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Co-constructed Narratives in Online, Collaborative Mathematics Problem-Solving

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Abstract. Our approach to the study of the narrative aspects of learning mathematical problem-solving extends the conception of narrative as the central artefact of interest, to include the process of collaborative dialog and emergent narratives. This perspective favours the conception of the dialogical aspects of interaction as shared achievements of co-participants and central meaning-making procedures. On the other hand, our qualitative analysis of transcripts of collaborative problem-solving interactions online revealed striking resemblances with the narrative form. Based on these observations we attempt to establish a link between the narrative and dialogical perspectives and explore relevant implications for the design of the Virtual Math Teams collaborative learning environment.

Truth is not to be found inside the head of an individual person, it is born between people collectively searching for truth, in the process of their dialogic interaction.
(Bakhtin, [1], p.110)

Introduction

Research in the field of Narrative Learning Environments (NLEs) is concerned with questions such as how to characterize the contribution of narratives and narration to learning, and how to use knowledge of narratives to design learning environments. As part of the Virtual Math Teams (VMT, see http://mathforum.org/wiki/VMT) research project, we have investigated talk-in interaction within the context of collaborative mathematical problem-solving online and have found similarities and differences between the narrative approach and a dialogical perspective on sense-making and interaction. Therefore, we propose to extend the idea narrative learning environments to encompass collaborative discourse-construction environments which, in addition to using narratives, offer also the possibility of collaboratively building meaning-centered narratives integrating a diverse set of linguistic and extra-linguistic objects (e.g. mathematical objects and their derivative properties).

In the following sections we present these perspectives and offer some reflections for future research and development. The next section briefly presents the problematic of narrative learning environments. Section 2 introduces the main ideas of the dialogical theory of Mikhail Mikhailovici Bakhtin and its relevance for narrative learning. Section 3 presents in detail a qualitative analysis of a chat transcript as part of the VMT project. Section 4 concludes with some implications for design and future research.

1. Narrative Learning Environments (NLE)

Theorists of the narrative aspect of cognition (e.g. Jerome Bruner [2],[3], Walter Fisher[4],[5], Roger Schank[6], etc.) argue that the narrative form is the primary means through which human beings create and convey meanings about the world. The interest in narrative that AI and Cognitive Science have shown revolves around the ability of narratives to structure and mediate knowledge [7]. As such, major areas of AI work include story understanding and generation as well as the development of interactive environments structured as narrative spaces. Research and development on Narrative Learning Environments (NLEs), a field of work at the intersection of AI, educational technologies and narratology, is concerned with intelligent learning environments where "narrative is approached and applied" to support learning and the construction of meaning [8]. According to the organizers of this workshop, a narrative learning environment is expected to promote three main kinds of activities for learners:

*(1) **co-construction**: [the ability to] participate in the construction of a narrative;*
*(2) **exploration**: engage in active exploration of the learning tasks, following a narrative approach and trying to understand and reason about an environment and its elements;*
*(3) **reflection**: engage in consequent analysis of what happened within the learning session.*

To date, research and development in the field of narrative learning environments has concentrated on the analysis and use of narrative elements such as virtual storytelling, interactive drama, and participatory narratives, mostly within the context of literacy development and language learning (e.g. [9]).

2. The dialogical perspective

Related to the narrative perspective but particularly concerned with the interactive and participatory aspects of joint talk in activity, the *dialogical* perspective pursues meaning-making as an interactional achievement of co-participants; more than a property of narratives or other linguistic objects. Theorists of the dialogical aspect of language and meaning (e.g. Mikhail M. Bakhtin [10],[11], Rom Harré [12], and conversation analyst such as Harvey Sacks [13],[14] and Emanuel Schegloff [15]) point to the features of talk as action, and of shared action in itself, as the core processes of human meaning-making. These socially shared procedures might suggest general sense-making strategies that are instantiated within particular domains (e.g. fictional storytelling, or mathematical problem-solving).

Bakhtin in particular, considers any human language related activity, be it in the form of oral speech or writing, as dialogic— i.e. containing more than one voice ([10], [11]). This perspective unifies speech and writing and allows us to see narratives also as dialogic. This is of no surprise if we realize that narratives, as interaction, contain not only the voice of the narrator but also, at least, the voice of the listener. When telling a story, the narrator anticipates for the listener, for instance possible aspects that might require elaboration (especially in learning contexts). Other voices might also be present including those referring other contributors of the current story being narrated.

From this perspective, narratives resemble, as well, processes of collaborative scientific discourse. The procedures utilized in structuring a narrative and, for example, writing a proof of a theorem, or presenting a solution for a problem exhibit significant similarities in their communicative structures. What is common to both narratives and theorem proving, or collaborative problem-solving is *the discourse*; the emergent coherence of the sequencing of

utterances generated with the aim of meaning making; of interacting with others and with meaningful artefacts. Furthermore, when we refer to these activities in the context of learning, it is interesting to note that "rather than speaking about <<acquisition of knowledge>>, many people prefer to view learning as *becoming a participant in a certain discourse*" [16], or of a certain narrative genre.

Participation in the learning processes is usually a social activity, language being an extremely important cultural artefact. As Vygotsky states in his concept of the Zone of Proximal Development [17], children's potential learning abilities are especially accessible within their interactions with others. As a consequence, we propose to extend the idea of using narrative-learning environments to the idea of collaborative discourse-construction environments. These should still offer the possibility of the usage of narratives, but may offer also the possibility of collaboratively building meaning that gets reified through narratives or even other forms of discourse ("speech genres", in Bakhtin's terminology [11]), like mathematical problem solving.

Participatory or interactive narratives offer opportunities for co-construction of meaning precisely based on the dialogic principle of interactivity, a point we seek to illustrate within the domain of collaborative mathematical problem-solving. Some of the narrative elements visible in this specific domain are attributable to the sequential unfolding of the problem-solving task, a condition that gets confounded with dialogical and interactional participation schemes adopted by the participants. We would like to suggest that elements intersecting the narrative and dialogical perspectives, discovered through our analysis, in the particular context of collaborative mathematics problem-solving, could represent an extended standpoint for the analysis and design of learning environments in general.

3. Virtual Math Teams (VMT)

The Virtual Math Teams (VMT) research program investigates the innovative use of online collaborative environments to support effective K-12 mathematics learning as part of the research and development activities of the Math Forum at Drexel University. The Math Forum, a leading center for mathematics education on the Internet and one of the most successful long-lasting virtual communities, offers a Problem of the Week (PoW) service through which students submit written solutions to non-routine mathematical problems and might receive asynchronous feedback. The Virtual Math Teams project extends the PoW service to provide synchronous collaborative sessions where small groups of students join together to solve mathematical problems online using a computer-supported collaborative learning environment which combines quasi-synchronous text-based communication (e.g. chat) and a shared whiteboard among other interaction tools. At the core of VMT research is the premise that primarily, group knowledge arises in discourse and is preserved in linguistic artefacts, whose meaning is interpreted within group processes (Stahl [18],[19]). Key issues addressed by the VMT include the design challenge of structuring the online collaborative experience in a meaningful and engaging way, and the closely related methodological challenge of finding appropriate theoretical approaches to study the forms of collaboration and reasoning that take place.

As part of the initial exploratory phase of research, the VMT offered more than 20, 1-hour online sessions in which small groups of students used AOL Instant Messenger© technologies to interact and collaboratively attempt to solve a mathematical problem provided. Through these events we have collected a corpus of chat transcripts that constitute our main source of data. The VMT implements a multidisciplinary approach to the analysis of these transcripts,

which integrates quantitative modelling of students' interactions as well as ethnographic and conversation analytical studies of collaborative problem solving.

3.1. Collaborative Problem-solving: Co-construction, exploration and reflection.

Several researchers have explored the interdependencies between narratives and mathematics (Cocking & Chipman [20]) as well as the role of narrative in mathematics learning (Burton, [21],[21]). Our approach to the study of the narrative aspects of learning mathematical problem-solving extends the conception of narrative as an artefact by favouring the conception of the dialogical aspects of interaction as shared achievements of co-participants. On the other hand, our analysis of transcripts of collaborative problem-solving interactions online revealed striking resemblances with the narrative elements. This empirical finding may be explained by Bakhtin's dialogic proposition, discussed in the previous section: It seems to follow as a natural consequence that similarities among narratives, mathematical meaning-making and collaborative, natural language-based problem solving find their base on their common dialogic nature, even if their apparent structure does not appear so (e.g. when there are no explicit voices as in traditional narratives)

The following analysis illustrates these ideas by using data from one of the online transcripts of a VMT collaborative problem-solving session. The session presented here has three main participants, SKI, YAG and GOH. "*Press for Time*" is the problem assigned for the session, which by virtue of its presentation as a word problem, could contribute to the display of narrative elements in the dialogical interactions among participants:

> *The Rational Reader, a popular daily newspaper, has to be printed by 5 a.m. so that it can be distributed. Late one night, a major story broke and the front page had to be rewritten, which delayed the start of the printing process until 3 a.m. To try to get the printing done on time, the Reader used both their new printing press and their old one. The new press is three times as fast as the old one, and with both of them running, the printing was finished exactly on time. How long does it take to print a normal edition of the paper using only the new press?*

As it turns out, at least two of the participants (SKI and YAG) had worked on the problem prior to their joint participation in the online collaborative session, and as a result, in addition to the orientation to the narrative structure mediated by the presentation of the problem, the participants orient themselves to an "expository[1]" mode of interaction in which reports of "ways" to solve the problem are offered in the form of story-like narrations. The form in which a way of solving a problem is then made accessible during this collaborative problem solving interaction is, to a certain extent, similar to that of the narration of a story. The process of narrating and the resulting narrative, however, are to be considered as an interactional achievement of all the participants despite the apparent fact of an established narrator voice or the references made by participants to the authorship of particular ways of proceeding with their joint work. An interactive narrative within the speech genre [11] of mathematics problem

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[1] The use of the "expository" participation framework here differs slightly from Mercer's [23] conception of the three kinds of inter-subjective of talk: *disputational*, *cumulative*, and *exploratory*. In Mercer's framework, *disputational talk* is characterized by the speakers being concerned with defending their own selves, at the possible expense of any attempt at a solution or an approach to truth. In *cumulative talk*, each speaker seeks to support the other's self but fail to explore facts and solutions. *Exploratory talk*, acording to Mercer occurs when speakers "engage critically but constructively with each other's ideas" (p.98).

solving, however, has specific characteristics that govern the space of possible transformations of the different "episodes" of a story. Let us illustrate an example in which participants allude to this:

1. 7:26:10	SKI	i started and solved with a system
2. 7:26:12	SKI	of equations
3. 7:26:14	YAG	let SKI explain...
4. 7:26:24	SKI	lets just say x is the time for the old machine and y is for the new
5. 7:26:29	GOH	ok
6. 7:26:35	SKI	our first equation is like this
7. 7:26:41	SKI	if we atke the recip of x
8. 7:26:45	YAG	*choughSHOWOFFchough*
9. 7:26:55	YAG	:P
10. 7:26:57	YAG	:-D
11. 7:26:59	SKI	thats how much of the job the old one does in one hour
12. 7:27:02	YAG	yep
13. 7:27:12	SKI	and the reciprocal of y is how much of the job the new one does in one hour
14. 7:27:16	YAG	recip [of] y is the new one
15. 7:27:24	SKI	ok
16. 7:27:29	SKI	recip=reciprocal
17. 7:27:33	SKI	anyways
18. 7:27:38	YAG	and, recip y+ recip x = 1/2
19. 7:27:43	SKI	we add 1/x and 1/y
20. 7:27:48	SKI	ya
21. 7:27:50	SKI	what YAG said
22. 7:27:53	SKI	1/2
23. 7:27:56	YAG	in hours and fraction of work
24. 7:28:04	YAG	needed to be done
25. 7:28:05	SKI	cuz they together get half the job done in one hour
26. 7:28:09	YAG	:P
27. 7:28:13	SKI	are u getting our first equation?

As can be seen in this excerpt, even in this "expository" orientation, co-participants take active roles in co-constructing the explanation. Even though SKI initiates his story-like report with the form of a first person narrative ("*i started and solved with a system of equations"),* the shared narrative space of this dialogical form gets transformed after YAG and GOH's interactional acceptance of SKI's narrator voice (lines 3 and 5). As a result, we see a transformation in the form of SKI's narrative to the first person plural (*"our first equation is like this"*) and subsequently we can observe how SKI and YAG share the narrator role by completing each other postings or interjecting new ones. SKI and YAG have, at this point, constituted themselves as a recognizable collectivity (Lerner [24]) oriented towards the task of producing an intelligible narrative explanation for GOH (e.g. line 27).

On the other hand, by virtue of the interactional nature of the conversation being produced, GOH is by no means restricted to a passive audience role. One of the interesting peculiarities of our attempt to intersect the framework of narratology and the domain of collaborative mathematical problem-solving, results in a unique instantiation of the idea of "possible worlds." The complex world of linguistic and mathematical objects that SKI, YAG and GOH both access and co-construct (e.g. the proposition *"The new press is three times as fast as the old one"* included in the problem statement, and SKI's posting *"the reciprocal of y is how much of the job the new one does in one hour"*), their individual perspectives, and the transformations that they exert on such objects (e.g. SKI use of "cuz" - *because* - on line 25)

are governed not by strict logical laws (as is sometimes assumed in narrative semantics) but by the local sense-making procedures of the co-participants and their orientation to joint-activity. For, instance, when SKI in line 27 asks GOH, indirectly but unequivocally, for an assessment of her state of participation, GOH eventually requests a clarification of the current state of the co-constructed narrative which, as can be seen in the following excerpt, is also co-produced and results in further re-organization of the meaning of the mathematical and narrative objects so far established (e.g. 1/x, *"the old one,"* *"how much of the job they do together in one hour,"* *etc.*):

1.	7:29:38	GOH	*how come 1/x and 1/y added equal 1/2?*
2.	7:29:42	SKI	*ok*
3.	7:29:47	YAG	*ummm*
4.	7:29:50	YAG	*pure luck!*
5.	7:29:51	SKI	*1/x is how much the old one does in one hour*
6.	7:29:57	GOH	*right.*
7.	7:29:58	SKI	*how much of the job it does in an hour*
8.	7:30:01	YAG	*(frac of job done)*
9.	7:30:03	SKI	*1/y is for the new machine*
10.	7:30:08	GOH	*right*
11.	7:30:11	SKI	*add those up*
12.	7:30:18	YAG	*and since they do it together at 3-5*
13.	7:30:20	SKI	*thats how much of the job they do together in one hour*
14.	7:30:22	YAG	*it took 2 hrs*
15.	7:30:25	SKI	*ya*
16.	7:30:29	SKI	*listen to [YAG]*
17.	7:30:38	YAG	*so 1/2 =0.5*
18.	7:30:42	YAG	*:P*
19.	7:30:44	SKI	*ya*
20.	7:30:47	SKI	*u getting that?*
21.	7:30:52	YAG	*slow*
22.	7:30:53	GOH	*I think so....*
23.	7:30:54	YAG	*down*
24.	7:30:55	SKI	*hmm*
25.	7:30:57	YAG	*[S-K-*
26.	7:30:58	SKI	*i will*
27.	7:30:59	YAG	*I]*
28.	7:31:06	SKI	*the whole job took 2 hours*
29.	7:31:14	YAG	*with both machines*
30.	7:31:19	SKI	*so in one hour they did 1/2 of the job*
31.	7:31:34	YAG	*and in the 2nd hour they did the other half*
32.	7:31:54	GOH	*Okay, I got it. 1/2 is how much of the job they do together in one hour*
33.	7:31:58	SKI	*rite*
34.	7:32:00	YAG	*yepyepyep*
35.	7:32:06	SKI	*u know what x and y represent rite?*

In addition to the co-construction of the narrative explanation, the dialogical participatory orientation opens the space for the exploration of possibilities of the local world of mathematical objects and, what is perhaps even more interesting as far as learning is concerned, to anticipate the intelligibility of the co-constructed narrative. In line 35, SKI's question to GOH seems to represent, both an orientation towards a prerequisite for the intelligibility of the mathematical narrative being produced, as well as an anticipation of a potential problem of understanding. It is in these instances of dialogical interaction where we are able to observe the power of what Feurenstein [25] has labelled the "mediated learning experience" where mediators "select, change, amplify and interpret both the stimuli that come

to the learner and the learner's responses" so as to produce a type of experience that leads to cognitive change. Needless to say this role is also shared among co-participants.

Although we have refered to this context as collaborative problem solving, it is clear from the interactions and the references made by the participants that the work being done is closer to an "explanation" than to co-construction of knowledge, and yet the participants, perhaps influenced by the very nature of dialogic interactions, make such explanations interactive and participatory for all members of the group. The outcome of this approach being that there is a constant interchange between first person singular and third person plural narration and a consequent change in agency and authorship embedded with objects of unique mathematical characteristics: "my way" (e.g *I started and solved with a system of equations")* contrasted to "your way" (e.g. "*YAG its kinda hard to understand ur way*"), and sometimes becoming "our way" (e.g. *"so 8 hours is 480 minute[s], divide by 3, to get 160 minutes our answer!!!!"*). Interactionally, a "way" might be equivalent to a trajectory of constituting collective understanding. There might be methods to start a way, to stop it, to abandon it, to "start over", etc., there might be interactional features that make a way "work", and there might possibly be "degrees" of achieving the constituting of such collective understanding which open up the space for different things to happen, such as following a different narrative and problem-solving trajectory.

We have seen that two of the central elements proposed for narrative learning environments: co-construction and exploration are clearly visible in the dialogical interactions illustrated through the transcript presented. The third characteristic element of a narrative learning environment, that of *reflection* or engagement in "*consequent analysis of what happened within the learning session"*[8] seems to present itself differently in the un-moderated experiences captured in our data, a fact that would suggest a potential area where explicit support from a pedagogical environment might be specially fruitful. Having access to, at least, a partial record of the interaction in the same way that we as researchers have had through the analysis presented here, might be a unique advantage of an electronic environment. In addition, we are interested in fostering reflection, particularly, at the community level, i.e. at the level where the activity of small-groups gets reified into one diverse and collective narrative, a narrative or dialogues. Although we have not explicitly used AI techniques to shape the learning environment where the interactions presented here take place, we expect that such methods might play a role at this level, in areas such as automated narrative summarization and intelligent indexing with the specific intent of facilitating the re-usability of collaborative problem-solving dialogs for specific learning purposes.

4. Implications for design, future research.

The analyses presented in the previous section illustrate how certain narrative elements emerge from the dialogical interactions of co-participants engaged in joint problem-solving. Although this is a selective case, we find it promising to pursue some of the central questions for the design of learning environments from the enhanced perspective of this intersection of the narrative and dialogical aspects of interaction. Engagement, participation, and ultimately, learning might be emergent aspects of the distributed activity systems that offer rich opportunities for the learner to construct meaning through language and interaction in dialogical contexts. Further research and development is necessary to integrate, in the design of future learning environment, theories of sense-making that account for the narrative and dialogical aspects of individual, small-group and community interactions.

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