Bennett S. LeBow College of Business

Probabilistic Equilibria for Evolutionarily Stable Strategies

Abstract

This comment suggest that an equilibrium framework may be retained, in an evolutionary model such as Gintis' and with far more realistic results, if rationality is relaxed in a slightly different way than he proposes: decisions are assumed to be related to rewards probabilistically, rather than with certainty. This relaxed concept of rationality gives rise to probabilistic equilibria.

Herbert Gintis' essay concedes that the rational-action core of game theory will be a difficulty for many scholars. On the whole, Gintis' strategy is to introduce beliefs as an autonomous factor in decisions along with preferences and constraints, and to suggest that well-known empirical anomalies in rational action theory can be isolated as errors in beliefs. Gintis goes further in relaxing the rational-action model, suggesting that Nash Equilibrium is too narrow and that the broader game theoretic concept of rationalizability is sufficient for his purposes. However, there is a difficulty here that suggests a logical inconsistency, in that an evolutionarily stable strategy, a central concept in evolutionary game theory, is a Nash equilibrium that satisfies some other conditions as well. Rationalizability is applicable to one-off play in which there is no repetition or learning while evolutionary game theory is largely based on models of repeated matching and can be a model of social learning. This comment suggests that the equilibrium framework may be retained, with far more realistic results, if rationality is relaxed in a slightly different way: decisions are assumed to be related to rewards probabilistically, rather than with certainty. This relaxed concept of rationality gives rise to probabilistic equilibria (e.g. McKelvey, R. D. and T. R. Palfrey 1995, Chen, Hsiao-Chi, James W. Friedman and Jacques-Franciose Thisse, 1997).

Suppose that an agent is to choose between two courses of action, A and B, where B pays zero and the payoff to A varies from –4 to +4. Suppose then that the probability that the agent will choose strategy i=A, B is given by $p_i = \frac{Y_i^\theta}{\sum_j Y_j^\theta}$, where Y_j is the payoff to strategy j. Then the probability that i is chosen increases with the relative payoff Y_i. This is shown in Figure 1 for several values of the exponent. As Figure 1 suggests, the exponent θ can be thought of as an index of relative rationality, in that the choice of the higher-payoff strategy is more probable, on the whole, when θ is larger.

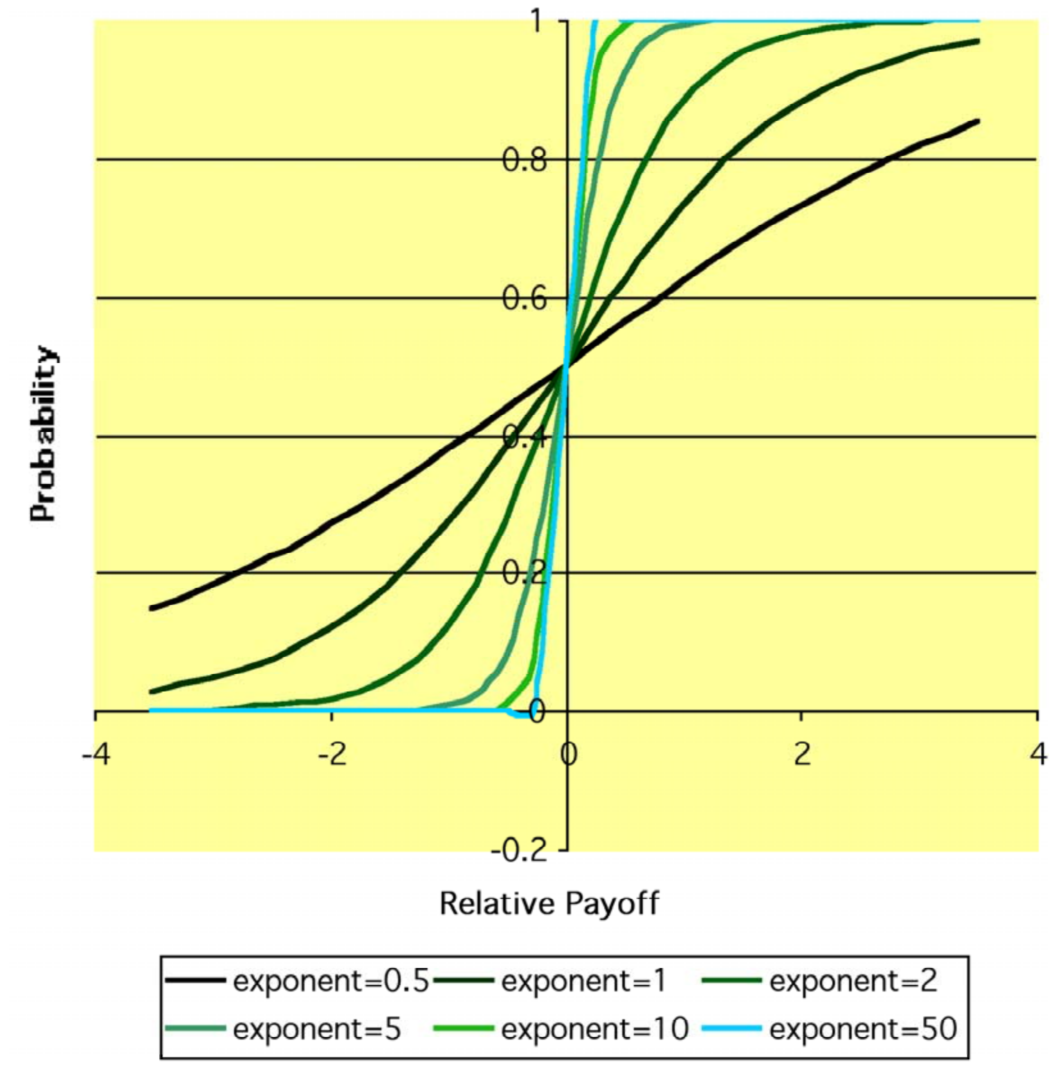

Figure 1. Probability of the Choice of Strategy A

If we consider a game-like interaction between two or more agents, each must consider the strategy choice of the other as a probability distribution and base his own choice of strategies on the expected values of payoffs from his own strategy options. A probabilistic equilibrium then is a set of probability distributions over strategy choices that are mutually consistent in that each is an approximately best response to the other. Consider, in particular, the small centipede game shown in Figure 2. This game can also be represented in normal form, using the contingent strategies shown in Table 1. We can compute a probabilistic equilibrium for this game by numerical methods (McCain 2003). Assuming a value for θ of 2 (Anderson et. al. p. 1044) and computing a probabilistic equilibrium based on the strategies in table 1 we obtain the probabilities for the nine possible strategy combinations as shown in Table 2. We note that the probability of a simple noncooperative equilibrium is about 65% in this example.

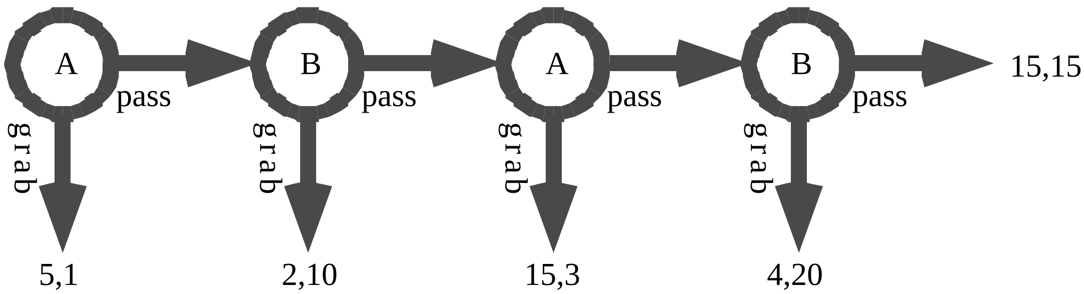

Figure 2. A Small Centipede

Table 1. Contingent Strategies for the Small Centipede

1	grab
2	pass, and if Bob passes once, grab
3	pass, and if Bob passes once , pass again
	Bob
1	If Al passes once, then grab
2	If Al passes once, then pass and, if Al passes twice, then grab
3	If Al passes once, then pass and, if Al passes twice, then pass

Table 2. Probabilities of the Strategy Combinations in an Example of Probabilistic Equilibrium in the Centipede

		Al 1	Al 2	Al 3
Bob	1	0.655	0.152	0.018
Bob	2	0.077	0.018	0.002
Bob	3	0.062	0.014	0.002

Gintis stresses the importance of non-self-regarding motives. While there is little precedent in the literature, it is quite simple in principle to introduce non-self-regarding motives into a probabilistic equilibrium model. We need not be concerned whether a non-self-regarding act generates a “warm feeling” that increases the person’s utility or not, nor whether people are in some ultimate sense self-interested even when their actions are self-regarding. We simply posit that the probability of choosing a strategy is influence by some non-self-regarding considerations as well as the payoffs.

To continue with the example of the centipede game, suppose motives of reciprocity influence the probabilities of strategies in this case. To represent reciprocity we need some reference values, so that (for example) when an agent’s payoff is less than the reference value he retaliates (negative reciprocity) and conversely. As Gintis stresses these reference values may depend on social norms, but it is possible by examining some games to make a plausible guess. In this case assume that the reference payoffs are the payoffs the agent would get if he were to grab at his first opportunity, i.e. 5 for Al and 10 for Bob. Whatever the probability of strategies 2,1 and 3,1 (Al passes and Bob grabs) this would give Al reason for negative reciprocity amounting to a shortfall of 3 and so reduce the probability of his choosing strategies 2 or 3. This is just one illustration. In general indicate reciprocity by $(Y-Y_r)(Z-Z_r)$, where Y and Z are the payoffs to Al and Bob respectively and Y_r, Z_r their reference payoffs. In place of Y_i^θ in the formula for the probability of strategy i write $Y_i^\theta + \omega \text{signum}((Y-Y_r)(Z-Z_r))\sqrt{\text{ABS}((Y-Y_r)(Z-Z_r))}$ where ω is a nonnegative weight representing the importance of reciprocity motives to the individual. Taking the exponent 2 as above and ω=0.333 for a single example, we have in Table 3 the probabilities for the nine possible strategy combinations. In this case, for example, we seen the overall probability of Al choosing the cooperative strategy 3 is 0.35, by comparison with 0.022 in the previous case. It should be noted that reciprocity can lead in some cases to multiple equilibria that reinforce both cooperative and noncooperative outcomes.

Table 3. Probabilities of the Strategy Combinations in an Example of Probabilistic Equilibrium in the Centipede with Reciprocity

		Al 1	Al 2	Al 3
Bob	1	0.036	0.404	0.237
Bob	2	0.009	0.105	0.061
Bob	3	0.008	0.088	0.052

We see that the probabilistic equilibrium concept admits of a larger and more plausible range of outcomes in this case than Nash equilibrium does, particularly when non-self-regarding motives are introduced in a natural way. In Summary, this conception of equilibrium has three major advantages in the context of Gintis' program:

1) It allows rationality to be a relative concept.

2) While probabilistic equilibria for some games closely approximate deterministic Nash equilibria, in some other cases, including the centipede, they can be

quite different and more plausible.

3) Non-self-regarding motives are easily introduced.

References

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