Inventory Management and Supply Chain Coordination Mechanisms

Mikhail M. Sher

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ABSTRACT

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This dissertation is on inventory management and supply chain coordination mechanisms within an economic order quantity framework. Specifically, this research focuses on modeling optimal order policies and coordination mechanisms for a supply chain involving items which experience probabilistic failure during storage. These items are common types of manufactured items which, nonetheless, require specialized order policy considerations due to their unique characteristics. We first develop the solution for the buyer's problem through the use of an economic order quantity (EOQ) model incorporating item failure. We then proceed to model the manufacturer's problem through the use of an economic production quantity (EPQ) model. Finally, we consider mechanisms to promote mutually-beneficial cooperation between the supplier and n buyers in service of coordinating the entire supply chain.

While prior research has focused on items which can be repaired or sold at a discount upon failure, such models are inappropriate for systems where repair costs exceed or are equivalent to item costs and imperfect items are unacceptable. Examples of industries featuring these inventory conditions include the medical, defense, and electronics industries where defective items are largely useless. First, our EOQ model considers a buyer-supplier relationship featuring delivery and stocking of items which experience probabilistic failure in storage. Thereafter, our EPQ model considers in-house production of such items. Collectively, our EOQ and EPQ models provide methods for developing optimal order policies necessary to achieve practicable supply chain coordination. In order to validate the necessity of the developed models, we include an empirical analysis of item reliability for some common mechanical components used in the defense industry, thereby identifying items which fail in the manner modeled in this dissertation.

Having considered optimal order policies for both buyers and suppliers, we next develop an optimal solution for a coordinated supply chain. The proposed solution allows the manufacturer to coordinate a supply chain consisting of n buyers in order to achieve a common replenishment time. Through this optimization framework, we minimize total system-wide costs and derive the cost savings associated with our coordinated solution.

Numerical examples are then used to demonstrate the magnitude of cost savings achievable through our coordination framework.

We conclude by proposing several mechanisms for leveraging the resulting cost savings to induce mutually-beneficial cooperation between the supplier and multiple buyers. Given the lack of buyer-supplier cooperation noted in empirical research related to supply chain coordination, our identification of specific mechanisms useful for inducing mutually-beneficial cooperation between buyers and suppliers represents an important practical contribution to the supply chain coordination literature. These models are accompanied by a thorough overview and discussion of economic order quantity theory, optimal order policies, and supply chain coordination mechanisms.

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Dedication

To my parents, Miron and Alla, for their unconditional love and support. They have always believed in me and encouraged me to be my best in everything I do in life.

To my late grandfather, Berko Grinfeld, who made immense contributions to the aviation industry through his work as an aircraft development engineer and taught Materials Science and Mechanical Engineering at Kaliningrad Military Aviation Technical School (KVATU). He is my role model as a person, scholar and educator.

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Chapter 1

Introduction

1.1 Inventory Management and Supply Chain Coordination Mechanisms

In this dissertation, we consider economic order quantity (EOQ) models for products and markets with characteristics that do not comply with the operating environment assumptions necessary for application of the basic EOQ model as described by Harris (1913). Inventory control is an increasingly important area affecting the cost of operations due to the increasingly competitive global marketplace for manufactured products and resulting cost pressures. The essential inventory control problem addressed through the EOQ model involves the balancing of holding and order costs to minimize total inventory related costs. While the basic EOQ model provides an intuitive and useful framework for understanding this critical tradeoff, its practical applicability is limited by its numerous, often unrealistic assumptions. By relaxing these assumptions, we are able to extend the basic EOQ model to determine optimal order policies for common types of inventory items with unique production and demand characteristics.

In this study, we develop both EOQ and EPQ models for use in determining optimal order policies for items which fail, rather than deteriorate, during storage according to a known probability of failure and which are not subject to rework or disposal via lot discounting. An item is subject to probabilistic failure while in storage despite having been of perfect quality at the time of delivery. This is particularly true in scenarios where cycle length is relatively long or storage environments are inadequate for preserving item quality. High fixed order costs (*e.g.*, transportation costs) resulting in large order quantities and high levels of inventory on-hand might also lead to item failure during storage. While deteriorating items may be assumed to lose a defined proportion of their value as they deteriorate, items which experience probabilistic failure are assumed to retain their full value until failure at which point they are assumed to lose all value. For example, medical professionals make use of a wide range of sterile supplies which experience probabilistic failure, including gauze pads, saline solution, small implantable silicone devices, and others. These items can experience failure during storage through loss of sterile conditions for various reasons such as tampering, unexpected changes in the storage environment, or package destruction. Sterile supplies tend to be sufficiently inexpensive such that replacement costs are less than re-sterilization costs, thus rendering the failed items worthless. Alternatively, re-sterilization may be prohibited by regulation or feasibility (i.e. saline solution cannot be recaptured and repackaged once its storage bag has been compromised). The total loss of value assumed in our model, therefore, can be due to an inability to perform rework or the expense of rework relative to purchasing a new item.

The inventory level of such items decreases due to item failure even in the presence of zero demand. The decreasing pattern is characterized by a failure rate defined by the reliability (or survival) function $S(t) = Qe^{-pt}$, where $S(t)$ is the number of good items at time t, Q is the original order quantity and p is the item failure rate. It is assumed that items which fail during the storage period have no value and the cost of repair is comparable to the cost of obtaining a new item.

A number of practical applications for such a model can be found in common use. Medical supplies are often subject to rigorous requirements which make imperfect quality intolerable. Silicone bands for scleral buckling, for example, are an example of a product which cannot be used if deteriorated in any way or reworked/repaired if broken (Roldán-Pallarés, Sanz, Susi and Refojo 1999). Single-use medical devices, such as certain classes of surgical equipment, can also be less expensive to repurchase than to reprocess for future use (Tessarolo, Caola, and Nollo 2011). One-time use may also be mandated by law, meaning that violating their sterility would render them worthless. Small electronic devices, such as individual circuits or switches, used in many products are also representative of products which may have no value upon failure, depending on whether or not they are repairable and the relative cost to repair. This model is accompanied by an empirical investigation of item failure rates which is intended to validate the failure rate assumptions utilized in the model.

We also develop an economic production quantity (EPQ) model for use in developing optimal production policies for items which fail during storage according to a known probability of failure and which are not subject to rework or disposal via lot discounting. Developed in 1918 by E. W. Taft as an extension of the EOQ model, the basic EPQ model considers production and item delivery which occur incrementally throughout the inventory cycle rather than in periodic lots. This model addresses inventory control among firms which choose to produce such items in-house rather than obtaining them through an outside supplier. As such, it represents an important model which can inform make-or-buy decisions for electronics and medical supply and device companies.

1.2 Contributions to the Literature

This dissertation, therefore, makes several contributions to the literature. One contribution relates to the type of items being modeled. As discussed earlier, defective and deteriorating items are not fully analogous to items which experience probabilistic failure during storage. Defective items are assumed in prior literature to be salvageable through single-lot disposal or reworkable at a known, specified cost. Items which have failed, by contrast, are not assumed to be salvageable or reworkable in our model due to excessive repair cost relative to replacement cost, impracticality or legal requirements. Additionally, we consider a probabilistic, rather than a deterministic, function of deterioration for products which prior literature has not considered. Rather than deterioration occurring at a constant rate or as a strict function of time, failure in our model occurs randomly. As such, we model products which may, but will not definitively, fail over time. Our study, by combining items which experience probabilistic failure with items that experience a complete loss of value upon failure provides innovative insights into optimal order quantities of items common to several large global industries: medical supplies and equipment, defense and electronics.

Similarly, the consideration of in-house production of items experiencing probabilistic failure during storage within the EPQ framework provides novel insights into optimal production policies for such items. Identifying sources of cost savings within supply chains creates significant opportunities for mutually-beneficial cooperation which can support a variety of supply chain coordination mechanisms. This model will provide a foundation for future research into supply chain coordination mechanisms and represents an important extension of the basic EPQ model.

1.3 Organization of the Dissertation

This dissertation is organized in seven chapters. Chapter 2 provides an overview of EOQ theory and prior literature which extends the basic EOQ model by relaxing its underlying assumptions. Chapter 3 develops an EOQ model for items which experience probabilistic failure in storage and which cannot be reworked or sold at a discount. Chapter 4 demonstrates the behavior of this EOQ model through the use of numerical sensitivity analyses and graphical representations. The second part of Chapter 4 summarizes the results of an empirical study of item reliability and failure rates for common electrical components. This empirical research is included in order to provide support for the importance of the probabilistic failure framework introduced in Chapter 3. Chapter 5 develops an EPQ model for items which experience probabilistic failure in storage and which cannot be reworked or sold at a discount. Chapter 6 outlines several mechanisms for utilizing the total cost savings obtained from the EOQ and EPQ models developed in Chapter 3 through 5 to induce mutually beneficial, sustainable cooperation. Chapter 7 concludes the dissertation and provides directions for future research.

Chapter 2

Literature Review

2.1 Introduction

This chapter provides an overview of the literature related to economic order quantity (EOQ) model and supply chain coordination within the EOQ framework. This review provides a basis for understanding the concepts introduced later in this dissertation.

Two major sections are developed within this chapter. First, Section 2.2 provides an overview of EOQ theory in order to describe the inventory management framework which underlies this dissertation. Section 2.3 examines EOQ models which expand the basic single-buyer assumption to multiple buyers.

2.2 Economic Order Quantity (EOQ) Theory

In this section, we provide an overview of the basic EOQ model developed by Harris (1913), including a brief explanation of its intuition, critical assumptions, and extensions. Section 2.2.1 defines the total inventory cost function and optimal order quantity as defined within the EOQ framework. Section 2.2.2 and its subsections outline basic EOQ model assumptions which have been relaxed and examined in detail in prior literature, including item quality, disposal, and failure assumptions.

2.2.1 Basic EOQ Model

Since the idea of what quantity to make at once was first published by Harris (1913), the economic order quantity (EOQ) model has achieved widespread acceptance in academic journals and has been used extensively for practical business applications. Harris (1913) described a simple formula for calculating order sizes for parts used in manufacturing which balances ordering and holding costs in order to minimize total inventory costs. The EOQ model adopts a number of simplifying assumptions which, while not fully reflective of true production environments, are useful for estimating inventory needs. These assumptions include:

- A single buyer obtains a single product treated autonomously from a single seller
- The rate of demand for modeled items is constant and known
- Order and holding costs are constant on a per-unit basis and known
- Purchase prices are constant with no discounts available
- Order lead times are fixed
- Inventory replenishment is instantaneous and orders are delivered in full
- All delivered items are of perfect quality

The basic EOQ model is derived from a total cost function which includes all purchase, ordering, and holding costs for an order as described in Equation (2.1):

$$Total\ Costs = cD + \frac{DK}{Q} + \frac{HQ}{2} \tag{2.1}$$

where

Q: order quantity
c: unit purchase cost
K: order placing cost per order
H: holding cost per unit per year
D: yearly demand

Total cost minimization is achieved through differentiation, with the optimal order quantity calculated as the derivative of the total cost with respect to Q. Equation (2.2) provides the first order optimality condition used for calculating the optimal order quantity, hereafter identified as Q^*:

$$Q^* = \sqrt{\frac{2DK}{H}} \tag{2.2}$$

2.2.2 EOQ Model and Item Quality Assumptions

While the basic EOQ model is effective at describing the intuition behind inventory management decisions (i.e. the balancing of ordering and holding costs within inventory management systems), it is limited in its practical applicability by the rigid assumptions used in its construction. Numerous modifications to the EOQ model have been proposed, therefore, which improve its practical applicability by relaxing one or more of the basic EOQ model's assumptions. A number of researchers has focused on relaxing the assumption that all items produced are perfect quality (Wright and Mehrez 1998). This section provides an overview of EOQ research dealing with item quality, including methods of dealing with imperfect item disposal and item failure rates.

2.2.2.1 Imperfect Item Quality and Failure Rates

Several frameworks have been introduced for relaxing the assumption of perfect item quality included in the basic EOQ model of Harris (1913).

One prominent framework, developed by Salameh & Jaber (2000), has been used extensively to model situations where production processes are prone to control issues according to a fixed probability or with a known probability distribution. Salameh & Jaber's (2000) model introduces a 100% screening mechanism for separating perfect quality items from imperfect quality items. Orders have a fixed probability of containing imperfect quality items, with such items being successfully identified through screening upon delivery. Their model also introduces a two-tier pricing system for items based on quality, where perfect quality items can be sold at full price and imperfect quality items can be sold at a discounted price. These mechanisms allow Salameh & Jaber's (2000) model to account for imperfect item quality and related profit losses when calculating optimal cycle time and order quantity, generally leading to larger orders and less frequent cycle times when inventoried items are allowed to be imperfect at some known rate.

Salameh & Jaber's (2000) framework has been extensively modified by researchers seeking to model inventory control decisions for items with imperfect quality. A number of these modification focus on the rate at which imperfect quality items enter the inventory system. Eroglu & Ozdemir (2007), for example, extend the EOQ model for defective quality items by introducing the assumptions that the item defect rate is a uniformly distributed random variable and defective items are sold as a single lot at a discounted price (hereafter referred to as single-lot disposal). Erogly & Ozdemir (2007) also allow for shortages to be backordered, with perfect quality items delivered during each cycle used to fulfill backordered inventory. Tsou (2007) models item quality using a normal distribution, with each lot's proportion of imperfect quality items being normally distributed. El-Kassar (2009), by contrast, allows for fixed probabilities of imperfect items and continuous demand of both perfect and imperfect items rather than the previous model of single-lot disposal of defects. This generalization of Salameh & Jaber's (2000) model accounts for scenarios where imperfect items enter inventory via channels other than manufacturer delivery. Given the potential for item stockouts in the presence of variable yield rates for ordered items (i.e. non-fixed rates of imperfect quality items within orders), Maddah et al. (2010) proposed a reordering mechanism whereby reorders are placed at the point where remaining inventory is merely sufficient to meet demand during the screening period. This mechanism, described as order overlapping in the literature, allows item demand to be fulfilled from current inventory while delivered items are screened, increasing holding costs while generally avoiding stockouts. Profitability losses associated with increased holding costs is offset by superior customer service associated with avoiding stockouts, suggesting this model is most appropriate when stockouts are more harmful to profitability than overstock scenarios.

Maddah & Jaber (2008) model the proportion of imperfect items in delivered lots as being a random fraction of delivered items. Optimal lot sizing is larger under these conditions relative to the basic EOQ model provided that yield rate variability is sufficiently low. Huang (2004) derives optimal order policies for a supply chain with an unreliable process that produces defective items at a random rate according to a known probability distribution where numerous defects appear in a single lot at unpredictable intervals. This model both utilizes a random variable to model item defect rates and incorporates warranty costs for imperfect item disposal. Papachristos & Konstantaras (2008) develop a similar model involving a uniformly distributed random defect rate for ordered items, suggesting that this type of distribution is relatively deterministic in practice despite being stochastic from a theoretical standpoint. Jaber, Goyal, & Imran (2008) use empirical data analysis to adjust the item defect rate assumption of Salameh & Jaber (2000) for learning effect, demonstrating that defect rates decline as a function of learning curve gains in the production process. Wahab & Jaber (2010) combine learning curve gains on yield rates with a two-tiered holding cost system based on item quality (i.e. different holding costs for imperfect and perfect quality items) in order to develop optimal order quantity and cycle time policies for inventory systems featuring imperfect quality items. Jaber, Zanoni and Zavanella (2013) adjust the model of Salameh & Jaber (2000) for an entropic production environment with imperfect quality item production. This model uses a random uniform distribution of imperfect items and a single-lot disposal mechanism to account for disposal of defective units.

Other models focus on items which deteriorate according to a known probability function rather than being defective immediately after production. Nahmias (1982) provided a review of early research in perishable inventory theory. Yano and Lee (1995) provide a separate review of models which account for lot sizing in the presence of random production or procurement yields (*i.e.*, random defect rates). Yield uncertainty rates covered within this review include binomial, Bernoulli, stochastically proportional, interrupted geometric, capacity-related randomness, and increasing failure rate based on batch size. Wee et al. (2006) introduce a model which includes both deteriorating items and imperfect quality. This model assumes that item quality is independent of deterioration and that defective items are identified and removed from inventory during screening. These assumptions limit the applicability of this model in industries where deterioration leads to irreparable failure after screening but before usage, such as may occur in the electronics, medical, and defense industries. Jaggi, Goel and Mital (2011) derive a model which accounts for items which could both be defective and deteriorate. As in prior models involving defective items, such items are disposed of via single-lot discounting. This model, however, does not assume that items experiencing deterioration cannot be reworked, replaced, or sold at a discount. Khanra, Ghosh and Chaudhuri (2011) describe items which have a constant deterioration rate with time-dependent demand and which cannot be repaired within the cycle. Optimal cycle times and order quantities for these items are derived under two credit policies, one where the credit period is less than or equal to the cycle time and another where the credit period is greater than the cycle time.

Dye (2013) models the rate of item deterioration as a non-instantaneous, concave time-dependent rate which can be altered via investment in technology (*i.e.*, refrigeration or storage). Uthayakumar and Rameswari (2012) describe an EOQ model for items which deteriorate at a constant rate and decrease in value over time, thereby accounting for the time value of money for both payments and inventory valuation over a fixed planning horizon. Optimal order policies are derived under conditions where backlogging is either allowed or not allowed. Similarly, Bose, Goswami, & Chaudhuri (1995) develop a model of items which deteriorate at a constant, known rate and which accounts for the effects of time-dependent demand rates and time discounting. Madhavi, Rao and Lakshminrayana (2011) model deteriorating items which can be sold at certain discounts rather than having to be disposed of or scrapped. Items within this model are assumed to have a random lifetime and a probabilistic deterioration rate. Thangam and Uthayakumar (2011) and Sana (2011) both model deterioration as a constant proportion of inventory on hand rather than being time-dependent or probabilistic. While Thangam & Uthayakumar (2011) focused on optimal order policies for deteriorating item based on the availability of various types of trade credits, Sana (2011) considered such items under demand conditions where demand decreases according to a quadratic function with price.

While prior research has focused a great deal of attention on items which deteriorate according to a deterministic rate, such items are not fully analogous to items which experience probabilistic failure during storage. From a practical perspective, items may fail independent of time: violation of sterile storage conditions for medical supplies, for example, may happen randomly rather than at regular intervals or over time. Unlike prior literature, our model uses a probabilistic survival function with a known probability density. Barlow, Marshall and Proschan (1963) suggest that this type of “hazard rate” has considerable practical application for describing the reliability of items, specifically for modeling items which wear out or fail randomly. While Moon and Yun (1993) model item deterioration as a function of a probabilistic inventory cycle length, deterioration is assumed at the end of the inventory cycle rather than within it. Halim, Giri and Chaudhuri (2008) model deteriorating items as having a fuzzy rate of deterioration within a specified interval. Halim et al.

(2008) also consider partial backlogging and stochastic demand in the context of deteriorating items based on the aforementioned fuzzy deterioration rate.

2.2.2.2 Imperfect Item Disposal

Researchers have proposed a number of methods for modeling imperfect quality item disposal within the EOQ framework. A number of models assume that imperfect quality items cannot be sold and, instead, must be reworked at additional cost to restore item quality for sale. Porteus (1986) highlights the impact of larger lot sizes on the quantity of defects within a process where control issues are only detected between lots, proposing that smaller lot sizes may promote lower defect rates due to a lower probability that process control issues will go undetected. Additional costs are incurred for rework when imperfect quality items are produced, thereby creating incentives for process control improvement investments. Similarly, Rosenblatt and Lee (1986) introduced both rework and restoration costs into the total cost function along with inspection policy costs and consider optimal order policies for inventory systems featuring items of perfect and imperfect quality. Lee and Rosenblatt (1987) develop a model which jointly determines cycle time and inspection schedules in order to minimize total costs, including inspection and rework costs. Gerchak, Vickson, & Parlar (1988) allow for variable item yield and imperfect quality items within an EOQ framework. Mabini, Pintelon and Gelders (1992) consider two types of items involving imperfect items that must be reworked in order to be sold: one where a single item has a fixed, known scrapping rate and sufficient capacity to facilitate repairs and another where multiple items share limited repair resources. Urban (1998) considers imperfect quality items in which demand varies as a function of shelf-space allocation. Unlike these works, Salameh and Jaber (2000) developed an EOQ model where defective items are kept in stock and sold at a discounted price instead of being reworked or scrapped. In recent years, this paper has received considerable attention and been widely extended by many researchers.

Other EOQ models have proposed selling processes for items of imperfect quality. A number of the papers discussed in Section 2.2.2.1 allow for single-lot disposal, a method of selling all defective items as a single lot at a specified discount. Konstantaras et al (2007) extend research on imperfect items by allowing for either rework of such items or single-lot disposal. Item screening is used to separate perfect and imperfect item quality, after which perfect items are routed to the work-in-process inventory and imperfect items are either reworked or disposed via single-lot disposal. Importantly, reworked items are treated as perfect quality items for selling purposes. Maddah & Jaber (2008) allow imperfect items to be screened out and held for several periods in order to minimize holding and shipment costs related to both imperfect and perfect quality items, an important consideration given the random yield rate associated with deliveries. Chan, Ibrahim, & Lochert (2003) develop a model which fully integrates imperfect item reject, rework, and disposal based on quality parameters. Tsou (2007) provides for imperfect quality items to be disposed of based on the extent of imperfections, with below specification items being scrapped and items within tolerance specifications being sold at a discount calculated using Taguchi & Wu's (1985) quality loss function. Tsou, Hejazi, & Barzoki (2009) derives optimal order policies for a production process which yields perfect, imperfect, and defective quality items which can be sold at full cost, sold at discount, and reworked, respectively.

While the papers referenced above assume that defective items yielded from imperfect processes are either reworked or scrapped, such an approach appears impractical for implementation in industries where high levels of accuracy are mandatory and imperfect quality items are unacceptable. Such industries include medical supplies and electronic components, where production and repair costs for defective units of items such as electric components or sterile medical products are comparable.

2.3 EOQ Models Involving Multiple Buyers

While the basic EOQ model of Harris (1913) assumes a single buyer sourcing items from a single vendor, modern supply chain relationships require vendors to respond quickly to the demands of multiple buyers with different demand characteristics. Not only are multiple external buyers a general feature of an increasingly global marketplace, but modern organizational structures can also create circumstances where suppliers must manage the needs of multiple internal buyers. Prior research has explored more complex supply chains in order to identify opportunities for cost savings and cooperation within supply chains. In their paper entitled "Coordination of a single-manufacturer/multi-buyer supply chain with credit option," Sarmah, Acharya, and Goyal (2008) focus their attention on a supply chain model where a single manufacturer sells a product to multiple buyers. The authors cite the need for such a model by referencing the rarity of scenarios where single manufacturers supply a product to a single buyer within modern production environments. Using a two-stage supply chain, the manufacturer supplies a product to multiple buyers located in different geographic areas. Given that prior research has identified that approximately sixty three percent of annual logistics costs can be tied to transportation, it is not surprising that consolidation of deliveries results in significant savings (Schaefer 2011).

Sarmah et al. (2008) develop their model using two transportation cost scenarios. The first is an ex-site delivery condition, where transportation costs are included in the product price and each buyer's order is handled independently. The second case is an ex-factory case where the cost of transportation is borne by the buyers. In both scenarios, coordinated product delivery at fixed intervals to multiple buyers sharing a common carrier reduces associated manufacturer and customer costs. Manufacturer can induce buyers to accept deliveries at fixed, rather than the buyer-preferred uneven, intervals through the provision of trade credit.

Li and Liu (2006) develop a model featuring a quantity discount mechanism useful for facilitating supply chain coordination. This model is developed within a single product, multi-period setting where customer demand is probabilistic. In contrast to the model developed in our paper, Li and Liu (2006) model a supply chain with a single buyer and manufacturer. The authors identify bounds within which the quantity discount results in increased profit and, as a result, enables supply chain coordination. Additionally, the authors develop a method for apportioning increased profits between the buyer and manufacturer and derive the optimal discount level under that method.

Siajadi, Ibrahim, & Lochert (2006) consider scenarios involving two or more buyers and derive a function for minimizing joint total relevant costs. This model highlights the importance of two different ratios in the minimization of system-wide costs: the ratio of production rate to demand rate and the ratio of transportation costs to holding costs. The authors propose a multiple shipment policy which is successful at reducing total system-wide costs assuming that the production rate is greater than the sum of the buyers' demand rates: a finding which is contrasted with Banerjee's (1986) policy under which a vendor produces to order for a purchaser on a lot-for-lot basis under deterministic conditions.

Woo, Hsu, and Wu (2001) presents a single-vendor, multiple-buyer supply chain where vendors and buyers cooperate to reduce ordering costs and, by extension, joint costs. Planned ordering costs are first used to develop optimal order cost reduction investment policies. Optimal order policies are then derived based on reduced order costs. Banerjee & Banerjee (1992) propose a similar type of coordination based on electronic data interchange, whereby a common replenishment time is achieved through the use of a vendor-managed inventory (VMI) system. Suppliers achieve cost minimization by making all replenishment decisions on behalf of multiple buyers. Zhang, Liang, Yu, & Yu (2007) also proposes VMI as a way to minimize system-wide costs in a two-echelon inventory system, describing a system where joint costs are minimized through the use of VMI technology. Ordering cycles are assumed to be heterogeneous and multiple replenishments are allowed within an order cycle, allowing demand to be better forecast with additional information during each inventory cycle. Cost minimization is achieved through a combination of superior demand information, technology investment, information sharing, and production planning. Yao & Chiou (2004) also address the problem of replenishment coordination, setting vendor total annual cost minimization as the objective function constrained by the buyers' willingness to incur costs. The authors propose a heuristic solution to this problem, demonstrating that the optimal cost curve is piece-wide convex with respect to the vendor's production and buyer's replenishment schedule.

Wang (2002) considers a supply chain where a single vendor provides items for multiple buyers with heterogeneous demand schedules. This model first considers the use of quantity discounts to induce buyers to conform to the vendor's desired delivery and production schedule and uses a game theoretic approach to describe parameters useful for developing quantity discount policies. Optimal order policies are then derived using non-linear programming subject to the optimal discount policy. Wang (2004) later extends this framework to consider the relative effectiveness of integer-ratio policies, as compared to quantity discounts, for inducing cooperation among heterogeneous buyers. Integer-ratio policies require that buyers place their orders at intervals which are some integer-related ratio of the vendor's optimal replenishment interval. Wang (2004) finds that, while both quantity discounts and integer-ratio policies can achieve cost savings, integer-ratio policies are more effective at achieving coordination and reducing system-wide costs.

Bylka (1999) considers a dynamic model for determining optimal order policies for a single-vendor, multiple-buyer supply chain. Both buyer and vendor demand are modelled as periodic sequences, with vendor demand determined by the sum of buyers ordering sequences. The authors propose a vendor production schedule which uses periodic buyer orders and aggregate demand forecasts to develop turnpikes, or order cycles which minimize average cost per period, within the supply chain. The optimal order policy, therefore, is developed as an average cost minimization function within the EOQ framework based on optimal turnpike policies and buyer demand parameters.

2.4 Research Opportunities

Though prior EOQ models have considered imperfect item quality for a number of item types, these models do not fully account for the broad range of items produced by manufacturers. EOQ models assuming that items remain in the condition in which they are delivered are inapplicable to items which arrive in perfect condition but become imperfect during storage. Whether an item remains in its purchased condition or deteriorates during storage is important because screening mechanisms fail to properly identify imperfect quality items when item quality changes after delivery. EOQ models on perishable items partially account for items which lose value during storage. Such models, however, generally assume item value loss to be time-dependent. EOQ models for perishable inventory are of limited value in developing optimal order policies for items which fail during storage but at times that are nondeterministic. Additionally, perishable inventory models, by assuming a gradual decrease in item value, are also of limited usefulness in modeling items where any imperfection in quality renders the item worthless.

The model developed in Chapter 3 of this dissertation is thus intended to address items which fail during storage according to a time-independent rate. This model represents an important extension of the EOQ framework in that it considers non-decaying item failure which occurs after order delivery but at a certain moment during storage rather than gradually. Previous studies which consider post-delivery item failure do so using relatively simple failure rate approximations. Hauptman (1996) models item failure rates using a binomial distribution and evaluate common cause failure for impulse pilot valves used in nuclear power plants. Leemis (2006) utilizes a Bernoulli failure rate in the context of single-component and multiple component systems in order to model independent component failure for inventory and maintenance planning purposes. Each of these papers considers item quality in binary terms as being either of good quality (i.e. compliant with specifications) or not good quality (i.e. defective). By utilizing a probabilistic, rather than discrete, failure rate, however, our study allows for the consideration of more complex continuous cases of post-delivery item failure within the EOQ framework.

Additionally, our model accounts for the immediate loss of item value following a change in item quality. Items with these characteristics, as discussed in Chapter 1, are widely used across a variety of industries. Sterile supplies used within the medical industry, for example, exhibit probabilistic failure during storage in that an item can arrive in perfect condition but become imperfect during storage if sterile conditions are violated. Such supplies are of no value if they become unsterile as standard medical practice and laws prohibit the use of unsterile supplies. Concerns over item sterility are so significant, in fact, that many healthcare organizations have a policy of discarding sterile-packed supplies if even the outer, non-sterile packaging is exposed to a potential contaminant (Perl 2013). Schierholz and Beuth (2001) highlight the risks associated with re-using sterilizable implantable materials if contaminated prior to implantation: risks which have led hospitals to discard contaminated items which could potentially be re-sterilized to avoid the increased infection risk.

Similarly, electronic components used in the defense and electronics industry may fail during storage after being delivered in perfect conditions.

While rework may be possible for imperfect electronic components, the low replacement cost of these items often makes rework impractical and essentially eliminates all value from imperfect quality items. This is particularly true for products containing electronic components with low tolerance requirements. Villasenor and Tehranipoor (2013) cite the dearth of electronic component rework and recycling in such products as calculators and remote controls, noting that “given the low cost of electronics parts for those products, such reuse wouldn’t usually be worth the trouble.”

Chapter 3

An Economic Order Quantity Model for Items Experiencing Failure in Storage

3.1 Introduction

In this chapter, we develop an economic order quantity (EOQ) model for items experiencing failure in storage. The remainder of this chapter is organized as follows: Section 3.2 provides an overview of the model, including the notation utilized to describe the model and the probabilistic failure function used to model item failure. Sections 3.3.1 and 3.3.2 derives optimal order quantity and cycle time policies for the model, respectively. Section 3.3.3 further derives the penalty cost function associated with the developed model.

3.2 Model

To develop the model, we make the following assumptions: (1) the demand rate, setup/order cost and inventory holding costs are known and deterministic, (2) orders are replenished instantaneously at the beginning of each cycle, with no shortage allowed, (3) a 100 % screening is performed when the lot is delivered to separate the defective items, which are to be replaced at supplier's cost, and (4) lots have a fixed rate of failure with known probability density function.

The following notation is adopted:

Q	order quantity
Q^*	optimal order quantity
c	unit purchase cost
K	order placing cost per order
H	holding cost per unit per year
s	selling price per unit
T	cycle length
b	inspection cost per unit (items are inspected when the lot is delivered)
D	yearly demand
$S(t)$	number of items in stock at time t

p	item failure rate

As previously mentioned, inventory level in this model decreases not only by demand, but also by item failure during the storage period as shown below in Figure 1. Note that the inventory level is depleted faster when both demand and failure rate are considered as opposed to when demand rate is considered alone. The latter case is indicated by the dotted line. The inventory cycle time is also affected and shortened when the failure rate is taken into account. The possibility of items experiencing failure in storage inherently requires shorter ordering interval (or inventory cycle), which implies increased ordering costs. Given that the inventory cycle will be shorter when items experience probabilistic failure, items will be held in inventory for shorter periods of time. As such, holding costs have a smaller impact on optimal cycle time and profit compared to the standard EOQ model. This dictates that managers will allocate resources toward developing and improving logistics related to ordering cost minimization. Identifying the optimal cycle time is a critical consideration for both cost minimization and profit maximization. Utilizing an optimal ordering schedule has the effect of limiting both ordering and holding costs, facilitating cost minimization. Shortage cases may result from using a non-optimal cycle time, especially when significant lead time exists between order placement and fulfillment.

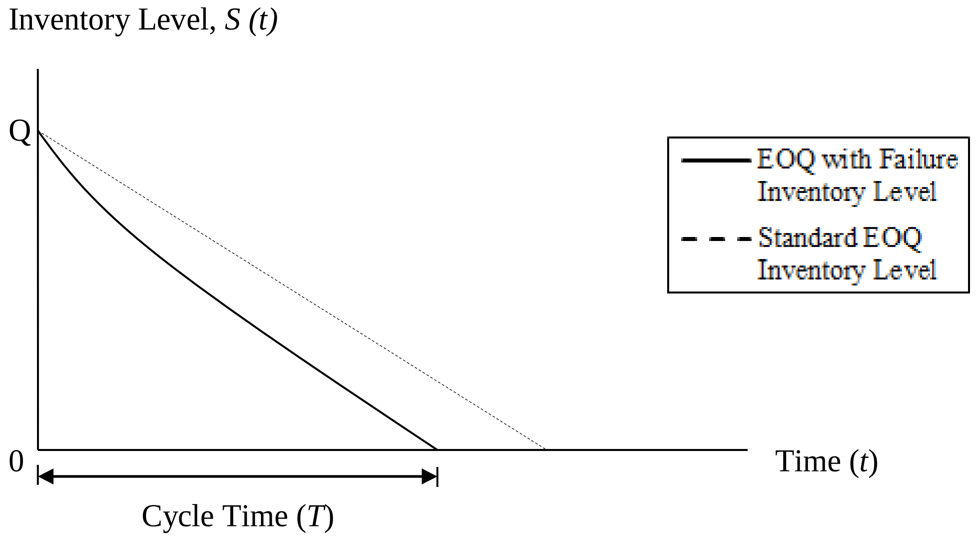

Figure 3.1: Inventory Level over Cycle Time

In Figure 3.1, T is the cycle time and Q is the initial inventory position at the time the system receives the order quantity from the supplier. As mentioned earlier, the inventory level at time t is described as $S(t) = Qe^{-pt} - Dt$, where p is the failure rate of the item per year and D is the annual demand rate. The inventory level continues to diminish during the cycle time, reaching 0 at the end of the cycle time T. Thus, $\frac{1}{T}\int_0^T (Qe^{-pt} - Dt)dt$ represents the average amount of inventory in stock during the cycle time. We assume that failed units are discovered upon attempted use and are discarded when failure is observed. Inventory holding costs are computed based on the average amount of inventory per cycle.

3.3 Optimization

The total revenue (TR) and costs (TC) per cycle are defined as

$$TR = sDT \tag{3.1}$$

$$TC(Q,T) = K + cQ + bQ + H\frac{1}{T}\int_0^T (Qe^{-pt} - Dt)dt \tag{3.2}$$

The total profit (TP) per cycle is given as

$$TP(Q,T) = TR - TC = sDT - K - cQ - bQ - H\frac{1}{T}\int_0^T (Qe^{-pt} - Dt)dt \tag{3.3}$$

The total profit per year, TPY, is

$$TPY(Q,T) = \frac{TP}{T} = \frac{sDT - K - cQ - bQ - H\frac{1}{T}\int_0^T (Qe^{-pt} - Dt)dt}{T}$$

$$= \frac{sDT - K - cQ - bQ - H\frac{1}{T}(-\frac{1}{p}Qe^{-pT} - \frac{DT^2}{2} + \frac{Q}{p})}{T}$$

$$= \frac{sDT - K - cQ - bQ + \frac{H}{Tp}Qe^{-pT} + \frac{HDT}{2} - \frac{HQ}{Tp}}{T} \tag{3.4}$$

3.3.1 Optimal Order Quantity

We begin with integrating the total cost function as previously described in Equation 3.2 as follows:

$$TC(Q,T) = K + cQ + bQ + H\frac{1}{T}(-\frac{1}{p}Qe^{-pT} - \frac{DT^2}{2} + \frac{Q}{p}) \qquad (3.5)$$

The total cost per year (TCY) is:

$$TCY(Q,T) = \frac{1}{T}\Big[K + cQ + bQ + \frac{H}{T}(-\frac{1}{p}Qe^{-pT} - \frac{DT^2}{2} + \frac{Q}{p})\Big] \qquad (3.6)$$

To find the order quantity Q which minimizes the total cost per year, we first determine the derivative $\frac{dTCY}{dQ}$.

$$\frac{dTCY}{dQ} = \frac{d(\frac{1}{T}\Big[K+cQ+bQ+\frac{H}{T}(-\frac{1}{p}Qe^{-pT}-\frac{DT^2}{2}+\frac{Q}{p})\Big]}{dQ}$$

$$= \frac{d}{dQ}\Big[\frac{1}{T}(K + cQ + bQ)\Big] + \frac{d}{dQ}\Big[\frac{1}{T}(\frac{H}{T}\Big(-\frac{1}{p}Qe^{-pT} - \frac{DT^2}{2} + \frac{Q}{p}\Big))\Big]$$

$$= \frac{(K+cQ+bQ)'T-(K+cQ+bQ)T\prime}{T^2}$$

$$+ \frac{H\left(-\frac{1}{p}Qe^{-pT} - \frac{DT^2}{2} + \frac{Q}{p}\right)' T^2 - H(-\frac{1}{p}Qe^{-pT} - \frac{DT^2}{2} + \frac{Q}{p})[T^2]'}{T^4}$$

$$= \frac{[(c+b)T-(K+cQ+bQ)T\prime]T^2}{T^4}$$

$$+ \frac{H\Big(-\frac{1}{p}e^{-pT}+Qe^{-pT}T'-DTT'+\frac{1}{p}\Big)T^2-H\Big(-\frac{1}{p}Qe^{-pT}-\frac{DT^2}{2}+\frac{Q}{p}\Big)2TT\prime}{T^4} \qquad (3.7)$$

Next, we set the derivative $\frac{dTCY}{dQ}$ equal to 0 and solve for Q.

$$[(c+b)T-(K+cQ+bQ)T']T^2+\left(Qe^{-pT}T'-DTT'+\frac{1}{p}-\frac{1}{p}e^{-pT}\right)HT^2$$

$$+\left(\frac{1}{p}Qe^{-pT}+\frac{DT^2}{2}-\frac{Q}{p}\right)2HTT'=0 \tag{3.8}$$

3.3.2 Optimal Cycle Time using MacLaurin Series

As shown in Equation (3.4), the optimal EOQ can be found when the cycle time is known. Given that it is dependent on both the demand and failure rates, a closed form solution for the cycle time in this paper cannot be obtained as easily as for the common case (*i.e.*, by dividing the lot size Q by the annual demand rate *D).* We will, therefore, describe an appropriate method to determine optimal cycle length.

Given that cycle time is the time duration required for inventory to decrease from Q to 0 (as illustrated by figure 1), we solve $S(t)=Qe^{-pt}-Dt$ for t to find the cycle time (T). To solve $Qe^{-pt}-Dt$ for t, we use the MacLaurin series approximation of second degree as follows:

$$f(x)\approx f(0)+f'(0)x+\frac{f''(0)}{2!}x^2$$

$$Dt\approx Q(1-pt+\frac{1}{2}p^2t^2)$$

$$\frac{1}{2}Qp^2t^2-Qpt+Q-Dt=0$$

$$\frac{1}{2}Qp^2t^2-(Qp+D)t+Q=0$$

$$T = \frac{Qp + D - \sqrt{(Qp + D)^2 - 4(\frac{1}{2}Qp^2Q)}}{2(\frac{1}{2}Qp^2)}$$

$$= \frac{Qp + D - \sqrt{(Qp + D)^2 - 2Q^2p^2}}{Qp^2}$$

Therefore, the approximate optimal cycle time is:

$$T \approx \frac{Qp + D - \sqrt{(Qp + D)^2 - 2Q^2p^2}}{Qp^2}. \tag{3.9}$$

As we could not obtain a closed form solution for Q, we introduce a computational approach in order to obtain a numerical solution and provide illustrative examples. In order to do so, we substitute the cycle time calculated in Equation (3.9) into the annual total profit function described in Equation (3.4), we then optimize Equation (3.4) with respect to the order quantity; thereby selecting the order quantity that maximizes yearly profit for each set of problem parameters.

3.3.3 Cost Benefits of the Model

The comparative cost savings achieved through the use of the developed model can be calculated using a penalty cost function. We begin the total cost per year (TCY) as described in Equation (3.6). To find the order quantity Q which minimizes the total cost per year, we first determine the derivative $\frac{dTCY}{dQ}$ using the quotient rule and set it equal to zero as described in Equation (3.8).

Inventory failure combined with sales leads to diminishing inventory levels throughout the inventory cycle, with total inventory reaching zero at the end of the cycle time T. We therefore specify the following condition which reflects item inventory position throughout the cycle:

$$Qe^{-pT} - DT = 0$$

$$Qe^{-pT}T' - DTT' = (Qe^{-pT} - DT)T' = 0$$

Simplifying Equation (3.8) yields the following equation:

$$[(c+b)T - (K + cQ + bQ)T']T^2$$

$$+\frac{1}{p}(1 - e^{-pT})HT^2 + \left[\frac{Q}{p}(e^{-pT} - 1) + \frac{DT^2}{2}\right]2HTT' = 0 \qquad \Rightarrow$$

$$(c+b)T^3 - KT^2T' - Q(c+b)T^2T' + \frac{1}{p}(1 - e^{-pT})HT^2 + \frac{Q}{p}(e^{-pT} - 1)2HTT'$$

$$+DT^3T'H = 0 \tag{3.10}$$

Solving for Q, we obtain the following expression

$$Q\left[\frac{2}{p}(e^{-pT} - 1)HTT' - (c+b)T^2T'\right] = KT^2T' - (c+b)T^3$$

$$-\frac{1}{p}(1 - e^{-pT})HT^2 - DT^3T'H$$

$$Q = \frac{KT^2T' - (c+b)T^3 - \frac{1}{p}(1 - e^{-pT})HT^2 - DT^3T'H}{\frac{2}{p}(e^{-pT} - 1)HTT' - (c+b)T^2T'} \tag{3.11}$$

In Equation (3.11), however, Q is modeled as a function of T' In order to evaluate Q, therefore, we obtain the closed-form expression for T':

$$Qe^{-pT} = DT$$

We first take the natural logarithm of both the left side and right side of the expression above

$$\ln(Qe^{-pT}) = \ln(DT)$$

$$\ln(Q) - pTln(e) = \ln(D) + \ln(T)$$

$$\ln(Q) - pT = \ln(D) + \ln(T)$$

$$\ln(Q) - \ln(D) = pT + \ln(T) \qquad (3.12)$$

We next differentiate both sides of Equation (3.12) as follows:

$$\frac{d}{dQ}[\ln(Q) - \ln(D)] = \frac{d}{dQ}[pT + lnT]$$

$$\frac{1}{Q} = pT' + \frac{T'}{T}$$

$$\frac{1}{Q} = T'(p + \frac{1}{T})$$

We may now obtain a closed-form expression for *T’* as described below:

$$T' = \frac{1}{Q(p + \frac{1}{T})}$$

Or $T' = \frac{T}{Q(pT+1)}$

We then substitute T' into Equation 3.10 as follows:

$$(c+b)T^3 - \frac{KT^3}{Q(pT+1)} - \frac{(c+b)T^3}{pT+1} + \frac{1}{p}(1-e^{-pT})HT^2 + \frac{2}{p}\frac{(e^{-pT}-1)HT^2}{(pT+1)} + \frac{DT^4H}{Q(pT+1)} = 0$$

$$\frac{DT^4H}{Q(pT+1)} - \frac{KT^3}{Q(pT+1)} = \frac{(c+b)T^3}{pT+1} - (c+b)T^3 - \frac{1}{p}(1-e^{-pT})HT^2 - \frac{2}{p}\frac{(e^{-pT}-1)HT^2}{(pT+1)} \quad \Rightarrow$$

$$\frac{1}{Q}\left[\frac{DT^4H}{(pT+1)} - \frac{KT^3}{(pT+1)}\right] = \frac{(c+b)T^3}{pT+1} - (c+b)T^3 - \frac{1}{p}(1-e^{-pT})HT^2 - \frac{2}{p}\frac{(e^{-pT}-1)HT^2}{(pT+1)}$$

Solving this expression for Q, we obtain:

$$Q = \frac{\dfrac{DT^4H - KT^3}{pT+1}}{\dfrac{(c+b)T^3}{pT+1} - (c+b)T^3 - \dfrac{1}{p}(1-e^{-pT})HT^2 - \dfrac{2}{p}\dfrac{(e^{-pT}-1)HT^2}{(pT+1)}}$$

Simplifying the expression above, we obtain:

$$Q = \frac{T^2(DT^2H - KT)}{T^2\left[(c+b)T - (c+b)T(pT+1) - \dfrac{(pT+1)}{p}(1-e^{-pT})H - \dfrac{2}{p}(e^{-pT}-1)H\right]}$$

$$= \frac{(DT^2H - KT)}{\left[(c+b)T(1-pT-1) - \dfrac{H}{p}[(pT+1)(1-e^{-pT}) - 2(e^{-pT}-1)]\right]}$$

$$= \frac{(DT^2H - KT)}{-pT^2(c+b) - \dfrac{H}{p}[(pT+1)(1-e^{-pT}) - 2(e^{-pT}-1)]}$$

$$= \frac{KT - DT^2 H}{pT^2(c+b) + \frac{H}{p}(1 - e^{-pT})(pT+3)} \tag{3.13}$$

where $T = \frac{Qp + D - \sqrt{(Qp+D)^2 - 2Q^2p^2}}{Qp^2}$

From the equation $Qe^{-pT} - DT = 0$, we also obtain the following conditions:

If $p = 0$, then $Q - DT = 0 \Rightarrow T = \frac{Q}{D}$ and we arrive at standard EOQ result without probabilistic failure. We therefore consider $p > 0$,

i.e. $\frac{1}{e^{pT}} < 1$

Then, from $DT = Qe^{-pT} < Q$, we obtain $T < \frac{Q}{D}$ for the modified case. Thus, $T_{modified\ model} < T_{standard\ model}$.

Standard model: $Q^* = \sqrt{\frac{2DK}{H}}$

$$T_{standard\ model} = \frac{Q^*}{D}$$

$$T_{modified\ model} < \frac{Q^*}{D} = T_{standard\ model}$$

The average inventory under the standard EOQ model is:

$$\frac{1}{2}Q^*$$

The average amount of inventory in stock under the modified EOQ model is:

$$Avg.Inv. = \frac{1}{T}\int_0^T (Qe^{-pt} - Dt)dt$$

$$= \tfrac{1}{T}\left| -\tfrac{1}{p}Qe^{-pt} - \tfrac{Dt^2}{2} \right|_0^T$$

$$= \tfrac{1}{T}\left[-\tfrac{1}{p}Qe^{-pT} - \tfrac{DT^2}{2} - \left(-\tfrac{Q}{p}\right) \right]$$

$$= \frac{1}{T}\left(-\frac{1}{p}Qe^{-pT} - \frac{DT^2}{2} + \frac{Q}{p}\right) \tag{3.14}$$

Q^* and T^* are optimal order quantity and cycle time as determined using the basic EOQ model. Q and T are optimal order quantity and associated cycle time determined using our modified EOQ model incorporating probabilistic failure. As the standard EOQ model does not account for item failure, optimal order quantities obtained through the use of that model will be higher compared to those provided by the modified model introduced in this dissertation when items are subject to probabilistic failure during storage. This follows the intuition of the inequality $T_{modified\,model} < T_{standard\,model}$ derived earlier in this section as shorter cycle times necessitate smaller order quantities.

The comparatively smaller optimal order quantity and a shorter cycle time obtained using our modified EOQ model support the Just-In-Time (JIT) philosophy. This philosophy centers around smaller, more frequent order quantities and enables companies to reduce inventory, minimize waste and better respond to customer demand and market conditions.

The penalty cost function for our modified EOQ model allows us to account for simultaneous changes in holding and ordering costs associated with the developed model. Given that average inventory declines along with optimal order quantity and cycle times through the adoption of our modified order policies, cost savings are achieved in our model through the reductions in holding costs. These savings can be calculated by subtracting annual holding costs under the modified EOQ from annual holding costs under the basic EOQ model. The holding cost component of the penalty cost function is calculated as follows:

$$\left[\frac{1}{(T^*)}\left(\frac{Q^*}{p}-\frac{Q^*}{p}e^{-pT^*}-\frac{DT^{*2}}{2}\right)-\frac{1}{T}\left(\frac{Q}{p}-\frac{Q}{p}e^{-pT}-\frac{DT^2}{2}\right)\right]H$$

$$=\left[\frac{Q^*}{pT^*}-\frac{Q^*}{pT^*}e^{-pT^*}-\frac{DT^*}{2}-\frac{Q}{Tp}+\frac{Q}{Tp}e^{-pT}+\frac{DT}{2}\right]H$$

$$=\left[\frac{1}{p}\left(\frac{Q^*}{T^*}-\frac{Q}{T}+\frac{Q}{T}e^{-pT}-\frac{Q^*}{T^*}e^{-pT^*}\right)+\frac{DT}{2}-\frac{DT^*}{2}\right]H \tag{3.15}$$

Similarly, smaller order quantities and shorter cycle times increase annual ordering costs, offsetting the previously described holding cost savings. We can therefore calculate the ordering cost increases that occur in our model by subtracting annual ordering costs under the modified EOQ model from annual ordering costs as determined using the basic EOQ model. The ordering cost component of the penalty cost function is calculated as follows:

$$\left(\frac{K}{T^*}+\frac{cQ^*}{T^*}+\frac{bQ^*}{T^*}\right)-\left(\frac{K}{T}+\frac{cQ}{T}+\frac{bQ}{T}\right)$$

$$= K\left(\frac{1}{T^*} - \frac{1}{T}\right) + c\left(\frac{Q^*}{T^*} - \frac{Q}{T}\right) + b\left(\frac{Q^*}{T^*} - \frac{Q}{T}\right) \tag{3.16}$$

Combining the holding and ordering cost components yields the full penalty cost function:

$$PF(Q,T) = \left[\frac{1}{p}\left(\frac{Q^*}{T^*} - \frac{Q}{T} + \frac{Q}{T}e^{-pT} - \frac{Q^*}{T^*}e^{-pT^*}\right) + \frac{DT}{2} - \frac{DT^*}{2}\right]H + K\left(\frac{1}{T^*} - \frac{1}{T}\right) + c\left(\frac{Q^*}{T^*} - \frac{Q}{T}\right) + b\left(\frac{Q^*}{T^*} - \frac{Q}{T}\right) \tag{3.17}$$

While we are able to derive the penalty cost function for our modified EOQ model, this function is problematic for practical implementation. The lack of a closed form solution for Q, as described in Section 3.3.2, yields a penalty cost function which is unwieldy due to its recursive nature. As such, it is infeasible to use Equation 3.13 to systematically determine simultaneous parameter changes in our modified EOQ model. We therefore devote Chapter 4 to numerical examples designed to test the sensitivity of our model to parameter changes.

Chapter 4

Numerical Examples and Empirical Analysis of Item Reliability

4.1 Introduction

In this chapter, we utilize equations 3.4 and 3.9 in order to obtain optimal order quantities and related yearly profits for five distinct sets of relevant parameters. We first calculate cycle lengths for a given failure rate, demand values, and a range of optimal order quantities. We then utilize the specified set of parameters (i.e. failure rate, order cost, holding cost, etc.) for each case in order to calculate total profit per year for a range of possible order quantities (*Q)* and identify the value of *Q* which yields the highest yearly profit.

Each case is accompanied by three tables: two in the body of the paper and a third included in the appendix. The first table lists the utilized parameters while the second table summarizes and contrasts the optimal order quantities and associated profits under both the model introduced in this paper as well as under the standard EOQ model. The third table illustrates how changes in order quantities affect the results under both models. Each case is accompanied by a brief discussion, with particular emphasis placed on how individual parameter changes influence profitability increases resulting from the implementation of our model. Additionally, we conduct an analysis of item reliability using failure rates for components produced for the defense industry. This analysis supports the practical necessity and applicability of the model developed in Chapters 3 and 4.

Section 4.2 derives optimal order quantities under both the basic EOQ model and the modified EOQ model which incorporates the probabilistic failure framework. Section 4.3 reports a variety of numerical sensitivity analysis designed to test the relative importance of specific parameters within the probabilistic failure framework. Section 4.4 contains our empirical item reliability analysis results, while Section 4.5 provides a summary of our numerical analysis and concludes the chapter.

4.2 Failure Rate Comparisons

We compare the results obtained using our model to results obtained via the basic EOQ model which does not incorporate the probabilistic failure framework. The optimal order quantity of 100 units obtained using the standard EOQ framework significantly exceeds the optimal order quantity of 48 obtained using our modified EOQ model. Most importantly, the use of the modified EOQ ordering quantity in the case above resulted in a more than 16% increase in annual profits. The longer-than-optimal cycle length obtained using the standard EOQ model may lead to shortage cases, impacting both current profits and, possibly, future sales.

c	Unit purchase cost	25
K	Order placing cost	100
H	Holding cost	5
s	Selling price	40
b	Inspection cost per item	2
D	Yearly demand	250
p	Item failure rate (yearly)	0.25

Table 4.1: Parameters for Case 1

	Optimal Order Quantity	Profit Per Year
Original EOQ Quantity	100	\$1,739.69
Modified EOQ Quantity (incorporating failure rate)	48	\$2,018.52

Table 4.2: Optimal order quantities and annual profit under standard and modified EOQ models for Case 1

<u>Profit increase resulting from model implementation</u>: 16.03%

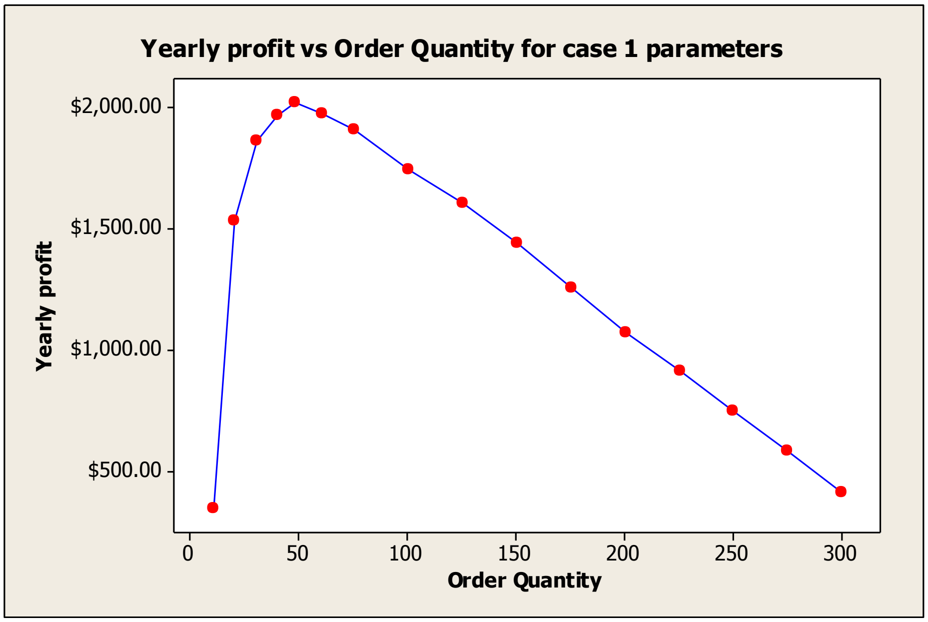

Figure 4.1: Yearly Profit vs Order Quantity for Case 1 Parameters

4.3 Sensitivity Analysis

In this section, we conduct four separate sensitivity analyses in order to test the sensitivity of our model to changes in various parameters. First, we consider changes in holding costs in Section 4.3.1. We then consider changes in order costs in Section 4.3.2. Section 4.3.2 considers changes in demand, while Section 4.3.4 examines changes in item failure rate.

4.3.1 Holding Cost

In this section, we consider the impact of doubling holding costs from $5 to $10 per unit per year. This parameter change reduced the optimal order quantity under the standard EOQ framework by 29% (from 100 to 71) while the optimal order quantity obtained using the modified EOQ framework with probabilistic failure decreased by only 4.17% (from 48 to 46). This example illustrates that the modified EOQ model with probabilistic failure is less sensitive to changes in holding cost compared to the standard EOQ model. In this case, the use of the Modified EOQ framework presented in this paper resulted in a 4.11% increase in yearly profits.

Figure 4.3 then provides the visual representation of yearly profit's sensitivity to changes in holding cost for optimal levels of order quantity.

c	Unit purchase cost	25
K	Order placing cost	100
H	Holding cost	10
s	Selling price	40
b	Inspection cost per item	2
D	Yearly demand	250
p	Item failure rate (yearly)	0.25

Table 4.3: Parameters for model with holding cost changes

	Optimal Order Quantity	Profit Per Year
Original EOQ Quantity	71	$1660.53
Modified EOQ Quantity (incorporating failure rate)	46	$1728.79

Table 4.4: Optimal order quantities and annual profit under standard and modified EOQ models with holding cost changes

<u>Profit increase resulting from model implementation</u>: 4.11%

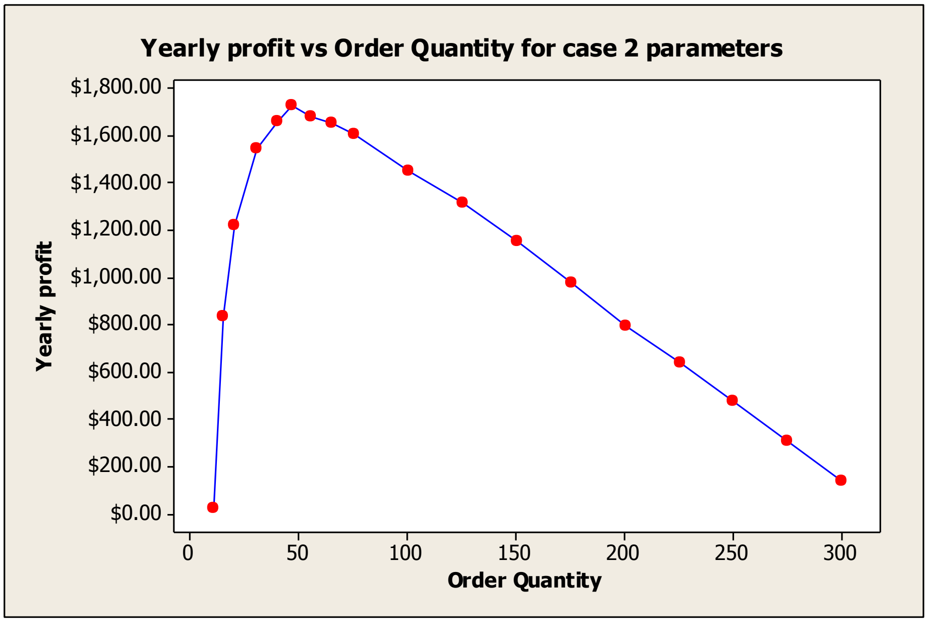

Figure 4.2: Yearly Profit vs Order Quantity for Case 2 Parameters

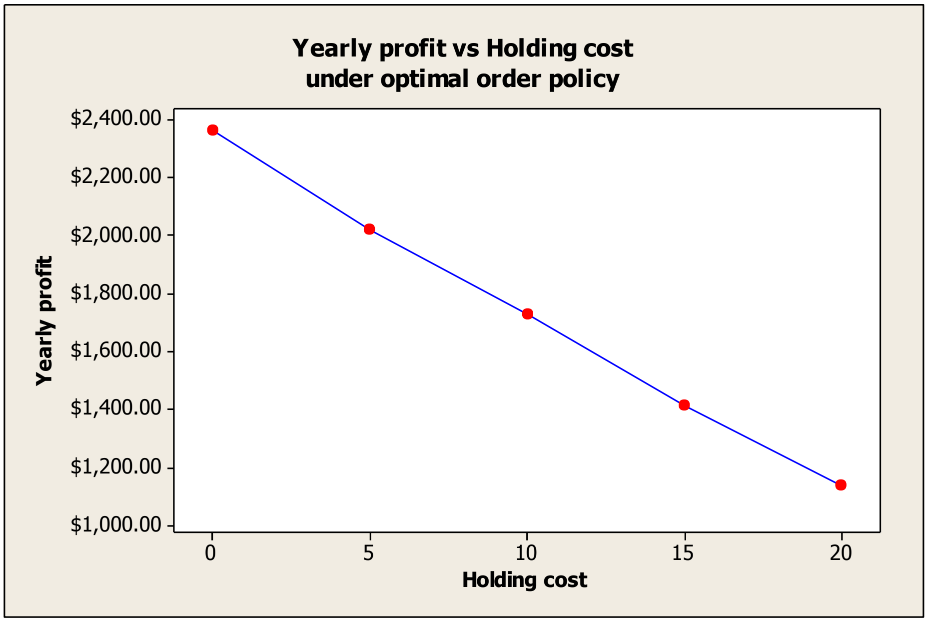

Figure 4.3: Yearly Profit vs Holding cost under Optimal Order Policy

As shown in Figure 4.3, the relationship between holding cost and yearly profit under our modified EOQ model is negative and approximately linear.

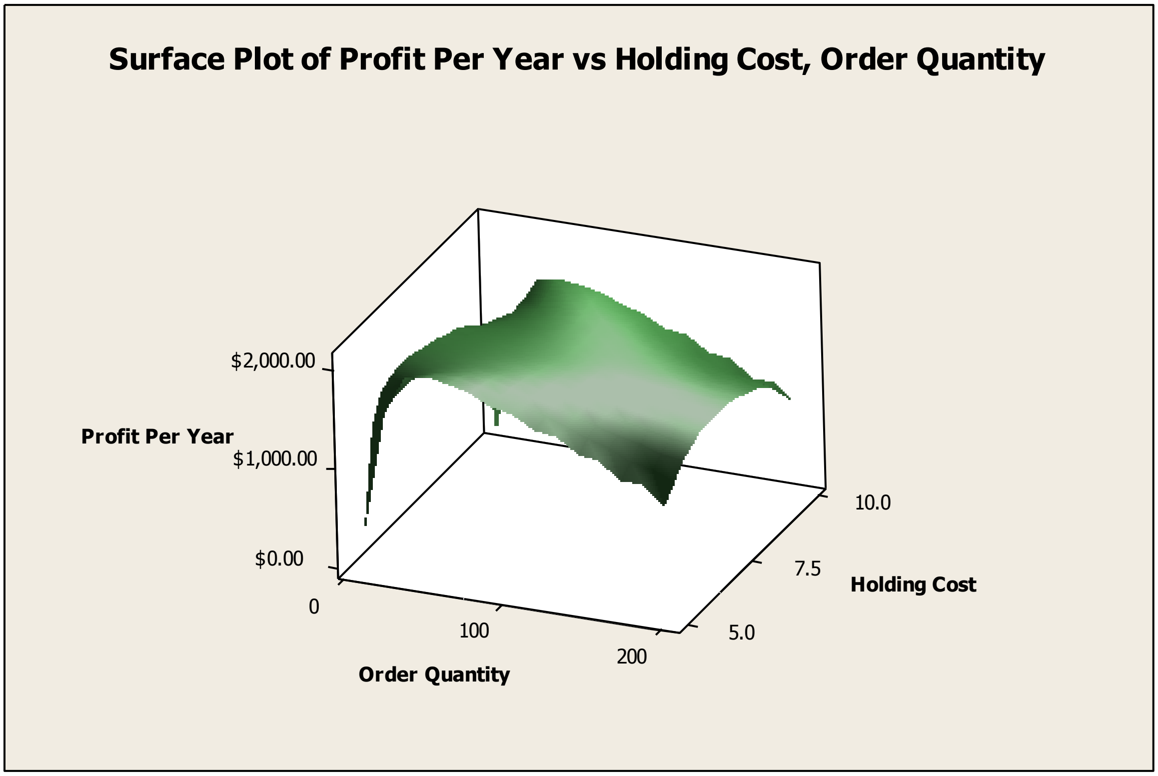

Figure 4.4: Yearly Profit vs Holding Cost and Order Quantities under Optimal Order Policies

Figure 4.4, a three-dimensional surface plot of yearly profit, holding cost and order quantity values (using table 4.3 parameters for unit purchase cost, order placing cost, holding cost, selling price, inspection cost and yearly demand), demonstrates the negative effect an increase in holding cost has on both optimal order quantities and yearly profit.

4.3.2 Order Cost

In this section, we consider the impact of doubling order placing costs from $100 to $200. These higher order placing costs resulted in a 48% increase (from 48 to 71) in the optimal order quantity under the modified EOQ model with probabilistic failure and a 42% increase in the optimal order quantity for the standard EOQ framework. Use of our modified EOQ model resulted in a 21.8% increase in yearly profits over the standard EOQ quantity.

Figure 4.5 then provides the visual representation of yearly profit's sensitivity to changes in order placing cost for optimal levels of order quantity. As was the case with holding cost, there is a negative linear relationship between order placing cost and yearly profits. Whereas a doubling of holding costs from 10 to 20 led to a reduction in yearly profit of approximately 33%, a similar doubling in order placing cost from 50 to 100 reduced profits by approximately 19%. As intuition suggests, given the loss of inventory value during storage, yearly profit is shown to be more sensitive to holding cost than order placing cost.

c	Unit purchase cost	25
K	Order placing cost	200
H	Holding cost	5
s	Selling price	40
b	Inspection cost per item	2
D	Yearly demand	250
p	Item failure rate (yearly)	0.25

Table 4.5: Parameters for model with order placing cost changes

	Optimal Order Quantity	Profit Per Year
Original EOQ Quantity	142	\$1,308.36
Modified EOQ Quantity (incorporating failure rate)	71	\$1,594.18

Table 4.6: Optimal order quantities and annual profit under standard and modified EOQ models for Case 3

<u>Profit increase resulting from model implementation</u>: 21.85%

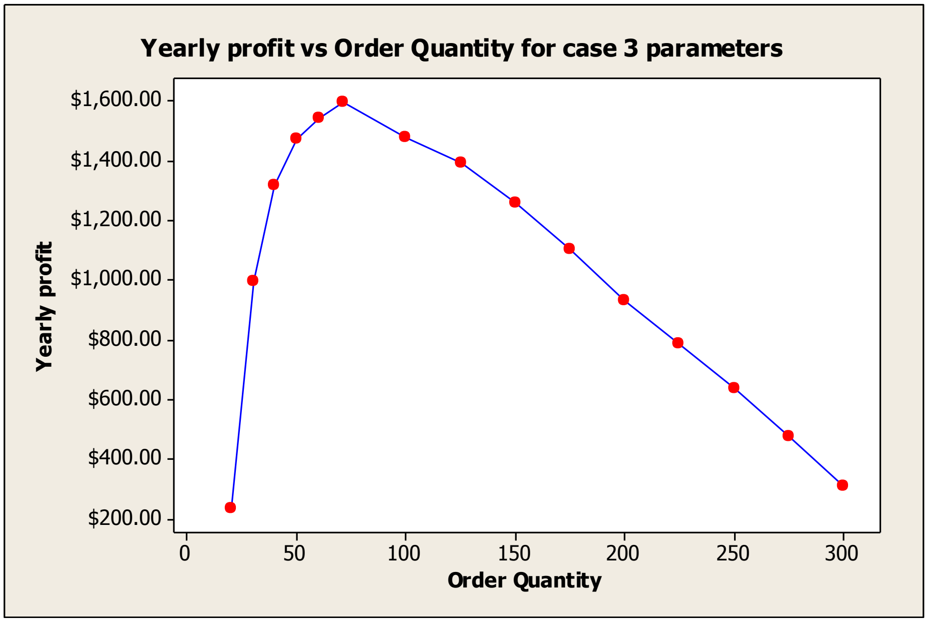

Figure 4.5: Yearly Profit vs Order Quantity for Case 3 Parameters

As shown in Figure 4.5, the relationship between order placing cost and yearly profit under our modified EOQ model is negative and close to linear, although the negative relationship is not as strong as the negative relationship between the yearly profit and holding cost we observed in Figure 4.3.

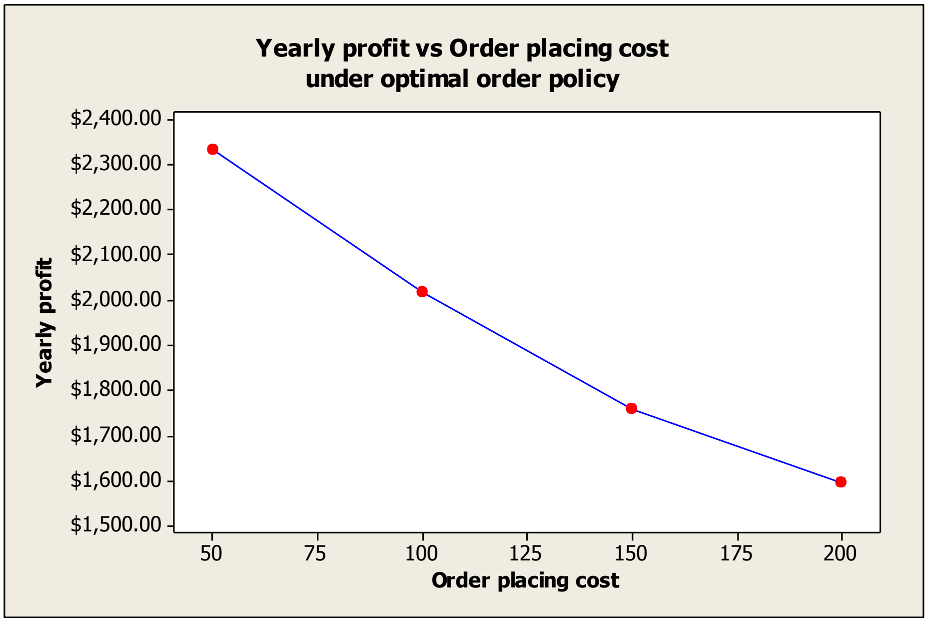

Figure 4.6: Yearly Profit vs Order placing cost under Optimal Order Policy

Figure 4.7, a three-dimensional surface plot of yearly profit, order cost and order quantity values (using table 4.5 parameters for unit purchase cost, order placing cost, holding cost, selling price, inspection cost and yearly demand), demonstrates the strong negative effect an increase in order cost has on both optimal order quantities and yearly profit.

4.3.3 Demand and Holding Costs

A simultaneous increase in both yearly demand and holding cost by a factor of 2 resulted in a 50% (48 to 72) increase in optimal order quantity for the modified EOQ model with probabilistic failure. Optimal order quantity for the standard EOQ model remained unchanged, as proportional changes in demand and holding cost do not change the optimal order quantity obtained using that framework. The use of the optimal order quantity suggested in our modified EOQ model resulted in a 5.67% increase in yearly profits over the standard EOQ quantity.

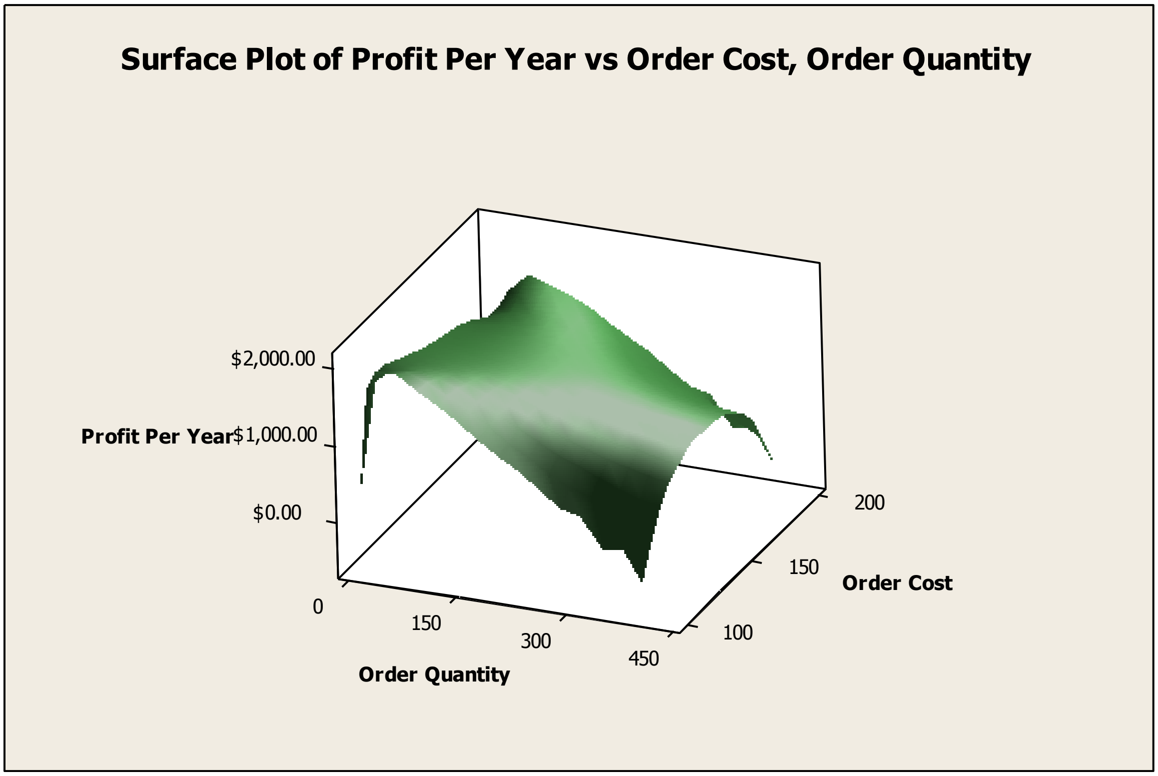

Figure 4.7: Yearly Profit vs Order Cost and Order Quantities under Optimal Order Policies

c	Unit purchase cost	25
K	Order placing cost	100
H	Holding cost	10
s	Selling price	40
b	Inspection cost per item	2
D	Yearly demand	500
p	Item failure rate (yearly)	0.25

Table 4.7: Parameters for model with demand and holding cost changes

	Optimal Order Quantity	Profit Per Year
Original EOQ Quantity	100	\$3.890.49
Modified EOQ Quantity (incorporating failure rate)	72	\$4,111.25

Table 4.8: Optimal order quantities and annual profit under standard and modified EOQ models for Case 4

Profit increase resulting from model implementation: 5.67%

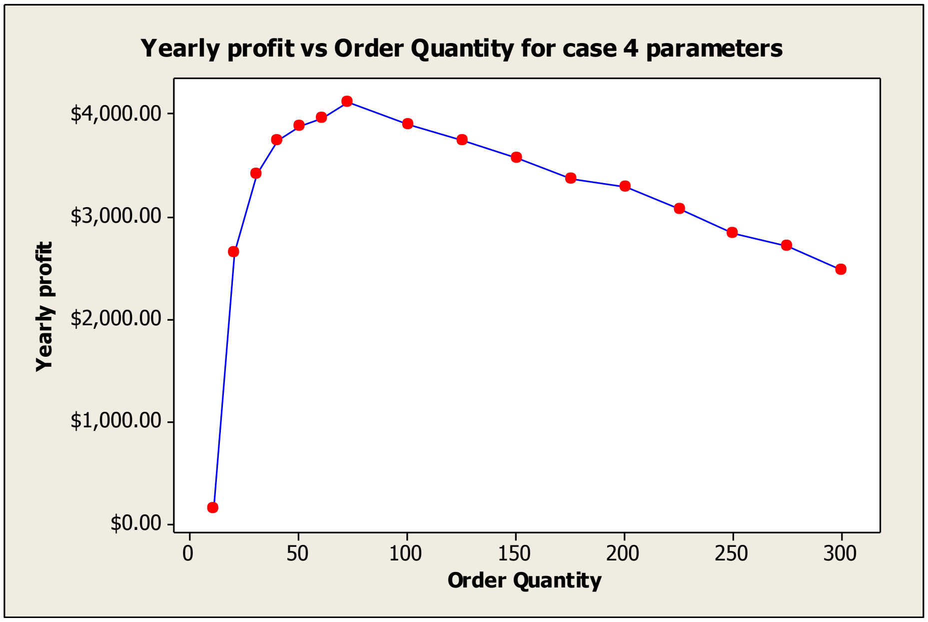

Figure 4.8: Yearly Profit vs Order Quantity for Case 4 Parameters

4.3.4 Failure Rate

Increasing the item failure rate from .25 to .50 resulted in a 31% decrease (from 48 to 33) in optimal order quantity under the modified EOQ model with probabilistic failure framework. Our model increases profitability by more than 155% under this scenario. Clearly, the benefit of using a model which makes adjustments related to item failure rises dramatically as item failure rates increase.

c	Unit purchase cost	25
K	Order placing cost	100
H	Holding cost	5
s	Selling price	40
b	Inspection cost per item	2
D	Yearly demand	250
p	Item failure rate (yearly)	0.50

Table 4.9: Parameters for Model with Increased Failure Rate

	Optimal Order Quantity	Profit Per Year
Original EOQ Quantity	100	$569.30
Modified EOQ Quantity (incorporating failure rate)	33	$1,452.95

Table 4.10: Optimal order quantities and annual profit under standard and modified EOQ models with Increased Failure Rate

<u>Profit increase resulting from model implementation</u>: 155.22%

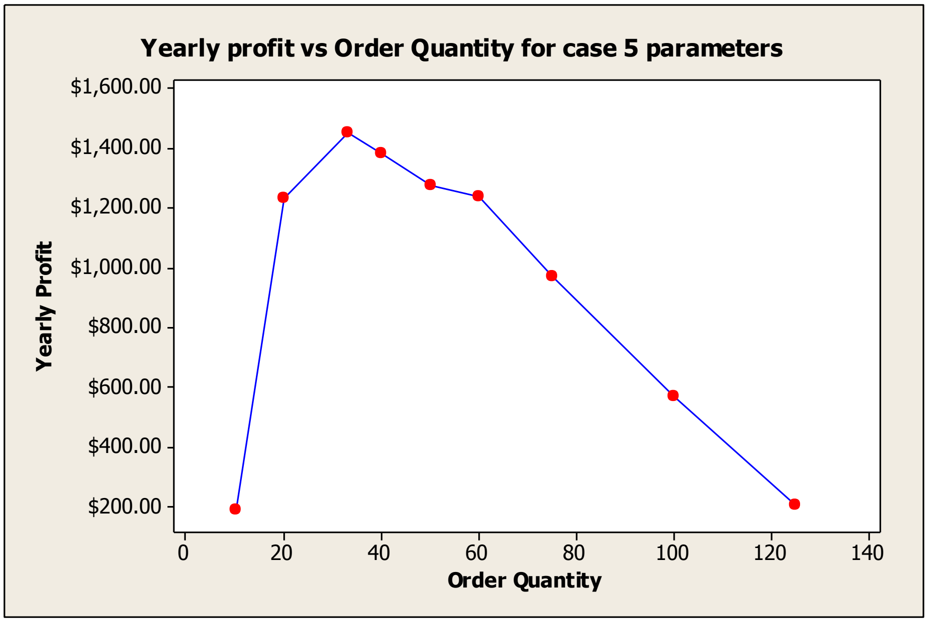

Figure 4.9: Yearly Profit vs Order Quantity for Case 5 Parameters

Figure 4.10, a three-dimensional surface plot of yearly profit, failure rate and order quantity values (using table 4.9 parameters for unit purchase cost, order placing cost, holding cost, selling price, inspection cost and yearly demand), highlights a strong negative effect an increase in the probabilistic failure rate has on both optimal order quantities and yearly profit.

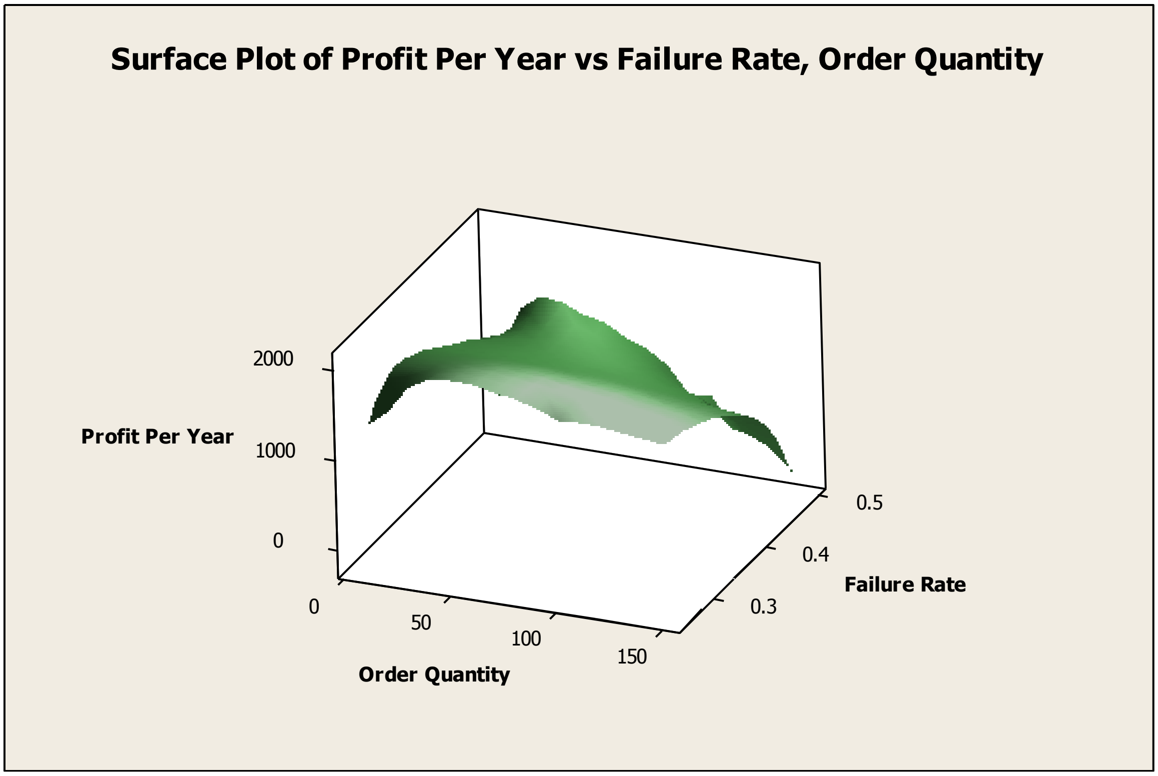

Figure 4.10: Yearly Profit vs Failure Rate and Order Quantities under Optimal Order Policies

4.4 Empirical Analysis of Item Reliability

4.4.1 Data

Data for this empirical analysis is taken from two sources. The first source is the *Nonoperating Reliability Databook*, a compilation of component testing and failure rate data prepared by the Department of Defense Information Analysis Center. This data includes testing and failure rate data for a variety of electronic and non-electronic components commonly used in military aircraft and Naval ships, including a variety of resistors, microcircuits, switches, tubes, and relays. The data contained in this document was derived from military and commercial equipment used in non-operating field or storage environments. The second source of item testing data is the *Military Handbook: Reliability Prediction of Electronic Equipment.* This handbook includes testing and failure rate data for electronic components used within ballistic missile systems among other data. The long storage periods for ballistic missiles makes them particularly useful for examining failure rates in storage.

4.4.2 Methodology

Failure rates data are summarized in tables as mean point estimates expressed in failures per million non-operating hours. The annual failure rate, which is discussed at length in Section 3.2.2 and indicated in our model as the parameter p, is obtained by first dividing the cumulative number of failures by the total part hours, where part hours are presented in millions of hours. Given that one million hours is the equivalent of 114.08 years, the annual failure rate (p) is then obtained by dividing the failure rate per million hours by 114.08.

Analysis of annual failure rates for five components is presented in Section 5.4. These components were selected to provide a cross-section of electronic and non-electronic components for analysis. The first component selected is a general motor generator set, a device which converts low voltage current to high voltage current. The second component selected is a vacuum tube, a device which controls electric current through a vacuum in a sealed container. The third component selected, a turbine generator, is a device which creates electrical current from mechanical energy obtained from wind, water, or steam (among other sources) in Naval settings. The fourth component, a hydraulic fluid piston, is a device which acts as a hydraulic pump for ballistic missiles. The fifth component, an accumulator diaphragm, is a device which utilizes compressibility of a gas to store hydraulic energy for use within a ballistic missile. The hydraulic fluid piston and accumulator diaphragm are of particular applicability to our model given their use within ballistic missiles, a deterrent weapon which is generally subject to very long storage times. The Minuteman III ICBM System, for example, had been in place for 40 years before replacement plans were developed in 2013 (Vanderschuere 2013). While other components presented within the data set for ballistic missile systems may have failure rates that are much lower than those of the two selected components, the unusually long storage period for these missiles supports the applicability of our model to such components.

4.4.3 Results

Table 4.11 presents annual failure rate calculations for each of the five components used in this analysis. Cumulative part hours (in millions of hours), total failures, failure rate per million hours, and annual failure rate for each component are calculated and included in this table. These empirical results all provide support for our use of a failure rate to model probabilistic item failure during storage and in non-operational stages of use. The selected components are exemplars of military components which fail at rates similar to those utilized within the numerical examples in this chapter.

Component Name	**Cumulative Part Hours (in millions)**	**Number Failed**	**Failure Rate per Million Hours**	**Annual Failure Rate (*p*)**
General Motor Generator Set	0.499	28	56.1122	0.492
Vacuum Tube	1.427	14	9.81108	0.086
Turbine Generator	0.078	3	38.4615	0.337
Hydraulic Fluid Piston	0.149	1	57.077	0.5
Accumulator Diaphragm	0.526	13	24.733	0.217

Table 4.11: Failure Rate Data for Selected Components

4.5 Concluding Remarks

In Chapters 3 and 4, we develop an Economic Order Quantity (EOQ) model for items that are subject to probabilistic failure while in storage despite having been of perfect quality at the time of delivery. The model presented in this paper is applicable to systems with large fixed order costs and lengthy inventory holding periods. Examples of such systems include the medical and national security industries, neither of which accepts defective items for delivery. Our modified EOQ model illustrates that investment in system process improvement (to reduce fixed ordering costs) generates more benefits than investment aimed at minimizing holding costs for such systems. We contribute to the literature on EOQ models for items of imperfect quality by modeling items experiencing probabilistic, rather than deterministic, failure during storage period. Additionally, our modeling of items which experience failure without possibility of rework or salvage represents an additional contribution of our model.

Chapter 5

INVENTORY POLICIES FOR AN ECONOMIC PRODUCTION QUANTITY MODEL WITH ITEM FAILURE IN STORAGE

5.1 Introduction

In this chapter, we develop an economic production quantity (EPQ) model for items experiencing failure in storage according to a probabilistic failure rate. This model extends the EOQ model developed in Chapters 3 and 4 by considering cases where companies choose to produce such items in-house rather than purchase them through an outside supplier.

As such, the developed EPQ model makes use of the same exponential failure function as was utilized by the EOQ model developed in the previous chapters of this dissertation.

The EPQ model was developed as an extension of the EOQ model with the similar goal of determining optimal inventory policies to minimize total inventory costs. The EPQ model, however, assumes that the company will either produce their own items or will receive shipments of the items from an external supplier during, rather than at the end of, the production cycle. This difference in model assumptions has several implications which significantly impact optimal inventory policies. First, setup costs are often considered in place of, or in addition to, fixed ordering costs since most EPQ models are used to model internal production policies. Also, items from single product lots are assumed to be delivered incrementally during the production run rather than in complete lots at the end of the run. Thus, the maximum level of inventory is held at some point during the inventory cycle rather than at the beginning of the cycle, with inventory increasing during production and depleting to the end of the period through sales and, in our model, failure.

While the EOQ model developed in Chapters 3 and 4 is useful in considering optimal order policies for buyers of items which experience probabilistic failure in storage, the types of items considered are often produced in-house rather than from third parties. Components for defense products, such as those examined in Chapter 4, are often required by law to be produced internally rather than through an extensive supply chain in order to maintain control over proprietary technology and maintain national security. In countries such as India, China, and Russia, for example, large state-owned companies produce defense-related components directly for the government (Bitzinger 2009). Similarly, many computer and electronics companies choose to produce components rather than outsource them in order to improve efficiency and reduce quality deficiencies. Lenovo, for example, has developed a strong source of competitive advantage through its decision to maintain in-house production of its computer components and complete systems (Chao 2012). Manufacturers have an incentive to produce components in-house when such components are to be used in larger systems rather than sold independently. Given many such users for the types of items considered in the EOQ model previously developed in this dissertation, we also consider an EPQ model to determine optimal production policies for similar items when they are produced in-house.

Additionally, the following chapter provides insight into mechanisms for achieving supply chain coordination for items which experience probabilistic failure during storage. The models developed in this dissertation highlight the potential benefits to be achieved from adopting a cooperative solution to inventory management for such items given the risk of inventory losses during storage. To make coordination practicable, a coordination framework must be developed which specifies mutually beneficial methods of cooperation. In order to support the model developed in this paper, we suggest and analyse methodological approaches to supply chain coordination between a manufacturer and buyers which result in total system cost minimization for both the production and distribution processes. It is noteworthy that this type of cooperation need not be limited to a supplier and their external buyers, but also extends to suppliers and internal buyers such as manufacturing cost centers providing items for use solely within the intra-organizational supply chain.

The remainder of this chapter is organized as follows: Section 5.2 provides an overview of the model, including notation and base equations. Section 5.3 describes the optimization process for the developed model. Chapter 6 then provides insight into coordination mechanisms which may be used to induce mutually beneficial cooperation between suppliers and buyers in both inter- and intra-organizational contexts.

5.2 Model

To develop the model, we make the following assumptions: (1) the demand rate, setup/order cost and inventory holding costs are known and deterministic, (2) production of items is continuous and at a constant rate during the production run, (3) inventory is manufactured incrementally during the production period, with maximum inventory levels achieved at the end of the production period, (4) a 100 % screening is performed when the lot is delivered to separate the defective items, which are to be replaced at supplier's cost, and (5) lots have a fixed rate of failure with known probability density function.

The following notation is adopted:

Q	order quantity
Q^*	optimal order quantity
c	unit variable production cost
K	setup cost per setup
H	holding cost per unit per year
s	selling price per unit
T	cycle length
D	yearly demand
$S(t)$	number of items in stock at time t
p	item failure rate
G	yearly production rate, $G > D$

The basic EPQ model is derived from a total cost function which includes all production, setup, and holding costs for an order as described in Equation (5.1):

$$Total\ Costs = cD + \frac{DK}{Q} + \frac{HQ}{2}(1 - \frac{D}{G}) \qquad (5.1)$$

Total cost minimization is achieved through differentiation, with the optimal production quantity calculated as the derivative of the total cost with respect to Q. Equation (5.2) provides the optimal order quantity obtained by deriving the first order optimality condition for Equation (5.1), hereafter identified as Q*:

$$Q^* = \sqrt{\frac{2DK}{H(1-\frac{D}{G})}} \qquad (5.2)$$

Figure 5.1 provides a graphical representation of inventory flow under the EPQ model, with the areas of triangles (i) and (ii) indicating inventory levels at all times during the inventory cycle. Maximum inventory is calculated as a function of both production and demand, reflecting the fact that inventory is depleted during production. The area of the triangle labeled (i) is calculated as $\frac{1}{2}Q\left(1 - \frac{D}{G}\right)t_1$ and maximum inventory (at time t$_1$) is calculated as $(1 - \frac{D}{G})Q$. Calculating the area of the triangle labeled (ii) requires that we account for probabilistic failure of item inventory following the end of the production period. As in our previous model from

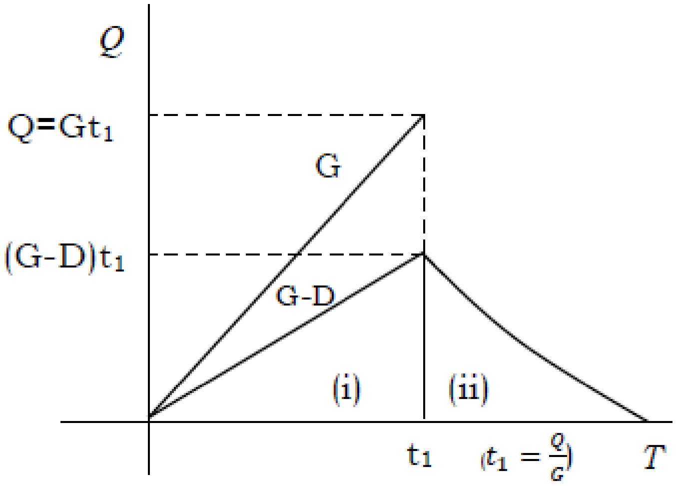

Figure 5.1 – Inventory Level of Cycle Time

Chapters 3 and 4, we utilize the following exponential failure rate to model probabilistic failure for our items:

$$Qe^{-pT} - DT$$

Thus, we use the following equations to calculate the area of the triangle in Figure 5.1 labeled (ii):

$$Area\ (ii) = \int_{t_1}^{T} \left[\left(1 - \frac{D}{G}\right) Qe^{-pt} - Dt\right] dt$$

Average inventory is calculated by dividing the area under the total inventory curve by total cycle length (T):

$$Avg.Inventory\ = \frac{\frac{1}{2}\left(1 - \frac{D}{G}\right) Qt_1 + \int_{t_1}^{T} \left[\left(1 - \frac{D}{G}\right) Qe^{-pt} - Dt\right] dt}{T}$$

In our EPQ model developed in this chapter, items are not moved into storage until the end of the production run despite being used to meet demand during production. As item failure occurs during storage rather than during production, item failure does not impact inventory levels until the production period ends. In the graph displayed in Figure 5.1, the production period runs from the start of the period (i.e. the point of origin for the graph) and through time t_1. The post-production period runs from time t_1 through Time T. The effects of failure can be seen in inventory levels beginning at time t_1, with the slope of the inventory line shifting from linear to curvilinear as inventory depletes faster once items are placed into storage.

5.3 Optimization and Results

5.3.1 Optimal Order Quantity

We can find the order quantity Q which minimizes the total cost function by setting the derivative $\frac{dTCY}{dQ}$ equal to 0 and solving for Q. The total cost function is set equal to 0 as follows:

$$\frac{dTCY}{dQ} = -\frac{D}{Q^2}K + \frac{d}{dQ}[Avg. Inventory\ ]H = 0 \qquad (5.3)$$

In order to derive the optimal value for Q, we first evaluate the equation for average inventory for insertion into Equation (5.1).

$$Avg.Inv. = \frac{\frac{1}{2}\left(1-\frac{D}{G}\right)Qt_1 + \int_{t_1}^{T}\left[\left(1-\frac{D}{G}\right)Qe^{-pt} - Dt\right]dt}{T}$$

$$= \frac{1}{T}\left\{\frac{1}{2}\left(1-\frac{D}{G}\right)Qt_1 + \left[\left(1-\frac{D}{G}\right)Q\left(-\frac{1}{p}\right)e^{-pt} - \frac{Dt^2}{2}\right]_{t_1}^{T}\right\}$$

$$= \frac{1}{T}\left\{\frac{1}{2}\left(1-\frac{D}{G}\right)Qt_1 + \left(1-\frac{D}{G}\right)Q\left(-\frac{1}{p}\right)e^{-pT} - \frac{DT^2}{2}\right.$$

$$\left. - \left[\left(1-\frac{D}{G}\right)Q\left(-\frac{1}{p}\right)e^{-pt_1} - \frac{Dt_1{}^2}{2}\right]\right\}$$

$$= \frac{1}{T}\left\{\frac{1}{2}\left(1-\frac{D}{G}\right)Qt_1 + \left[-\left(1-\frac{D}{G}\right)\frac{Q}{p}e^{-pT} - \frac{DT^2}{2} + \left(1-\frac{D}{G}\right)\frac{Q}{p}e^{-pt_1} + \frac{Dt_1{}^2}{2}\right]\right\}$$

$$= \frac{1}{T}\left\{\frac{1}{2}\left(1-\frac{D}{G}\right)Qt_1 + \left(1-\frac{D}{G}\right)\frac{Q}{p}\left(e^{-pt_1} - e^{-pT}\right) - \frac{D}{2}\left(T^2 - t_1{}^2\right)\right\},$$

where $t_1 = \frac{Q}{G}$ (5.4)

5.3.2 Optimal Cycle Time using MacLaurin Series

We first calculate the cycle time for the period from t_1 through T, or the storage period during which failure occurs. We then use that value to calculate total cycle time including both the storage and production periods. As was previously described in Section 3.3.2, we utilize a MacLaurin Series approximation to determine optimal cycle length due to the difficulty involved in obtaining a closed form solution for the cycle time during the storage period of our EPQ model.

Here, our cycle time is expressed as $t = T - t_1, t_1 = \frac{Q}{G}$. We use $\frac{Q}{G}$ to identify t₁ as the total quantity produced that is equal to the production rate multiplied by the length of the production period ($Q = G * t_1$).

$$\left(1 - \frac{D}{G}\right) Qe^{-pt} = Dt$$

$$Dt \approx \left(1 - \frac{D}{G}\right) Q \left(1 - pt + \frac{1}{2}p^2t^2\right)$$

$$\frac{1}{2}\left(1 - \frac{D}{G}\right) Qp^2t^2 - \left(1 - \frac{D}{G}\right) Qpt - Dt + \left(1 - \frac{D}{G}\right) Q = 0$$

$$t = \frac{\left(1 - \frac{D}{G}\right) Qp + D - \sqrt{\left[\left(1 - \frac{D}{G}\right) Qp + D\right]^2 - 4 \times \frac{1}{2}\left(1 - \frac{D}{G}\right) Qp^2 \left(1 - \frac{D}{G}\right) Q}}{2 \times \frac{1}{2}\left(1 - \frac{D}{G}\right) Qp^2}$$

$$t = T - t_1 = T - \frac{Q}{G} = \frac{\left(1 - \frac{D}{G}\right) Qp + D - \sqrt{\left[\left(1 - \frac{D}{G}\right) Qp + D\right]^2 - 2\left(1 - \frac{D}{G}\right)^2 Q^2p^2}}{\left(1 - \frac{D}{G}\right) Qp^2}$$

In this section, we obtain the optimal production quantity for our EPQ model for items which experience probabilistic failure during storage. We first calculate our optimal production run length t, we then use the equation $t = T - t_1$ to calculate our total cycle length T. This result allows us to determine the optimal production quantity, thereby allowing us to obtain optimal production policies which maximize annual profits.

Chapter 6

SUPPLY CHAIN COORDINATION MECHANISMS

6.1 Coordinated Solution

We next develop an optimal policy mechanism for use in conjunction with the Economic Production Quantity model developed earlier in this chapter. This mechanism will allow the manufacturer to coordinate a supply chain consisting of n buyers in order to achieve a common replenishment time. We denote the manufacturer's production quantity for the coordinated case as Q_K (where Q_K is a positive integer multiple of the manufacturer's lot size).

We obtain the expected level of inventory for each buyer i using each buyer's inventory level expressed as follows:

$$Q_i e^{-pT_i} = D_i T_i \;\Rightarrow\; Q_i = D_i T_i e^{pT_i}$$

The average inventory level for a single buyer is expressed in Equation (3.14). In order to obtain average inventory levels for each buyer and derive optimal policies for the coordinated case, we substitute individual values for $Q_i$into Equation (3.14).

$$Average\ Inventory = \frac{1}{T_i}\left(-\frac{1}{p}D_iT_i - \frac{D_iT_i^2}{2} + \frac{Q_i}{p}\right)$$

$$= \frac{1}{T_i}\left(-\frac{1}{p}D_iT_i - \frac{D_iT_i^2}{2} + \frac{D_iT_ie^{pT_i}}{p}\right)$$

The total cost function for each of i buyers (hereafter denoted as TCB_i) is comprised of holding and ordering costs.

$$TCB_i = \frac{K_{bi}}{T_i} + \frac{H_{bi}}{T_i}\left(-\frac{1}{p}D_iT_i - \frac{D_iT_i^2}{2} + \frac{D_iT_ie^{pT_i}}{p}\right)$$

Simplifying the expression above, we obtain:

$$TCB_i = \frac{K_{bi}}{T_i} - \frac{H_{bi}D_iT_i}{pT_i} - \frac{H_{bi}}{T_i}\frac{D_iT_i^2}{2} + \frac{H_{bi}}{T_i}\frac{D_iT_ie^{pT_i}}{p}$$

$$= \frac{K_{bi}}{T_i} - \frac{H_{bi}D_i}{p} - \frac{H_{bi}D_iT_i}{2} + \frac{H_{bi}D_ie^{pT_i}}{p}$$

We adopt the following additional notation:

A_i	Buyer i’s sum of holding and ordering costs before cooperation
T	Common order replenishment time

β_m Average inventory factor for the manufacturer, denoted as

$$\beta_m = (Q_K - 1) - (Q_K - 2)\frac{D}{G}$$

$$TCM(Q_K, T) = \frac{K_m}{Q_K T} + \frac{C_c}{T} + \frac{1}{2}H_m D T \beta_m + \sum_{i=1}^{n}\left\{\left(\frac{K_{bi}}{T} - \frac{H_{bi}D_i}{p} - \frac{H_{bi}D_i T}{2} + \frac{H_{bi}D_i e^{pT}}{p}\right) - A_i\right\}$$

(6.1)

$$\frac{\partial TCM}{\partial T} = -\frac{K_m}{Q_K T^2} - \frac{C_c}{T^2} + \frac{1}{2}H_m D \beta_m + \sum_{i=1}^{n}\frac{\partial}{\partial T}\left\{\left(\frac{K_{bi}}{T} - \frac{H_{bi}D_i}{p} - \frac{H_{bi}D_i T}{2} + \frac{H_{bi}D_i e^{pT}}{p}\right) - A_i\right\}$$

$$\frac{\partial TCM}{\partial T} = -\frac{K_m}{Q_K T^2} - \frac{C_c}{T^2} + \frac{1}{2}H_m D \beta_m + \sum_{i=1}^{n}\left(-\frac{K_{bi}}{T^2} - \frac{H_{bi}D_i}{2} + \frac{H_{bi}D_i p e^{pT}}{p}\right)$$

Setting $\frac{\partial TCM}{\partial T}$ equal to 0, we obtain the following expression:

$$-\frac{K_m}{Q_K T^2} - \frac{C_c}{T^2} + \frac{1}{2}H_m D \beta_m - \frac{1}{T^2}\sum_{i=1}^{n} K_{bi} - \frac{1}{2}\sum_{i=1}^{n} H_{bi}D_i + \sum_{i=1}^{n} H_{bi}D_i e^{pT} = 0$$

We next obtain a value for T by rearranging the terms of the previous expression to isolate all terms containing T^2 and solving first for T^2 and then T.

$$\frac{1}{2}H_m D \beta_m - \frac{1}{2}\sum_{i=1}^{n} H_{bi}D_i + \sum_{i=1}^{n} H_{bi}D_i e^{pT} = \frac{K_m}{Q_K T^2} + \frac{C_c}{T^2} + \frac{1}{T^2}\sum_{i=1}^{n} K_{bi}$$

$$T^2\left(\frac{1}{2}H_m D \beta_m - \frac{1}{2}\sum_{i=1}^{n} H_{bi}D_i + \sum_{i=1}^{n} H_{bi}D_i e^{pT}\right) = \frac{K_m}{Q_K} + C_c + \sum_{i=1}^{n} K_{bi}$$

$$T^2 = \frac{\frac{K_m}{Q_K} + C_c + \sum_{i=1}^{n} K_{bi}}{\frac{1}{2}H_m D\beta_m - \frac{1}{2}\sum_{i=1}^{n} H_{bi}D_i + \sum_{i=1}^{n} H_{bi}D_i e^{pT}}$$

$$T = \sqrt{\frac{\frac{K_m}{Q_K} + C_c + \sum_{i=1}^{n} K_{bi}}{\frac{1}{2}H_m D\beta_m - \frac{1}{2}\sum_{i=1}^{n} H_{bi}D_i + \sum_{i=1}^{n} H_{bi}D_i e^{pT}}}$$

For values of p and T ranging from $0 < p < 1$ and $0 < T < 1 \Rightarrow 1 < e^{pT} < e$. We select these bounds of interest due to the unlikelihood that the common replenishment time will be above one year within most practical contexts. Most organizations engage in budget planning, including product and raw materials ordering, on an annual basis rather than over longer periods of time due to the difficulties involved in long-range forecasting. Even in the unusual case where an organization has replenishment times which extend over several years, it would be unrealistic to coordinate such purchasing activity with other buyers. Bounding T at one year, therefore, reflects the relative rarity of organizations adopting a multi-year replenishment schedule, especially in environments involving supply chain coordination.

Using the derived values for T^2, the practical ranges of T and p values expressed in terms of their relationship to T^2 are as follows:

$$\frac{\frac{K_m}{Q_K} + C_c + \sum_{i=1}^{n} K_{bi}}{\frac{1}{2}H_m D\beta_m - \frac{1}{2}\sum_{i=1}^{n} H_{bi}D_i + \sum_{i=1}^{n} H_{bi}D_i e} < T^2 < \frac{\frac{K_m}{Q_K} + C_c + \sum_{i=1}^{n} K_{bi}}{\frac{1}{2}H_m D\beta_m - \frac{1}{2}\sum_{i=1}^{n} H_{bi}D_i + \sum_{i=1}^{n} H_{bi}D_i 1}$$

Note that we obtain the expression above by substituting e instead of e^{pT} on the left side of the interval and 1 instead of e^{pT} on the right side of the interval. Similarly, the practical ranges of T and p values expressed in terms of their relationship to T are as follows:

$$\sqrt{\frac{\frac{K_m}{Q_K}+C_c+\sum_{i=1}^{n}K_{bi}}{\frac{1}{2}H_m D\beta_m-\frac{1}{2}\sum_{i=1}^{n}H_{bi}D_i+e\sum_{i=1}^{n}H_{bi}D_i}} < T < \sqrt{\frac{\frac{K_m}{Q_K}+C_c+\sum_{i=1}^{n}K_{bi}}{\frac{1}{2}H_m D\beta_m-\frac{1}{2}\sum_{i=1}^{n}H_{bi}D_i+\sum_{i=1}^{n}H_{bi}D_i}}$$

Combining like terms in the denominators above, we obtain the simplified interval below:

$$\sqrt{\frac{\frac{K_m}{Q_K}+C_c+\sum_{i=1}^{n}K_{bi}}{\frac{1}{2}H_m D\beta_m+\left(e-\frac{1}{2}\right)\sum_{i=1}^{n}H_{bi}D_i}} < T < \sqrt{\frac{\frac{K_m}{Q_K}+C_c+\sum_{i=1}^{n}K_{bi}}{\frac{1}{2}H_m D\beta_m+\frac{1}{2}\sum_{i=1}^{n}H_{bi}D_i}}$$

Using these ranges, we can obtain values of T using iterative procedures which utilize the bisection method based on the intermediate value theorem.[1] The detailed application of this bisection method algorithm is shown in Appendix B of this dissertation. This method allows us to narrow the interval and obtain an approximate value of T based on the values of relevant problem parameters.

Taking the second derivative of Equation (6.1) with respect to T, while keeping Q_K fixed, we obtain the following expression:

$$\frac{\partial^2 TCM(Q_K,T)}{\partial T^2}=\frac{2K_m}{Q_K T^3}+\frac{2C_c}{T^3}+\sum_{i=1}^{n}\left(\frac{2K_{bi}}{T^3}\right)+\sum_{i=1}^{n}H_{bi}D_i p e^{pT}>0$$

---

[1] See http://www.sosmath.com/calculus/limcon/limcon07/limcon07.html for a description of this method.

Similarly, we can demonstrate that the equation (3.15) is also convex with respect to Q_K:

$$\beta_m = (Q_K - 1) - (Q_K - 2)\frac{D}{G}$$

$$= Q_K - 1 - Q_K\frac{D}{G} + 2\frac{D}{G}$$

$$= Q_K\left(1 - \frac{D}{G}\right) + \left(2\frac{D}{G} - 1\right)$$

We can now substitute this expression for β_m into the equation (6.1):

$$TCM(Q_K, T) = \frac{K_m}{Q_K T} + \frac{C_c}{T} + \frac{1}{2}H_m DT\left[Q_K\left(1 - \frac{D}{G}\right) + \left(2\frac{D}{G} - 1\right)\right]$$

$$+ \sum_{i=1}^{n}\left\{\left(\frac{K_{bi}}{T} - \frac{H_{bi}D_i}{p} - \frac{H_{bi}D_i T}{2} + \frac{H_{bi}D_i e^{pT}}{p}\right) - A_i\right\}$$

$$\frac{\partial TCM}{\partial Q_K} = -\frac{K_m}{Q_K^2 T} + \frac{1}{2}H_m DT\left(1 - \frac{D}{G}\right)$$

$$\frac{\partial^2 TCM(Q_K, T)}{\partial Q_K^2} = \left(-\frac{K_m}{T}\right)(-2)Q_K^{-3} = \frac{2K_m}{TQ_K^3} > 0$$

Thus, we determine that Equation (3.15) is convex with respect to both T and Q_K for all values of $T > 0$, thereby showing that our optimal value for the manufacturer’s total cost also represents a minimum solution for manufacturer’s costs.

6.2 Numerical Results for the Coordinated Solution

In this section, we provide numerical results for four cases with varying buyer demands, number of buyers, manufacturer's transportation costs and failure rates in order to illustrate the potential cost savings available under the coordinated solution as compared to the standard EPQ model. Each of these cases utilizes the following supplier production parameters:

G	7000 units	Manufacturer Production Rate (per year)
K_m	\$250	Manufacturer Setup Cost (per setup)
H_m	\$2	Manufacturer Holding Cost (per unit per year)
C_c	\$100	Manufacturer Transportation Cost (per delivery)

In the first case (hereafter referred to as Case 1), we use the following demand and cost parameters for each of 5 buyers:

Buyer	**Demand (D_i) (per year)**	**Ordering Cost (Kb_i) (per order)**	**Holding Cost (Hb_i) (per unit per year)**
1	300	20	3
2	550	15	3.3
3	350	6	3.6
4	200	10	3.6
5	700	18	2.5

We use these buyer demand and cost parameters to calculate common order replenishment times using the bisection method described in the previous section and Appendix A. Common order replenishment times (T) for two, three, four, and five buyers are listed in Table 6.1 below:

# of Buyers	**Buyers**	T
1	1	-
2	1 & 2	0.267
3	1, 2, & 3	0.247
4	1, 2, 3, & 4	0.241
5	1, 2, 3, 4, & 5	0.227

Table 6.1 – Case 1: Common Order Replenishment Times

Having calculated the common order replenishment times, we next calculate total system costs (TC) with and without coordination and determine the level of cost savings achieved through the use of a common order replenishment time (TCS). We report these results in Table 6.2 as follows:

	TC **(without coordination)**	TC **(with coordination)**	TCS **($)**	**TCS (%)**
2 buyers	$2,424.94	$1,587.93	$837.01	34.52%
3 buyers	$3,709.81	$1,895.18	$1,814.63	48.91%
4 buyers	$4,501.96	$2,083.16	$2,418.80	53.73%
5 buyers	$5,671.33	$2,545.67	$3,125.66	55.11%

Table 6.2 – Case 1: Total Cost and Total Cost Savings

As can be seen in Figure 6.1, the coordinated solution achieves total system cost savings for all cases with multiple buyers. Additionally, the percentage cost savings increase along with the number of buyers suggesting that the use of a common order replenishment time may be more beneficial for companies with larger numbers of buyers (as opposed to fewer).

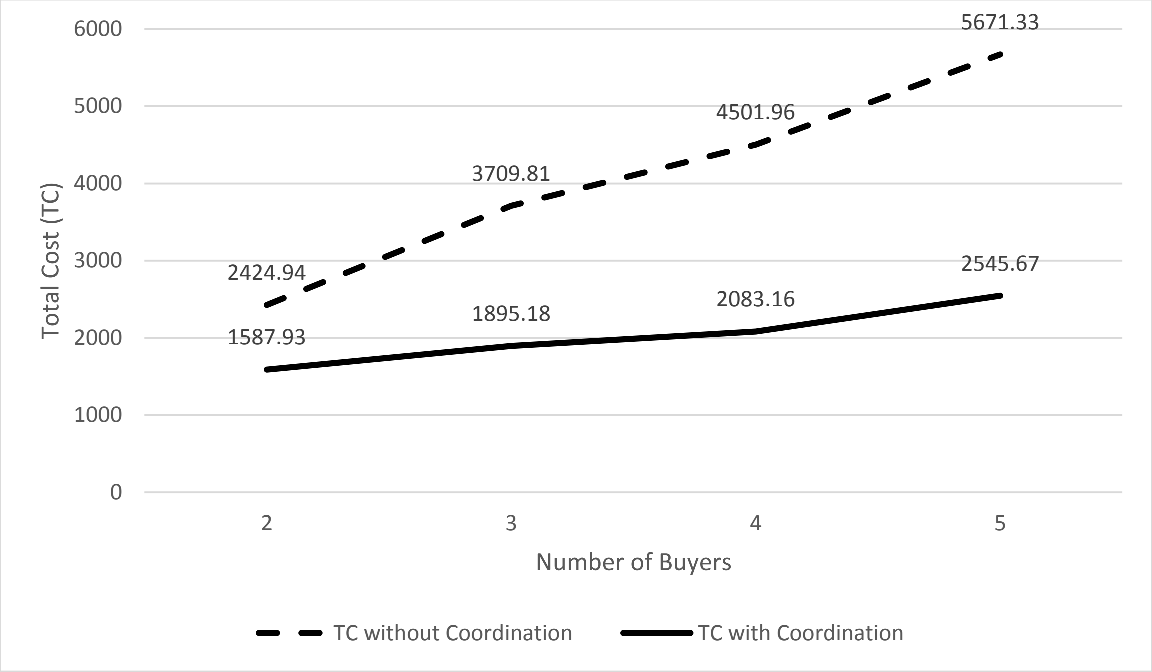

Figure 6.1 – Graph of Total Costs with and without Coordination

Buyer	# of deliveries without coordination	# of deliveries with coordination
1	4.7	4.4
2	7.8	4.4
3	10.2	4.4
4	6.0	4.4
5	7.0	4.4

Table 6.3 – Case 1: Average Number of Deliveries (per year) for n=5 buyers

Table 6.3 reports the number of deliveries for each buyer with and without coordination. A portion of the cost savings achieved through coordination is related to transportation cost savings. As such, we consider a second case in which buyer demand is higher in order to demonstrate the sensitivity of our cost savings model to changes in buyer demand. We utilize the following buyer parameters which feature a doubling of annual demand.

Buyer	**Demand (D_i) (per year)**	**Ordering Cost (Kb_i) (per order)**	**Holding Cost (Hb_i) (per unit per year)**
1	600	20	3
2	1,100	15	3.3
3	700	6	3.6
4	400	10	3.6
5	1,400	18	2.5

Again, we use the bisection method and updated buyer demand and cost parameters to calculate common order replenishment times, which are listed in Table 6.4 below

# of Buyers	**Buyers**	***T***
1	1	-
2	1 & 2	0.234
3	1, 2, & 3	0.198
4	1, 2, 3, & 4	0.186
5	1, 2, 3, 4, & 5	0.162

Table 6.4 – Case 2: Common Order Replenishment Times

We calculate total system costs with and without coordination and total cost savings under a common order replenishment time for scenarios ranging from 2 to 5 buyers and report these values in Table 6.5. Additionally, we calculate the number of deliveries per year for each buyer with and without coordination. As shown in Table 6.6, the number of deliveries increases by only 41% despite a doubling of demand. Total costs also increased by less than 50%, demonstrating the economies of scale associated with utilizing a common order replenishment time. These economies of scale with respect to both the number of buyers and individual buyers' demands are further illustrated by Figures 6.2 and 6.3. Our model continues to provide significant cost savings (over 50%) under conditions of increased demand and order frequency, thereby supporting the applicability of our framework to high demand items which experience failure during storage and which are ordered in large quantities. The medical industry, in particular, makes use a wide variety of sterile medical supplies which can be damaged or otherwise fail during storage. Our model is directly applicable to this type of product.

	TC (without coordination)	**TC (with coordination)**	**TCS ($)**	**TCS (%)**
2 buyers	$3,429.90	$2,182.12	$1,247.78	36.38%
3 buyers	$5,252.39	$2,645.29	$2,607.10	49.64%
4 buyers	$6,373.29	$2,919.18	$3,454.11	54.20%
5 buyers	$8,022.21	$3,582.17	$4,440.04	55.35%

Table 6.5 – Case 2: Total Cost and Total Cost Savings

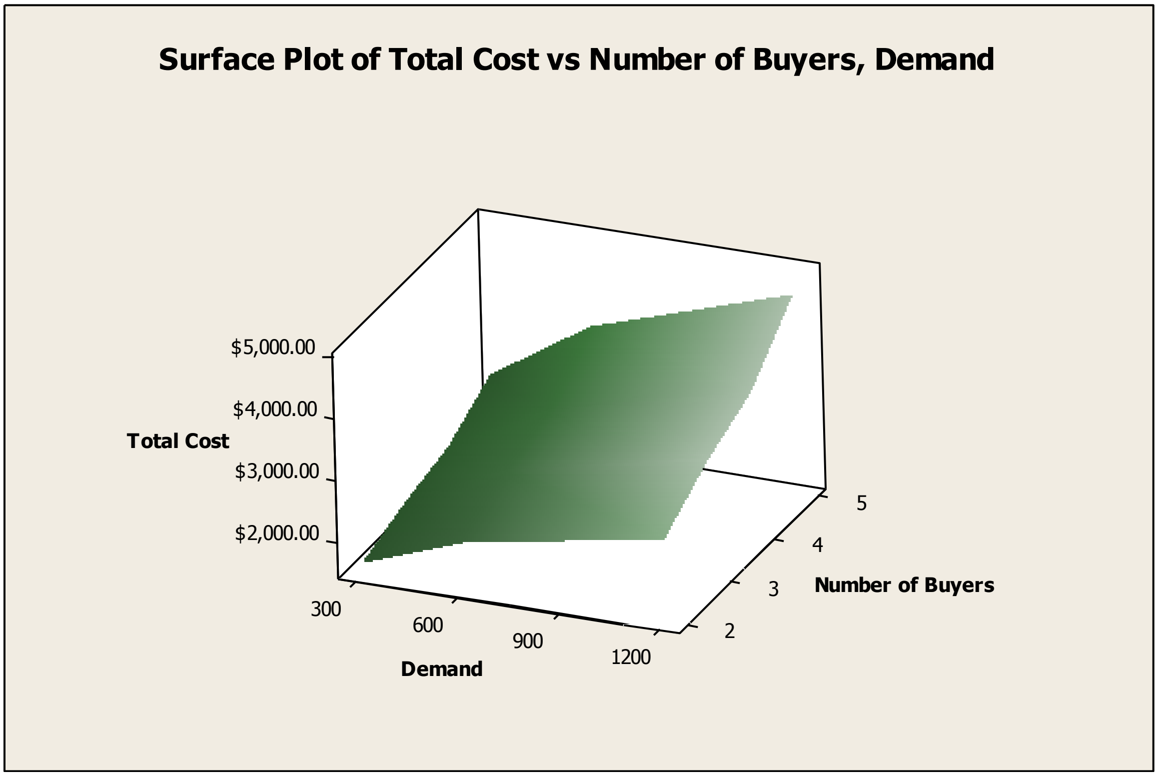

Figure 6.2: Total Cost (with coordination) vs Number of Buyers and Buyer 1's Demand (with demand of other buyers changing proportionally to buyer 1's Demand)

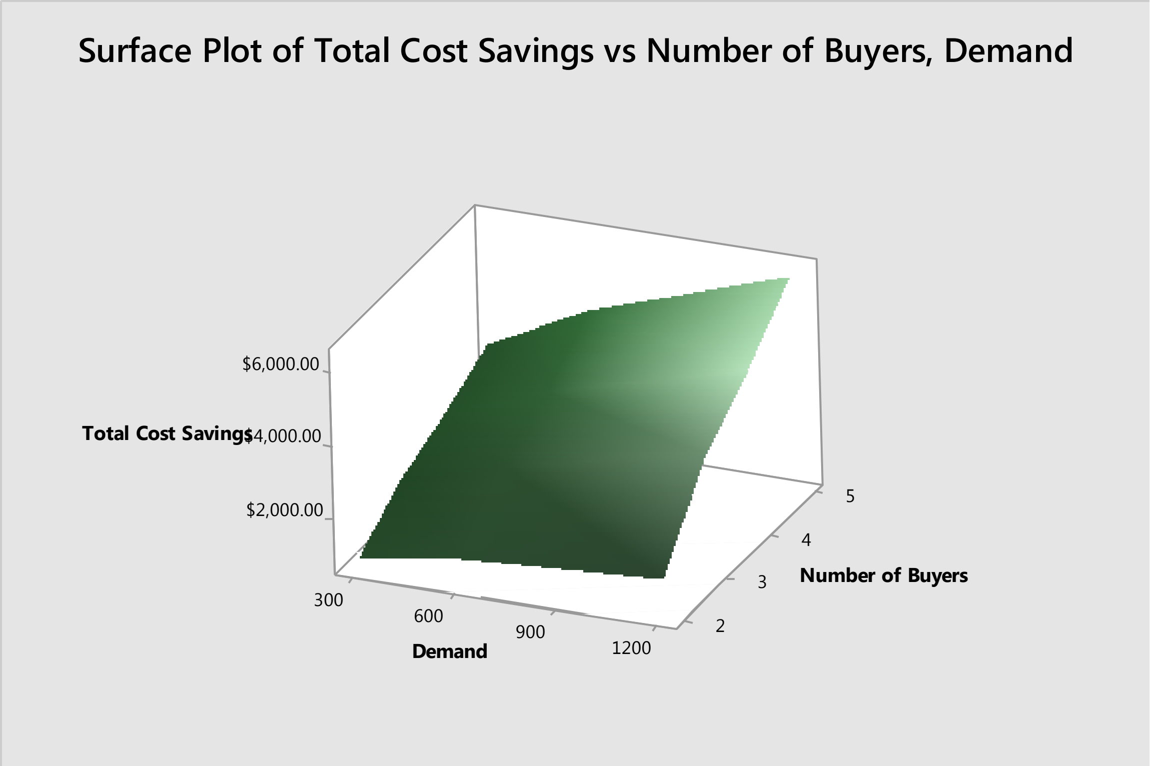

Figure 6.3: Total Cost Savings (obtained with coordination) vs Number of Buyers and Buyer 1's Demand (with demand of other buyers changing proportionally to buyer 1's Demand)

Buyer	**# of deliveries without coordination**	**# of deliveries with coordination**
1	6.7	6.2
2	11.0	6.2
3	14.5	6.2
4	8.5	6.2
5	9.9	6.2

Table 6.6 – Case 2: Average Number of Deliveries (per year) for n=5 buyers

In the third case (hereafter referred to as Case 3), we consider the effects of increasing the manufacturer's transportation costs from \$100 to \$200 per delivery (with the other parameters from Case 1 remaining constant):

# of Buyers	**Buyers**	**T**
1	1	-
2	1 & 2	.380
3	1, 2, & 3	.322
4	1, 2, 3, & 4	.302
5	1, 2, 3, 4, & 5	.261

Table 6.7 – Case 3: Common Order Replenishment Times

While we observe a very significant increase in common order replenishment times under coordinated scenario, the total cost savings resulting from the coordination framework remain almost as large as in the original scenario. These results showcase the benefits of the model in situations with high delivery costs that frequently occur in both the defense and medical industries and can require secure or sterile delivery considerations, respectively.

	TC (without coordination)	**TC (with coordination)**	**TCS (\$)**	**TCS (%)**
2 buyers	\$2,898.87	\$1,836.81	\$1,062.06	36.64%
3 buyers	\$4,183.74	\$2,215.95	\$1,967.79	47.03%
4 buyers	\$4,975.90	\$2,431.68	\$2,544.22	51.13%
5 buyers	\$6,200.70	\$2,954.84	\$3,245.86	52.35%

Table 6.8 – Case 3: Total Cost and Total Cost Savings

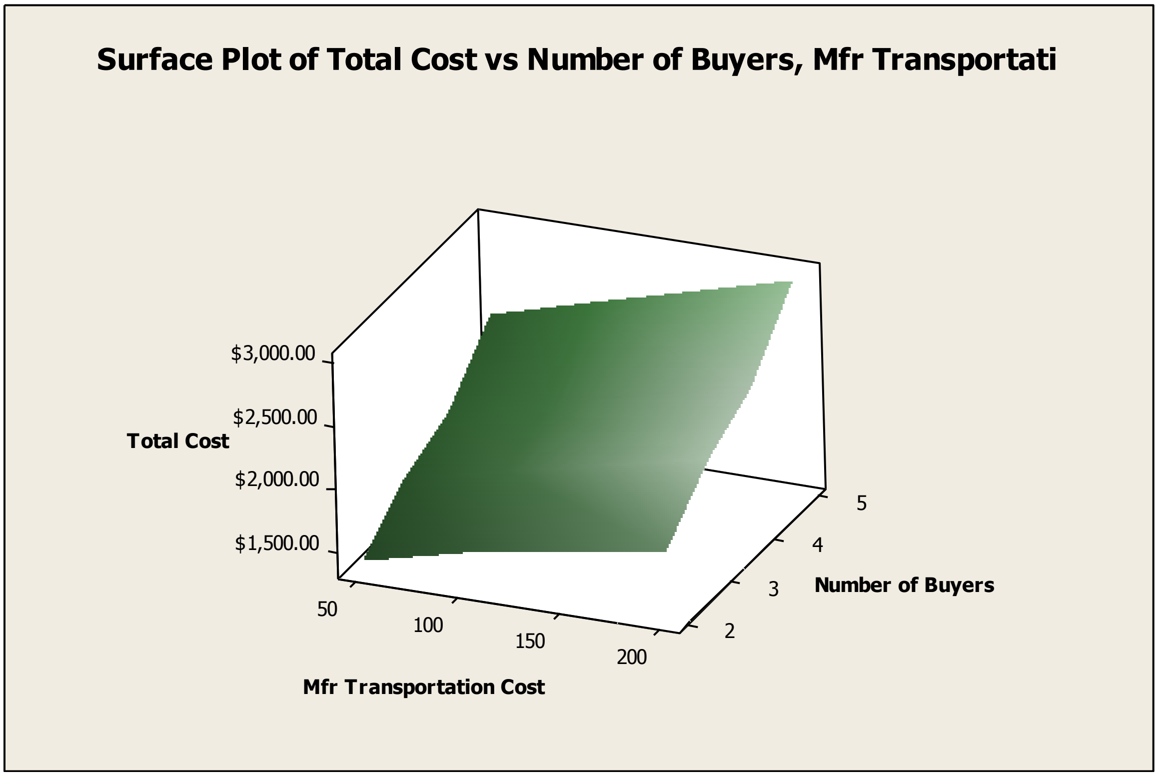

Figure 6.4: Total Cost (with coordination) vs Number of Buyers and Manufacturer's Transportation Cost (per delivery)

In the fourth and final case, we consider simultaneous changes in item failure rate and manufacturer's transportation cost for a wide range of both parameters (with the other parameters from Case 1 remaining constant). We observe that a reduction of manufacturer's transportation cost from 100 to 50 results in 9.93% to 10.36% drop in total cost for a range of failure rates between 0.25 and 1. Additionally, we observe that an increase in manufacturer's transportation cost from 100 to 200 results in 16.07% to 16.86% increase in total cost for the same range of failure rate values.

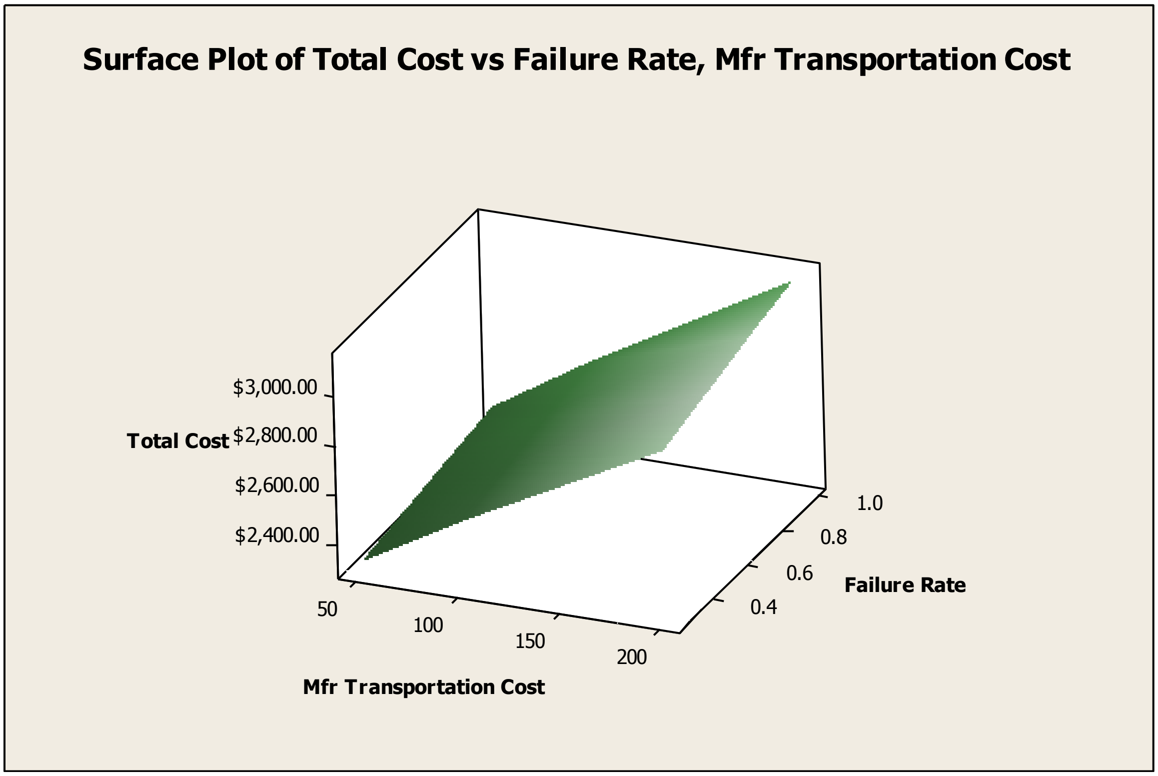

Figure 6.5: Total Cost (with coordination) vs Failure Rate and Manufacturer's Transportation Cost (per delivery)

The results of the numerical examples presented in this section demonstrate the dramatic reductions in total supply chain costs which can be achieved through our coordinated solution for a wide range of parameter values, further validating the practical contributions of our model.

6.3 Distribution of Cost Savings

Developing supply chain coordination mechanisms for inducing mutually beneficial cooperation between suppliers and buyers involves the consideration of both the supplier's and buyers' optimal inventory policies. Zimmer (2002) describes the problem of supply chain coordination as one of minimizing total system costs subject to the cost functions of both the supplier and buyer. Given the capital intensity of many manufacturing processes, suppliers tend to prefer larger order quantities and longer inventory cycles. Such policies allow the manufacturer to maximize efficiency while minimizing costs and excess capacity, thereby making the best use of fixed asset investments for producing items. Buyers, by contrast, generally prefer to have the flexibility to order inventory as needed in order to account for demand fluctuations. More flexible inventory policies allow order quantities to be demand driven, thereby avoiding stockouts and overstock situations which can result in lost profits.

Given the differences in preferred inventory policies between suppliers and buyers, achieving coordination requires that both parties cooperate to achieve the available cost savings and minimize total system costs. Cost savings achieved through cooperative inventory management, such as those generated through the adoption of our supply chain coordination model by buyers and suppliers, provide the basis for concessions designed to make cooperation mutually beneficial (Li & Wang 2007).[2]

Prior to determining what form such concessions will take, however, manufacturers must first determine how much of the achieved cost savings is appropriate to share with buyers. The following notation is utilized in order to facilitate the discussion of apportioning cost savings:

TCS	System-wide cost savings obtained through supply chain coordination
CS_m	Manufacturer's share of cost savings
CS_{bi}	Buyer i's share of cost savings
α	Proportion of system-wide cost savings retained by the manufacturer

Given that they hold private information related to the magnitude of total cost savings, manufacturers can choose to retain a portion of the surplus rather than distribute 100% of it when they are independent of buyers. The proportion of the cost savings suppliers are able to retain, however, is

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[2] Appendix D provides a graphical illustration of the viability of an uncoordinated solution to the problem described in this Chapter. The ability of the supply chain to function without coordination highlights the need for concessions to induce mutually beneficial cooperation between the supplier and buyer. This diagram shows that inventory always remains above zero, allowing the manufacturer to provide all orders at the buyer preferred times despite uneven ordering intervals.

likely a function of several factors. Dominant suppliers, or those with significantly greater market power relative to that of their buyers, are better able to retain higher proportions of any cost savings achieved through coordination solely on the basis of their market power. Inventory obsolescence or item failure risk may also be a factor considered when deciding on α. To the extent that buyers undertake higher levels of inventory risk when accepting larger quantities of items in their deliveries, buyers may refuse to cooperate with the supplier's coordination arrangements when offered low proportions of the achieved cost savings. This factor is of particular concern for the types of items we consider in our model (i.e. small electronic/defense/medical components which fail during storage). Additionally, the strength of the supplier-buyer relationship may influence the supplier's realized value of α. Relationships which are characterized by greater trust and cooperation may be associated with savings splits which are more favourable to buyers than relationships which are more fractious or uncooperative.

Assuming, therefore, that manufacturers retain α of the costs savings achieved through coordination, the remaining $(1-\alpha)$ of such savings can be distributed to buyers using a variety of decision rules.

1. Manufacturers may choose to share an equal amount of the remaining cost savings with each of i buyers. In this scenario, $CS_m = \alpha\, TCS$ and $CS_{bi} = \frac{TCS - CS_m}{n}$ where n is equal to the number of buyers within the system.

Suppose $\alpha = 0.5$. Given the prevalence of supply chains with a dominant supplier and the prevalence of information asymmetry between buyer and suppliers, high levels of α such as .5 or higher are both realistic and common. We calculate the distribution of cost savings using the Case 1 parameters from Section 6.2 and report the results in Table 6.9:

Total Savings (per Year)	$3,125.66
Manufacturer's share of Cost Savings	$1,562.83
Buyer 1's share of cost savings	$312.57
Buyer 2's share of cost savings	$312.57
Buyer 3's share of cost savings	$312.57
Buyer 4's share of cost savings	$312.57
Buyer 5's share of cost savings	$312.57

Table 6.9 - Distribution of Cost Savings Achieved through Coordination with Equal Sharing **($\alpha = 0.5$)**

Suppose $\alpha = 0.25$. This level of cost savings split is more common in systems featuring a less powerful supplier or dominant buyers. We calculate the distribution of cost savings using the Case 1 parameters from Section 6.2 and report the results in Table 6.10:

Total Savings (per Year)	\$3,125.66
Manufacturer's share of Cost Savings	\$781.42
Buyer 1's share of cost savings	\$468.85
Buyer 2's share of cost savings	\$468.85
Buyer 3's share of cost savings	\$468.85
Buyer 4's share of cost savings	\$468.85
Buyer 5's share of cost savings	\$468.85

Table 6.10 - Distribution of Cost Savings Achieved through Coordination with Equal Sharing **($\alpha = 0.25$)**

2. Manufacturers may choose to allocate cost savings among buyers in a way which recognizes their relative "importance" or market share. In this scenario, $CS_m = \alpha\, TCS$ and $CS_{bi} = \frac{TCS - CS_m}{\frac{D}{D_i}}$ where D_i is the demand of the i^{th} buyer and $D = \sum_{i=1}^{n} D_i$.

Suppose $\alpha = 0.5$. Table 6.11 reports the distribution of cost savings using the Case 1 parameters from Section 6.2:

Total Savings (per Year)	\$3,125.66
Manufacturer's share of Cost Savings	\$1,562.83
Buyer 1's share of cost savings	\$223.26
Buyer 2's share of cost savings	\$409.31
Buyer 3's share of cost savings	\$260.47
Buyer 4's share of cost savings	\$148.84
Buyer 5's share of cost savings	\$520.94

Table 6.11 - Distribution of Cost Savings Achieved through Coordination with Demand-Based Sharing **($\alpha = 0.5$)**

Suppose $\alpha = 0.25$. We calculate the following distribution of cost savings using the Case 1 parameters from Section 6.2:

Total Savings (per Year)	$3,125.66
Manufacturer's share of Cost Savings	$781.42
Buyer 1's share of cost savings	$334.89
Buyer 2's share of cost savings	$613.97
Buyer 3's share of cost savings	$390.71
Buyer 4's share of cost savings	$223.26
Buyer 5's share of cost savings	$781.42

Table 6.12 - Distribution of Cost Savings Achieved through Coordination with Demand-Based Sharing **($\alpha = 0.25$)**

It is noteworthy that Buyer 5 obtains cost savings equal to those achieved by the manufacturer under this cost sharing rule. Under a scenario with demand-based cost savings allocation and lower levels of α, it is possible for a dominant buyer's cost savings to exceed that of the manufacturer. It is clear, therefore, that the relative strength of buyers and suppliers is an important consideration for any cost savings rule considered.

3. Manufacturers may choose to allocate cost savings among buyers in a way which recognizes relative cost concessions. In this scenario, $CS_m = \alpha\, TCS$ and $CS_{bi} = \frac{TCS - CS_m}{\frac{C}{C_i}}$ where C_i is the increased cost assumed by the i^{th} buyer upon accepting a coordinated delivery schedule and $C = \sum_{i=1}^{n} C_i$.

In order to implement this decision rule, we first calculate each buyer's total cost before and after coordination using Case 1 parameters as follows:

Buyer	TCB_i before coordination	TCB_i with coordination
1	\$194.83	\$196.16
2	\$237.16	\$284.00
3	\$124.49	\$177.71
4	\$122.55	\$130.50
5	\$255.53	\$289.41

Table 6.13 –Total Buyer Cost with and without Coordination

By examining Tables 6.3 and 6.13 together, we can see that a buyer's total cost increases are proportionally larger, in comparison to other buyers, when their delivery frequency with coordination is farther from their delivery frequency without coordination. By contrast, smaller differences in delivery frequency between the coordinated and uncoordinated case result in relatively smaller increases in total buyer costs. This method of allocating cost savings recognizes that buyers who incur greater costs through coordination will likely require greater inducement to participate in a coordinated solution. Our decision rule, therefore, is designed to offer the largest proportion of cost savings to those buyers who incur the greatest levels of cost increases upon adopting a coordinated solution.

Suppose $\alpha = 0.5$. Table 6.14 reports the distribution of cost savings using the Case 1 parameters from Section 6.2:

Total Savings (per Year)	$3,125.66
Manufacturer's share of Cost Savings	$1,562.83
Buyer 1's share of cost savings	$14.51
Buyer 2's share of cost savings	$511.04
Buyer 3's share of cost savings	$580.77
Buyer 4's share of cost savings	$86.79
Buyer 5's share of cost savings	$369.72

Table 6.14 - Distribution of Cost Savings Achieved through Coordination with Cost Concession-Based Sharing **($\alpha = 0.5$)**

Suppose $\alpha = 0.25$. We calculate the following distribution of cost savings using the Case 1 parameters from Section 6.2 and report them in Table 6.15.

Total Savings (per Year)	$3,125.66
Manufacturer's share of Cost Savings	$781.42
Buyer 1's share of cost savings	$21.76
Buyer 2's share of cost savings	$766.57
Buyer 3's share of cost savings	$871.15
Buyer 4's share of cost savings	$130.19
Buyer 5's share of cost savings	$554.58

Table 6.15 - Distribution of Cost Savings Achieved through Coordination with Cost Concession-Based Sharing **($\alpha = 0.25$)**

Certain supply chain mechanisms described in this chapter rely on the manufacturer having information related to buyer cost and demand functions. As such, they are predicated on some level of information sharing between the supplier and buyers. This type of cooperation between suppliers and buyers can have a number of benefits throughout the supply chain (Fiala 2005). Cachon & Fisher (2000) find that full information sharing within a supply chain provides, on average, a 2.2% reduction in total supply chain costs. These cost savings tend to be higher when demand information is serially correlated over time and thus provide more precise predictive power (Hau, Kut, & Tang 2000). While information sharing may have been difficult prior to the development of sophisticated information technology, ERP and supply chain management systems (e.g. vendor-managed inventory) have substantially enabled information sharing within many supply chains (Kelle & Akbulut 2005; Sahin & Robinson 2002). Information sharing is, however, costly both in terms of relationship-specific investments information technology and the potential loss of control over proprietary information (Fiala 2005). An accurate assessment of the potential benefits to be obtained from information sharing is also critical, including considering the effects of product substitution, demand correlation among supply chain partners, and partial information sharing on the benefits derived from information sharing (Ganesh, Raghunathan, & Rajendran 2014). Patnayakuni, Raj, & Seth (2006) also point out that simply spending money on enabling technology is insufficient to induce information sharing and mutually-beneficial cooperation. Instead, suppliers and buyers tend to build such relationships over time in order to ensure that the appropriate information is shared and that each party can be trusted by the other (Li & Zhang 2008; Zhou & Benton 2007). In addition to trust, Wu, Chuang, & Hsu (2014) highlight the role of commitment, reciprocity, and relative power in building information sharing relationships. Thus, our coordination mechanisms assume a certain level of relationship depth which would help facilitate the implementation of a coordinated solution.

Cooperation between manufacturers and buyers can be induced by the manufacturer providing both a quantity discount and a constant reorder interval discount (Li & Liu 2006). In exchange for these concessions, the buyers agree to receive goods in equal size batches throughout the year according to the manufacturer's preferred inventory policies (Sarmah et al 2008). The manufacturer benefits from such an arrangement through reduced production costs achieved by eliminating potential demand spikes. Regular shipments also allow the manufacturer to better manage the production schedule, possibly leading to lower capacity requirements. Assuming the manufacturer sets discounts at appropriate levels, the buyers are able to benefit from such an arrangement by offsetting storage costs through the receipt of those discounts. Benton & Park (1996) provide a review of the academic literature on quantity discounts, concluding that such discounts promote deeper supplier-buyer relationships and economies of scale for both manufacturing and transportation. In a qualitative survey of manufacturers, Munson & Rosenblatt (1998) find that cost savings and economies of scale are the most often cited reasons for offering quantity discounts. We can assess the feasibility of cooperation in this scenario by comparing the highest amount the manufacturer is willing to pay and the lowest among the buyers are willing to accept. Sustainability of this cooperative solution requires the first amount to exceed the second. Prior literature provides numerous examples of these types of discounts. Taylor (2002) examines the use of rebates for inducing supply chain coordination and finds that coordination can be achieved when buyers have an influence on demand through sales effort. Cachon (2004) proposes the use of advance-purchase discount contracts to induce buyers to accept supplier-preferred terms and achieve supply chain coordination.

Cooperation can also be sustained through bargaining over lot sizing and delivery intervals rather than adopting one or the other's preferred inventory policies outright. Given the manufacturer's preferred order quantity, hereafter described as $Q_{Manufacturer}$, and the buyer's preferred order quantity (Q_{Buyer}), the parties can select a mutually agreeable Q which reduces costs for both parties. This quantity, hereafter denoted $Q_{Negotiated}$, is selected according to the following inequality:

$$Q_{Buyer} < \boldsymbol{Q_{Negotiated}} < Q_{Manufacturer}$$

Given the manufacturer's stated preference for fixed and equally spaced delivery intervals with large order quantities, and the buyer's stated preference for unequally spaced delivery intervals with demand-specified order amounts, it is logical that lot size and delivery interval would be areas of negotiation between the two parties. Such a solution would be sustainable through the use of manufacturer concessions as previously discussed. Technological advancements in inventory management have been shown to be useful in helping coordinate volume and delivery schedule concessions. Cheung and Lee (2002) highlight the effectiveness of shipment coordination between buyers and suppliers in achieving supply chain coordination. By investing in vendor-managed inventory (VMI) technology, suppliers are able to monitor buyer inventory levels and work with buyers to develop shipment schedules to achieve supply chain coordination. Dong & Xu (2002) find that VMI technology is effective in increasing the buyer's profit, though effects on supplier profitability vary in the short-run.

Additionally, individual firm characteristics have a significant impact on cooperative solutions such as these. Identification of delivery parameter specifications at which buyers and manufacturers are able to make mutually beneficial trade-offs between delivery size and schedule, for instance, requires an examination of relative cost structures between the buyer and manufacturer. It is clear that a manufacturer might have higher costs in certain areas as compared to buyers, whereas buyers might have cost advantages in other areas. Minimizing total system costs, therefore, will include shifting costs to the party which has a comparative cost advantage relative to that cost.

In the case of our developed models, which account for item failure, expense sharing may take the form of expense sharing for losses on items which fail after delivery while storage. More specifically, the use of a fixed payment (or discount) for anticipated item failures or a percentage payment for each failed item, negotiated prior to delivery, may be helpful in inducing buyers to accept larger order quantities and longer inventory cycle times. Other forms of expense sharing include cooperative advertising (Huang, Li, and Mahajan 2002; Yue, Austin, Wang, and Huang 2006) and other promotional activities (Krishnan, Kapuscinski, & Butz 2004), buybacks and operating subsidies (Cho and Gerchak 2005; Moses & Seshadri 2000), new product development cooperation (Petersen, Handfield, & Ragatz 2005), and/or risk-sharing contracts (Chen, Chen, and Chen 2006) where manufacturers and buyers share the risk of demand fluctuations between periods.

Conversely, revenue sharing may take the place in expense sharing. Rather than directly sharing expenses, a buyer may choose to share revenue with the supplier in order to minimize upfront costs and increase cooperation within the supply chain. Li, Zhu, and Huang (2009) highlight online marketplaces as an area where revenue sharing contracts are used extensively, with suppliers choosing delivery quantities and buyers setting revenue sharing percentages. Giannoccaro & Pontrandolfo (2004) derive a revenue sharing contract which allows for maximum cost savings through the adjustment of contract parameters through mutually-beneficial cooperation in a three-stage supply chain. Revenue sharing contracts are not a panacea, however. The administrative expense involved in maintaining such arrangements, however, may not yield cost savings over more simple methods of quantity discounting or expense sharing (Cachon & Lariviere 2005). Additionally, revenue sharing contracts do not work well for retailers who compete on price or who can influence demand through their actions (or inactions) (Cachon & Lariviere 2005).

In each of these cases, it is important to note that coordination need not be between arms-length parties. As noted in Chapter 5, in-house production can lead to internal item delivery for related parties. Examples of such situations include manufacturing cost centers within large organizations, where part of the company produces items and/or components for other areas and arranges for transfers of goods through mutual cooperation. Thus, this coordination framework applies not only to our EOQ model when suppliers have multiple buyers with different optimal order quantities, but also to our EPQ model where optimal production and order policies may differ between the manufacturer and customers. Similarly, this coordination framework might occur through the use of a third-party who aids in supply chain coordination as recently described by Masten & Kim (2015)

Chapter 7

Conclusions and Future Research

7.1 Conclusions

This dissertation has focused on inventory management and supply chain coordination mechanisms within the context of an economic order quantity framework. Specifically, this research involves modeling optimal order policies and supply chain coordination mechanisms for items and markets with characteristics which fall outside the bounds of the standard economic order quantity (EOQ) model. The items of interest are common types of manufactured items which, nonetheless, require specialized order policy considerations due to their unique characteristics.

This research involves the development of economic order quantity and economic production quantity (EPQ) models for items which experience probabilistic failure during storage. While prior research has focused on items which can be repaired or sold at a discount upon failure, such models are inappropriate for systems where repair costs exceed or are equivalent to item costs and imperfect items are unacceptable. Examples of industries featuring these inventory conditions include the medical, defense, and electronics industries where defective items are largely useless.

We begin with the development of a modified EOQ model for the presented framework. As previously mentioned, the determination of optimal cycle time holds important practical implications for firms. Optimal cycle time facilitates cost minimization by striking the optimal balance between holding and ordering costs within an inventory cycle. Similarly, optimal cycle times ensure that firms maximize revenues by avoiding item shortage during the inventory cycle. The model is then illustrated with a number of numerical examples, including sensitivity analyses designed to examine the effects of changes in parameter values on optimal order quantities and firm profitability.

Of particular note is the effect of simultaneous proportional changes in demand and holding costs illustrated in the numerical results provided in Chapter 4. Under the conditions assumed in the basic EOQ model, the optimal economic order quantity remains constant with proportional changes in demand and holding cost. Our model illustrates, however, that for items which experience probabilistic failure, the optimal economic order quantity increases with proportional changes in demand and holding costs. Thus, our modified EOQ model leads to higher profitability under these conditions, with increasing comparative benefits at the higher levels of order quantity. As such, our model is particularly beneficial to organizations and company cost centers that generally place larger orders, further validating the importance and applicability of our model to business operations.

A further implication of our results is the importance of identifying the appropriate method for modeling deterioration of an item for the purpose of calculating optimal order policies. Fercho and Ringer (1972) propose a number of statistical tests which may be used to determine whether an item fails at a constant or non-constant rate. These tests can be used to determine whether our model (or other models) is appropriate for use in modeling the specific items sold by a particular firm. Our results suggest that the type of rate assumed within an EOQ model has a practically significant effect on both cycle time and profit. As such, our results suggest that firms may realize substantial benefit from exerting the effort to properly model item deterioration within their system.

Additionally, the empirical results and analysis provided in Section 4.4 provide support for our use of a failure rate to model probabilistic item failure during storage and in non-operational stages of use. The selected components are exemplars of military components which fail at rates similar to those utilized within the numerical examples in this chapter.

Rossi (1987) provides further support for this finding, suggesting that manufacturers within the defense industry and military planners make use of exponential failure rates when modeling item failure for electronic components.

Then, we adapt the EPQ model, an extension of the EOQ framework, for use in modeling items which experience probabilistic failure during storage and which cannot be reworked or sold at a discount. In contrast to the EOQ model, our EPQ model considers in-house incremental production and delivery of such items rather than the periodic lot deliveries between suppliers and outside buyers. In Chapters 5 and 6, this model and related supply chain coordination mechanisms are developed which exploit system-wide cost savings to induce mutually beneficial cooperation between a single manufacturer and n buyers.

These models highlight the importance of continuing to test the assumptions of the basic EOQ model. Despite over 100 years having passed since its introduction, the EOQ framework maintains broad appeal and usage by both academics and practitioners due to its relative simplicity and generalizable conceptual underpinnings. As demonstrated in this dissertation, however, there continues to be room for modifications and extensions to the EOQ model which further our understanding of inventory management and optimal order policies. The model developed in Chapter 3 highlights the continued importance of item quality within the

EOQ framework. Item quality is increasingly important given the broad range of item quality and manufacturing processes available within the global marketplace (Peterson, Prayer, and Scannell 2000).

As a result of shorter inventory cycle in the model incorporating failure in storage, items are held in inventory for shorter periods of time. Consequently, holding costs have a smaller impact on optimal cycle time and profit in comparison to the base EOQ model. This dictates that effective managers will allocate more resources toward developing and improving logistics related to ordering cost minimization. The EPQ lot sizing model developed in Chapter 5, therefore, highlights the importance of inventory management for developing mechanisms through which supply chain coordination can be achieved between suppliers and buyers.

Having considered optimal order policies for both buyers and suppliers, we next develop an optimal solution for a coordinated supply chain in Chapter 6. The proposed solution allows the manufacturer to coordinate a supply chain consisting of n buyers in order to achieve a common replenishment time. Through this optimization framework, we minimize total system-wide costs and derive the cost savings associated with our coordinated solution. Numerical examples are then used to demonstrate the magnitude of cost savings achievable through our coordination framework.

We conclude by proposing several mechanisms for leveraging the resulting cost savings to induce mutually-beneficial cooperation between the supplier and multiple buyers. Given the lack of buyer-supplier cooperation noted in empirical research related to supply chain coordination, our identification of specific mechanisms useful for inducing mutually-beneficial cooperation between buyers and suppliers represents an important practical contribution to the supply chain coordination literature. These models are accompanied by a thorough overview and discussion of economic order quantity theory, optimal order policies, and supply chain coordination mechanisms.

This dissertation, therefore, both extends the EOQ and supply chain coordination literatures and emphasizes the need for continued research in both areas.

7.2 Future Research

This dissertation has generated a number of ideas for future research which would contribute to our understanding of the EOQ framework and supply chain coordination mechanisms. One potential direction for future research is the consideration of items with failure rates that vary across the item life cycle. Items may be subject to this type of variable failure rate due to inherent characteristics of the item or due to characteristics of the storage environment. For example, military ordnance (i.e. gunpowder, combustible materials) may remain stable for long periods of time before becoming gradually unstable over time. Unlike perishable items, however, these items retain their full value until the point of failure, at which point they lose all value. More precise order policies could be generated by accounting for differences in the failure rate over time.

Another area for potential future research is additional statistical analysis for items in other industries using items with failure characteristics similar to those described in our EOQ model in Chapter 3. The medical industry, for example, makes use of a wide variety of sterile supplies and equipment which experience probabilistic failure in storage. While reliability testing and failure rate analysis have been widely conducted in the defense industry, such analyses have not, to my knowledge, been conducted in the medical industry. Ascertaining the appropriate statistical distribution to be applied to failure rates in the medical industry will improve the applicability of our model to items within that industry, thereby benefiting both manufacturers and users of medical devices and supplies.

Additional opportunities for research may exist for applying the proposed model to reliability studies for series and parallel systems. Application of the model developed in Chapter 3 is appropriate where individual components experience failure in accordance with the framework presented in our paper. Such components are certainly present within parallel and series systems.

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Appendix A1: Comparison of annual profit under standard and modified EOQ models at various quantity levels for Case 1

Order Quantity	Profit per year under the model incorporating failure rate	Profit per Year in the Standard EOQ Scenario (no item failure)
5	-$2,200.58	-$1,262.50
10	$344.14	$1,225.00
15	$1,155.83	$2,045.83
20	$1,534.50	$2,450.00
30	$1,859.39	$2,841.67
40	$1,968.82	$3,025.00
45	$1,987.86	$3,081.94
48	**$2,018.52**	$3,109.17
50	$1,992.75	$3,125.00
55	$1,987.43	$3,157.95
60	$1,974.52	$3,183.33
65	$1,955.84	$3,202.88
75	$1,906.01	$3,229.17
100	**$1,739.69**	**$3,250.00**
125	$1,601.95	$3,237.50
150	$1,436.13	$3,208.33
175	$1,255.83	$3,169.64
200	$1,068.17	$3,125.00
225	$911.26	$3,076.39
250	$747.05	$3,025.00
275	$578.35	$2,971.59
300	$407.15	$2,916.67
350	$62.41	$2,803.57
400	-$279.96	$2,687.50
450	-$581.58	$2,569.44
500	-$638.69	$2,450.00
550	-$1,181.57	$2,329.55
600	-$1,475.35	$2,208.33

Appendix A2: Comparison of annual profit under standard and modified EOQ models at various quantity levels for Case 2

Order Quantity	Profit per year under the model incorporating failure rate	Profit per Year in the Standard EOQ Scenario (no item failure)
5	-$2,523.79	-$1,275.00
10	$22.71	$1,200.00
15	$836.18	$2,008.33
20	$1,216.61	$2,400.00
30	$1,545.00	$2,766.67
40	$1,657.89	$2,925.00
45	$1,678.65	$2,969.44
46	**$1,728.79**	$2,976.52
48	$1,706.91	$2,989.17
50	$1,685.25	$3,000.00
55	$1,681.62	$3,020.45
60	$1,670.40	$3,033.33
71	**$1,660.53**	**$3,042.89**
75	$1,606.89	$3,041.67
100	$1,448.73	$3,000.00
125	$1,312.36	$2,925.00
150	$1,149.75	$2,833.33
175	$973.72	$2,732.14
200	$790.93	$2,625.00
225	$635.25	$2,513.89
250	$473.16	$2,400.00
275	$307.25	$2,284.09
300	$139.30	$2,166.67
350	-$197.98	$1,928.57
400	-$532.13	$1,687.50
450	-$829.68	$1,444.44
500	-$1,126.13	$1,200.00
550	-$1,419.03	$954.55
600	-$1,706.76	$708.33

Appendix A3: Comparison of annual profit under standard and modified EOQ models at various quantity levels for Case 3

Order Quantity	Profit per year under the model incorporating failure rate	Profit per Year in the Standard EOQ Scenario (no item failure)
5	-$7,400.58	-$6,262.50
10	-$2,255.86	-$1,275.00
15	-$577.50	$379.17
20	$234.50	$1,200.00
30	$992.73	$2,008.33
40	$1,318.82	$2,400.00
45	$1,410.08	$2,526.39
48	$1,471.15	$2,588.33
49	$1,422.50	$2,607.09
50	$1,472.75	$2,625.00
55	$1,514.71	$2,703.41
60	$1,541.19	$2,766.67
65	$1,555.84	$2,818.27
71	**$1,594.18**	$2,868.27
75	$1,559.34	$2,895.83
100	$1,479.69	$3,000.00
125	$1,389.71	$3,037.50
142	**$1,308.36**	**$3,042.89**
150	$1,256.82	$3,041.67
175	$1,100.61	$3,026.79
200	$931.33	$3,000.00
225	$787.45	$2,965.28
250	$634.00	$2,925.00
275	$474.35	$2,880.68
300	$310.86	$2,833.33
350	-$21.46	$2,732.14
400	-$354.25	$2,625.00
450	-$649.11	$2,513.89
500	-$944.94	$2,400.00
550	-$1,238.72	$2,284.09
600	-$1,528.41	$2,166.67

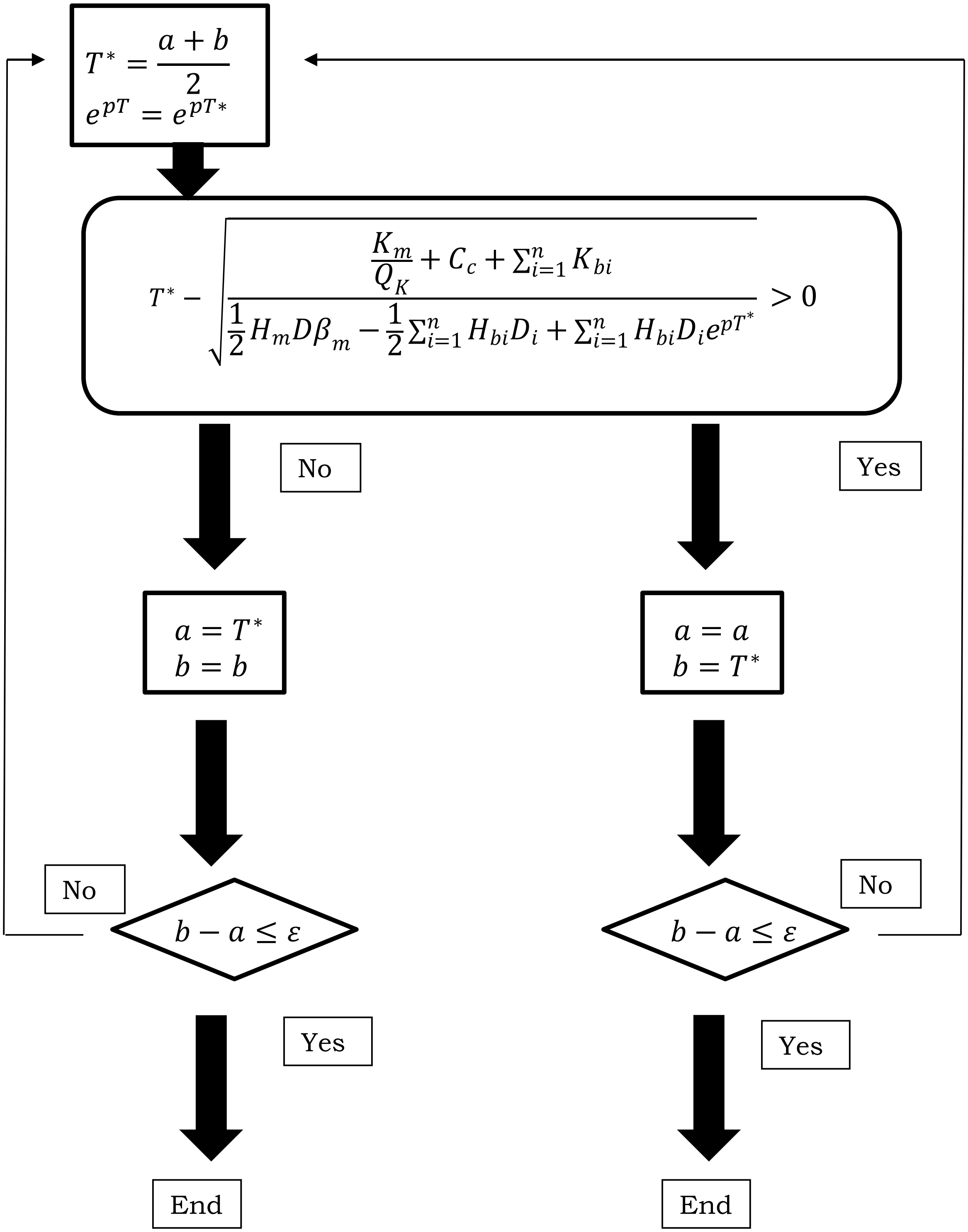
Appendix A4: Comparison of annual profit under standard and modified EOQ models at various quantity levels for Case 4

Order Quantity	Profit per year under the model incorporating failure rate	Profit per Year in the Standard EOQ Scenario (no item failure)
5	-$4,993.87	-$2,525.00
10	$152.42	$2,450.00
15	$1,832.18	$4,091.67
20	$2,645.43	$4,900.00
30	$3,405.70	$5,683.33
40	$3,733.22	$6,050.00
45	$3,824.97	$6,163.89
50	$3,887.99	$6,250.00
55	$3,930.14	$6,315.91
60	$3,956.67	$6,366.67
65	$3,971.23	$6,405.77
70	$3,976.41	$6,435.71
72	**$4,111.25**	$6,445.56
75	$3,974.12	$6,458.33
80	$3,965.77	$6,475.00
85	$3,952.48	$6,486.76
90	$3,935.07	$6,494.44
100	**$3,890.49**	**$6,500.00**
125	$3,741.06	$6,475.00
150	$3,560.45	$6,416.67
175	$3,363.45	$6,339.29
200	$3,288.03	$6,250.00
225	$3,062.31	$6,152.78
250	$2,836.96	$6,050.00
275	$2,710.31	$5,943.18
300	$2,478.82	$5,833.33
350	$2,102.67	$5,607.14
400	$1,718.71	$5,375.00
450	$1,394.30	$5,138.89
500	$1,003.06	$4,900.00
550	$666.89	$4,659.09
600	$327.46	$4,416.67

Appendix A5: Comparison of annual profit under standard and modified EOQ models at various quantity levels for Case 5

Order Quantity	Profit per year under the model incorporating failure rate	Profit per Year in the Standard EOQ Scenario (no item failure)
5	-$2,278.41	-$1,262.50
10	$189.95	$1,225.00
15	$926.72	$2,045.83
20	$1,231.88	$2,450.00
30	$1,413.93	$2,841.67
33	**$1,452.95**	$2,909.92
35	$1,414.59	$2,948.21
40	$1,385.92	$3,025.00
45	$1,338.23	$3,081.94
50	$1,277.69	$3,125.00
55	$1,208.20	$3,157.95
60	$1,241.53	$3,183.33
65	$1,153.03	$3,202.88
75	$969.98	$3,229.17
100	**$569.30**	**$3,250.00**
125	$203.00	$3,237.50
150	-$181.74	$3,208.33
175	-$568.81	$3,169.64
200	-$950.04	$3,125.00
225	-$1,291.40	$3,076.39
250	-$1,628.81	$3,025.00

Appendix B: Application of the Bisection Method



When desired precision
level is reached

Appendix C: Finding the Total Cost of Buyer i

The supply chain coordination model developed in Chapter 5.4.1 utilizes information regarding the total cost of each of n buyers within a supply chain. In order to determine the total cost of each buyer, we adopt the following methodology. The total cost function and related first-order conditions for buyer i are expressed as follows:

$$TCB_i = \frac{K_{bi}}{T_i} - \frac{H_{bi}D_i}{p} - \frac{H_{bi}D_iT_i}{2} + \frac{H_{bi}D_ie^{pT_i}}{p}$$

$$\frac{\partial TCB_i}{\partial T_i} = -\frac{K_{bi}}{{T_i}^2} - \frac{H_{bi}D_i}{2} + \frac{H_{bi}D_ipe^{pT_i}}{p}$$

We next set $\frac{\partial TCB_i}{\partial T_i}$ equal to 0 to obtain the optimal replenishment time for buyer i:

$$-\frac{K_{bi}}{T_i^2} - \frac{H_{bi}D_i}{2} + H_{bi}D_ie^{pT_i} = 0$$

$$-\frac{H_{bi}D_i}{2} + H_{bi}D_ie^{pT_i} = \frac{K_{bi}}{T_i^2}$$

$$T_i^2 = \frac{K_{bi}}{H_{bi}D_ie^{pT_i} - \frac{H_{bi}D_i}{2}}$$

$$T_i = \sqrt{\frac{K_{bi}}{H_{bi}D_i\left(e^{pT_i} - \frac{1}{2}\right)}} \qquad (*)$$

We then use the bisection algorithm from equation (*) to calculate each individual buyer's optimal replenishment time based on their demand and cost parameters. This iterative calculation method is procedurally similar to that used for determining common order replenishment times for the manufacturer and has been described in detail in both Chapter 6 and Appendix B.

Finally, we derive second-order conditions for TCB_i in order to assess the convexity of our solution with respect to T_i for all values of $T_i > 0$.

$$\frac{\partial^2 TCB_i}{\partial T_i^2} = \frac{K_{bi}}{T_i^3} + H_{bi} D_i p e^{pT_i} > 0$$

This demonstrates that our optimal value for the total cost of buyer i represents a minimum solution for buyer i’s costs.

Appendix D: Manufacturer Inventory Time Plot for Supply Chain Featuring Probabilistic Failure without Coordination

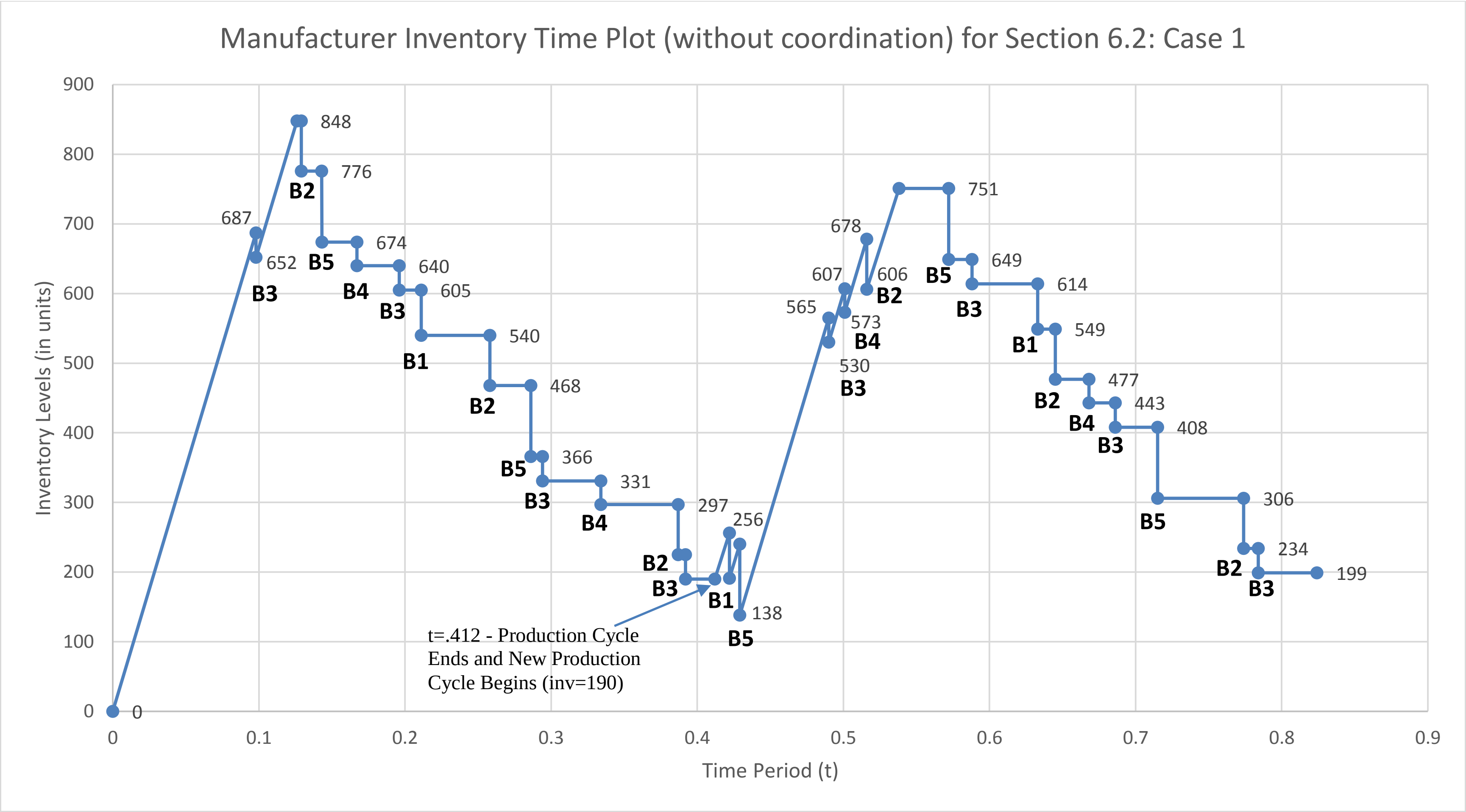
The goal of this appendix is to show that the uncoordinated solution for Case 1 described in Section 6.2 is feasible. We introduce the inventory time plot contained within this appendix in order to graphically demonstrate the viability of the uncoordinated solution. This diagram illustrates buyers receiving periodic shipments according to their optimal economic order quantities rather than adopting a coordinated shipment schedule with the manufacturer. By demonstrating the ability of the supply chain to function without coordination, we show the need to develop coordination mechanisms in order to induce mutually beneficial cooperation between the manufacturer and multiple buyers.

Using parameters from Case 1 contained in Section 6.2, we calculate the production cycle length of the manufacturer as .412. We then calculate and list the economic order intervals and quantities for each buyer as follows:

Buyer	Economic order intervals of the buyers and manufacturer's production cycle time	Buyer and Manufacturer Lot Sizes
1	0.211	65
2	0.129	72
3	0.098	35
4	0.167	34
5	0.143	102
Manufacturer	0.412	883

Based on the calculated order and production parameters for buyers and the manufacturer, we produce the inventory flow diagram for two manufacturer production cycles. In order for the solution to be feasible, the manufacturer must have sufficient inventory on-hand to fulfill buyer orders according to each buyer's preferred schedule. Despite our not continuing the diagram into future production periods, the solution clearly remains feasible in subsequent cycles as well.

While the uncoordinated solution is feasible, however, it is not optimal compared with the coordinated solution developed in Chapter 6. Consider the manufacturer's cycle time under both solutions. Without coordination, the manufacturer's cycle time for Case 1 in Section 6.2 is .412. Using the coordination mechanism developed in our model, the manufacturer's cycle time is equal to the common delivery cycle time of .227. In addition to system-wide cost savings achieved through coordination, the reduction in cycle time to match the common delivery cycle time offers a number of intangible benefits. The manufacturer achieves reduced idle time between the production and delivery periods, thereby achieving greater production efficiency. Additionally, manufacturers are better able to plan for inventory holding costs and capacity, further improving operating efficiency.



Manufacturer Inventory Time Plot (without coordination) for Section 6.2: Case 1

t=.412 - Production Cycle Ends and New Production Cycle Begins (inv=190)

Mikhail M. Sher

Contact Information

Gerri C. Lebow Hall 729
Decision Sciences
Drexel University
Philadelphia, PA 19104 USA

Voice: (412) 901-4312
Fax: (215) 895-2907
E-mail: Mikhail.M.Sher@drexel.edu

Education

Drexel University, Philadelphia, Pennsylvania USA
Ph.D. Decision Sciences, December 2015

- Dissertation Topic: “Inventory Management and Supply Chain Coordination Mechanisms”
- Advisor: Seung-Lae Kim

Columbia University, New York, New York USA
M.S. Operations Research, May 2006

Carnegie Mellon University, Pittsburgh, Pennsylvania USA
B.S. Economics, August 2005
B.S. Mathematics, August 2005

Research Interests

My main research interests lie in the areas of inventory control and supply chain management with a focus on optimal order policies and supply chain coordination.

Publications

Sher, M., & Kim, S.L. (2015). An Economic Order Quantity Model for Items Experiencing Failure in Storage. *International Journal of Operational Research*, 22(4).

Sher, M., & Kim, S.L. (2014). Supply Chain Coordination with Quantity Discount for Seasonal Demand. *International Journal of Supply Chain Management*, 3(3).

Sher, M. and Banerjee, A. (2014). Solving Unit Commitment Problem with Parallel Computing. *Proceedings of the Seventh International Conference on Management Science and Engineering Management.* Berlin, Germany: Springer Berlin Heidelberg.

Papers in Preparation

“An Empirical Investigation of Failure Rates for Certain Mechanical Components”
In this paper, I estimate failure rates and conduct reliability analysis for a sample of mechanical components using statistical techniques. This study will validate the practical necessity and proposed applications of the EOQ model developed in by Sher & Kim (2015).

“Corporate Charity Selection using Form 990 Data: An Application of Data Envelopment Analysis” – with Michael T. Paz
In this paper, several different DEA models are considered which focus on the question of how firms can identify the most efficient charitable organizations versus the most effective ones.

“Optimal Order Policies for Series and Parallel System Components”
This paper is an extension of the EOQ model proposed by Sher & Kim (2015) which develops an optimal order policy framework for components used in series and parallel systems.

Conference Presentations

Sher, M., & Kim, S.L. Supply Chain Coordination with Quantity Discount for Seasonal Demand

- Presented at Informs Annual Meeting 2014, San Francisco, CA, November 2014

Sher, M., & Banerjee, A. Solving Unit Commitment Problem with Parallel Computing

- Presented at the Seventh International Conference on Management Science and Engineering Management, Philadelphia, PA, November 2013

Sher, M., & Kim, S.L. An Economic Order Quantity Model for Items Experiencing Failure in Storage

- Presented at Informs Annual Meeting 2013, Minneapolis, MN, October 2013

Honors and Awards

Le Bow College of Business Dean's Fellowship Award
Milken Scholar
New York Governor's Committee Scholarship Award (College scholarship annually awarded to top New York State public high school graduates)
United States Chess Federation National Master

Teaching Experience

Instructor

Le Bow College of Business Undergraduate level courses:

- OPM 200: Operations Management, Fall 2013 – developed online course content currently used by several other instructors
- STAT 201: Introduction to Business Statistics, Spring 2014, Summer 2015
- OPM 200: Operations Management, Fall 2014
- OPM 325: Advanced Planning and Control of Operations, Summer 2015

Recitation Instructor

Recitation Instructor appointments for LeBow College of Business undergraduate level courses. Duties have included teaching weekly recitation sessions, preparing course content, holding student office hours, and guest lecturing.

Le Bow College of Business Undergraduate level courses:

- OPM 200: Operations Management, Fall 2011, Spring 2012, Summer 2012
- STAT 202: Business Statistics II, Fall 2011, Winter 2012

Teaching Assistant

Selected teaching assistant appointments for graduate-level courses (MBA and Ph.D.). Duties have included developing online course content, grading, holding student office hours, and guest lecturing.

- FIN 642: Business Conditions and Forecasting (MBA), Winter 2014
- OPR 601: Managerial Decision Models and Simulation (MBA), Summer 2012, Fall 2012, Summer 2012
- POM 601 Operations Management (MBA), Spring 2011, Summer 2011, Spring 2013
- POM 625: Supply Chain Management (MBA), Summer 2012, Summer 2013
- STAT 932: Statistics for Behavioral Science (Ph.D.), Fall 2011

Selected Teaching Assistant appointments for undergraduate-level courses. Duties have included preparing course content, grading, holding student office hours, and guest lecturing.

- OPM 200: Operations Management, Fall 2012, Winter 2013, Spring 2013
- OPM 321: Planning and Control of Operations, Winter 2012
- OPM 341: Supply Chain Management, Spring 2013
- OPR 320: Linear Models for Decision Making, Fall 2012
- STAT 201: Introduction to Business Statistics, Winter 2014
- STAT 202: Business Statistics II, Fall 2011, Winter 2012

Professional Affiliations

Institute for Operations Research and the Management Sciences (INFORMS) – Member
Institute for Supply Management (ISM) - Member
Decision Sciences Institute (DSI) - Member

Professional Service

International Journal of Electrical Power and Energy Systems - Referee
INFORMS Annual Meeting 2014, San Francisco, CA – Game Theory Session Chair

References

Seung-Lae Kim
Professor of Operations Management
Decision Sciences Department
Drexel University
kimsl@drexel.edu
215-895-2181

Avijit Banerjee
Professor of Operations Management
Decision Sciences Department
Drexel University
banerjea@drexel.edu
215-895-1449

Bruce McCullough
Professor of Statistics
Decision Sciences Department
Drexel University
bdmccullough@drexel.edu
215-895-2134