Variability and Scale-Dependency of Tire-Derived Aggregate

Patrick M. Strenk, M.ASCE[1]; Joseph Wartman, M.ASCE[2]; Dennis G. Grubb, M.ASCE[3];
Dana N. Humphrey, M.ASCE[4]; and Mark F. Natale[5]

Abstract: This paper presents a variability study of several engineering properties of tire-derived aggregate based on a comprehensive literature survey of experimental test programs. The dry compacted unit weight, cohesion intercept, friction angle, constrained modulus, and modified secondary compression index were evaluated and compared to the engineering parameter variability of natural soils. A series of regression analyses were performed to investigate the presence and significance of scale-dependency. The results of the variability analysis indicate that unit weight has the lowest value of coefficient of variation (COV) whereas the shear strength parameters, constrained modulus, and compression index have COV values that are substantially higher. Regression analyses indicated that unit weight and constrained modulus showed the greatest sensitivity to changes in maximum tire particle size. A nonstatistical investigation was used to further investigate the variability and scale-dependency of the shear strength parameters. Using Mohr-Coulomb failure criterion and assuming that cohesion is negligible, the analysis showed a scale-independent relationship which is consistent with the statistical findings for cohesion and friction angle.

DOI: 10.1061/(ASCE)0899-1561(2007)19:3(233)

CE Database subject headings: Tires; Recycling; Scale effects; Particle size; Uncertainty principle; Cohesion; Compressibility; Internal friction.

Introduction

The use of recycled materials in civil engineering applications has been increasing rapidly over the past decade. Owing to their light weight, favorable drainage characteristics, and widespread availability, scrap tires have become one of the more popular recycled materials used in geotechnical engineering applications such as roadway embankment construction, landfill drainage layers, retaining wall backfill, thermal insulation, and vibration attenuation media. In these applications, scrap tires are typically reduced to particle sizes ranging from 12 to 305 mm, and are classified as either *tire shreds* (50–305 mm) or *tire chips* (12–50 mm). Tire shreds and tire chips are collectively referred to as *tire-derived aggregate (TDA)*, a designation that reflects the processed nature of these materials (Humphrey 2004b). A third and less common size classification is *granulated rubber*, i.e., material having a particle size of 12 mm or less. Granulated (or crumb) rubber is used primarily in product manufacturing and roadway pavement construction. Recommendations for the processing, use, and quality of TDA in geotechnical applications are provided in ASTM standard D 6270, which addresses restrictions on: (1) on fine material occurring with the tire shreds; (2) free and exposed steel; (3) maximum shred layer thickness; and (4) limitations on tire shred exposure to air and water.

The shear strength, compressibility, and permeability characteristics of TDA are important parameters for geotechnical design and have been the primary focus of past studies (e.g., Ahmed 1993; Ahmed and Lovell 1993; Benda 1995; Bernal et al. 1996; Cosgrove 1995; Drescher and Newcomb 1994; Edil and Bosscher 1992, 1994; Heimdahl 1998; Humphrey and Sandford 1993; Manion and Humphrey 1992; Masad et al. 1996; Moo-Young et al. 2003; Tatilsoz 1996; Tatilsoz et al. 1997; Yang et al. 2002; Youwai and Bergado 2003; Zornberg et al. 2004). Overall, these studies have shown generally similar trends in the test data but often significant variation in the derived engineering properties, mainly due to differences in tire sources/suppliers, tire types, particle sizes, manufacturing (shredding) processes and the nonstandardized and/or modified laboratory testing methods and equipment used in the experimental programs. Moreover, because conventional soil testing equipment is designed for specimens having a limited particle size, most studies were performed on granulated rubber or tire chips, whereas much larger tire shreds are used in most field applications.

The principal objectives of this study were to investigate the variability and scale-dependency of select engineering properties of TDA. In particular, this work investigates if TDA has more variation in its engineering properties than conventional soil. Additionally, by studying the scale-dependency of TDA, this research considers the validity of using engineering parameters derived from smaller TDA to represent the behavior of the larger TDA used in most field applications. The approach was to first

---

[1] Graduate Student Researcher, Dept. of Civil, Architectural, and Environmental Engineering, Drexel Univ., 3141 Chestnut St., Philadelphia, PA 19104.

[2] Assistant Professor, Dept. of Civil, Architectural, and Environmental Engineering, Drexel Univ., 3141 Chestnut St., Philadelphia, PA 19104.

[3] Senior Associate, Schnabel Engineering North, LLC, 510 East Gay St., West Chester, PA 19380.

[4] Professor, Dept. of Civil and Environmental Engineering, Univ. of Maine, 5711 Boardman Hall, Orono, ME 04469.

[5] Graduate Student Researcher, Dept. of Civil, Architectural, and Environmental Engineering, Drexel Univ., 3141 Chestnut St., Philadelphia, PA 19104.

Note. Associate Editor: Louay N. Mohammad. Discussion open until August 1, 2007. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on August 30, 2005; approved on November 15, 2005. This paper is part of the ***Journal of Materials in Civil Engineering***, Vol. 19, No. 3, March 1, 2007. ©ASCE, ISSN 0899-1561/2007/3-233–241/$25.00.

develop a database of engineering properties from the available literature on experimental studies, then perform statistical analyses to assess variability in the properties of TDA and compare this to the inherent variability in parameters of traditional soils. To investigate the scale-dependency, the relationship between tire particle size and engineering parameter value was studied. This work is an expansion of an ASCE conference publication (Strenk et al. 2005) augmented with new experimental data (Natale 2005), a larger database that includes a wider range of TDA particle sizes, inclusion of additional engineering parameters, and additional analyses and interpretation.

Data Set Development

Preliminary data sets were created for five fundamental geotechnical engineering parameters typically used in design applications involving TDA: Compacted dry unit weight (γ), secant constrained modulus (M_{sec}), modified secondary compression index ($C_{\alpha\varepsilon}$), and the shear strength parameters cohesion (c), and angle of friction (ϕ). The TDA characteristics (e.g., particle sizes), laboratory testing method, and data interpretation procedures were also noted.

The final data sets for the five properties were developed using consistent criteria specific to each individual engineering parameter. The compacted unit weight data were collected from experimental programs that used impact energy laboratory compaction methods (standard and modified Proctor). For the shear strength properties, the final data sets consisted of friction angle and cohesion values taken from both triaxial compression and direct shear tests. As TDA is a strain-hardening material, the strength parameters were developed based on the shear strengths measured at 10% axial strain (for triaxial tests) or at displacements corresponding to a maximum of 10% of the shear box dimension (for direct shear tests). The shear strength parameters were derived from testing at normal stresses in the range of 1–100 kPa. At this stress range, the shear strength failure envelope is generally nonlinear, and hence c and ϕ are, in part, a function of the normal stress at which they are evaluated. As such, in using failure strain as the primary criterion, the c and ϕ data sets include differing normal stress ranges. For the constrained modulus, the data set consisted of *secant modulus* (M_{sec}) values calculated from one-dimensional (1D) constrained compressibility test data. This parameter serves as a useful index for characterizing the compressibility of TDA. For this study secant constrained modulus was defined as

$$M_{\mathrm{sec}} = \frac{\Delta\sigma_v}{\Delta\varepsilon_v} \qquad (1)$$

where $\Delta\sigma_v=50$ kPa and $\Delta\varepsilon_v$=change in vertical strain that occurs between 0 and 50 kPa. A vertical stress of 50 kPa was taken to be representative of a typical overburden stress in TDA for a variety of common geotechnical applications. For the modified secondary compression index, which describes time-dependent deformation of TDA (Wartman et al. 2007), the final data set consisted of long-term settlement data from 1D constrained compressibility tests. For this study, the modified secondary compression index was defined as

$$C_{\alpha\varepsilon} = \frac{\Delta\varepsilon_t}{\log \dfrac{t_2}{t_1}} \qquad (2)$$

where $\Delta\varepsilon_t$=time-dependent volumetric strain; t_1=initial time of time-dependent compression (assumed to be 1 day); and t_2=final time at which time-dependent compression is measured (typically taken as the time at the end of the test, which ranged from 5 to 100 days). Values of $C_{\alpha\varepsilon}$ were derived from tests performed over a range of stress levels (10–200 kPa); however, as long term compression in TDA is independent of stress level (Wartman et al.2007) this should not have had any effect on the results.

For the scale-dependency analyses, the engineering properties and tire particle size ranges were used. The average grain size (D_{50}) of soil is often used as a representative parameter to characterize a soil's particle size; however, few of the test studies provided this information. An alternative particle size parameter, the maximum particle size, $D_{\max}$, was cited in each study, and accordingly, this parameter was adopted to describe TDA particle size.

Statistical Analyses

Measures of central tendency (mean, median, and mode) and variation (standard deviation, variance) were estimated for each of the data sets assuming that they represented the entire population (n) rather than a sample ($n-1$). Using these results, variability was calculated in terms of the coefficient of variation (COV). Each of the data sets were then separated according to the three ASTM D 6270 size classifications (tire shreds, tire chips, and granulated rubber) and the mean, standard deviation, and COV were computed for each classification.

A series of linear regression models were then fitted to the five data sets. In all cases a linear regression model provided the best fit between the engineering parameter and maximum tire particle size. These regression models were used for residual analyses to confirm both the homoscedasticity of the data and the normality of the residuals (Draper and Smith 1966). The coefficient of correlation (ρ) and the coefficient of determination (R^2) were also determined for the regression models. T-tests evaluated at a 95% confidence level were also performed on ρ to determine whether or not the correlations were statistically significant (Draper and Smith 1966). The term "statistically significant" only implies that the correlation between the independent and dependent variables is unlikely to be a result of random inherent variability.

Results and Discussion

TDA Variability

Table 1 presents the calculated values of mean, standard deviation, and COV for the five engineering properties considered in this study. Cohesion and unit weight have the highest and lowest values of COV, respectively. The total variability associated with constrained modulus, friction angle, and modified secondary compression index are similar and slightly lower than that of cohesion.

Phoon and Kulhawy (1996, 1999a,b) attributed the variability of engineering properties of soils to several sources including inherent material variability, laboratory or in situ measurement

Table 1. Central Tendency and Variability for the Finalized Data Sets

	Statistical properties				
Property	Data set size (n)	Range (Min.–Max.)	Mean ($\bar{x}$)	Standard deviation (σ)	COV ($\sigma/\bar{x}$) (%)
γ (kN/m^3)	**22**	**4.71–6.41**	**5.79**	**0.5**	**8**
Granulated rubber	4	4.95–5.88	5.48	0.4	7
Tire chips	8	5.13–6.41	5.91	0.4	7
Tire shreds	10	4.71–6.30	5.83	0.5	9
c (kPa)[a]	**15**	**3.3–25.4**	**13.8**	**7.4**	**54**
Granulated rubber	1	21.6	21.6	0	0
Tire chips	7	3.3–25.4	17.2	8.1	47
Tire shreds	7	4.3–13.2	9.3	3.6	38
ϕ (degrees)[b]	**23**	**6–39**	**21.8[c]**	**8.5**	**39**
Granulated rubber	6	6–32	18.8[c]	8.7	46
Tire chips	9	11–38	19.1	8.0	42
Tire shreds	8	19–39	27.0	6.2	23
M_{sec} (kPa)	**17**	**130–485**	**280**	**95**	**34**
Granulated rubber	—	—	—	—	—
Tire chips	7	253–485	354	76	22
Tire shreds	10	130–373	229	70	31
$C_{\alpha\varepsilon}$	**14**	**0.0038–0.0128**	**0.0070**	**0.0021**	**30**
Granulated rubber	—	—	—	—	—
Tire chips	7	0.0051–0.0090	0.0068	0.0011	16
Tire shreds	7	0.0038–0.00128	0.0071	0.0028	39

Note: COV=coefficient of variation.

[a] Cohesion values reported for normal stresses <100 kPa.

[b] Majority of friction angle values reported at normal stress <100 kPa. One data point (granulated rubber, $\phi=6°$) from Masad et al. (1996) was evaluated at 150–350 kPa.

[c] Excluding Masad et al. (1996), total $\bar{x}$, σ, and COV change by +3, −6, and −9%; granulated rubber $\bar{x}$, σ, and COV change by +14, −17, and −27%.

error and transformation error from correlations. For TDA, the first two sources of variability are applicable. Inherent material variability includes differences in the source tires and different processing methods. The former is generally small since passenger and light truck tires comprise approximately 90% of discarded tires, as determined from statistics presented by the Rubber Manufacturers Association (2004). Although some tires are glass belted, these are generally diverted to other recycling options, leaving steel belted radial passenger and light truck tires as the predominant source material for TDA. However, differences in processing methods can lead to variability in particle shape and amount of steel belt exposed at the cut edges of the pieces. The high values of COV for constrained modulus and compression index of TDA may be the combined result of several factors: (1) differences in particle shape, exposed steel, and initial density (inherent variability); (2) modified existing testing methods and/or equipment to test a nonconventional engineering material (measurement error); and (3) the limited size of the data sets. The high values of COV for the shear strength parameters may be due to these factors, and owing to the nonlinearity in the shear strength failure envelope, to differences in the normal stress ranges used for the evaluation of c and ϕ.

Phoon and Kulhawy (1999a,b) identified soil property variability and COV ranges for several geotechnical parameters as a function of soil type, test method (laboratory and in situ), and empirical correlation. Baecher and Christian (2003) compiled published tables of reported COV values for a wide variety of soil properties. The COV values reported by Phoon and Kulhawy (1999a,b) and Baecher and Christian (2003) pertain to natural soils that are broadly classified as sand, silt or clay. Phoon and Kulhawy (1999a,b) make further distinctions between test method category (laboratory or in situ test); the COV values reported by Baecher and Christian (2003) are more generic with soil type distinctions only. Table 2 compares COV values of four comparable engineering properties of natural soils to TDA. Matching of the test methods used in property evaluation was possible for only a portion of the engineering properties considered. The modified secondary compression index was excluded since equivalent COV values for natural soils are not available.

Table 2 suggests that the COV values calculated for TDA herein are generally similar to those of traditional soils. For unit weight and constrained modulus, TDA has property variability that is comparable to that of natural soils. The COV values for TDA shear strength parameters are close to or slightly exceed the maximum values of the ranges reported for comparable natural

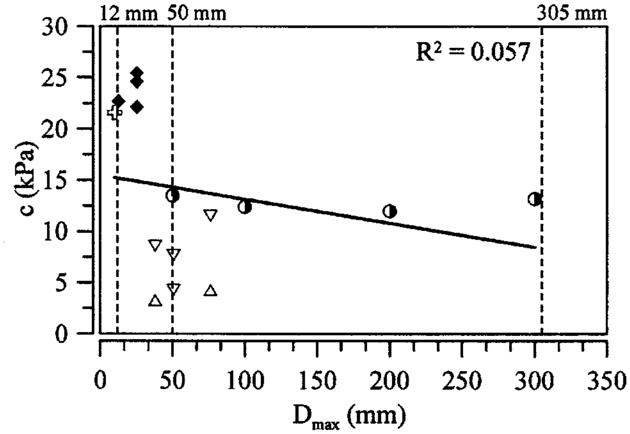
Table 2. Comparison of TDA and Soil Coefficient of Variation

		Natural soil			
TDA		Phoon and Kulhawy (1999a,b)		Baecher and Christian (2003)	
Property	COV (%)	Property	COV (%)	Property	COV (%)
γ	8	γ (lab)	<10	γ	1–10
c	54	s_u (lab)	10–55	s_u	20–50
ϕ	39	ϕ (lab)	7–20	ϕ	5–15
M_{sec}	34	E (in situ)	20–70	E	—

Note: COV=coefficient of variation.

Symbol	Reference(s)
◆	Ahmed 1993, Ahmed and Lovell 1993
◇	Benda 1995
▲	Bernal et al. 1996
△	Cosgrove 1995
●	Drescher and Newcomb 1994
○	Edil and Bosscher 1992, 1994
■	Heimdahl 1998
□	Humphrey et al. 1992
▼	Humphrey et al. 1993, Humphrey and Sandford 1993
▽	Manion and Humphrey 1992, Humphrey and Manion 1992
✕	Masad et al. 1996
◑	Moo-Young et al. 2003
◨	Natale 2005
+	Tatilsoz 1996, Tatilsoz et al. 1997
✙	Yang et al. 2002

(a)



(b)

Fig. 1. (a) Data source legend for Figs. 1(b) and 2–5 and (b) TDA cohesion (c) versus maximum particle size (D_{max})

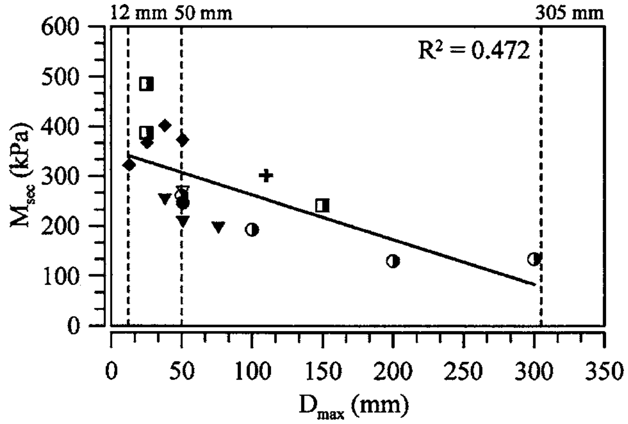


Fig. 3. TDA constrained modulus (M_{sec}) versus maximum particle size (D_{max})

soil parameters (s_u and ϕ). The elevated values of COV for the shear strength parameters are likely a result of the nonlinear failure envelope, or more specifically, the influence of normal stress on the evaluated values of c and ϕ (discussed further in a subsequent section). In spite of this, the overall similarities in variability between TDA and natural soils should provide designers with a reasonable degree of confidence during the design process.

TDA Scale-Dependency

Figs. 1(b) and 2–5 consider scale-dependency of the five engineering parameters as a function of maximum TDA particle size [see Fig. 1(a) for the data source legend]. Trends for c, γ, and M_{sec} decrease, while ϕ increases with increasing particle size [see Figs. 1(b) and 2–4]. The opposing trend between ϕ and c may be due to a nonlinear failure envelope, and/or the systematic decrease in normal stress with increased particle size. Fig. 5 suggests that $C_{\alpha\varepsilon}$ is not scale-dependent for TDA.

Table 3 summarizes the coefficient of correlation (ρ) and variability (R^2) for the linear regressions provided in Figs. 1(b) and 2–5. The correlation coefficient ($+1 \leqslant \rho \leqslant -1$) only implies the level of association (or disassociation) between two variables, not a causal relationship. For the linear regression model, the R^2 value reflects the portion of variation in the dependent variable

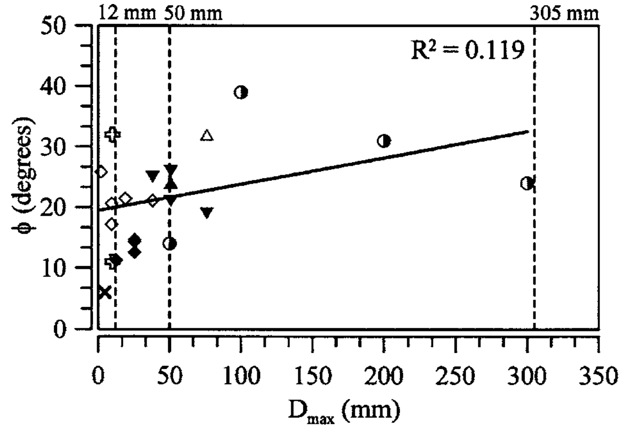


Fig. 2. TDA friction angle (ϕ) versus maximum particle size (D_{max})

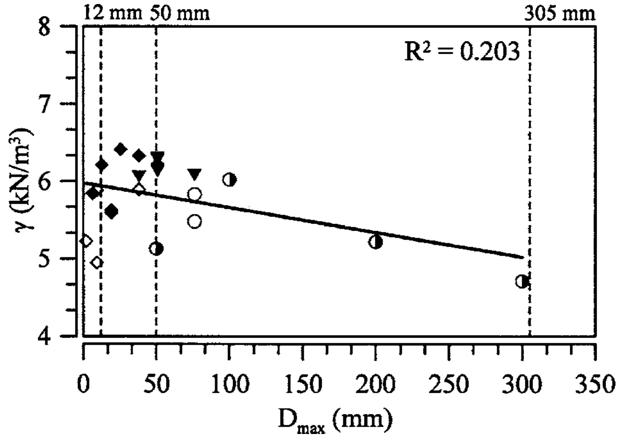


Fig. 4. TDA compacted unit weight (γ) versus maximum particle size (D_{max})

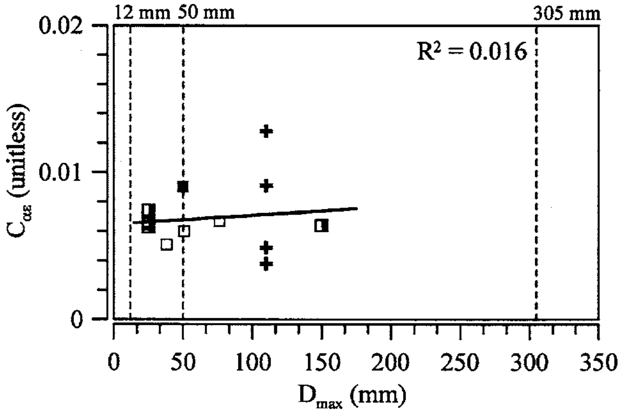


Fig. 5. TDA modified secondary compression index ($C_{\alpha\varepsilon}$) versus maximum particle size (D_{max})

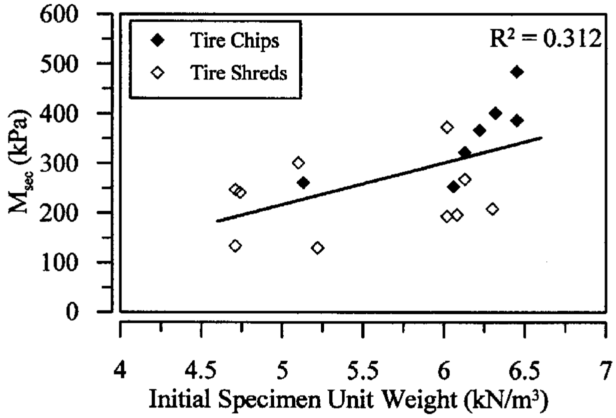


Fig. 6. Secant constrained modulus (M_{sec}) versus initial TDA specimen unit weight (γ)

($\gamma, c, \phi, M_{sec}, C_{\alpha\varepsilon}$) that is "explained" by (or attributed to) the independent variable D_{max}. The "unexplained" variation (or $1-R^2$) is the portion of variation in the dependent variable that is attributed to factors other than D_{max}. In practical terms, these other factors can be tire types (car/truck or glass/steel belted), particle geometry or aspect ratio, amount of exposed steel belting, differences in initial specimen density, differences in confining or normal stress, loading/boundary conditions or differences introduced by the nonstandardized and/or modified laboratory testing methods and equipment used in the test programs.

For cohesion intercept, friction angle, and modified secondary compression index, variability is explained by factors other than those directly accounted for by the linear correlation to D_{max}, and as such, these three properties appear to be relatively independent of the tire particle size. Based on the model trends observed for the shear strength parameters [Figs. 1(b) and 2], this conclusion may seem contrary, however a statistical interpretation suggests that the cohesion intercept and friction angle are unrelated to D_{max}. The results from the *t*-test performed at a 95% confidence level provide further evidence for the relationship suggested by the ρ and R^2 statistics for c, ϕ, $C_{\alpha\varepsilon}$, and D_{max}. Unit weight and constrained modulus appear more closely related to D_{max} than other factors as suggested by the ρ and R^2 statistics and the *t*-test results. However, neither parameter incorporates the particle size as the single dominating (or controlling) factor as evidenced by high the unexplained variability ($1-R^2$). Overall, it appears that γ and M_{sec} are more dependent (sensitive to) changes in D_{max} than c, ϕ, and $C_{\alpha\varepsilon}$.

As TDA is a manufactured product, the compaction characteristics of TDA can be influenced by the shredding process, tire type, reinforcement material, and particle geometry. Understanding these factors can provide insight into the mechanisms involved in the scale-dependency of compacted unit weight. For example, Humphrey et al. (1992) found that TDA from glass-belted tires had a lower specific gravity (G_s) than TDA from steel-belted tires. Practical experience of the writers suggests that processing issues such as shredder blade sharpness and repeated shredding of oversize particles can contribute to the quantity and length of exposed steel belting. Wartman et al. (2007) recognized a similar processing-related trend and moreover suggested that a greater content of exposed steel can lead to particle entanglement producing a "bridging effect" between particles. In terms of particle geometry, tire shreds tend to be elongated and flat with high aspect ratios and in combination with their larger size can produce larger, more frequent void spaces and thus a lower compacted unit weight. On the contrary, equidimensional tire chips with their smaller size and more uniform gradation tend to achieve more efficient packing, producing smaller, less frequent void spaces and thus a higher compacted unit weight.

Table 3. Linear Regression Results for the Finalized TDA Data Sets

	Statistical properties			
Property	Correlation coefficient (ρ)	Explained variability (R^2) (%)	Unexplained variability ($1-R^2$) (%)	Statistically significant? (*t*-test)
γ	**−0.45**	**20.3**	**79.7**	**Yes**
c	−0.24	5.7	94.3	No
ϕ	0.34	11.9	88.1	No
M_{sec}	**−0.69**	**47.2**	**52.8**	**Yes**
$C_{\alpha\varepsilon}$	0.08	1.6	98.4	No

Fig. 6 shows the relationship between initial specimen unit weight and the constrained modulus and implies that TDA size is inversely related to γ and M_{sec}. Initial specimen unit weight was excluded as part of the criteria for developing the data set in order to obtain a sufficiently large data set. Assuming that the trends shown in Fig. 3 are representative, the values of M_{sec} derived from small TDA will be inherently unconservative. That is, the field stiffness of tired shred TDA will be less than the measured laboratory stiffness of small TDA as supported by experimental results of Natale (2005) and Moo-Young et al. (2003). However, evidence from other research (Ahmed 1993; Humphrey and Sandford 1993; Manion and Humphrey 1992) and actual field behavior of TDA projects (Humphrey et al. 2000) conflict with the trends shown in Fig. 3.

Limitations of Statistical Analyses

Considering Figs. 1(b) and 2–5 as a whole, it becomes apparent that while correlation is suggested by high R^2 values that are also statistically significant (*t*-test), this does not assure a useful equation for design purposes. The unexplained variation can remain large, making estimations or predictions all together too inaccurate to be useful. Hence, it is important to keep in mind that statements such as explained or unexplained variation are not inferring causality, but rather the observed tendencies that are based solely on the nature of the regression model (Berthouex and

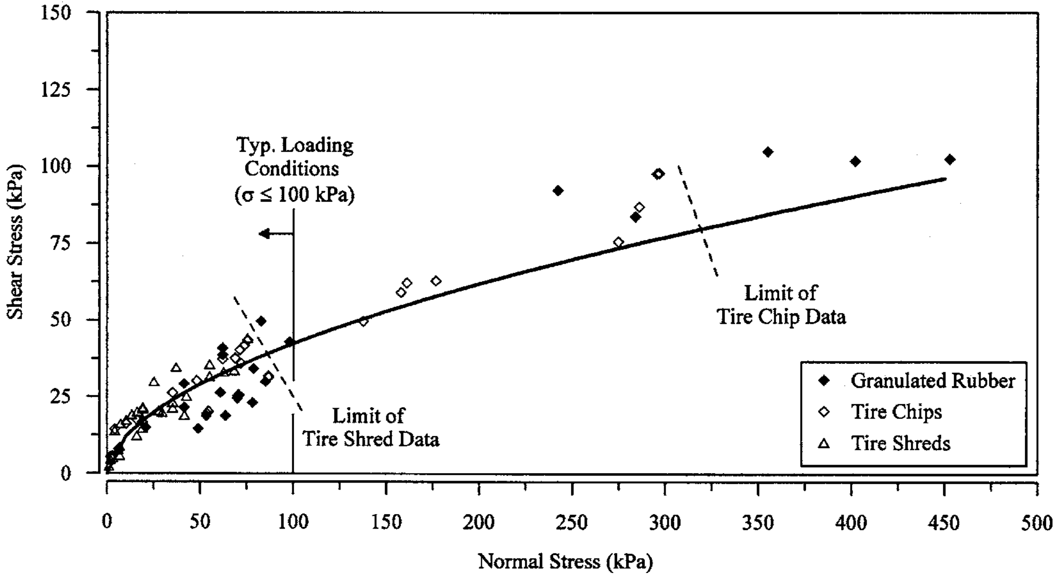


Fig. 7. Failure envelope plot from combined direct shear and triaxial tests results

Brown 1994). Inferring a causal relationship involves an understanding of the fundamental mechanisms under examination which are best elucidated by executing well-planned experiments with rigorous statistical design controls as opposed to conducting correlation or regression analyses on data from unassociated experiments.

As stated earlier, the normal stresses for the TDA shear strength parameters used in the variability and scale-dependency analyses are typically less than 100 kPa and are generally characterized by a nonlinear shear strength failure envelope where c and ϕ are a function of the normal stress conditions. Given this complex interdependent relationship, statistical-based analyses can "compress" the data to the point where important dependencies are not expressed in the result and have limited practical meaning. Accordingly, given the ambiguity that may arise from pure statistical analyses, the shear strength parameters of TDA were examined in a nonstatistical framework to provide clarity into the variability and scale-dependency trends observed in Table 1 and Figs. 1(b) and 2, respectively.

TDA Shear Strength Parameters

Data for this portion of the analysis were gathered from a combination of sources including interpolated values from published plots, backcalculated values from the literature, and selected final data set values [Fig. 1(a) references]. The failure envelope data include both direct shear and triaxial compression test data for failure strains of 10%. Fig. 7 shows the nonlinear relationship between normal and shear stress for applied normal stress in the range of 1–450 kPa.

For geotechnical applications of TDA it is important to properly anticipate the stress ranges to gauge compressibility effects. The design guidelines provided by ASTM D 6270 suggest a maximum tire layer thickness of 3 m (Class II Fill) to mitigate internal heating of the TDA. For a typical roadway embankment with a single encased layer of TDA and a 2 m cover of compacted soil, the maximum overburden stress at the bottom of the tire layer is approximately 50 kPa. Extending this boundary to 100 kPa, we obtain an upper bound on a reasonable overburden stress range that would be typical of most embankment configurations using TDA. As shown in Fig. 7, the tire shred data are limited to normal stresses of 75 kPa, whereas granulated rubber has been tested to under normal stresses as great as 450 kPa. This likely reflects testing equipment limitations that prevented the large-size tire shreds from being tested at higher confining stresses. This trend is particularly pertinent to the triaxial compression test as no shear strength data were available for tire shreds. As conventional equipment prevents tire shreds from being tested at high normal stresses, the tire shred data plot in the steeper portion of the nonlinear failure envelope (near the origin) which yield higher ϕ values and lower c values (as shown in Fig. 7). However, since granulated rubber can be tested to higher normal stresses with conventional soil testing equipment, the data will plot along a much wider range of normal stresses resulting in lower ϕ values and higher c values. The relationship just described is similar to the opposing trends observed in Figs. 1(b) and 2.

The data from the lower stress ranges of Fig. 7 (<100 kPa) were replotted to demonstrate the influence of Mohr-Coulomb failure envelope nonlinearity on shear strength parameter variability (Fig. 8). Unlike the statistical analysis of variability, the influence of normal stress can be isolated and the trends in c and ϕ can be examined. The data for the lower stress range (0–30 kPa) plot along the steep portion of the nonlinear failure envelope (high ϕ, low c) whereas the higher stress range data (30–100 kPa) plot on the more gentle portion (low ϕ, high c). The failure envelope for the lower stress range shows a stronger correlation ($R^2=0.664$) in comparison to the higher stress range ($R^2=0.286$). For the piecewise linear failure envelopes, the R^2 values can be used as an index to suggest that variability of c and ϕ would change depending on the normal stress used for their evaluation. Thus, it is likely that lower COV values may have been calculated for the shear strength parameter data sets had it been possible to use a consistent normal stress range for the statistical analysis.

The data from Fig. 8 (<100 kPa) was replotted to show the effect of test method on Mohr-Coulomb failure envelopes (Fig. 9). The triaxial test data yielded lower strengths as compared with the direct shear test data. Similar trends are typically observed for soils (Bardet 1997). The Mohr-Coulomb failure envelope for the

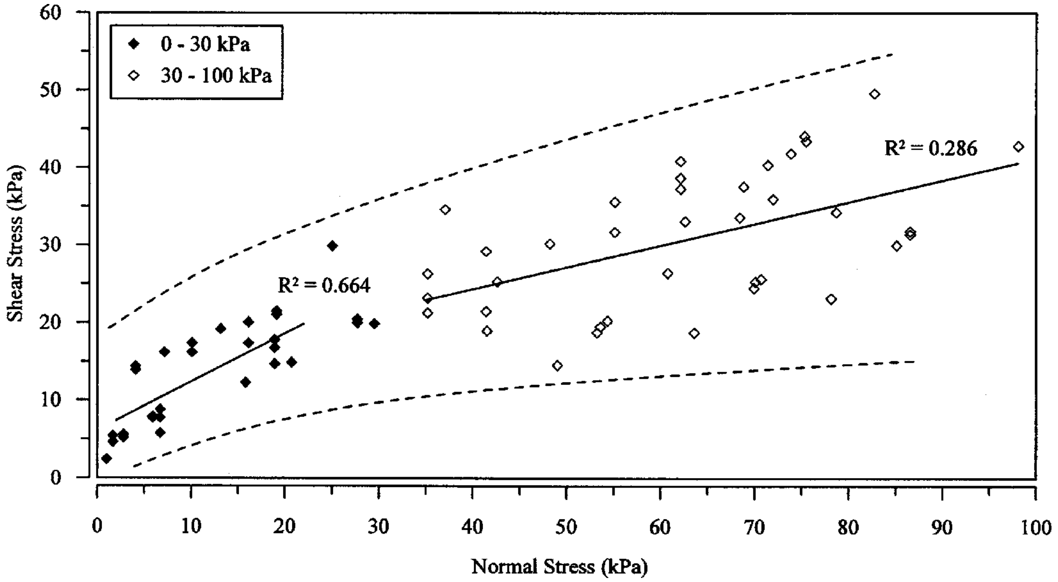


Fig. 8. Mohr-Coulomb failure envelope for TDA according to normal stress ranges

direct shear data shows a very strong correlation ($R^2=0.876$) yielding $\phi=28°$ and the distribution of data along the trend line appears uniform for all three size classifications. This suggests a scale-independent relationship for the shear strength parameters. The failure envelope for the triaxial test data is not correlated as strongly as the direct shear data ($R^2=0.488$). The data scatter for the triaxial test seems to indicate scale independence as well; however, this conclusion is limited since tire shred data were unavailable.

It is apparent from Figs. 8 and 9 that the Mohr-Coulomb framework provides more clarity and insight into the factors that influence c and ϕ. In other words, a statistical approach alone does not always tease out the dependencies in a population set that would be self-evident by other evaluation techniques or frameworks.

Practical Implications and Conclusions

The variability of five basic engineering properties $(\gamma, c, \phi, M_{\mathrm{sec}}, C_{\alpha\varepsilon})$ of TDA were considered in this study. Statistical analyses indicated the highest and lowest COV values were associated with cohesion (54%) and unit weight (8%). Secant constrained modulus, friction angle, and modified compression index all had similar COV values that were in between this range (34, 39, and 30%). The COV values for shear strength parameters ϕ and c were on the high end of the ranges reported for natural soils, whereas the other engineering properties had COV values comparable to soil. Additional insight into the variability of the shear strength parameters was made by analysis of shear strength failure envelopes. Trends suggest that variability of c and ϕ is highly dependent on normal stress range at which they are evalu-

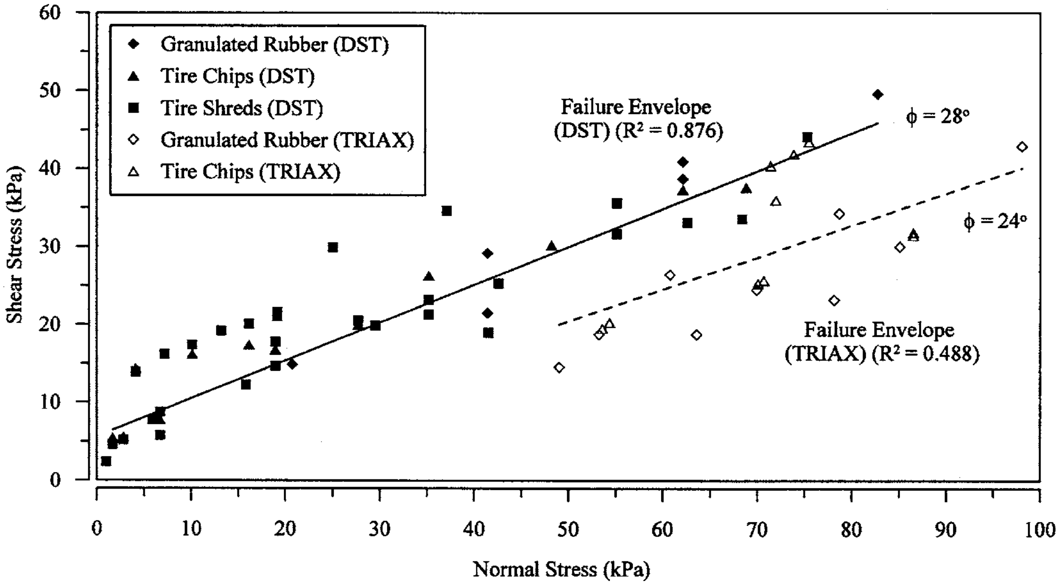


Fig. 9. Mohr-Coulomb failure envelope for TDA based on separated direct shear (DST) and triaxial (TRIAX) test data

Table 4. Proposed Design Parameters for Tire Shreds

Property	Mean value ($\bar{x}$)[a]	Design value	Justification
γ (kN/m^3)	5.83	7.86	-[b]
c (kPa)	9.3	0	Fig. 9: high R^2
ϕ (degrees)	27	25	Fig. 9 high R^2
$C_{\alpha\varepsilon}$ (unitless)	0.0071	0.008	Table 1: $\bar{x}$ and low R^2

[a] Mean value from Table 1 (reported for tire shreds only).

[b] This is a default design value that should be adjusted to obtain the compressed, in-place unit weight (Humphrey 2004a).

ated and that COV values in Table 1 are likely to be lower. Overall, the COV values determined for the engineering properties indicate a total variability that is for practical purposes equivalent to natural soils. Thus designers should realize that the level of variability of TDA, a manufactured product, is quite similar to that of natural soils.

The regression modeling indicated that unit weight and constrained modulus are related to tire particle size. Cohesion, friction angle, and modified secondary compression index do not appear to vary with particle size; that is, tests on small scale specimens can be taken to be representative of larger TDA particles. It should be noted that these conclusions are based on statistical quantities (specifically ρ, R^2, and the t-test for correlation), that do not imply causality between tire particle size and engineering properties. Further insight into the scale-dependency relationships of the shear strength parameters was made by critical analysis of shear strength failure envelopes. The trends of the failure envelope suggested a scale-independent relationship for normal stresses less than 100 kPa.

Design value(s) for four of the engineering properties considered in this study are proposed in Table 4. The design value for $C_{\alpha\varepsilon}$ is a conservative value based on the mean value developed for the data set (Table 1). Based on Fig. 9, a strong correlation was found between direct shear test data (c and ϕ) and all three size classifications using the Mohr-Coulomb model. The recommended design value of 25° (Table 4) is proposed based on the mean value in Table 1 as well as the findings shown in Fig. 9 (corrected/lowered for differences between direct shear and triaxial testing and for data set limitations). Although a relatively modest level of cohesion was considered in this study, this strength component is typically neglected ($c=0$) in practice owing to its small contribution to overall strength and because large strains are usually required to mobilize peak values of cohesion (Humphrey and Sandford 1993). The proposed cohesion intercept value in Table 4 is based on this rationale. The scale-independence reflected in the failure envelopes of Figs. 7 and 9 is consistent with the statistical findings for the cohesion intercept and friction angle.

It is important to note that the data set of unit weights used in the statistical analyses consists of laboratory-derived "uncompressed" compacted unit weight values, not actual design values. To obtain design values of unit weight it is necessary to consider the compressibility of TDA, as the in-place or "compressed" unit weight of TDA increases with overburden pressure. Humphrey (2004a) describes an iterative procedure that accounts for the compressibility of TDA when estimating the in-place unit weight. In this procedure, compressibility of a tire shred layer under its own self-weight and under the weight of overlying material is incorporated by utilizing laboratory-measured 1D constrained compressibility data (e.g., Manion and Humphrey 1992; Humphrey and Manion 1992; Humphrey et al. 1993; Humphrey and Sandford 1993; Nickels 1995; Natale 2005). The evaluation of in-place, compressed unit weight is typically undertaken on a project-specific basis; however, field experience has shown that an in-place unit weight of 7.86 kN/m^3 (50 lb/ft^3) can be assumed for the preliminary design of TDA structures with maximum particle sizes between 152 and 305 mm (ASTM 1998; Dickson et al. 2001; Humphrey 2004a).

Due to the complex nature of constrained modulus, data set limitations, and conflicting evidence from other researchers, it is difficult to make definitive conclusions regarding the scale-dependency of this engineering property. Additional research in the form of a well-designed experimental testing program is warranted to properly isolate variables and examine the dependencies of this elastic parameter.

Acknowledgments

Financial support for this research was provided in part by the National Science Foundation under Grant No. CMS-0134370. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the writers and do not necessarily reflect the views of the National Science Foundation. Additional financial support for this research was provided in part by the Koerner Fellowship at Drexel University, Philadelphia.

Notation

The following symbols are used in this paper:

$C_{\alpha\varepsilon}$ = modified secondary compression index;
COV = coefficient of variation;
c = cohesion (or cohesion intercept);
$D_{\max}$ = maximum TDA particle size;
D_{50} = average TDA (or soil) particle size;
E = Young's modulus;
G_s = specific gravity;
$M_{\sec}$ = secant constrained modulus;
n = number of values in the data set;
R^2 = coefficient of determination;
s_u = undrained shear strength;
t_i = time;
$\bar{x}$ = mean;
γ = dry compacted unit weight;
$\Delta\varepsilon_t$ = time-dependent strain;
$\Delta\varepsilon_v$ = vertical strain;
$\Delta\sigma_v$ = vertical stress;
ρ = coefficient of correlation;
σ = standard deviation; and
ϕ = angle of internal friction.

References

Ahmed, I. (1993). "Laboratory study on properties of rubber soils." *Rep. No. FHWA/IN/JHRP-93/4*, Dept. of Civil Engineering, Purdue Univ., West Lafayette, Ind.

Ahmed, I., and Lovell, C. W. (1993). "Rubber soils as lightweight geomaterial." *Transportation Research Record. 1422*, National Research Council, Transportation Research Board, Washington, D.C., 61–70.

ASTM. (1998). "Standard practice for use of scrap tires in civil engineering applications." *D 6270-98*, West Conshohocken, Pa.

Baecher, G. B., and Christian, J. T. (2003). *Reliability and statistics in geotechnical engineering*, Wiley, New York.

Bardet, J. P. (1997). *Experimental soil mechanics*, Prentice-Hall, Upper Saddle River, N.J.

Benda, C. C. (1995). "Engineering properties of scrap tires used in geotechnical applications." *Rep. No. 95-1*, Materials and Research Division, Vermont Agency of Transportation, Montpelier, Vt.

Bernal, A., Lovell, C. W., and Salgado, R. (1996). "Laboratory study on the use of tire shreds and rubber-sand in backfills and reinforced soil applications." *Rep. No. FHWA/IN/JHRP-96/12*, Dept. of Civil Engineering, Purdue Univ., West Lafayette, Ind.

Berthouex, P. M., and Brown, L. C. (1994). *Statistics for environmental engineers*, Lewis, Boca Raton, Fla.

Cosgrove, T. A. (1995). "Interface strength between tire shreds and geomembrane for use as a drainage layer in a landfill cover." *Proc., Geosynthetics '95*, Industrial Fabrics Association, St. Paul, Minn., 3, 1157–1168.

Dickson, T. H., Dwyer, D. F., and Humphrey, D. N. (2001). "Prototype tire-shred embankment construction." *Transportation Research Record. 1755*, National Research Council, Transportation Research Board, Washington, D.C., 160–167.

Draper, N. R., and Smith, H. (1966). *Applied regression analysis*, Wiley, New York.

Drescher, A., and Newcomb, D. E. (1994). "Development of design guidelines for use of shredded tires as lightweight fill in road subgrade and retaining walls." *Rep. No. MN/Rc-94/04*, Minnesota Dept. of Transportation, St. Paul, Minn.

Edil, T., and Bosscher, P. (1992). "Development of engineering criteria for shredded waste tires in highway applications." *Research Rep. No. GT-92-9*, Wisconsin Dept. of Transportation, Madison, Wis.

Edil, T. B., and Bosscher, P. J. (1994). "Engineering properties of tire chips and soil mixtures." *Geotech. Test. J.*, 17(4), 453–464.

Heimdahl, T. C. (1998). "Deformability of shredded tires." MSCE thesis, Univ. of Minnesota, St. Paul, Minn.

Humphrey, D. N. (2004a). "Civil engineering applications of tire shreds." *California Integrated Waste Management Board*, California Environmental Protection Agency, Sacramento, Calif.

Humphrey, D. N. (2004b). "Tire derived aggregate—A new geomaterial for your toolbox." *Geo-Strata-Geo Institute of ASCE*, 5(2), 20–22.

Humphrey, D. N., and Manion, W. P. (1992). "Properties of tire chips for lightweight fill." *Proc., Grouting, Soil Improvement, and Geosynthetics*, ASCE, New York, 1344–1355.

Humphrey, D., and Sandford, T. (1993). "Tire chips as lightweight subgrade fill and retaining wall backfill." *Proc., Symp. on Recovery and Effective Reuse of Discarded Materials and By-Products for Construction of Highway, Facilities*, Federal Highway Administration, Denver.

Humphrey, D. N., Sandford, T. C., Cribbs, M. M., Gharegrat, H. G., and Manion, W. P. (1992). "Tire chips as light-weight backfill for retaining walls—Phase I." *A Study for the New England Transportation Consortium*, Dept. of Civil Engineering, Univ. of Maine, Orono, Me.

Humphrey, D. N., Sandford, T. C., Cribbs, M. M., and Manion, W. P. (1993). "Shear strength and compressibility of tire chips for use as retaining wall backfill." *Transportation Research Record. 1422*, National Research Council, Transportation Research Board, Washington, D.C., 29–35.

Humphrey, D. N., Whetten, N., Weaver, J., and Recker, K. (2000). "Tire shreds as lightweight fill for construction on weak marine clay." *Proc., Int. Symp. on Coastal Geotechnical Engineering in Practice*, Balkema, Rotterdam, The Netherlands, 611–616.

Manion, W. P., and Humphrey, D. N. (1992). "Use of tire chips as lightweight and conventional embankment fill, Phase I—Laboratory." *Tech. Paper 91-1*, Technical Services Division, Maine Dept. of Transportation, Augusta, Me.

Masad, E., Taha, R., Ho, C., and Papagiannakis, T. (1996). "Engineering properties of tire/soil mixtures as lightweight fill material." *Geotech. Test. J.*, 19(3), 297–304.

Moo-Young, H., Sellasie, K., Zeroka, D., and Sabnis, G. (2003). "Physical and chemical properties of recycled tire shreds for use in construction." *J. Environ. Eng.*, 129(10), 921–929.

Natale, M. F. (2005). "Compressibility characteristics of shredded tire-sand mixtures." MSCE thesis, Drexel Univ., Philadelphia.

Nickels, W. F., Jr. (1995). "The effect of tire shreds as subgrade fill on paved roads." MSCE thesis, Univ. of Maine, Orono, Me.

Phoon, K. K., and Kulhawy, F. H. (1996). "On quantifying inherent soil variability." *Uncertainty in the geologic environment*, C. D. Shackelford, P. P. Nelson, and M. J. S. Roth, eds., Geotechnical Special Publications No. 58, ASCE, Madison, Wis., 326–340.

Phoon, K. K., and Kulhawy, F. H. (1999a). "Characterization of geotechnical variability." *Can. Geotech. J.*, 36(4), 612–624.

Phoon, K. K., and Kulhaway, F. H. (1999b). "Evaluation of geotechnical property variability." *Can. Geotech. J.*, 36(4), 625–639.

Rubber Manufacturers Association (RMA). (2004). *U.S. scrap tire markets—2003 edition*, Washington, D.C.

Strenk, P. M., Wartman, J., Grubb, D. G., and Humphrey, D. N. (2005). "Variability and scale-dependency of recycled rubber tire materials." *Proc., Recycled Materials in Geotechnics*, *Geotechnical Special Publication No. 127*, ASCE, Reston, Va., 105–122.

Tatilsoz, N. (1996). "Using tire chips in earthen structures." MSCE thesis, Univ. of Wisconsin, Madison, Wis.

Tatilsoz, N., Benson, C. H., and Edil, T. B. (1997). "Effect of fines on mechanical properties of soil-tire chip mixtures." *Testing soil mixed with waste or recycled materials*, STP 1275, M. Wasemiller and K. Hoddinott, eds., ASTM, West Conshohocken, Pa., 93–108.

Wartman, J., Natale, M. F., and Strenk, P. M. (2007). "Immediate and time-dependent compression of tire derived aggregate." *J. Geotech. Geoenviron. Eng.*, 133(3), 245–256.

Yang, S., Lohnes, R. A., and Kjartanson, B. H. (2002). "Mechanical properties of shredded tires." *Geotech. Test. J.*, 25(1), 44–52.

Youwai, S., and Bergado, D. T. (2003). "Strength and deformation characteristics of shredded rubber tire-sand mixtures." *Can. Geotech. J.*, 40(1), 254–264.

Zornberg, J. G., Cabral, A. R., and Chardphoom, V. (2004). "Behavior of tire shred-sand mixtures." *Can. Geotech. J.*, 41(1), 227–241.