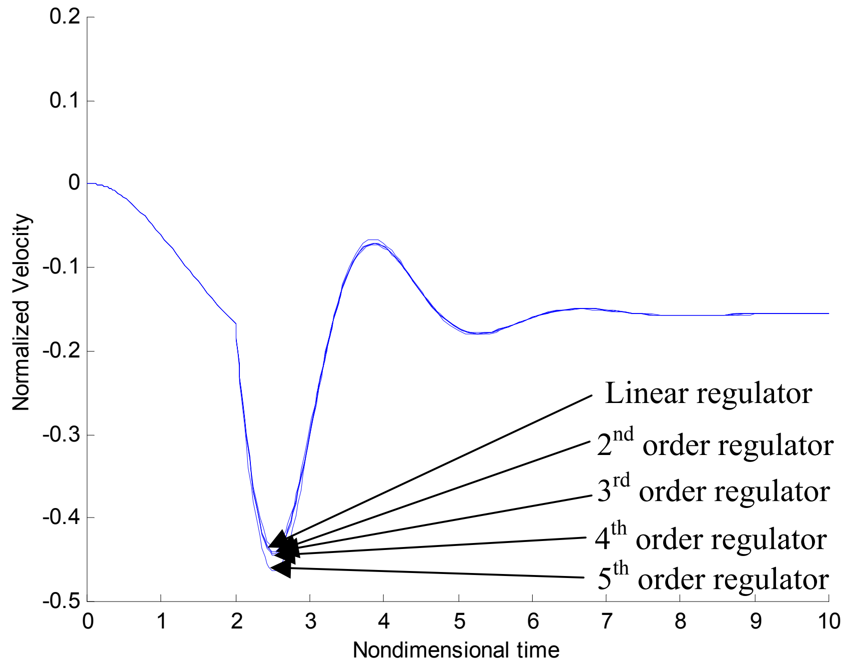


Figure 5.2. Normalized velocity elevator failure $\delta_e = -.02$, Switching time $\tau = 2$

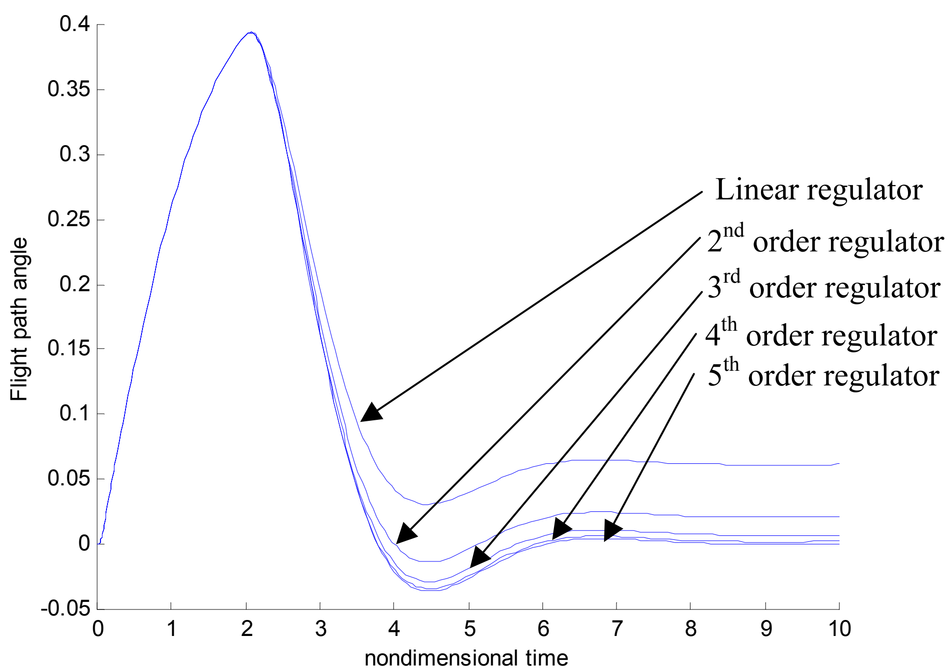


Figure 5.3. Flight path angle for elevator failure $\delta_e = .02$, Switching time $\tau = 2$

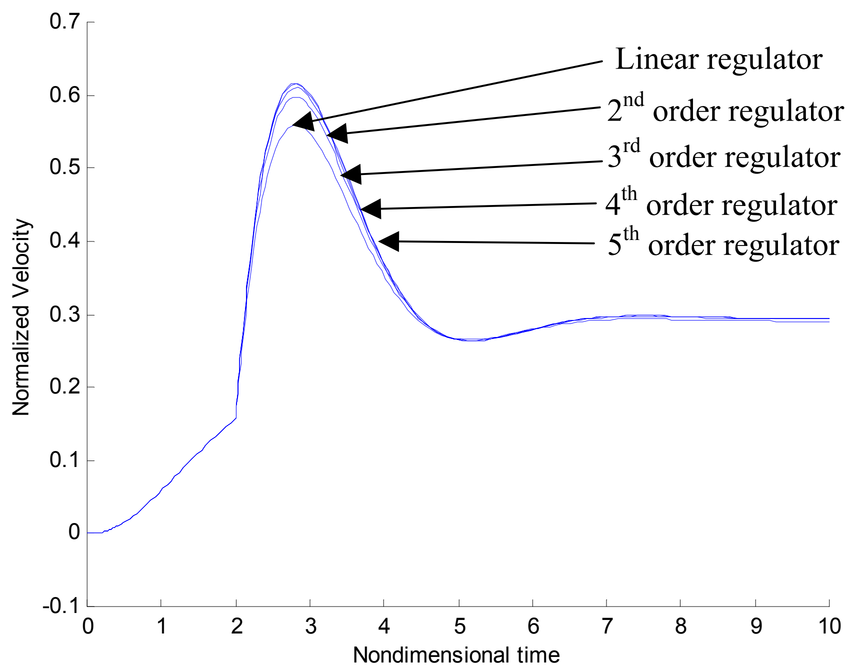


Figure 5.4. Normalized velocity for elevator failure $\delta_e = .02$, Switching time $\tau = 2$

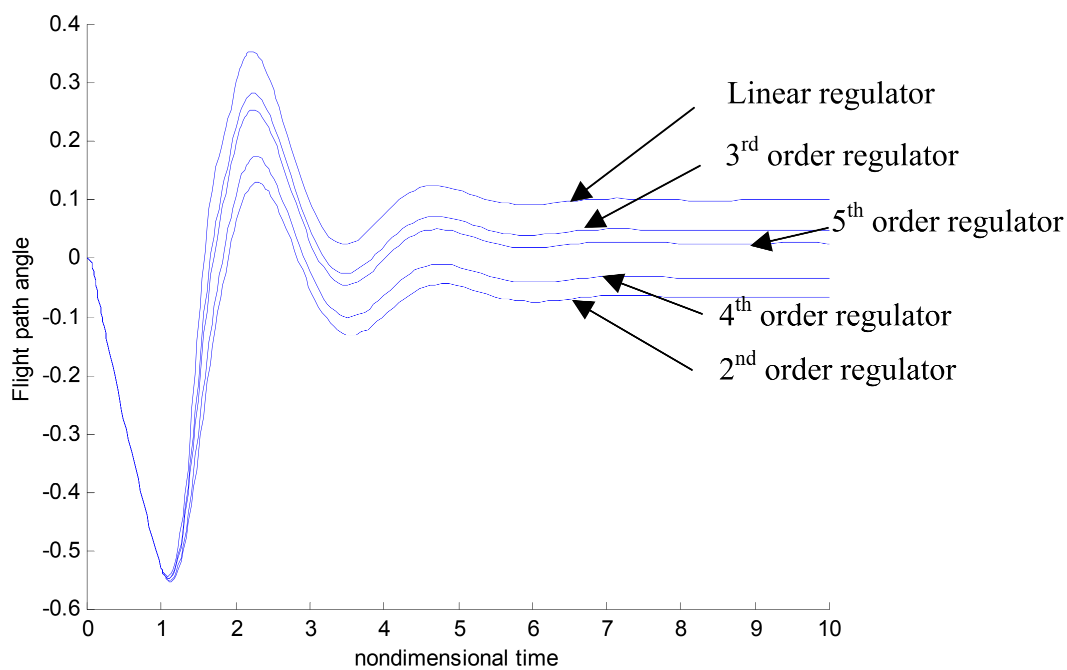


Figure 5.5. Flight path angle for elevator failure $\delta_e = -.04$, Switching time $\tau = 1$

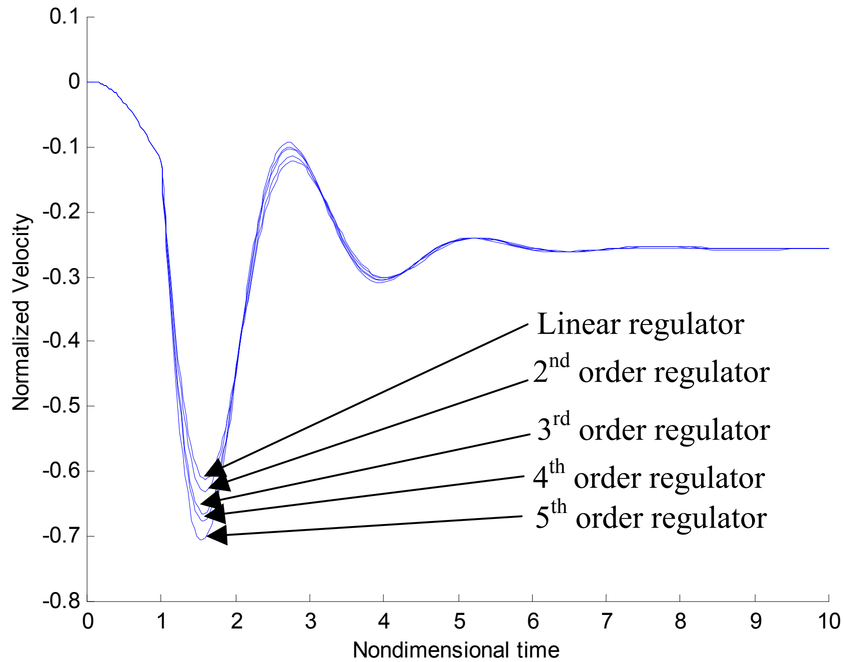


Figure 5.6. Normalized velocity for elevator failure $\delta_e = -.04$, Switching time $\tau = 1$

5.5. Conclusions

Nonlinear design may be desirable when the functions associated with the either the failed control; the functioning control or the system contains significant nonlinearities. As seen in the example as the magnitude at which the failure takes place becomes larger the nonlinear design achieves significantly better regulation than the linear design. Since the nonlinear design has a larger domain of conversion than the linear design it offers a larger window of safety in face due to the delays in the failure detection mechanism. In the simulation the increase in regulation accuracy with the increase in the order of the controller is clearly demonstrated.

Chapter 6. Use of variable structure controllers

A class of systems with discontinuous feedback control, called variable structure systems developed initially in the former USSR has evolved as a useful design technique for control systems [24,58, 79, 84]. The important class of these systems gives rise to motion referred to as the sliding mode, which is motion along the governing "switching" surface of the discontinuity [79]. Since the design of output regulators, hence, the problem of predesign of controllers for jammed actuators involves stabilization to a zero error manifold a parallel can be drawn between the two problems.

6.1. Variable structure servomechanisms.

Consider then the linear time invariant systems of the form

$$\dot{x} = Ax + Bu \tag{6.1}$$

where $x \in R^n$, $u \in R^r$ and A and B are matrices of appropriate dimensions.

The failure models can therefore be written as

$$\dot{x} = Ax + B_e u_e + B_f u_f \tag{6.2a}$$

with $$\dot{u}_f = 0 \tag{6.2b}$$

Let the number of failed controls, i.e. size of u_f be q and the remaining effective controls be, i.e. u_e be m

Let the regulated variables be represented by an l-dimensional vector given by

$$z = C_1 x + D_{11} u_e + D_{12} u_f \tag{6.2c}$$

Also, consider the measurements be given by an s-dimensional vector and given by

$$y = C_2 x + D_{21} u_e + D_{22} u_f \tag{6.2d}$$

The objective again is to regulate the output to zero without directly measuring the disturbance due to the stuck actuators u_f.

It is assumed that (A, B_{2e}) is controllable and (A, C_2) is observable. These conditions along with the existence of two matrices W and U used to define the steady state x_{ss} and control input u_{ess} as linear functions of the failed control u_f.

Moreover,

The calculation for W and U can be accomplished by solving

$$AW + B_f + B_e U = 0 \tag{6.3a}$$

$$C_1 x + D_{11} U + D_{12} = 0 \tag{6.3b}$$

The design procedure for the variable structure controller to address regulation conveniently separates into four distinct parts:

1. Determination of the steady state solutions, i.e. for matrices W and U.
2. Solution of an $n-m$ dimensional state feedback problem
3. Solution of a nonlinear state feedback stabilization problem with m states and m controls and finally
4. the design of the observer.

6.2. Details of the design

As shown earlier W and U can be calculated by solving the coupled matrix equations given in the last section. First we define the motion of the plant state and the control input relative to their steady state values.

$$\Delta x \equiv x - x_{ss} \tag{6.4a}$$

$$\Delta u_e \equiv u_e - u_{ess} \tag{6.4b}$$

The feedback law when it is implemented uses the estimates from a composite observer for the states and the stuck controls, i.e. $\hat{x}$ and $\hat{u}_f$. The dynamics of the observer are given by

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{w}} \end{bmatrix} = \begin{bmatrix} A - L_1 C_2 & B_f - L_1 D_{21} \\ -L_2 C_2 & -L_2 D_{21} \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} + \begin{bmatrix} B_e - L_1 \\ -L_2 D_{22} \end{bmatrix} u + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} y \tag{6.5}$$

The matrices L_1 and L_2, can be designed given the observability assumption earlier, such that $\hat{x}(t) \to x(t)$ and $\hat{w}(t) \to \hat{w}(t)$ as $t \to \infty$.

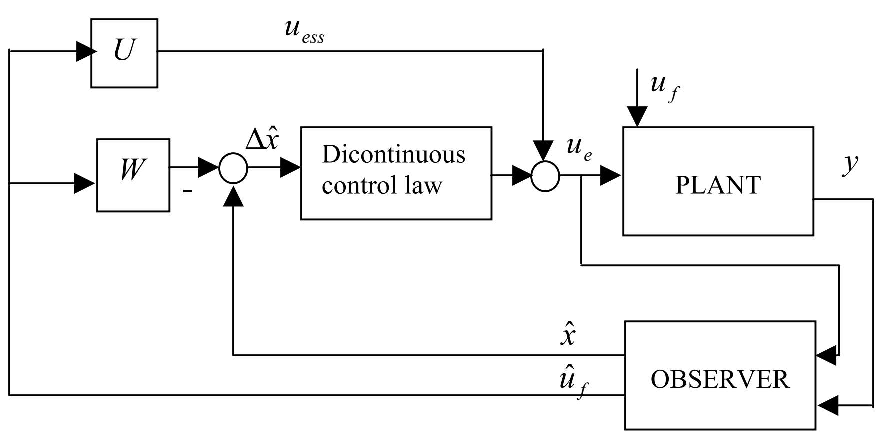
Let Δu_e be a discontinuous feedback control

$$\Delta u_{ei}(\Delta\hat{x}) = \begin{cases} \Delta u_{ei}^{+}(\Delta\hat{x}) & s_i(\Delta\hat{x}) > 0 \\ \Delta u_{ei}^{-}(\Delta\hat{x}) & s_i(\Delta\hat{x}) < 0 \end{cases} \tag{6.6}$$

with

$$s(\Delta\hat{x}) = G\Delta\hat{x} \tag{6.7}$$

The servomechanism structure is as shown in the figure.



6.1. Variable structure servomechanism

The idea in the design of the hyperplane is to get acceptable sliding dynamics. The motion assuming sliding occurs is described by the equations.

$$\begin{bmatrix} \dot{\Delta\hat{x}_s} \\ x-\hat{x} \\ w-\hat{w} \end{bmatrix} = \begin{bmatrix} MAN & M[L_1 - WL_2]C_2 & M[L_1 - WL_2]C_2 \\ 0 & A - L_1C_2 & B_f - LD_{12} \\ 0 & -LC_2 & -L_2D_{12} \end{bmatrix} \begin{bmatrix} \Delta\hat{x}_s \\ x-\hat{x} \\ w-\hat{w} \end{bmatrix} \tag{6.8}$$

where $\Delta\hat{x}_s \equiv M\Delta\hat{x}$, the columns of the matrix M^T span the null space of $B_2{}^T$,

i.e. $B^T M^T = 0$,

and the columns of the matrix N span the null space of G,

i.e. $GN = 0$

The only thing left is to make the hyperplane globally attracting and this is done by using discontinuous feedback law in manner that the vector field at all neighborhoods about the hyperplane is pointing towards the hyperplane. These part of design is enabled the discontinuity.

The discontinuity in the feedback functions

$$\Delta u_{ei}(\Delta\hat{x}) = \begin{cases} \Delta u_{ei}{}^{+}(\Delta\hat{x}) & s_i(\Delta\hat{x}) > 0 \\ \Delta u_{ei}{}^{-}(\Delta\hat{x}) & s_i(\Delta\hat{x}) < 0 \end{cases} \tag{6.9}$$

allows shaping of the vector field towards the manifold

$G\Delta x = 0$

Under the hypotheses of the above results the variable structure mechanism regulates the system error to zero for arbitrary persistent disturbances generated by the failed controls u_f.

6.3. Conclusion

In this chapter the design of variable structure mechanisms for the predesign of controllers was presented. The variable structure servomechanism can be used as an alternate design methodology for the design of fault-tolerant systems.

Chapter 7. Conclusions and further research

The thesis has addressed the predesign of controllers for sensor and actuator failures. Sensor failures are seen not to be as severe as the actuator failures. The predesign for actuator failures has been addressed in available literature using *ad hoc* techniques or using adaptive control techniques. In this thesis, important connection between the predesign for actuator failure problem and the regulator problem with internal stability is made. Since both necessary and sufficient conditions for the design of regulators for linear systems are fairly well understood this leads to ways of assessing available control redundancy in systems. This analysis can be used for designing systems with designs capable of tolerating faults. The connection with the regulator theory also allows generalizing the result to the state feedback control of nonlinear systems. In addition to the above the final chapter of the thesis addresses design of variable structure servomechanisms and these can be utilized to design controller for actuator failures.

Many aspects of the thesis call for further investigation. Chief among them is the coupling of the state feedback nonlinear design with nonlinear observers to solve problem with output feedback. The design of nonlinear observers is an active research field just like the output regulation problem. The extension of ideas presented in the final chapter of the thesis, i.e., those of using alternative techniques to design servomechanisms to address failures calls for further investigation in order to generalize the result to nonlinear systems.

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Appendix A

Notation

$\in$	belongs to
$:=$	is defined as
$\rightarrow$	tends to
R	field of real numbers
C	field of complex numbers
R^n	n-dimensional Euclidean space
R_+	set of non-negative real numbers
x^T	transpose of vector x
A^T	transpose of matrix A
$A^\perp$	pseudo-inverse of a matrix A
a^*, A^*	the complex conjugate transpose of the complex vector a, matrix A
$\langle a,b \rangle$	inner product of two vectors a and b
$[a,b]$	closed interval on the real line from a to b

Appendix B

This appendix reviews some definitions and theorems associated with the thesis.

Let for a matrix A with a spectral list $\{\lambda_k\}_{k=1}^{n}$, $\{e_k\}_{k=1}^{n}$ represent the basis of right eigenvectors associated with the spectral list defined by the solution of the equation $Ae_k = \lambda_k e_k$ for all $k = 1,2,...n$. $\{\eta_k\}_{k=1}^{n}$ represent the basis of left eigenvectors associated with the spectral list defined by the solution to the equation $\eta_k^* A = \lambda_k \eta_k^*$ for all $k = 1,2,...n$.

Theorem B1: Consider the autonomous linear differential system $\dot{x} = Ax$, $x(0) = x_0$ where $t \in R$, $x \in R^n$, the solution can be expressed as $\sum_{k=1}^{n} \langle \eta_k, x_0 \rangle \exp(\lambda_k t) e_k$.

Proof: [45]

N.B.: So if we think of initial state vector as a linear combination of eigenvectors, then the resulting motion of the states is a linear combination of very simple motions given in *Theorem B1*. These motions are called *modes* and form a basis of linear space of solutions of the autonomous differential system given in *Theorem B1*.

Stability

The states of dynamical system $\dot{x} = Ax$ are referred to as *asymptotically stable* if and only if for all $x_0 \in R^n$

a. the solution is bounded for all $t \geq 0$

b. the solution tends to zero as $t \to \infty$

additionally, those modes that do not go to zero as $t \to \infty$ are *unstable modes.*

Definition B.1: The dynamical system $\dot{x} = Ax + Bu$, $y = Cx + Du$, where $x \in R^n$, $u \in R^m$ and $y \in R^p$, also, A, B, C and D are constant matrices of appropriate dimensions, or equivalently, the pair (A, B), is said to be *state controllable,* for any initial state $x(0) = x_0$, any time $t_1 > 0$ and any final state x_1, there exists an input $u(t)$ such that $x(t_1) = x_1$. Otherwise the system is said to be *state uncontrollable.* In addition, the modes that are controllable are referred to as *uncontrollable modes.*

Definition B.2: The dynamical system $\dot{x} = Ax + Bu$, $y = Cx + Du$, where $x \in R^n$, $u \in R^m$ and $y \in R^p$, also, A, B, C and D are constant matrices of appropriate dimensions, (or the pair (A, C)) is said to be *state observable*, if for any time $t_1 > 0$ the initial state $x(0) = x_0$ can be determined from the time history of the input $u(t)$ and output $y(t)$ in the interval $[0, t_1]$. Otherwise the system, (or (A, C)), is said to be *state unobservable.* Additionally, if the system is unobservable there are states in the system that remain undetermined and are referred to as *unobservable states.*

Definition B.3: Consider the time-invariant system given in *Definition B.1 and B.2*. Let $\sigma[A] := \{\lambda_k\}$, $k = 1,2...n$, be the spectrum of $A \in R^{n \times n}$ with the corresponding eigenspaces. An *uncontrollable hidden mode at* λ_k exists if an only if the generalized eigenvector associated with λ_k uncontrollable. An *unobservable hidden mode at* λ_k exists if and only if there exists a generalized eigenvector at λ_k that is unobservable.

Definition B.4: A system is said to be *state stabilizable* or simply *stabilizable* if all the uncontrollable modes are stable. A system is *state detectable* or simply *detectable* if all unstable modes are state observable.

R1: Remark on Kalman Decompositions

Using transformations which use basis-vectors from the A-invariant subspaces we can construct transformations that put a linear system into the following form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & A_{22} & 0 & A_{24} \\ 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & 0 & A_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & C_2 & 0 & C_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + Du$$

The above form explicitly reveals the controllability, observability structure of a control system. The coordinates x_1, x_2 correspond to the controllable subspace and x_2, x_4 correspond to the observable subspace.

Definition B.5: Second order modes

Consider the dynamical system $\dot{x} = Ax + Bu$, $y = Cx$, where $x \in R^n$, $u \in R^m$ and $y \in R^p$, also, A, B and C are matrices of appropriate dimensions. The controllability Gramian is defined by

$$W_c(t) \equiv \int_0^t e^{A\tau} BB^T e^{A^T\tau} d\tau$$

and the observability Gramian is defined by

$$W_o(t) \equiv \int_0^t e^{A^T\tau} C^T C e^{A\tau} d\tau$$

The eigenvalues for $W_c W_o$ are called the second order modes of the system.

R.2: Steady state response of a Nonlinear System

Steady state response of a system intuitively is that particular response which any initial condition converges to as time increases. More rigorously, consider

$$\dot{x} = f(x,u) \tag{B.1}$$

with state x defined in the neighborhood U of the origin in R^n and the input $u \in R^m$, assume that $f(0,0) = 0$, and let $x(t, x^0, u(\bullet))$ denote the value of state achieved at the time $t > 0$ under the effect of the input $u(\bullet)$, starting from the initial state x^0 at time $t = 0$. Let $u^*(\bullet)$ be a specific input function and suppose these exists an initial state x^* with the property that

$$\lim_{t \to \infty} \left\| x(t, x^0, u^*(\bullet)) - x(t, x^*, u^*(\bullet)) \right\| = 0 \tag{B.2}$$

for every x^0 in some neighborhood U^* of x^*. If this is the case, then the response

$$x_{ss}(t) = x(t, x^*, u^*(\bullet)) \tag{B.3}$$

is called the steady state response of (B.1) to the specific input $u^*(\bullet)$.

This notion of steady state response is particularly useful in the analysis of responses of system to inputs, which are "persistent" in time, as is the case with failed actuators. Usually, these inputs can be thought of as being "generated" by a suitable dynamical system modeled by the equations of the form.

$$\dot{w} = s(w)$$

$$u = p(w)$$

whose state w is defined in a neighborhood W of the origin in R^r and in which $s(0) = 0$ and $p(0) = 0$. To impose that the inputs generated by such a system are bounded, it is sufficient that the point $w = 0$ is a stable equilibrium (in the ordinary sense of Lyapunov) of the vector field $s(w)$ and to choose the initial condition at time $t = 0$ in some appropriate neighborhood $W^0 \subset W$. To impose that the inputs are persistent in time, it is convenient to assume that every point w^0 of W^0 is Poisson stable. Poisson stability implies that the trajectory $w(t)$, which originates in w^0 passes arbitrarily close to w^0 for arbitrarily large times, in forward and backward direction. For convenience we call the both the conditions i.e. those of origin being stable in the ordinary sense and that of Poisson stability in the neighborhood of origin as neutral stability.

If the dynamics of w are neutrally stable and the equilibrium $x = 0$ of $\dot{x} = f(x,0)$ is stable in the first approximation then there exists a mapping $x = \pi(w)$ defined in a neighborhood $W^0 \subset W$ of the origin, with $\pi(0) = 0$, which satisfies

$$\frac{\partial \pi}{\partial w} s(w) = f(\pi(w), p(w))$$

for all $w \in W^0$. Moreover, for each $w^* \in W^0$, the input

$$u^*(t) = p(\Phi_t^s(w^*))$$

produces a well-defined steady state response, which is given by

$$x_{ss}(t) = x(t, \pi(w^*, u^*(\bullet)).$$

This is the sufficient condition for the existence of well-defined steady state response and is used to develop the regulator equations.

A differentiable manifold is a set of points, which is locally equivalent to a Euclidean space. We will make this concept precise.

Definition B.6: An *m-dimensional manifold* is a set M together with a countable collection of subsets $U_i \subset M$ and one-to-one mappings φ_i: $U_i \rightarrow V_i$ onto open subsets V_i of R^m, each pair (U_i, φ_i) called a *coordinate chart* , with the following properties:

a) the coordinate charts cover M,

$$\bigcup_i U_i = M$$

b) on the overlap of any pair of charts $U_i \cap U_j$ the composite map

$$f = \varphi_j \circ \varphi_i^{-1} : \varphi_i(U_i \cap U_j) \rightarrow \varphi_j(U_i \cap U_j)$$

is a smooth function.

c) if $p \in U_i$ and $\bar{p} \in U_j$ are distinct points of M, then there are neighborhoods, W of $\varphi_i(p)$ in V_i and $\overline{W}$ of $\varphi_j(\overline{p})$ in V_j such that

$\varphi_i^{-1}(W) \cap \varphi_j^{-1}(\overline{W}) = \emptyset$

The coordinate charts provide the set M with a topological structure so that the manifold is a topological space. Condition c) of the definition is a form of the so-called Hausdorff separation axiom so that these manifolds are Hausdorff topological spaces. The coordinates in R^m $(x_1,..,x_m)$ of the image $\varphi(p)$ of a point $p \in M$ are called the *coordinates of p*. A chart (U,φ) is called a *local coordinate system*. If the overlap functions $f = \varphi_j \circ \varphi_i^{-1}$ are k-times continuously differentiable, then the manifold is called a C^k*-manifold*. If $k = \infty$, then the manifold is said to be *smooth*.

VITA

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Major publications include

- G. Bajpai and B. C. Chang, "Decoupling of failed actuators in flight control systems", Proceedings of American Control Conference, Arlington VA, 2001, pp. 1836-1840.

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