Integrated Production, Inventory and Pricing Decisions in Two-Echelon Supply Chains

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Dedications

I dedicate this thesis to my family.

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Abstract

Integrated Production, Inventory and Pricing Decisions
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This thesis investigates the optimal decisions for maximizing the expected profit level of a supply chain when market demand is price-sensitive. We examine two-echelon supply chains consisting of a single manufacturer and one or more retailers, where the organizations simultaneously determine the retail price, production lot size and inventory replenishment schedules to maximize the profit of the entire supply chain. In particular, we first develop a model for single-manufacturer single-retailer (SMSR) supply chains, assuming deterministic market demand. We then extend the SMSR model to supply chains with multiple-retailers (i.e. SMMR supply chains). Finally, we examine both SMSR and SMMR supply chains in stochastic market demand environments. We show that supply chain cooperation/centralization brings higher profits for the manufacturer and the retailer(s), and benefits retail consumers with a lower retail price. We propose efficient algorithms for our models and illustrate them through numerical examples. Managerial implications and future research directions are also discussed.

1 Introduction

1.1 Motivation

With rapid developments in market globalization and information technology, supply chain management has become one of the most important strategic aspects of organizations and plays an increasingly important role in their business success. Today's competitive environment requires companies to provide better products with lower operational costs for consumers with heightened expectations, which forces companies to continuously find ways to improve their supply chain management practices. Companies have to find effective ways to coordinate the production of goods and/or services, to manage their inventories, and to distribute goods and/or services timely. Many companies, especially in the manufacturing (e.g. Toyota) and the retail sectors (e.g. Wal-Mart), are frequently cited for their excellence in managing their supply chains[1], which contribute significantly towards their overall business success.

Supply chain management is defined by Mentzer et.al (2001) as "the systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole" (Mentzer et.al., 2001, p.18). As this definition indicates, integration of business functions within a company and coordination of businesses within a supply chain are two important ways to improve the overall performance of the entire supply chain.

[1]For example, http://www.businessweek.com/adsections/2005/pdf/0515_supply.pdf

In fact, there is a growing trend in practice towards integrated supply chain management. According to a report by Deloitte Consulting, “extending the supply chain is number one priority” for many large manufacturers and retailers in North America[2] . Organizations increasingly find that they must rely on effective and integrated supply chains to succeed in the global market. Moreover, due to the developments in information and communication technologies, companies are able to exchange production, inventory and market information in a timely manner with each other, which facilitates supply chain integration. Therefore, more and more companies are moving towards building business partnership with other supply chain members, while integrating their own business functions.

As mentioned above, companies can integrate business functions in two directions: cooperating closely over the supply chain (vertical direction) and integrating different business functions within the company (horizontal direction). On the one hand, companies can work closely with other parties within the supply chain to reduce costs and increase the profitability of the whole supply chain. Then, through a coordination mechanism, every party in the supply chain can share the resulting benefits. On the other hand, companies can integrate their decision processes in their business functions, such as production, inventory, and pricing, to reduce the operational costs and increase their own profitability.

Supply chain integration issues have been extensively studied by researchers in the field. In particular, researchers examine supply chains consisting of a manufacturer (vendor) and one or several retailers (buyers), where an item is produced by the manu-

[2]Deloitte Consulting, Energizing the Supply Chain: Trends and Issues in Supply Chain Management, Report, Deloitte Consulting LLC, 1999.

facturer and sold in a price-sensitive market via the retailer(s). One aspect of research on vertical direction integration (i.e., for a given retail price and market demand, how the manufacturer and the retailers work in a cooperative manner to reduce the production and distribution costs for the entire supply chain) is the joint economic lot-sizing problem (JELP). Whereas some research on horizontal direction integration issues (i.e., how the retailer(s) can make joint decisions on its/their pricing and inventory polices to maximize its/their profits) has focused on the joint pricing and inventory control problem. Both these two streams of research have been extensively developed to cover various realistic aspects in real-world supply chain management, such as examining more than two-echelon supply chains, incorporating production capacity, considering delivery lead time, etc. A comprehensive review of these two streams of research is provided in the second section of this thesis.

It is widely demonstrated that both the integration of business functions and the integration (cooperation) over a supply chain can improve the profitability of the entire supply chain. In contrast, if each party in a supply chain works in a non-cooperative manner and tries to maximize its own profit, or if decisions on different business functions are made in isolation, lower profit levels accrue to the supply chain. Thus, researchers have proposed integration methods in vertical and horizontal directions respectively, to ensure higher profit levels for the entire supply chain.

Although research on both vertical and horizontal directions of supply chain integration has been well developed, surprisingly little attention has been paid to integration in these two directions simultaneously. In practice, organizations not only can integrate their own business functions but also can cooperate with each other in the supply chain.

Particularly, in a supply chain consisting of a manufacturer and several retailers facing price-sensitive market demand, the manufacturer and the retailers can work in a cooperative manner to make decisions on production, inventory control and retail pricing simultaneously to ensure higher profitability for the entire supply chain. The traditional JELP models do not address this situation since they assume that market demand is exogenous and is not price dependent. The existing research on the joint pricing and inventory control problems, which focuses on an individual company, fails to take into account the cooperation between the manufacturer and the retailers.

It is likely that integrating the supply chain in these two directions simultaneously can further improve the profit level of the entire supply chain. Therefore, it is meaningful to fill this gap pertaining to the two research streams of supply chain integration and examine the issues of how companies can simultaneously integrate the supply chain in two directions, i.e., integrating business functions within a particular company and cooperating with other parties within the supply chain. Research in this respect is likely not only to contribute to the literature of supply chain integration, but also to provide effective integration methods with important managerial implications for real-world supply chain management practice. Furthermore, in future, this work can be extended in various directions, and become a building block for designing more effective and realistic supply chains.

1.2 Objectives and Research Questions

The main objective of this dissertation is to develop efficient methods to simultaneously integrate a supply chain in both vertical and horizontal directions. In particular, we consider the situation where an item is produced by a manufacturer in lots and each lot is delivered via several shipments to one or several retailers that face a price-sensitive market demand. We attempt to address the issue of how the manufacturer and the retailer(s) can work in a cooperative manner (i.e., a centralized supply chain) to maximize the profit of the entire supply chain by making joint decisions on production lot size (i.e., the number of items to produce in each production batch), delivery schedule, inventory control, and retail price. We compare our results with a supply chain where the manufacturer and the retailers(s) work in a non-cooperative manner (i.e., a decentralized supply chain) in order to show the beneficial effects of centralized decisions.

In the real world, a particular item produced by a manufacturer may be delivered to a single or multiple retailers. Moreover, demands of some items are more predictable, while others are less predictable with higher variability. Thus, it is necessary to examine a number of cases of demand, in order to address the complex concerns in the real world. In this dissertation, we start our analysis with a single-manufacturer single-retailer supply chain, where market demand is deterministic and dependent on the retail price. The major research questions of interest in this case are: (1) How to determine the production lot size, the number of shipments per production lot, and the retail price to maximize the profit of the entire supply chain? (2) What are the effects of such integrated decisions on channel profit, the profit of the manufacturer, the profit of the retailer(s), and the resulting consumer benefit?

When there is more than one retailer in the supply chain, they may have different cost parameters, and, thus, may desire different inventory replenishment policies.

Replenishment schedules of the various retailers should be coordinated and schedule feasibility must be guaranteed, while maximizing the profit of the entire supply chain. Therefore, in addition to the issue of how the manufacturer and the retailers can co-operate with each other to maximize the profit of the entire supply chain, some other interesting questions are also addressed: (1) How the manufacturer can coordinate the inventory replenishment schedules of the retailers? (2) How do the single-retailer and multiple-retailer supply chains differ?

When market demand is stochastic and highly unpredictable, joint decisions involving production, inventory control and pricing become much more complex. When making their decisions, the manufacturer and the retailers must balance the holding costs of safety stocks and possible shortage costs associated with stochastic market demand. Therefore, when market demand is stochastic, the production lot size and the stock replenishment quantity are not constant but vary according to realized market demand in previous periods. We propose a production and inventory policy and develop the methods to find the optimal solution under the proposed policy.

In summary, this dissertation aims to provide analytical models and methods to solve the problem of integrated production, inventory and pricing decisions under various situations. In term of theoretical research, this dissertation attempts to fill a gap in the existing literature and serve as a building block for future research in this important area. In term of practice, this dissertation intents to provide some useful managerial insights and guidelines for action to supply chain managers.

1.3 Contributions

The problem of integrated production, inventory and pricing decisions is simple to state but difficult to solve. As we point out in section §2, there exists little academic work on this problem, although extensive research has been done on the problem of integrated inventory and pricing decisions and the problem of joint production and inventory decisions (i.e. joint economic lot-sizing problem). To our best knowledge, Jokar and Sajadieh (2009) provide the only existing work directly addressing this problem. However, they assume a lot-for-lot policy and thus significantly simplify the problem. Besides, they provide only an approximate solution for single-manufacturer single-retailer supply chains under deterministic market demands.

We believe that one reason for the lack of existing research in this respect involves analytical challenges. The first challenge is that the objective function in the problem, the centralized total channel profit, is not simply convex or concave on the decision variables. Thus it is difficult to derive the optimal solution from the optimality conditions. Particularly, since the number of shipments is limited to be positive integers, the maximization problem becomes more difficult to solve. A second challenge lies in the extension to the scenario of multiple retailers and the stochastic demand environment. Most of existing research focuses on the single manufacturer and single retailer case under deterministic demand. Although some of the existing literature deals with multiple retailers or stochastic demand, most of it is focused on the problem of joint production and inventory decisions or the problem of integrated inventory and pricing decisions. The analyses are usually developed under restrictive assumptions, providing limited insights on how to extend the problem to multiple retailers, as well as to stochastic market demand. As a pioneering work in this area, this dissertation attempts to contribute to both the literature and the managerial practice.

This study contributes to the literature by developing analytical models to address the problem of integrated production, inventory and pricing decisions in two-echelon supply chains. These models fill a gap between the existing works concerning integrated inventory and pricing decisions and those that deal with the joint economic lot-sizing problem. We not only develop a model for the single-manufacturer single-retailer case, but also extend it to single-manufacturer multiple-retailer supply chains, as well as supply chains with stochastic demand. We also discuss potential future research on this area based on our models in developed section §6.

Another potential contribution of this dissertation involves the efficient methods developed for solving the proposed models. We outline some propositions of the maximization problems, which enable us to develop binary-search algorithms to arrive at the optimal retail price. In addition, we provide analytical procedure for determining the remaining decision variable values based on the search-based optimal retail price. Our suggested algorithms to search the optimal retail price are efficient, having a relatively low complexity of $O(log_2 N)$, and can be easily implemented in widely available computational software packages, such as Excel, R and Matlab, which are commonly used in business environments.

In terms of managerial practice, this dissertation offers useful managerial insights on supply chain cooperation and centralized decisions. We show the advantages of a centralized supply chain in term of total channel profit and consumer benefit. We suggest ways to incentivize the manufacturer and the retailers to cooperate with each other, and provide important insights on channel integration and market regulation. For example, our analysis shows that the centralization of supply chains with a relatively powerful manufacturer and a weak retailer brings higher extra social benefit than the centralization of supply chains with a powerful retailer and a weak manufacturer. In other words, channel integration is more beneficial for supply chains dominated by the manufacturer. Additional managerial implications are discussed in section §6.

The rest of this study is organized as follows: in section §2, we provide a comprehensive review of the two major streams of research relevant to this dissertation. We then develop models for centralized and decentralized supply chains, respectively, with deterministic demand for the single-manufacturer single-retailer case in section §3. We extend our analysis to the single-manufacturer multiple-retailer case in section §4, followed by stochastic demand models in section §5. Finally, in section §6, we conclude this thesis by summarizing our findings and providing discussions on the managerial implications of our research, and point to possible future research directions. The notational schema used in this thesis are listed in Appendix 6.2.3. The MATLAB codes used for analytical purpose are listed in Appendix 6.2.3. All the necessary mathematical proofs pertaining to sections 3, 4, and 5 are presented in Appendices 6.2.3, 6.2.3 and 6.2.3, respectively.

2 Literature Review

The two major streams of research mentioned earlier related to this thesis have been extensively studied by academicians. One of them pertains to the joint economic lot-sizing problem (JELP) in supply chain management; and the other deals with the problem of joint pricing and inventory control. The former focuses on integration across the supply chain members (the manufacturer and the retailer); whereas, the latter addresses the integration of business functions (i.e. pricing and inventory control) within an individual supply chain member (e.g. the retailer).

As mentioned before, the Joint Economic Lot-sizing Problem (JELP) concerns supply chain integration in the "vertical direction", i.e., via cooperation between different business parties within the supply chain, where the manufacturer and the retailer cooperate with each other in order to reduce the production and distribution costs for the whole supply chain. Basic JELP models attempt to minimize supply chain costs, including production, inventory holding, ordering, and shipment costs, for the entire system. Typically, an item is produced by a manufacturer in lots and each lot is shipped to the retailer via several deliveries. The production lot-size and the shipment schedule are decided jointly by both the manufacturer and the retailer in a cooperative manner. In this stream of research, market demand is usually assumed to be exogenous and the retail price, being fixed, is not a decision variable.

Another stream of the research concerns supply chain integration in the "horizontal direction", i.e., the joint pricing and inventory control problem for an individual member of the supply chain, such as a retailer or a manufacturer which sells its products directly to consumers (DTC). market demand is usually assumed to be price-sensitive, and the retail price is an important decision variable. Within an individual company, the joint decisions on pricing and inventory control can be made to maximize the expected profit of the firm. Two types of research are included in this stream. One is the retailing problem, where a retailer makes joint decisions on its ordering and pricing policies. The other is the manufacturing problem, i.e., a DTC manufacturer decides the production and pricing policies simultaneously. For a retailer, the decision variables involve its own inventory replenishment policy, as well as the retail price; whereas a DTC manufacturer is concerned with its production policy and the product's price charged to the consumer.

2.1 Joint Economic Lot-sizing Problem (JELP)

A considerate amount of attention has been paid to the research on the single-product JELP, which generally deals with the following scenario: in a two-echelon supply chain consisting of a manufacturer and a retailer, the retailer faces deterministic demand for an item and places orders from the manufacturer; the manufacturer produces the item in lots of a fixed quantity with a set-up cost for each lot. Each production lot is delivered to the retailer in several shipments with a fixed order/delivery cost per shipment; the holding costs per unit per time unit for the manufacturer and the retailer are known. In JELP models, the manufacturer and the retailer work in a cooperative manner to determine the production lot-size and the delivery schedule based on the minimization of the integrated total cost function for the whole supply chain, rather than optimizing their cost functions individually. Some additional assumptions are commonly used in developing basic JELP models, which are (1) shortages are not allowed; (2) the planning horizon is infinite; and (3) the production rate is greater than the demand rate.

The literature on basic JELP models evolved from a simple model with infinite production rate and lot-for-lot policy (Goyal, 1977) to a united framework with finite production rate which was used to compare various complex shipment policies (Ben-Daya, Darwish, & Ertogral, 2008). Several types of shipment policies have been proposed: lot-for-lot policy, equal-sized shipment policy, geometric policy, and a combination of equal-sized and geometric shipments policies.

Goyal (1977) is one of the first papers dealing with the JELP problem. He develops an integrated inventory model for a single supplier-single buyer problem assuming an infinite production rate and a lot-for-lot delivery policy. That is, each production lot is delivered to the retailer in a single shipment. Banerjee (1986a) generalizes Goyal's (1977) model by relaxing the infinite production rate assumption while retaining the lot-for-lot delivery policy. He examines a joint total relevant cost model for a single vendor, single buyer production inventory system, and concludes that joint determination of the economic lot size for the vendor and buyer can substantially reduce the total relevant cost for the whole supply chain.

Goyal (1988) extends Banerjee's (1986a) model by relaxing the lot-for-lot delivery policy assumption and proposes a delayed equal-sized shipment policy, i.e., each production lot is delivered to the retailer in several equal-sized shipments, but shipments are not allowed until the entire production lot has been produced. Banerjee and Kim (1995), as well as Lu (1995), further extend Goyal's (1988) model and propose non-delayed equal-sized shipment policies. Shipments are allowed during the production process. Equal-sized shipment models are also proposed in other studies, such as Ha and Kim (1997) and Kim and Ha (2003).

Compared with the lot-for-lot policy, the equal-size shipment policy may deliver products to the retailer more frequently with smaller delivery sizes, thus reducing the holding cost for both the manufacturer and the retailer. Although this may substantially increase transportation costs, the reduction in holding costs can be sufficiently large to compensate for the increase in transportation costs. It has been shown that equal-size shipment policy can achieve a lower total relevant cost as compared with the lot-for-lot policy (Banerjee, 1986a).

Researchers further suggest that successive shipments from a production lot are not necessary equal-sized but can increase with a growth factor λ. Such a policy is called a geometric policy. Goyal (1995) is one of the first to propose such a policy, but he simplifies the growth factor λ to be the ratio of production rate to the demand rate, P/D. Hill (1997) relaxes this restriction, treating λ as a decision variable, and develops a heuristic method to search for the value of λ and the number of shipments. He numerically shows that the geometric policy is superior to the equal-sized shipment policy.

Note that the growth factor λ in a geometric policy is assumed to be constant during the production cycle. Hill (1999) relaxes this assumption and shows that a structure of geometric-then-equal sized shipments provide the optimal solution. For each production lot, the first m shipments increase in size with a growth factor of λ and then the remaining shipments are equal-sized. The production lot size, total number of shipments per production lot, as well as m and λ, are determined by a proposed solution method in order to minimize the total relevant supply chain cost.

Although Hill's (1999) model can determine the optimal solution, it is too complicated to effectively take into account other practical considerations such as product quality, lead time control, and set-up cost reduction. Also, in practice, it is not easy to manage the shipments using the structure proposed by Hill (1999). In order to overcome these limitations, Goyal and Nebebe (2000) propose a simplified geometric-then-equal sized policy, i.e., the first shipment is small and each of the remaining shipment sizes is equal to P/D times the first shipment size. In other words, it is a special case of Hill (1999) with $m = 2$ and $\lambda = P/D$.

To study and summarize the performance of different shipment policies, Ben-Daya, Darwish and Ertogral (2008) provide a comprehensive review of JELP models and further proposed a unified framework for the basic JELP model to accommodate and compare all the policies proposed by previous researchers. They find that the simplified geometric-then-equal sized policy proposed by Goyal and Nebebe (2000) provides good solutions which are very close to the optimal solution proposed by Hill (1999) [3].

The basic JELP model has been extended in several aspects by researchers to better address the reality of supply chain management. These extensions are outlined below:

(1) Consideration of stochastic demand (Ben-Daya & Hariga, 2004; Ouyang, Wu, & Hu, 2004), stock-dependent demand (Sajadieh, Thorstenson, & Akbari-Jokar, 2009) or price-dependent demand (Jokar & Sajadieh, 2009). For example, Ben-Daya and Hariga (2004) consider the single vendor, single buyer integrated production inventory

[3] In their examples, simplified geometric-then-equal sized policy is only about 0.6% inferior to the respective optimal solutions.

problem, and assume that market demand is stochastic and that delivery lead time varies linearly with the lot size. They propose a simple procedure to obtain an approximate solution for a continuous review inventory policy under this scenario, and illustrate it with numerical examples.

(2) Extension to the one-manufacturer multiple-retailers situation (Affisco, Nasri, & Paknejad, 1991; Affisco, Pakejad, & Nasri, 1993; C. Chan & Kingsman, 2007; Hoque, 2011; Joglekar & Tharthare, 1990; Lu, 1995; H. Siajadi, R. Ibrahim, & P. Lochert, 2006; Viswanathan & Piplani, 2001; Yau & Chiou, 2004). For example, Siajadi, Ibrahim and Lochert (2006) consider a situation when one type of item from a single manufacturer is distributed to multiple retailers. They propose an exact solution for the two-retailer case, but only an approximate solution procedure for the more than two-retailer case.

(3) Incorporating some practical aspects, such as setup & order cost reduction(Chang, Ouyang, Wu, & Ho, 2004; Nasri, Paknejad, & Affisco, 1991), production quality (Darwish, 2005; Huang, 2002), and lead time control (Chang, et al., 2004; Hoque & Goyal, 2006; Pan & Yang, 2002). In some studies, several realistic factors are examined simultaneously. For example, Hoque (2011) relaxes several unrealistic assumptions in the existing literature, including unlimited capacities of the transport equipment and retailer's storage, insignificant set up and transportation times, and unlimited lead time and batch sizes (Hoque, 2011).

(4) Extensions to more than two-echelon supply chains (Ben-Daya & As'ad, 2006; Khouja, 2003; Lee, 2005; Muson & Rosenblatt, 2001; Nikandish, Eshghi, & Torabi, 2009). Most of the extensions in this direction consider a three-echelon supply chain

including a supplier, a manufacturer and a retailer. The supplier provides raw materials to the manufacturer which converts raw materials into a final product and sends them to the retailer using an equal-size shipment policy. Some of these studies (e.g. Nikandish, Eshghi, & Torabi, 2009) further investigate the case of a single supplier, several manufacturers and multiple retailers.

Although it is shown by Hill (1999) that the geometric-then-equal sized shipment policy is the optimal structure for the JELP model, almost all the extensions to the basic JELP model are based on the lot-for-lot or equal-sized shipments policies. There are several reasons for this. First, analysis of the extension to the geometric policy or the geometric-then-equal sized shipment models, or even to the simplified geometric-then equal sized shipment models, is cumbersome, and the results are usually too complex to be used by practicing managers.

Secondly, the equal-sized shipment policies provide closed-form solutions which are close to the optima. Ben-Daya, Darwish and Ertogral (2008) show that solutions to the equal-sized shipment policies are close to optimal. In their examples, when the parameters of the system (such as holding and setup costs) increase to large values, the inferiority of the equal-sized shipment policy may decrease to less than 2%.

Finally, the equal-sized shipment policy is much easier to use in practice, such as designing warehouse capacities or truck load sizes (for example, Kim and Ha (2003) show that equal-size shipment policy can easily standardize the size of the transportation vehicle). In contrast, the geometric-then-equal sized shipment policy, changing the shipment sizes over time, are usually impractical. In fact, equal-size shipment policies are much more widely used in real world manufacturing and retailing than the geometric policy and the geometric-then-equal sized shipment policies.

In other words, an equal-sized shipment policy provides good approximation to the optimal solution. Under such a policy, it is much easier to incorporate a number of practical considerations, which is useful in management practice. Therefore, we adopt the equal-sized shipment policy in this thesis.

2.2 Joint Pricing and Inventory Control

The other stream of research related to this thesis is the research on retailer's problems in joint pricing and inventory control. This stream of research considers the following scenario: a retailer faces a price-sensitive demand from consumers; it orders goods from the manufacturer and sells them to the consumers. The planning horizon may be a single period, a finite number of periods, or infinite; and the retailer decides the ordering and pricing policies at the beginning of each period to maximize its expected profit. Extensive research has been done in this area involving various scenarios and assumptions, but can be largely characterized by the assumed demand type. Chan, Shen, Simchi-Levi and Swann (2004) provide a comprehensive review of this area of research. Here we provide a review of the literature most relevant to this thesis, especially those studies published after the Chan et. al. (2004)' review.

Two demand types are adopted in the literature: deterministic demand and stochastic demand. The proto models with deterministic demand are extended from the EOQ model by considering price as a decision variable (Whitin, 1955). Most of these models assume that demand is a linear function of price (e.g. Whitin, 1955; Pekelman, 1974;

Lai, 1990), a concave function of price (e.g. Biller, Chan and Simchi-Levi, 2002), or a general decreasing function of price (e.g. Smith & Achabal, 1998).

Most of recent research on the joint pricing and inventory control problems focuses on the stochastic demand environment rather than the deterministic one. The proto models with stochastic demand are extended from the classical news-vendor model, by considering price as a decision variable. Most of such models assume that demand has a Poisson distribution, while a few of them assume that demand has a general distributional form. These models are based on either continuous review or periodic review control policies.

Chen and Simchi-Levi (2004 b) develop a periodic review model over an infinite horizon and show that an (s, S, p) policy is optimal under the "symmetric k-convexity" condition[4] of the profit function, where s is the reorder point, S is the order-up-to level and p is the retail price which is dependent on the inventory level at the beginning of each period. They then extend their model to the continuous review case and show that an (s, S, p) policy is optimal without the condition of symmetric k-convexity (Chen & Simchi-Levi, 2006).

The (s, S, p) policy is also shown by Chen and Simchi-Levi (2004 a), as well as Chen, Ray and Song (2006), to be optimal in the periodic review finite horizon setting when the demand function is additive. That is, the demand consists of two components, a deterministic part which is a function of the price and an additive random perturbation. However, when the demand function is not additive, the (s, S, p) policy is not

[4]"A real-valued function of f is called symmetric k-convex for $k \geq 0$, if for any $x_0 \leq x_1$ and $\lambda \in [0, 1]$, $f((1-\lambda)x_0 + \lambda x1) \leq (1-\lambda)f(x_0) + \lambda f(x_1) + max\{\lambda, 1-\lambda\} \cdot k$" (Chen & Simchi-Levi, 2004a, p.891)

necessarily optimal and it is extremely cumbersome to analyze the resulting maximization problem. They used the concept of symmetric k-convexity to prove that an (s, S, A, p) policy [5] is optimal in this case.

Subsequent to the work of Chen and Simchi-Levi (2004 a, b; 2006), several studies on the optimality of (s, S, p) type policies have been conducted. For example, Song, Ray, and Boyaci (2006) deal with the case of multiplicative demand model with lost sales. They proved the optimality of the (s, S, p) policy. Chao and Zhou (2006) investigate an infinite-horizon continuous review system with a Poisson demand process. Yin and Rajaram (2007) extend the results of Chen and Simchi-Levi (2004 a, b) to a finite horizon and periodic review system with Markovian demand. For the additive demand model, they show the existence of an optimal (s, S, p) type feedback policy. Huh and Janakiraman (2008) show the optimality of (s, S, p) policy in periodic review systems under both backordering and lost-sales assumptions.

Some recent research (e.g. Chen, Wu, & Yao, 2010; Wei, 2010; Zhang & Chen, 2006) further relax the assumption of a constant price for each period in the traditional joint pricing and inventory control problems, allowing the retail price to change dynamically, i.e., dynamic pricing with consideration of inventory control. For example, Zhang and Chen (2006) study a periodic review model over a finite horizon. The demand is assumed to be stochastic and have a distribution dependent on the retail price, but "one or more parameters of the demand distribution are unknown with a known prior distribution that is chosen from the natural conjugate family" (Zhang & Chen,

[5] "(Define) a set $A_t \in [s_t, (s_t + S_t)/2]$, possibly empty depending on the problem instance. When the inventory level x_t at the beginning of period t is less than s_t or $x_t \in A_t$, an order of size $S_t - x_t$ is placed. Otherwise, no order is placed" (Chen & Simchi-Levi, 2006, p.324)

2006). They provide a Bayesian dynamic program formulation and characterize the structure of an optimal policy.

2.3 Integration of Pricing, Production and Inventory Decisions

As mentioned above, research on the JELP focuses on the cooperation of the manufacturer and the retailer(s) to minimize the total cost for the supply chain, representing vertical integration within the supply chain. Research on joint pricing and inventory control problem, on the other hand, focuses on the maximization of profit for the retailer via integration of the pricing and inventory decisions, representing horizontal integration of the supply chain. However, very few models are formulated to fill the gap between these two streams of research and describe how the manufacturer and the retailer(s) can cooperate with each other and simultaneously make decisions concerning production lot size, delivery schedule, and retail pricing, in order to maximize the total profit for the whole supply chain.

To the best of our knowledge, Jokar and Sajadieh (2009) are among the first to explore this issue. They propose a JELP model with a price-sensitive market demand. Using an analytical approach, they obtain a closed-form approximate solution for determining the order quantity and the retail price. Nevertheless their work is limited by the adoption of a lot-for-lot production/delivery policy. This setting makes the production lot equal to the retailer's order quantity, which significantly simplify the problem. However, as shown by previous studies (Banerjee & Kim, 1995; Lu, 1995 et. al.), the equal-sized shipment policy is superior to the lot-for-lot delivery policy. Therefore, it would be worthwhile for us to adopt an equal-sized shipment policy and reexamine this problem.

There are some studies providing useful insights into this issue, although they do not examine it directly. For example, Zahir and Sarker (1991) analyze the pricing, economic ordering policies, and the production lot-sizing policy for a supply chain with a single manufacturer and multiple regional wholesalers. The products are assumed to be delivered on a lot for lot basis. They obtain the optimum EOQs for the wholesalers first. And then they propose that, to minimize its cost, the manufacturer would encourage the wholesalers to order in quantities different from their respective EOQs by providing compensation to offset the wholesalers' increased costs. Although this study finds ways to maximizes the wholesalers' profits and minimize the producer's cost respectively, it does not provide solutions to maximize the profit of the whole supply chain. Instead of making decisions simultaneously in a cooperative manner, the wholesalers and the producer make decision sequentially and focus on their own costs and profit.

Another example is the work of Boyaci and Gallego (2002). They analyze the integration of pricing and inventory replenishment policies for a supply chain consisting of one wholesaler and one or more geographically dispersed retailers. They show that "a solution that maximizes channel profits can be interpreted as consignment selling "[6] (Boyaci & Gallego, 2002, p.95). They also show that when the demand rate is large enough, separating the pricing and lot sizing decision is near optimal. However, they assume that the wholesaler either obtains the product from other producers or produces it at an infinite rate. Therefore, they fail to cover the production process and incorporate

[6] The retailers pay the wholesaler only as and when the products are sold. And, the wholesaler takes back all the unsold products from the retailers.

the production lot size decision in their model.

3 Single-Manufacturer Single-Retailer (SMSR)

Single-manufacturer single-retailer supply chains are common in the business environment, especially for small businesses. For example, the AGAS Manufacturing Group is a local clothing manufacturer which distributes its product via its own distribution center in Philadelphia. In such a supply chain, the manufacturer and the retailer, being the same corporate entity, naturally cooperate with each other. Other single-manufacturer single-retailer supply chains are not naturally centralized because the manufacturer and the retailer are different organizations. Whether supply chains are naturally centralized or not, supply chain managers are concerned about making the centralized decisions. Especially, for supply chains which are not naturally centralized, it is important for the managers to examine both the centralized and decentralized decisions and, thus, resolve profit sharing issues via negotiation.

In this chapter, we focus on a single product, single-manufacturer single-retailer, infinite horizon model, assuming that demand is price-dependent and deterministic. We first describe the modeling assumptions and notation in section 3.1. In section 3.2, the model of a centralized supply chain, where the manufacturer and the retailer work in a cooperative manner to maximize the channel profits for the entire supply chain, is developed, followed by a proposed solution algorithm. We next analyze the decentralized supply chain in section 3.3. In section 3.4, we discuss revenue sharing between the manufacturer and the retailer in a centralized supply chain and the social benefit of centralization associated with different channel structures. We provide some numerical examples and sensitivity analysis to illustrate the proposed algorithm and compare the results of centralized and decentralized supply chains in section 3.5.

3.1 Assumptions

We consider the following scenario in this chapter: a manufacturer produces a product in discrete lots at a production rate P with a setup cost of S per batch and a production cost of C per unit. For each production lot, the manufacturer delivers the product to the retailer in n equal-sized shipments with a fixed cost of F per shipment plus a variable cost of V per unit. The retailer pays the transportation cost, in addition to an ordering cost of O per shipment. The objective is to determine the production lot size Q, equal shipment sizes of q, the number of shipments per production cycle, n, and the retail price, p_r, in order to maximize the profit of the entire supply chain.

The following assumptions are made in this chapter:

(1) $D = a – bp_r$; where the parameters $a, b > 0$ are known;

(2) The planning horizon is infinite;

(3) Shortages are not allowed;

(4) The production rate P is greater than the demand rate D;

(5) The holding cost per unit per time unit of the retailer (H_r) is not less than the holding cost per unit per time unit of the manufacturer (H_m);

(6) Delivery lead time is zero or negligible.

Assumption (1) is widely used in the literature to represent the relationship between the retail price and market demand (as discussed in section 2.2 of this thesis). To simply our analysis, we use a linear demand function in this thesis. However, our analysis could be easily extended to accommodate other retail demand functions. Assumption (2) is commonly used in JELP models. Assumptions (3) and (4) are made to ensure that all the demand can be fulfilled, to guarantee consumer satisfaction. Assumption (5) is reasonable, since the holding cost usually increases as a product moves down the supply chain, due to the value added at each echelon. Assumption (6) is used for presentation and analysis simplicity only. Including lead time in our model does not make any differences in the qualitative results and the properties derived in this research.

3.2 Centralized Supply Chain

3.2.1 Model Formulation

Note that D is a function of p_r. Thus, the decision variables in our model are q, n, p_r, Q and the production cycle length, T. Once three of these variables are chosen, the other two will also be determined, since we have $Q = nq$ and $T = Q/D$. Therefore, in this section, we analyze how the manufacturer and the retailer can work in a cooperative manner to decide n, q, and p_r to maximize the profit of entire supply chain. To do so, we first examine the inventory levels of both parties in the supply chain, so that we can develop their inventory holding costs. We then formulate the profit function for both parties as well as for the entire supply chain, and provide solutions to the profit maximization problem for the entire supply chain.

Figure 3.1 illustrates the inventory patterns for the manufacturer, the retailer and the

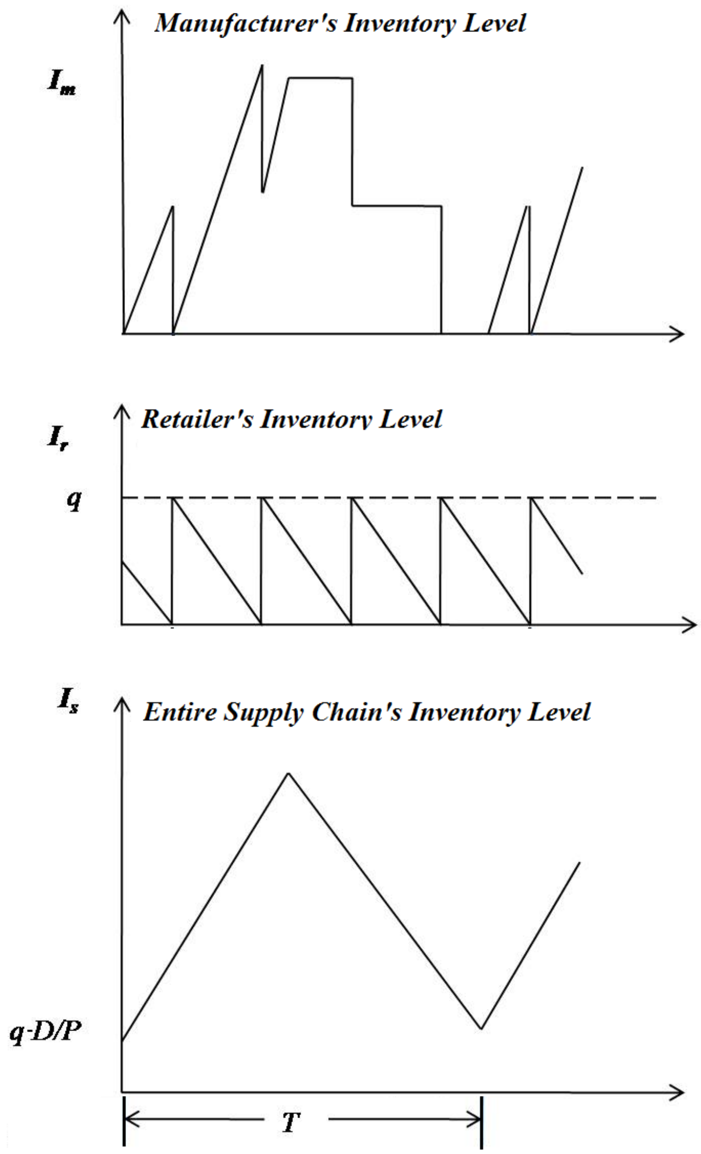

Figure 3.1: Inventory pattern in a single-manufacturer single-retailer environment

entire supply chain in an inventory cycle. We focus on the inventory level of the entire supply chain first, which is the sum of the inventory levels of the manufacturer and the retailer. Note that the manufacturer has to begin its production cycle at q/P time unit before the retailer depletes its inventory (to have q unit ready when the retailer depletes its inventory and needs replenishment). Thus, the total inventory of the whole supply chain system (held by the retailer) is at the beginning of each production lot. This is the inventory left over at the retailer when the manufacturer starts to produce a batch. Note that the inventory of the system increases at a rate of $(P - D)$ during the production process, which has duration of $n \cdot q/P$. Thus, the maximum inventory level of the whole system (at the end of the production process) is $(P - D) \cdot n \cdot q/P$ as shown in Figure 3.1. Therefore, we have the average inventory of the system is

$$I_s = \frac{D \cdot q}{P} + \frac{(P - D) \cdot n \cdot q}{2P}. \tag{3.1}$$

As shown in Figure 3.1, the retailer has inventory level of q at the beginning of each order cycle and replenishes when it depletes its inventory. Thus, average inventory level for the retailer is

$$I_r = \frac{q}{2}. \tag{3.2}$$

The average inventory level of the manufacturer, which is the difference of those of the entire supply chain and the retailer, can be written as

$$I_m = \frac{D \cdot q}{P} + \frac{(P - D) \cdot q \cdot n}{2P} - \frac{q}{2}. \tag{3.3}$$

Based on the inventory pattern analysis above, we now develop the profit functions for the retailer, the manufacturer and the entire supply chain respectively. The retailer has a sale's revenue of $D \cdot p_r$ per year, and it has to pay for the purchase cost of the products $D \cdot p_w$, ordering cost of $O \cdot D/q$, transportation cost of $F \cdot D/q + V \cdot D$, and inventory cost of $H_r \cdot I$. Thus, the profit of the retailer per year can be written as

$$\pi_r = D \cdot (p_r - p_w - V) - \frac{(O + F) \cdot D}{q} - \frac{H_r \cdot q}{2}. \tag{3.4}$$

The manufacturer has a sale's revenue of $D \cdot p_w$ per year, and it has to pay for the production cost of $D \cdot C$, setup cost of $S \cdot D/(n \cdot q)$, and inventory cost of $H_m \cdot I_m$. Using equation (3.3), we can write the profit of the manufacturer per year as

$$\pi_m = D \cdot (p_w - C) - \frac{S \cdot D}{n \cdot q} - H_m \left\{ \frac{D \cdot q}{P} + \frac{(P - D) \cdot q \cdot n}{2P} - \frac{q}{2} \right\}. \tag{3.5}$$

Thus, the profit of the entire supply chain, which is the sum of the profits of the retailer and the manufacturer, is

$$\pi_s = D \cdot (p_r - C - V) - \frac{D \cdot [S + n \cdot (O + F)]}{q \cdot n} - \frac{q}{2} \left\{ H_m \left[\frac{(2 - n) \cdot D}{P} + n - 1 \right] + H_r \right\}. \tag{3.6}$$

In a centralized supply chain, the manufacturer and the retailer work in a cooperative manner to maximize the profit of the entire supply chain by controlling n, q and p_r. Therefore, the optimization problem for the centralized supply chain can be formulated as follows:

$$\begin{aligned} &\underset{n,q,p_r}{Maximize} && \pi_s \\ &s.t. && D = a - bp_r \\ &&& D < P \\ &&& D, p_r, P > 0 \\ &&& n \in \{1, 2, 3, ...\} . \end{aligned} \tag{3.7}$$

To solve the above maximization problem, we first relax the constraint of n to be a positive integer. We will incorporate this constraint later. For any given D, q and n must satisfy the following first order conditions:

$$\frac{\partial \pi_s}{\partial q} = \frac{D}{q^2}\frac{[S + n \cdot (O + F)}{n} - \frac{H_m}{2}\left[\frac{(2-n) \cdot D}{P} + n - 1\right] - \frac{H_r}{2} = 0 \tag{3.8}$$

$$\frac{\partial \pi_s}{\partial n} = \frac{D \cdot S}{q \cdot n^2} - \frac{q \cdot H_m}{2}\left[1 - \frac{D}{P}\right] = 0. \tag{3.9}$$

From equation (3.8), we have

$$\frac{D \cdot S}{q \cdot n} + \frac{D \cdot (O + F)}{q} = \frac{q}{2}\left\{H_m\left[\frac{(2-n) \cdot D}{P} + n - 1\right] + H_r\right\}. \tag{3.10}$$

The left hand side of equation (3.10) is the sum of the manufacturer's set up cost and the retailer's ordering and fixed transportation costs; whereas the right hand side is the inventory cost of the entire supply chain. This means that when the optimal solution is achieved, the delivery lot size, q, will be adjusted to “balance” the costs on the left hand side, which is decreasing with q, and the inventory costs on the right hand side, which is increasing with q.

From equation (3.9), we have

$$\frac{D \cdot S}{q \cdot n} = \frac{n \cdot q \cdot H_m}{2} \left[1 - \frac{D}{P}\right]. \tag{3.11}$$

Note that the left hand side of equation (3.11) is the manufacturer’s setup cost, whereas the right hand side is a part of the manufacturer’s inventory holding cost, which is affected by n. Thus adjusting n only does not affect the cost of the retailer, but an increase in n will lower the manufacturer’s average setup cost, simultaneously increasing the production lot size and, thus, increase its inventory holding cost. Again, when the optimal solutions are achieved, n will be adjusted to “balance” the costs on both sides of equation (3.11).

Since n and q are positive, equations (3.8) and (3.9) have a unique solution as follows:

$$\hat{n} = \sqrt{\frac{S}{O+F}\frac{H_r + H_m\left(\frac{2D}{P} - 1\right)}{H_m\left(1 - \frac{D}{P}\right)}}, \tag{3.12}$$

$$\hat{q} = \sqrt{\frac{2D \cdot (O+F)}{H_r + H_m\left(\frac{2D}{P} - 1\right)}}. \tag{3.13}$$

Note that, for any given D, ($\hat{n}$, $\hat{q}$) is the unique solution obtained from the first order optimality conditions, as shown by previous work (e.g. Kim and Ha, 2003, p.5). Also, the total cost of the entire supply chain is jointly convex in n and q, hence, ($\hat{n}$, $\hat{q}$) is the optimal solution, when D is given. Once D (as a function of p_r) is determined, the corresponding optimal n and q are unique and determined. Note that the market demand D is determined by the retail price p_r. Therefore, the maximization problem 3.7 on page 29 can be rewritten as a maximization problem with only decision variable p_r. Substituting (3.10), (3.12) and (3.13) into the objective function, we have an equivalent objective function with only variable p_r, i.e.

$$\begin{aligned} \pi_s^c(p_r) &= (a - b \cdot p_r)\left[p_r - C - V\right] - \sqrt{2(a - b \cdot p_r) \cdot S \cdot H_m \left(1 - \frac{a - b \cdot p_r}{P}\right)} \\ &\quad - \sqrt{2(a - b \cdot p_r) \cdot (O + F) \cdot \left[H_r + H_m \left(2\left(\frac{a - b \cdot p_r}{P}\right) - 1\right)\right]}. \end{aligned} \tag{3.14}$$

Thus, the problem now becomes choosing p_r in order to maximize π_s^c. To better describe the properties of this maximization problem, and also for notational simplicity, we define:

$$\begin{aligned} \delta &= \frac{a}{b} - C - V - 2\sqrt{\frac{(O + F) \cdot H_m}{P}}, \\ \eta &= \sqrt{1 - \frac{2}{P}\left(\frac{b^2 \cdot S \cdot H_m}{4}\right)^{1/3}}. \end{aligned}$$

Note that the first term in δ, a/b, is the maximum feasible retail price, which reflects market profitability; i.e. the higher the a/b value, the higher the market profitability. The other terms in δ describe part of the unit cost of selling the product to the market.

Therefore, δ in fact reflects the profitability of the product. Based on the analysis in Appendix 6.2.3, we develop following propositions.

Proposition 1. *The product is not profitable if $\delta \leq \frac{P}{b}\left(1-\eta^3\right)$. If this condition holds, the entire supply chain cannot achieve a positive profit.*

The idea behind Proposition 1 is that the supply chain would not be able to generate any positive profit from a product, if the retail price is less than the average unit cost. To better interpret Proposition 1, we can rewrite the condition as follows:

$$\frac{a}{b} \leq C+V+2\sqrt{\frac{(O+F)\cdot H_m}{P}}+\frac{P}{b}\left(1-\left(\sqrt{1-\frac{2}{P}\left(\frac{b^2\cdot S\cdot H_m}{4}\right)^{1/3}}\right)^3\right).$$

Note that the left hand side, a/b, is the maximum feasible retail price, and the right hand side reflects the average per-unit cost, where C and V are fixed parameters that are independent of the supply chain decisions. The remaining terms on right-hand side of the above expression represent sum of the other relevant per unit costs minimized by adjusting the supply chain decisions. Thus, to some extent, the right-hand side represents the minimum feasible per-unit cost. Hence, Proposition 1 indicates that, if the maximum feasible retail price is not greater than the minimum feasible per-unit cost, the supply chain profit cannot be positive.

Supply chain managers may use Proposition 1 in an approximate but more intuitive way. Practitioners are usually able to estimate an approximate range of each cost factor. For example, a manager may be able to state with some degree of confidence that the holding cost would be about 10% to 15% of the total production cost. Thus, he/she can compare the estimated per-unit cost with the maximum feasible price, a/b, for ascertaining the profitability of the product in question.

Proposition 1 shows that, when the product profitability is lower than or equal to a certain level, the best strategy for the manufacturer and the retailer is to not deal with this product. In contrast, if the product profitability is very high, the manufacturer and the retailer should be willing to provide as much of the product as possible to the market to increase their respective profits. This characteristic is shown and described in following proposition.

Proposition 2. *If* $\delta \geq \frac{P}{b}(1+\eta^3)$, *the profit of the entire supply chain is maximized by choosing* $p_r = \frac{a-P}{b}$. *That is, the manufacturer will keep producing the product continuously without any interruptions. In this case, optimal* $n \mapsto \infty$*and* $q = \sqrt{\frac{2P\cdot(O+F)}{H_r+H_m}}$

.

When $p_r = \frac{a-P}{b}$, $D \longmapsto P$, the manufacturer has to use all its production capacity to produce a single product. In this case, the manufacturer will keep producing and never have any idle time. Thus, the production lot size and n will both be infinite. It is easy to derive the optimal q, using equation (3.13). The corresponding supply chain profit under this condition is given by

$$\hat{\pi_s} = P \cdot \left(\frac{a-P}{b} - C - V\right) - \sqrt{2(O+F)\cdot P \cdot (H_m + H_r)}. \qquad (3.15)$$

When the product profitability δ is neither as low as the condition stated in Proposition 1, not as high as the condition stated in Proposition 2, the manufacturer and the retailer have to determine the retail price in order to maximize the profit of the entire supply chain. In this situation, we outline following proposition.

Proposition 3. *If* $\frac{P}{b} \leq \delta < \frac{P}{b}(1+\eta^3)$, *the objective function is strictly concave in* $\left(\frac{a}{b} - \frac{P\cdot(1+\eta)}{2b}, \frac{a}{b} - \frac{P}{2b}\right]$; *If* $\frac{P}{b}(1-\eta^3) < \delta \leq \frac{P}{b}$, *the objective function is strictly concave in* $\left[\frac{a}{b} - \frac{P}{2b}, \frac{a}{b} - \frac{P\cdot(1-\eta)}{2b}\right)$. *In these situations, the maximization problem has a unique inner local maximum solution,* $\bar{p_r}$. *Specifically,*

$$\begin{cases} \frac{a}{b} - \frac{P}{2b} < \bar{p_r} < \frac{a}{b} - \frac{P}{2b}(1-\eta) & if\ \frac{P}{b}(1-\eta^3) < \delta < \frac{P}{b} \\ \bar{p_r} = \frac{a}{b} - \frac{P}{2b} & if\ \delta = \frac{P}{b} \\ \frac{a}{b} - \frac{P}{2b} > \bar{p_r} > \frac{a}{b} - \frac{P}{2b}(1+\eta) & if\ \frac{P}{b}(1+\eta^3) > \delta > \frac{P}{b} \end{cases}$$

Based on the above propositions, a binary-search algorithm are developed in section 3.2.2 to search the optimal retail price. Once the optimal retail price is reached, as discussed above, the objective function is concave on the number of shipments n and delivery lot size q. Thus, the corresponding optimal q is determined by equation (3.13); and the optimal n is an integer around $\hat{n}$ in equation (3.12), either $\lfloor \hat{n} \rfloor$ or $\lceil \hat{n} \rceil$.

3.2.2 Solution Algorithm

Based on propositions 1, 2 and 3, we propose a procedure for finding a solution to the maximization problem (3.7). We first calculate δ and η. Based on the calculated δ and η, we apply the corresponding proposition. If $\delta \geq \frac{P}{b}(1+\eta^3)$ or $\delta \leq \frac{P}{b}(1-\eta^3)$, an optimal solution can be obtained immediately. Otherwise, based on proposition 3, a binary search method is used to search for optimal retail price in the corresponding interval. In particular, the following steps are used in this algorithm:

Step 1. Calculate δ and η. If $\delta \geq \frac{P}{b}\left(1+\eta^3\right)$, then $p_r^* = \frac{a-P}{b}$, $n^* \mapsto \infty$, $q^* = \sqrt{\frac{2P\cdot(O+F)}{H_r+H_m}}$; If $\delta \leq \frac{P}{b}\left(1-\eta^3\right)$, then stop; If $\delta = \frac{P}{b}$, then $\bar{p_r} = \frac{a-P/2}{b}$ and go to step 4; If $\frac{P}{b}\left(1-\eta^3\right) < \delta < \frac{P}{b}$, then go to step 2; otherwise, go to step 3;

Step 2. Do a binary search over $\left[\frac{a}{b} - \frac{P}{2b}, \frac{a}{b} - \frac{P\cdot(1-\eta)}{2b}\right)$ to search $\bar{p_r}$ such that $2\bar{p_r} - \sqrt{S \cdot H_m}\frac{\left(P-2\bar{D}\right)}{\sqrt{2\bar{D}\cdot P\left(P-\bar{D}\right)}} = \frac{2a}{b} - \delta$, where $\bar{D} = a - b\cdot \bar{p_r}$; go to step 4;

Step 3. Do a binary search over $\left(\frac{a}{b} - \frac{P\cdot(1+\eta)}{2b}, \frac{a}{b} - \frac{P}{2b}\right]$ to search $\bar{p_r}$ such that $2\bar{p_r} - \sqrt{S \cdot H_m}\frac{\left(P-2\bar{D}\right)}{\sqrt{2\bar{D}\cdot P\left(P-\bar{D}\right)}} = \frac{2a}{b} - \delta$, where $\bar{D} = a - b\cdot \bar{p_r}$; go to step 4;

Step 4. Calculate $\bar{D} = a - b\cdot\bar{p_r}$, $\tilde{n} = \sqrt{\frac{S}{O+F}\frac{H_r+H_m\left(\frac{2\bar{D}}{P}-1\right)}{H_m\left(1-\frac{\bar{D}}{P}\right)}}$, $\bar{q} = \sqrt{\frac{2\bar{D}\cdot(O+F)}{H_r+H_m\left(\frac{2\bar{D}}{P}-1\right)}}$; using equation (3.6), if $\pi_s\left(\bar{p_r}, \lfloor\tilde{n}\rfloor, \bar{q}\right) \geq \pi_s\left(\bar{p_r}, \lceil\tilde{n}\rceil, \bar{q}\right)$, then $\bar{n} = \lfloor\tilde{n}\rfloor$, otherwise, $\bar{n} = \lceil\tilde{n}\rceil$; calculate corresponding $\bar{\pi}_s = \pi_s\left(\bar{p_r}, \bar{n}, \bar{q}\right)$; go to step 5;

Step 5. If $\bar{\pi}_s \geq max\left\{\hat{\pi}_s, 0\right\}$, then $p_r^* = \bar{p_r}$, $n^* = \bar{n}$, $q^* = \bar{q}$; if $\bar{\pi}_s \leq 0$, then stop; otherwise, $p_r^* = \frac{a-P}{b}$, $n^* \mapsto \infty$, $q^* = \sqrt{\frac{2P\cdot(O+F)}{H_r+H_m}}$.

3.3 Decentralized Supply Chain

The discussion in the previous section is based on the assumption that the manufacturer and the retailer work in a cooperative manner to maximize the profit of the entire supply chain. In reality, the manufacturer and the retailer are often most concerned about their own individual profits and not necessarily about that for the whole system, especially when one of them has the negotiating power to impose its own independently derived optimal policy. In this section, we discuss the situation when they work in a non-cooperative manner. For presentation clarity, we use superscripts c and d to denote the parameters for a centralized or a decentralized supply chain, respectively.

3.3.1 Model Formulation

Clearly, the wholesale price, p_w, affects the profit share of the entities in the supply chain. The manufacturer would prefer a higher p_w, whereas the retailer wishes it to be as low as possible. The manufacturer and the retailer may negotiate with each other to determine the wholesale price, p_w, to decide their individual profit levels. Once p_w is established, the retailer has to decide on the retail price and its economic order size q; and the manufacturer needs to determine its production lot size Q or the number of shipment per batch, n. Note that the retailer's decisions are independent of the manufacturer's decisions, whereas the latter has to make its decisions based on the retailer's choices. The retailer faces the following maximization problem

$$\begin{aligned} \underset{q,\,p_r}{Max} \quad & (a - b \cdot p_r) \cdot (p_r - p_w - V) - \frac{(O+F) \cdot (a - b \cdot p_r)}{q} - \frac{H_r \cdot q}{2} \\ s.t. \quad P > \quad & a - b \cdot p_r > 0 \\ p_r > \quad & 0 \\ q > \quad & 0. \end{aligned} \tag{3.16}$$

From (3.16), the optimal q for any given p_r can be written as

$$q = \sqrt{\frac{2\,(O+F) \cdot (a - b \cdot p_r)}{H_r}}. \tag{3.17}$$

Thus, based on equation (3.17), maximization problem (3.16) is equivalent to

$$\begin{aligned} \underset{p_r}{Max}\ \pi_r^d\left(p_r\right) \quad &= \quad (a-b\cdot p_r)\cdot(p_r-p_w-V)-\sqrt{2H_r\cdot(O+F)\cdot(a-b\cdot p_r)} \\ s.t.\quad p_r > \quad & \frac{a-P}{b} \\ p_r < \quad & \frac{a}{b}. \end{aligned} \tag{3.18}$$

In Appendix 6.2.3, we provide the details of an algorithm to quickly solve the maximization problem above. The manufacturer will then make its decision on the production lot size Q (or the shipment schedule $n = Q/q$) via solving the following problem:

$$\begin{aligned} \underset{n}{Max}\ \pi_m^d \quad &= \quad (a-b\cdot p_r)\cdot\left\{p_w-c-\frac{S}{n\cdot q}-\frac{q\cdot H_m}{2}\left[\frac{(2-n)}{P}+n-1\right]\right\} \\ s.t.\quad n\in \quad & \{1,2,...\}\,. \end{aligned} \tag{3.19}$$

Since $\frac{\partial^2\pi_m}{\partial n^2} = -2\frac{(a-b\cdot p_r)S}{n^3 q} < 0$, π_m is concave in n, and is maximized at either $\lfloor\bar{n}\rfloor$or $\lceil\bar{n}\rceil$, where

$$\bar{n} = \frac{\sqrt{2\left(a-b\cdot p_r\right)\cdot S}}{q\cdot\sqrt{H_m\left(1-\frac{a-b\cdot p_r}{P}\right)}}. \tag{3.20}$$

3.3.2 Comparison with Centralized Supply Chain

As discussed above, the manufacturer and the retailer cooperate with each other to maximize the total channel profit in the centralized supply chain, whereas, in the decentralized supply chain, they only focus on their own respective profits. Under this mechanism, the centralized supply chain has higher total channel profit than the decentralized supply chain. In the former case, the retail price is determined by maximizing (3.14) while the retail price in the latter case is determined by the maximization problem (3.18). Denote the optimal retail price in centralized supply chain and decentralized supply chain by p_r^c and p_r^d respectively. Note that, as shown in (3.21), π_s^c in (3.14) can be reorganized and written as the sum of the retailer's profit, π_r^d, in (3.18) and a decreasing function of p_r, $f\left(p_r\right)$, as follows:

$$\pi_s^c\left(p_r\right) = \pi_r^d\left(p_r\right) + f\left(p_r\right), \tag{3.21}$$

where $f\left(p_r\right) = \left(a - b \cdot p_r\right)\left\{p_w - C - \sqrt{2H_m}\left[\sqrt{\frac{S}{a-b\cdot p_r} - \frac{S}{P}} + \sqrt{\left(2\left(\frac{O+F}{P}\right) - \frac{O+F}{a-b\cdot p_r}\right)}\right]\right\}$. It is easy to verify that

$$\begin{aligned} f(p_r) &\geq 0, \\ \frac{\partial f(p_r)}{\partial p_r} &< 0. \end{aligned}$$

Consequently, we outline the following proposition on the relationship between p_r^c and p_r^d.

Proposition 4. *The optimal retail price in a centralized supply chain, p_r^c, is not greater than the retail price in a decentralized supply chain, p_r^d. Moreover, the difference between p_r^d and p_r^c, $p_r^d - p_r^c$, is increasing with the wholesale price, p_w.*

Proof. If $p_r^c > p_r^d$, then $f\left(p_r^d\right) > f\left(p_r^c\right)$. Note that $\pi_r^d\left(p_r^d\right) \geq \pi_r^d\left(p_r^c\right)$. Therefore, if $p_r^c > p_r^d$, $\pi_s^c\left(p_r^d\right) = \pi_r^d\left(p_r^d\right) + f\left(p_r^d\right) > \pi_r^d\left(p_r^c\right) + f\left(p_r^c\right) = \pi_s^c\left(p_r^c\right)$, which contradicts the fact that p_r^c maximizes $\pi_s^c(p_r)$. Moreover, p_r^c is independent of p_w; whereas p_r^d is increasing with p_w. Thus, $p_r^d - p_r^c$ increases with p_w.

3.4 Revenue Sharing and Social Benefit

Based on the above discussion, the centralized supply chain has a higher total channel profit than the decentralized supply chain. Both the manufacturer and the retailer have an incentive to cooperate with each other in order to gain higher profits. But, it remains unclear how the manufacturer and the retailer share the benefit from centralizing the supply chain decisions. To address this issue, we develop the Nash bargaining equilibrium in section 3.4.1, followed by a further discussion on the social benefit issues in section 3.4.2.

3.4.1 Nash Bargaining Equilibrium

Various solutions have been proposed to solve the bargaining game between two players (i.e. the manufacturer and the retailer in our case). The bargaining game in cooperative game theory considers the following problem. There are some feasible outcomes if an agreement can be reached. If an agreement cannot be reached, a given disagreement outcome is the result. As discussed earlier, the centralized supply chain has higher total profit than the decentralized supply chain. Thus, there is an incentive for the manufacturer and the retailer to negotiate with each other. In this section, we discuss the bargaining game between the manufacturer and the retailer.

One of the most important approaches for solving the bargaining game without consideration of negotiation power is the Nash bargaining equilibrium. John Nash (Nash, 1951) proposed that a solution to the bargaining game should satisfy certain axioms: (1) invariant to affine transformations or invariant to equivalent utility representations; (2) Pareto optimality; (3) independence of irrelevant alternatives; and (4) symmetry. Based on these axioms, the game players will seek to maximize the Nash product, which is defined as $(u_1 - d_1) \cdot (u_2 - d_2)$ where u_1 and u_2 are the bargaining outcomes for the two game players respectively and d_1 and d_2 are outcomes if they cannot reach an agreement.

In our case, if the two game players, the manufacturer and the retailer, reach an agreement and thus operate in a centralized supply chain environment, the bargaining outcomes for them are π_m^c and π_r^c respectively. If they cannot reach an agreement, that is, the supply chain is decentralized, their outcomes are π_m^d and π_r^d respectively. In other words, the Nash product is $\left(\pi_m^c - \pi_m^d\right) \cdot \left(\pi_r^c - \pi_r^d\right)$. Thus, the Nash bargaining problem can be written as

$$\begin{aligned} Max \quad & \left(\pi_m^c - \pi_m^d\right) \cdot \left(\pi_r^c - \pi_r^d\right) \\ s.t. \quad & \pi_m^c + \pi_r^c \leq \pi_s^c. \end{aligned} \tag{3.22}$$

The Nash bargaining problem above has a unique solution as follows:

$$\begin{aligned} \pi_m^c &= \pi_m^d + \frac{\pi_s^c - \pi_s^d}{2} \\ \pi_r^c &= \pi_r^d + \frac{\pi_s^c - \pi_s^d}{2} \end{aligned}$$

Therefore, when the manufacturer and the retailer have equivalent negotiation power, they equally share the difference between π_s^c and π_s^d, which is the extra channel benefit resulting from centralization.

Note that the above discussion does not consider the negotiation power. To develop generalized Nash bargaining equilibrium with consideration of negotiation power, we normalize the negotiation power of the manufacturer and the retailer to be α and β, where $\alpha + \beta = 1$. Following previous literature (e.g., Roth, 1979), the generalized Nash bargaining problem can be summarized as

$$\begin{aligned} Max \quad & \left(\pi_m^c - \pi_m^d\right)^\alpha \cdot \left(\pi_r^c - \pi_r^d\right)^\beta \\ s.t. \quad & \pi_m^c + \pi_r^c \le \pi_s^c. \end{aligned} \tag{3.23}$$

This generalized Nash bargaining problem above has a unique solution as follows:

$$\begin{aligned} \pi_m^c &= \pi_m^d + \alpha\left(\pi_s^c - \pi_s^d\right) \\ \pi_r^c &= \pi_r^d + \beta\left(\pi_s^c - \pi_s^d\right). \end{aligned}$$

Both the Nash bargaining equilibrium and the generalized Nash bargaining equilibrium assumes that all parties (the manufacturer and the retailer) are rational. However, in reality, the manufacturer and/or the retailer might have fairness concerns and, thus, are boundedly rational. There is some empirical evidence that supply chain coordination efforts sometimes fail due to fairness concerns in the supply chain (Lim & Ho, 2007; Ho & Zhang, 2008). This issue of fairness concern has received some attention in the recent supply chain literature. For example, Cui, Raju and Zhang (2007) examine the effect of fairness considerations in supply chain coordination, and Su (2008) studies some aspects of supply chain contracts that arise from the notion of bounded rationality. In addition, Hu (2008) examines a fair and equitable profit sharing mechanism which allocates supply chain profit according to each party's contribution (i.e., relevant cost involved). Chen (2011) investigates the fair sharing of costs and revenue in supply chains under stochastic demand. The methods used by these two researchers can be applied and extended to our setting for sharing total supply chain profits in proportion to each party's contribution.

It seems important to consider which mechanism, a Nash bargaining equilibrium or a fair profit allocation, is more reasonable and realistic. This issue, however, is somewhat controversial. Nevertheless, we believe that the choice depends on the fairness preference and the relative negotiation power of the parties involved. Obviously, the stronger the fairness preference, the more probable that a fair profit allocation is implemented. Also, when the negotiation powers of the supplier and the retailer are roughly at parity, a fair profit allocation would be more likely. In contrast, if there is a significant power imbalance, it is hard to imagine that a fair allocation process would be resorted to.

3.4.2 Social Benefit

According to proposition 4, centralized decisions in supply chain not only increase the total channel profit but also benefit the consumers in the form of a lower retail price. In other words, the manufacturer and the retailer gain more profit in a centralized supply chain by satisfying more market demand with a lower retail price. Thus, from the perspective of social benefit, centralization in a supply chain is not only beneficial to the manufacturer and the retailer, but also is beneficial to the consumers of the product.

Further more, the wholesale price affects the difference between the optimal centralized retail price and the decentralized retail price, and, thus, affects the extra channel benefit resulting from centralization. When the retailer is more powerful than the manufacturer in a decentralized supply chain, the wholesale price will be relatively low and the decentralized retail price will be closer to the centralized retail price. On the other hand, if the manufacturer is more powerful than the retailer, the wholesale price will be relatively high and the decentralized retail price will be further away from the centralized retail price. Note that the further the decentralized retail price is from the centralized retail price, the stronger would be the effect of centralization in realizing extra channel and consumer benefits. Therefore, from the perspective of supply chain regulation, it is more necessary to centralize supply chains dominated by the manufacturer, for enhancing the resulting social benefit.

3.5 Numerical Example and Sensitivity Analysis

In this section, we use a numerical example to illustrate the proposed algorithm for the centralized supply chain, as well as the solution for the decentralized supply chain. This example is based on data from a shoe factory in south China. A type of shoes is produced by this factory and distributed to a retailer for sale. The supply chain parameters are given as follows: C=\$10/pair; S=\$800/setup; O=\$50/order; F=\$100/shipment; V=\$1/pair; H_m=\$10 per pair per year; H_r=\$11 per pair per year; P=20,000 pairs per year; a=20,000; b=800.

3.5.1 Centralized Supply Chain

In this section, we consider the situation when the shoe factory and the retailer cooperate with each other to maximize the total supply chain profit, i.e. the supply chain is centralized. The supply chain parameters are computed as

$$\begin{aligned}\delta &= \frac{a}{b} - C - V - 2\sqrt{\frac{(O+F)\cdot H_m}{P}} \\ &= \frac{20,000}{800} - 10 - 1 - 2\sqrt{\frac{(50+100)\times 10}{20,000}} \\ &= 13.225,\end{aligned}$$

$$\begin{aligned}\eta &= \sqrt{1 - \frac{2}{P}\left(\frac{b^2\cdot S\cdot H_m}{4}\right)^{1/3}} \\ &= \sqrt{1 - \frac{2}{20,000}\left(\frac{800^2\cdot 800\cdot 10}{4}\right)^{1/3}} \\ &= 0.9442.\end{aligned}$$

Thus, $\frac{P}{b}\left(1-\eta^3\right) < \delta < \frac{P}{b}$. Our solution algorithm yields the following results (the MATLAB code is provided in Appendix 2-1):

$$\begin{aligned}n* &= 2 \\ q* &= 501 \text{ pairs} \\ p_r* &= \$18.54 \text{ per pair}\end{aligned}$$

The algorithm takes only 6 iterations for obtaining the optimal solution, which is extremely efficient. The annual demand is 5169 pairs per year and the maximum profit for the entire supply chain is about $28033 per year.

3.5.2 Decentralized Supply Chain

If the shoe factory and the retailer do not cooperate with each other, but make independent decisions to maximize their own profits, then according to our earlier discussion on decentralized supply chain, we have the following results for this scenario under different wholesale prices.

Table 1: Results of SMSR decentralized supply chain under different p_w

p_w	p_r	π_r^d	π_m^d	π_s^d
12	19.21	24855	2645	27500
13	19.72	20428	6316	26744
14	20.23	16409	9185	25594
15	20.75	12800	11249	24049
16	21.26	9604	12505	22109
17	21.76	6814	13074	19888
18	22.31	4458	12557	17015
19	22.85	2520	11315	13835
20	23.40	1017	9153	10170

As expected, the centralized supply chain yields higher profits for the entire supply chain than the decentralized case for various levels of p_w. Figure 3.2 below shows the shares of the supply chain profits under different wholesale prices; where the dashed line represents the profit level for the centralized supply chain.

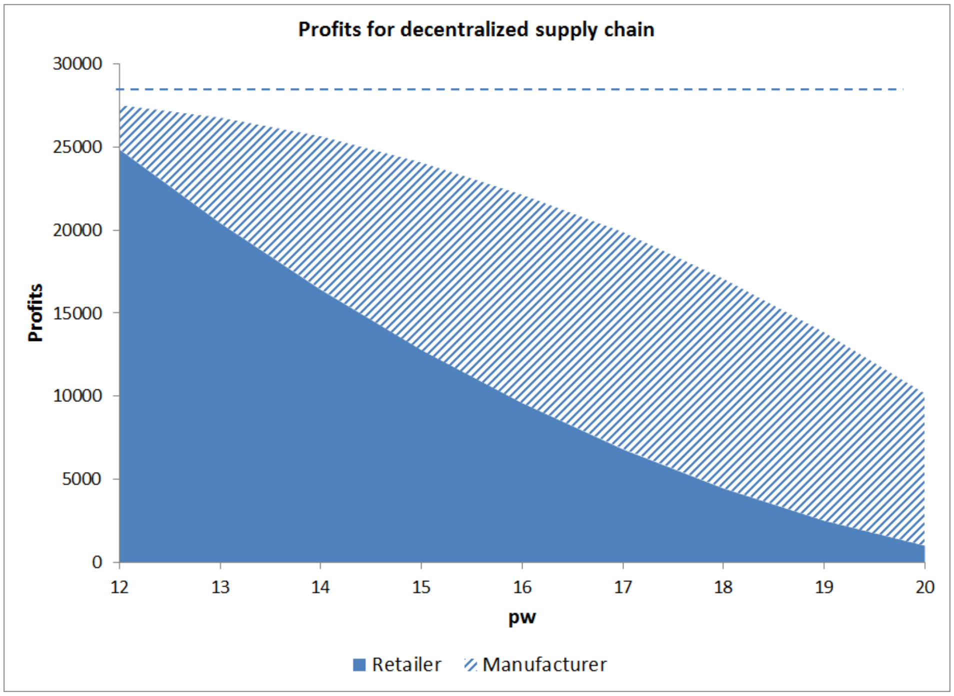

Figure 3.2: Profits for SMSR decentralized supply chain

As shown above, in a decentralized supply chain, the profit of the entire supply chain decrease with increasing wholesale price. Consequently, when the retailer is the more powerful party and is able to ask for a lower wholesale price, the performance of the entire supply chain improves. The rationale for this is that, a lower wholesale price allows the retailer more flexibility to set the retail price. As we can see in Figure 3.2, when the wholesale price is sufficiently low, the profit of the decentralized supply chain is close to that of the centralized supply chain.

One important question here is the effect of centralization on the retail price. Table 1 shows that the retail price in a centralized supply chain is always lower than that of a decentralized supply chain. It indicates that the centralized supply chain yields higher consumer benefit, since it offers lower retail price with higher market demand. As shown in this example, compared with the traditional non-cooperative setting, the cooperation of the manufacturer and the retailer will lower the retail price, increase market demand and improve the profit of the entire supply chain. Moreover, for the decentralized supply chain, the entire supply chain is much more efficient when the retailer is the leader than when it is the follower. In other words, the more powerful the retailer, the more efficient is the decentralized supply chain.

3.5.3 Sensitivity Analysis

The results of the centralized decisions in a supply chain are dependent on various factors, including the market demand parameters a and b, production capacity P, and cost parameters C, O, S, V, F, H_m and H_r. undoubtedly, improving (decreasing) the values of the cost parameters will increase the total profit of the supply chain. But it is not so obvious as to how the market factors (i.e. the demand function parameters, a and b) and the production capacity affect the centralized retail price and the total profit of the supply chain. In this section, we conduct sensitivity analysis to examine this issue.

(1) Changes in the market factors

To examine the effect of the market factors, we fix the production capacity and other cost parameters but change the market factors. We examine the retail price and total supply chain profit for three levels of b and various values of a. The following figures 3.3 and 3.4 show the effect of market factors on the retail price and total supply chain profit respectively.

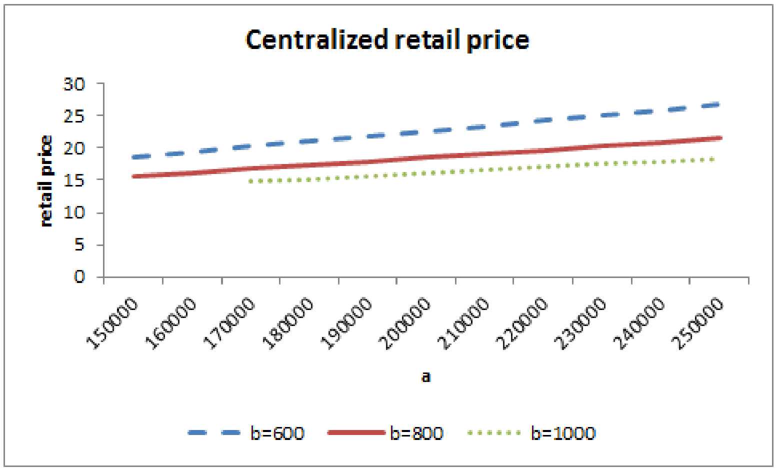

Figure 3.3: Effect of market factors on the retail price

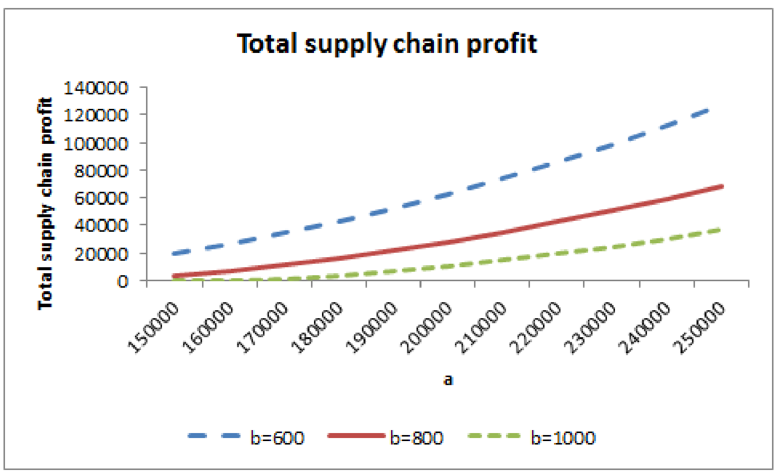

Figure 3.4: Effect of market factors on total supply chain profit

As expected, the higher the value of parameter a, the higher are the retail price and the total supply chain profit. The higher the value of parameter b, the lower are the retail price and the total supply chain profit. Note that, market factor a represents the maximum possible market size. Obviously, the higher the possible market size, the more profitable the supply chain. Thus, when a is higher, the centralized retail price and the total supply chain profit are higher. While for any given a, the lower the value of b, the more price sensitive are the consumers. Hence, under such a situation, the manufacturer and the retailer are forced to lower the retail price, resulting in lower profit level.

(2) Changes in production capacity

To examine the effect of the production capacity, we fix the market factors and the cost parameters but change the production capacity. We examine the retail price and total supply chain profit for various values of production capacity, P. Following figures 3.5 and 3.6 below show the effect of production capacity on the retail price and total supply chain profit respectively.

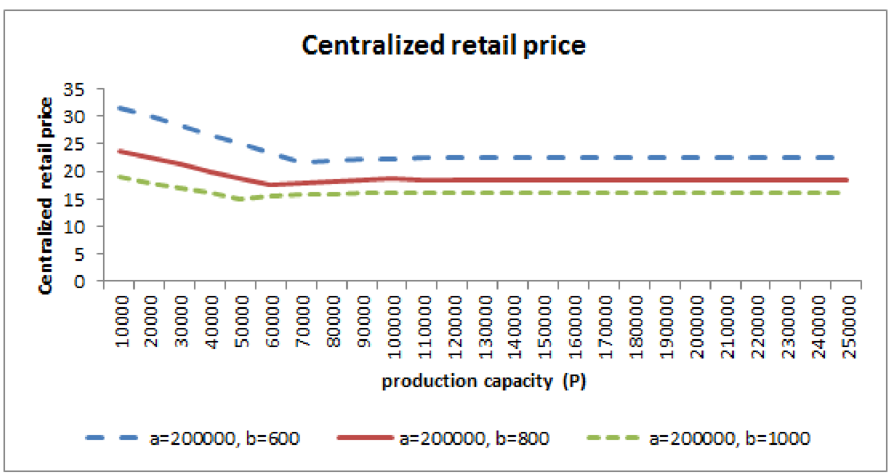

Figure 3.5: Effect of production capacity on the retail price

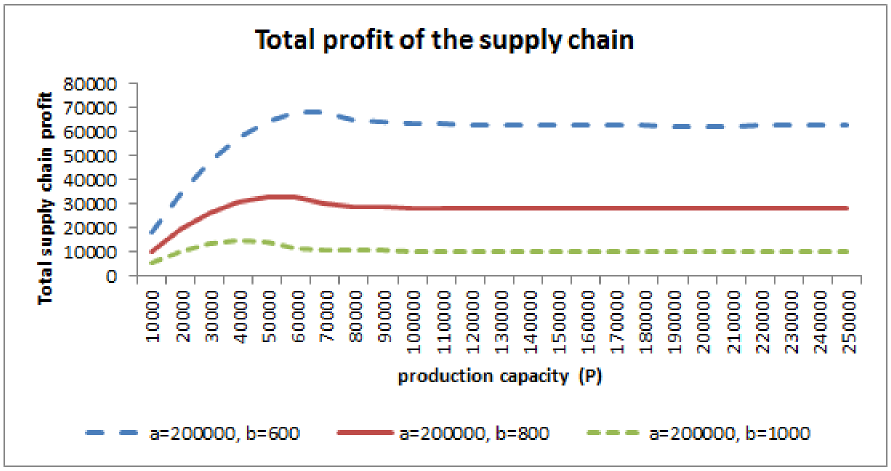

Figure 3.6: Effect of production capacity on total supply chain profit

These figures indicate that a high production capacity does not necessarily bring high profit in the centralized supply chain. Considering the market factors and the cost parameters, it is possible that the resulting market demand is much lower than the production capacity. In this case, improvement of the production capacity does not generate additional profit.

4 Single-Manufacturer Multiple-Retailer (SMMR)

The situations considered in most of the existing literature assume that the demand for an item is from a single buyer (retailer). In practice, it is not uncommon that a manufacturer produces an item and distributes it to several retailers. For example, Apple Inc. manufactures the iPhone and sells it via its distribution centers all over the world. In this situation, the manufacturer and the retailers are naturally centralized. While in some other cases, the supply chain is not necessarily naturally centralized. For example, Sony manufactures PS3 game station and distributes it via some retailers such as Bestbuy.com, Wal-mart, Target and so on. In this chapter, we consider the case where there is one manufacturer and R retailers in the supply chain, facing price-sensitive market demand.

Some attention has been paid by previous researchers to the one-manufacturer multiple-retailer case, but most of them assume a deterministic and exogenous market demand. Lal and Staelin (1984) develop a quantity discount schedule for a manufacturer supplying multiple homogeneous retailers. However, they assume that the manufacturer's production policy is not affected by changes in the retailer's order quantities. Joglekar (1988) points out that the order sizes of the retailer affect not only the manufacturer's revenue, but also its manufacturing cost, which is ignored by Lal and Staelin (1984). Since the decisions made by the manufacturer and the retailers affect each other's profits, it is important to study how the manufacturer and retailers can work in a cooperative manner to maximize the entire supply chain's profit.

Zahir and Sarker (1991) examine the single manufacturer and multiple retailers case under price dependent demands. They assume that the market demands from retailers are functions of their respective retail prices, then they obtain optimum EOQs for the retailers. They also study how the manufacturer can minimize the production cost by compensating the retailers and encouraging them to order quantities different from their EOQs. Although this paper is meaningful from the standpoint of pricing and inventory management practice, it has two shortcomings. First, it does not incorporate the multiple lot shipment policy, i.e, each production lot is delivered in one shipment; secondly, it does not maximize the profit of the entire supply chain, but just find a way for the manufacturer to minimize its cost based on the possible reaction of the retailers. Hence the manufacturer and the retailers do not work in a cooperative manner.

To overcome the limitations of Zahir and Sarker (1991)'s work, Siajadi, Ibrahim and Lochert (2006) propose a single manufacturer and multiple retailers model to minimize the joint total relevant cost (JTRC) for the manufacturer and the retailers. They propose an exact solution for the two-retailer case, as well as an approximate solution for the case of more than two retailers. However, they assume that the product's demand is known and is deterministic. Hoque (2011) further improves this model by incorporating some realistic factors, including limited capacities of transport vehicles and buyer's storage space, significant setup and transportation times, limited lead times and batch sizes. They propose a common optimal solution technique to their models.

Some researchers also examined the single-manufacturer multiple-retailer case from other perspectives. For example, Lu (1995) proposes that it may be difficult for the manufacturer to have information about the retailers' holding and ordering costs. He considers a situation where the objective is to minimize the manufacturer's annual total cost, subject to maximum costs which the retailers are prepared to incur. In order to use this model, the manufacturer only needs to know retailers' annual demands and previous order frequencies. Lu (1995) then proposes a heuristic procedure to obtain an approximate solution for his model. Based on Lu's model, Yao and Chiou (2004) argue that the optimal cost curve of this model is piece-wise convex. Based on this finding, they propose a search algorithm which is claimed to be more efficient than other heuristics and yields the global optimal solutions for the 20 experimental problems used in their paper.

Research on the single-manufacturer multiple-retailer case has been extended in several directions. One of the important extensions involves a three stage supply chain consisting of a single raw material supplier, single or multiple manufacturers, and multiple retailers. Kim, Hong, and Chang (2006) propose an interesting model with a supplier, a manufacturer and multiple retailers, where the retailers may request different types of items. They propose a heuristic procedure to find the production sequence of multiple items, the common production cycle length, and the delivery frequencies and quantities to minimize the average total cost. Nikandish, Eshghi and Torabi (2009) extend the model with a single supplier and multiple manufacturers, each supplying multiple retailers. They propose an analytical solution procedure, as well as an efficient heuristic solution method.

Another important extension is to relax the assumption of deterministic demand and examine the model in a stochastic environment. For example, Banerjee and Banerjee (1994) develop an analytical model based on a common cycle time approach, for a single product, single-manufacturer, multiple-retailer supply chain under stochastic conditions. They present an iterative algorithm for determining the operating parameters, and show that it can be beneficial to all parties in the supply chain to implement their model.

In this chapter, our aim is to improve existing models concerning the single-manufacturer multiple-retailer case by incorporating the pricing decisions (or determining market demand by adjusting the retail price). In this case, market demand is unknown but depends on the retail price. It is not uncommon in practice that an item is produced by a manufacturer and sold in the market by several retailers. Any individual retailer does not have enough market power to significantly affect or determine the market price of this item. In this case, an equilibrium market price will be reached depending on the total quantity of this item in the market. We investigate how the manufacturer and the retailers can work in a cooperative manner to maximize supply chain profit by determining the production sequence, delivery lot sizes and frequencies, as well as the retail price of the product.

The assumptions and notation used in this chapter are presented in the next section. In section 4.2, we derive an analytical model for a centralized supply chain, and propose a solution method for this model. In section 4.3, we discuss the equilibrium retail price in a decentralized supply chain and corresponding maximization problems for the retailers and the manufacturer. We then illustrate our solution procedure through some numerical examples in section 4.4, followed by a discussion of our findings.

4.1 Assumptions

On the basis of existing work in this area (e.g. Banerjee & Banerjee, 1994; Siajadi, Ibrahim, & Lochert, 2006; Kim, Hong, & Chang, 2006; Nikandish, Eshghi, & Torabi, 2009), we assume that the manufacturer coordinates the deliveries to the retailers by employing a common cycle approach. There is a common inventory replenishment cycle time, T, for the manufacturer and all the retailers. In each cycle, the manufacturer has one production setup, and sends n_i equal size shipments of the item to retailer i. Note that the shipment sizes may differ for the retailers in accordance with their relevant individual cost parameters and market shares.

We further assume that production is organized in such a way that the first shipment for all retailers is made at the same time. There are two reasons to organize production in this manner. First, this ensures that the item enters the market via all the retailers at the same time, and none of the retailers will have an advantage by stepping into the market earlier than the others. Secondly, the production sequence proposed by previous researchers may have infeasible schedules. Our model guarantees that the schedule is feasible by organizing it in this way.

We use subscript i to denote the parameters pertaining to retailer i. Consistent with the last chapter and most of the existing literature, we assume an infinite planning horizon, a single product and no shortages in our model. We also assume that, in the centralized supply chain, the market will reach a unique equilibrium price p_r, such that, all the retailers will have the same retail price. Furthermore, each retailer i has a known and fixed market share α_i of this item. Thus, $D_i = \alpha_i \cdot D$ and $\sum_{i=1}^{R} \alpha_i = 1$. The total market demand is confined to no more than P and is dependent on the retail price defined by a linear function, $D = a - b \cdot p_r$.

4.2 Centralized Supply Chain

To understand clearly the implication of the inventory levels at the various stocking points of a single-manufacturer multiple-retail supply chain, Figure 4-1 shows for illustrative purposes, these inventory levels against time for three retailers and a manufacturer.

We first examine the inventory holding costs for the manufacturer and the retailers. The average inventory level for retailer i is

$$I_{r,i} = \frac{q_i}{2} = \frac{T}{2}\frac{D_i}{n_i} = \frac{\alpha_i}{2}\frac{TD}{n_i}. \tag{4.1}$$

Next, we focus on the average inventory level for the whole supply chain. The minimum inventory level for the entire supply chain occurs when the manufacturer starts the production process. At this time, the inventory held by the retailers, which will be replenished by the first set of deliveries, can be expressed as

$$I_{s,min} = D \cdot \sum_{i=1}^{R} \frac{q_i}{P} = \frac{T \cdot D^2}{P} \sum_{i=1}^{R} \frac{\alpha_i}{n_i}.$$

where $\sum_{i=1}^{R} \frac{q_i}{P}$ is the time needed by the manufacturer to produce the necessary quantity of the item for delivering the first shipment to all the retailers. The minimum inventory level can be seen as the "safety stock" for the system to ensure the feasibility of the

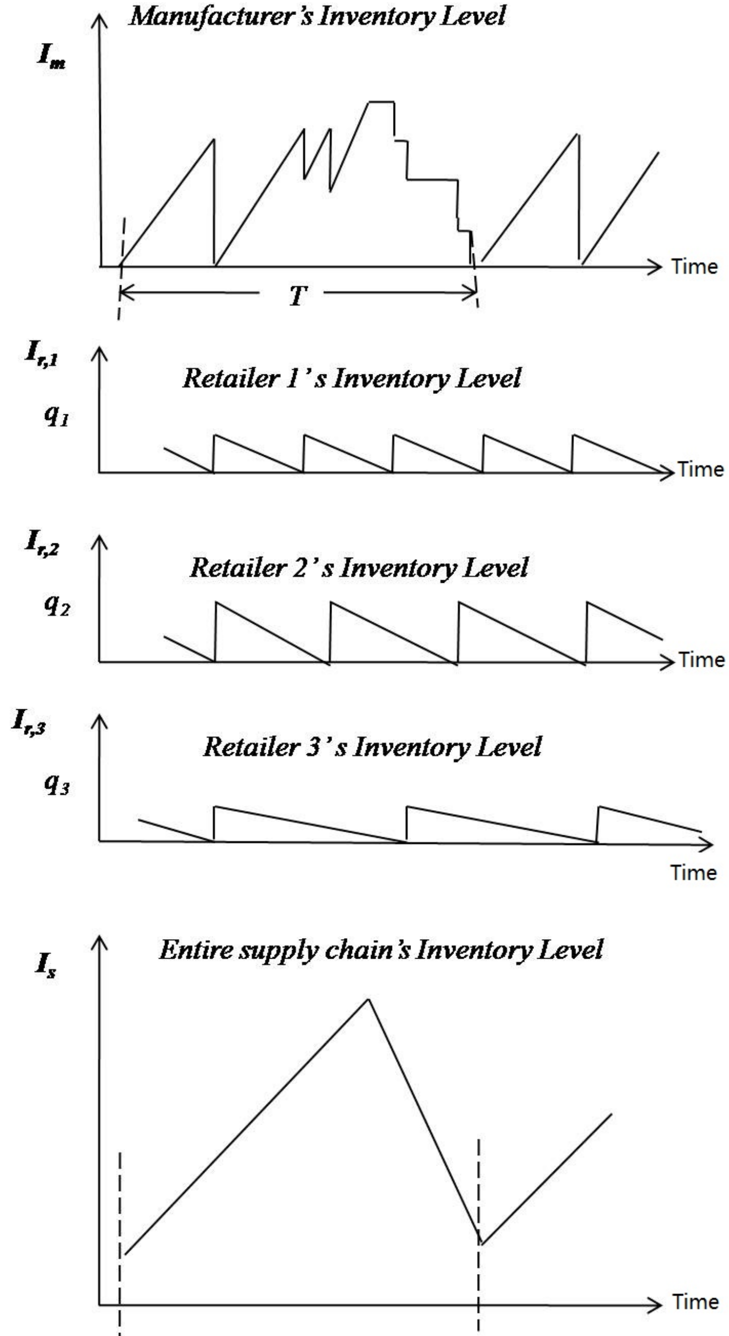

Figure 4.1: Inventory pattern of a supply chain consisting of a manufacturer and 3 retailers

replenishment schedule.

The maximum inventory level for the entire supply chain is the minimum level plus the inventory built up during the production cycle. The rate of increase is the difference between the production rate and the overall market demand rate, $P - D$, and the production time is the production lot size divided by the production rate, i.e. DT/P.

Thus, the maximum inventory level can be written as

$$\begin{aligned} I_{s,max} &= I_{s,min} + \frac{D}{P} \cdot T \cdot (P - D) \\ &= \frac{T \cdot D^2}{P} \left(1 - \sum_{i=1}^{R} \frac{\alpha_i}{n_i} \right) + \frac{D}{P} \cdot T. \end{aligned}$$

Consequently, the average inventory level of the entire supply chain is

$$\begin{aligned} I_s &= I_{s,min} + \frac{1}{2} \left(I_{s,max} - I_{s,min} \right) \\ &= \frac{T \cdot D^2}{P} \sum_{i=1}^{R} \frac{\alpha_i}{n_i} + \frac{D}{2P} \cdot T \cdot (P - D) \,. \end{aligned} \tag{4.2}$$

The average inventory level of manufacturer is

$$\begin{aligned} I_m &= I_s - \sum_{i=1}^{R} I_{r,i} \\ &= \frac{T \cdot D^2}{P} \sum_{i=1}^{R} \frac{\alpha_i}{n_i} + \frac{D}{2P} \cdot T \cdot (P - D) - \sum_{i=1}^{R} \left(\frac{\alpha_i}{2} \frac{TD}{n_i} \right). \end{aligned} \tag{4.3}$$

4.2.1 Model Formulation

The inventory holding cost for the entire supply chain system is

$$\begin{aligned} & H_m \cdot I_s + \sum_{i=1}^{R} (H_{r,i} - H_m) \cdot I_{r,i} \\ = \; & H_m \cdot \left[\frac{D^2}{P} \cdot T \cdot \sum_{i=1}^{R} \frac{\alpha_i}{n_i} + \frac{D}{2P} \cdot T \cdot (P - D) \right] + \sum_{i=1}^{R} \left[(H_{r.i} - H_m) \cdot \frac{\alpha_i}{2} \cdot \frac{T \cdot D}{n_i} \right]. \end{aligned}$$

The sum of the production setup, the buyers' ordering and the fixed shipment costs per year can be written as

$$\frac{S + \sum_{i=1}^{R} [n_i \cdot O_i]}{T}.$$

The transportation cost per year consists of the fixed costs and associated variable costs:

$$\frac{\sum_{i=1}^{R} [n_i \cdot F_i]}{T} + D \sum_{i=1}^{R} (V_i \cdot \alpha_i).$$

Therefore, the profit for the entire supply chain can be expressed as

$$\begin{aligned} \pi_s = \; & D \cdot (p_r - C) - D \cdot \sum_{i=1}^{R} (V_i \cdot \alpha_i) - \frac{S + \sum_{i=1}^{R} [n_i \cdot (O_i + F_i)]}{T} \\ - \; & \left\{ H_m \cdot \left[\frac{D^2}{P} \cdot T \cdot \sum_{i=1}^{R} \frac{\alpha_i}{n_i} + \frac{D}{2P} \cdot T \cdot (P - D) \right] + \sum_{i=1}^{R} \left[(H_{r.i} - H_m) \cdot \frac{\alpha_i}{2} \cdot \frac{T \cdot D}{n_i} \right] \right\}. \end{aligned}$$

In summary, we have following optimization model

$$\begin{aligned}
\underset{D,T,n_i}{Max}\pi_s &= D\cdot\left(\frac{a-D}{b}-C\right)-D\cdot\sum_{i=1}^{R}(V_i\cdot\alpha_i)-\frac{S+\sum_{i=1}^{R}[n_i\cdot(O_i+F_i)]}{T}\\
&- \left\{H_m\cdot\left[\frac{D^2}{P}\cdot T\cdot\sum_{i=1}^{R}\frac{\alpha_i}{n_i}+\frac{D}{2P}\cdot T\cdot(P-D)\right]+\sum_{i=1}^{R}\left[(H_{r.i}-H_m)\cdot\frac{\alpha_i}{2}\cdot\frac{T\cdot D}{n_i}\right]\right\}\\
s.t. \quad & D,T>0\\
& D<P\\
& n_i\in\{1,2,3,...\} \text{ for } i=1,2,...,R. \qquad (4.4)
\end{aligned}$$

To solve this problem, we first relax the requirements of n_i to be positive integers. These will be considered later. For any given D, T and n_i must satisfy the first order optimality conditions as shown below.

$$\begin{aligned}
\frac{\partial\pi_s}{\partial T} &= \frac{1}{T^2}\left\{S+\sum_{i=1}^{R}[n_i\cdot(O_i+F_i)]\right\}\\
&- \left\{H_m\cdot\left[\frac{D^2}{P}\cdot\sum\frac{\alpha_i}{n_i}+\frac{D}{2P}\cdot(P-D)\right]+\sum_{i=1}^{R}\left[(H_{r.i}-H_m)\cdot\frac{\alpha_i}{2}\cdot\frac{D}{n_i}\right]\right\}\\
&= 0. \qquad (4.5)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\pi_s}{\partial n_i} &= -\frac{(O_i+F_i)}{T}+H_m\left[\frac{D^2}{P}\cdot T\cdot\frac{\alpha_i}{n_i^2}\right]+(H_{r,i}-H_m)\cdot\frac{\alpha_i}{2}\frac{T\cdot D}{n_i^2}\\
&= 0. \qquad (4.6)
\end{aligned}$$

From these, we can derive the optimal T and n_i values as follows:

$$T = \sqrt{\frac{2 \cdot S \cdot P}{H_m \cdot D \cdot (P - D)}} \tag{4.7}$$

$$n_i = T \cdot \sqrt{\frac{\left[H_m \cdot \frac{D}{P} + \frac{H_{r,i} - H_m}{2}\right] \cdot D \cdot \alpha_i}{(O_i + F_i)}}. \tag{4.8}$$

Substituting (4.7), (4.8) and $D = a - b \cdot p_r$ into the objective function, we convert the original optimization model (4.4) to a single variable maximization problem involving p_r, i.e.

$$\begin{aligned} \underset{p_r}{Max} \pi_s &= (a - b \cdot p_r)\left[p_r - C - \sum_{i=1}^{R}(V_i \alpha_i)\right] - 2\sqrt{\frac{H_m \cdot (a - b \cdot p_r) \cdot (P - a + b \cdot p_r) \cdot S}{2P}} \\ &- 2\sum_{i=1}^{R}\sqrt{\left[H_m \cdot \frac{(a - b \cdot p_r)}{P} + \frac{H_{r,i} - H_m}{2}\right] \cdot (a - b \cdot p_r) \cdot \alpha_i \cdot (O_i + F_i)}. \end{aligned} \tag{4.9}$$

For simplicity, we define the following parameter:

$$\bar{\delta} = \frac{a}{b} - C - \sum_{i=1}^{R} \alpha_i V_i - 2\sum_{i=1}^{R}\sqrt{\frac{\alpha_i (O_i + F_i) \cdot H_m}{P}}.$$

With this parameter, our analysis (shown in Appendix 3) leads to the propositions below. All the propositions of the single-manufacturer single-retailer centralized supply chain as outlined in Chapter 3 can be extended to single-manufacturer multiple-retail centralized supply chain by replacing δ with $\bar{\delta}$.

Proposition 5. *The product is not profitable if* $\bar{\delta} \leq \frac{P}{b}\left(1 - \eta^3\right)$ *. If this condition holds, the entire supply chain cannot achieve a positive profit.*

As discussed in chapter 3, $\bar{\delta}$ reflects the product's profitability. Proposition 5 shows that, when the product profitability is lower than or equal to a certain level, the best strategy for the manufacturer and the retailer is to not deal with this product. In contrast, if the product profitability is very high, the manufacturer and the retailer should be willing to provide as much of the product as possible to the market to increase their respective profits. This characteristic is shown and described in following proposition.

Proposition 6. *If* $\bar{\delta} \geq \frac{P}{b}(1+\eta^3)$, *the profit of the entire supply chain is maximized by choosing* $p_r = \frac{a-P}{b}$. *That is, the manufacturer will keep producing the product continuously without any interruptions. In this case, optimal* $n_i \mapsto \infty$*and* $q_i = \sqrt{\frac{2P\cdot(O_i+F_i)}{H_{r,i}+H_m}}$

.

The corresponding supply chain profit under this condition is given by

$$\hat{\pi_s} = P \cdot \left(\frac{a-P}{b} - C - \sum_{i=1}^{R} \alpha_i V_i\right) - \sum_{i=1}^{R} \alpha_i \sqrt{2(O_i+F_i)\cdot P \cdot (H_m + H_{r,i})}. \quad (4.10)$$

When the product profitability $\bar{\delta}$ is neither as low as the condition stated in Proposition 5, and not as high as the condition stated in Proposition 6, the manufacturer and the retailer have to determine the retail price in order to maximize the profit of the entire supply chain. In this situation, we state following proposition.

Proposition 7. *If* $\frac{P}{b} \leq \bar{\delta} < \frac{P}{b}(1+\eta^3)$, *the objective function is strictly concave in* $\left(\frac{a}{b} - \frac{P\cdot(1+\eta)}{2b}, \frac{a}{b} - \frac{P}{2b}\right]$; *If* $\frac{P}{b}(1-\eta^3) < \bar{\delta} \leq \frac{P}{b}$, *the objective function is strictly concave in* $\left[\frac{a}{b} - \frac{P}{2b}, \frac{a}{b} - \frac{P\cdot(1-\eta)}{2b}\right)$. *In these situations, the maximization problem has a unique inner local maximum solution,* $\bar{p_r}$. *Specifically,*

$$\begin{cases} \frac{a}{b} - \frac{P}{2b} < \bar{p_r} < \frac{a}{b} - \frac{P}{2b}\left(1-\eta\right) & if\ \frac{P}{b}\left(1-\eta^3\right) < \bar{\delta} < \frac{P}{b} \\ \bar{p_r} = \frac{a}{b} - \frac{P}{2b} & if\ \bar{\delta} = \frac{P}{b} \\ \frac{a}{b} - \frac{P}{2b} > \bar{p_r} > \frac{a}{b} - \frac{P}{2b}\left(1+\eta\right) & if\ \frac{P}{b}\left(1+\eta^3\right) > \bar{\delta} > \frac{P}{b}. \end{cases}$$

Based on above propositions, a binary-search algorithm is developed in section 4.2.2 to search for the optimal retail price. Once the optimal retail price is determined, the objective function is concave in the number of shipments $n_i, \forall i$, and T. Thus, the corresponding optimal T is determined by equation (4.7); and the optimal n_i is an integer around $\hat{n_i}$ in equation (4.8), either $\lfloor \hat{n_i} \rfloor$ or $\lceil \hat{n_i} \rceil$.

4.2.2 Solution Algorithm

The single-manufacturer single-retailer model can thus be seen as a special case of single-manufacturer multiple-retailer model. Following the same line of reasoning as in the last chapter, we propose the following algorithm to solve this optimization problem. We first calculate $\bar{\delta}$ and η. Based on the calculated $\bar{\delta}$ and η, we apply corresponding propositions. If $\bar{\delta} \geq \frac{P}{b}\left(1+\eta^3\right)$ or $\bar{\delta} \leq \frac{P}{b}\left(1-\eta^3\right)$, an optimal solution can be determined immediately. Otherwise, based on Proposition 7, a binary search method is used to find the optimal retail price within the corresponding interval. The following steps are used in this algorithm:

Step 1. Calculate $\bar{\delta}$ and η. If $\bar{\delta} \geq \frac{P}{b}\left(1+\eta^3\right)$, then $p_r^* = \frac{a-P}{b}$, $n_i^* \mapsto \infty$, $q_i^* = \sqrt{\frac{2P\cdot(O_i+F_i)}{H_{r,i}+H_m}}$; If $\bar{\delta} \leq \frac{P}{b}\left(1-\eta^3\right)$, then stop; If $\bar{\delta} = \frac{P}{b}$, then $\bar{p_r} = \frac{a-P/2}{b}$ and go to step 4; If $\frac{P}{b}\left(1-\eta^3\right) < \bar{\delta} < \frac{P}{b}$, then go to step 2; otherwise, go to step 3;

Step 2. Perform a binary search over the interval $\left[\frac{a}{b}-\frac{P}{2b},\frac{a}{b}-\frac{P\cdot(1-\eta)}{2b}\right)$ to search for a $\bar{p_r}$ such that $2\bar{p_r}-\sqrt{S\cdot H_m}\frac{\left(P-2\bar{D}\right)}{\sqrt{2\bar{D}\cdot P\left(P-\bar{D}\right)}}=\frac{2a}{b}-\bar{\delta}$, where $\bar{D}=a-b\cdot\bar{p_r}$; go to step 4;

Step 3. Perform a binary search over the interval $\left(\frac{a}{b}-\frac{P\cdot(1+\eta)}{2b},\frac{a}{b}-\frac{P}{2b}\right]$ to search for a $\bar{p_r}$ such that $2\bar{p_r}-\sqrt{S\cdot H_m}\frac{\left(P-2\bar{D}\right)}{\sqrt{2\bar{D}\cdot P\left(P-\bar{D}\right)}}=\frac{2a}{b}-\bar{\delta}$, where $\bar{D}=a-b\cdot\bar{p_r}$; go to step 4;

Step 4. Calculate $\bar{D}=a-b\cdot\bar{p_r}$, $\bar{T}=\sqrt{\frac{2\cdot S\cdot P}{H_m\cdot\bar{D}\cdot\left(P-\bar{D}\right)}}$, $n_i=\bar{T}\cdot\sqrt{\frac{\left[H_m\cdot\frac{D}{P}+\frac{H_{r,i}-H_m}{2}\right]\cdot D\cdot\alpha_i}{(O_i+F_i)}}$; using equation (4.9), if $\pi_s\left(\bar{p_r},\lfloor\tilde{n_i}\rfloor,\bar{T}\right)\geq\pi_s\left(\bar{p_r},\lceil\tilde{n_i}\rceil,\bar{T}\right)$ (comparing all possible combinations), then $\bar{n}=\lfloor\tilde{n}\rfloor$, otherwise, $\bar{n_i}=\lceil\tilde{n_i}\rceil$; calculate corresponding $\bar{\pi_s}=\pi_s\left(\bar{p_r},\bar{n_i},\bar{T}\right)$; go to step 5;

Step 5. If $\bar{\pi_s}\geq max\left\{\hat{\pi_s},0\right\}$, then $p_r^*=\bar{p_r}$, $n_i^*=\bar{n_i}$, $T^*=\bar{T}$; if $\bar{\pi_s}\leq 0$, then quit; otherwise, $p_r^*=\frac{a-P}{b}$, $n_i^*\mapsto\infty$, $q_i^*=\sqrt{\frac{2P\cdot(O_i+F_i)}{H_{r,i}+H_m}}$.

4.3 Decentralized Supply Chain

The decentralized multiple-retailer model is quite different from the decentralized single-retailer model. Unlike a centralized supply chain, in a decentralized supply chain, the retailers do not work in a cooperative manner, but compete with each other. Numerous studies have been done to examine the competition among multiple retailers in a supply chain. Most of the existing literature, nevertheless, does not incorporate the production process of the manufacturer and its production capacity (e.g. Cachon, 2000). Since the production decision and capacity are incorporated in this thesis, conclusions and solution methods from the earlier studies cannot be applied.

4.3.1 The Nash Equilibrium

The competition among the retailers will result in a Nash equilibrium retail price, p_r. According to the market clear condition, the total market demand D can be expressed as $D = a + b \cdot p_r = \sum_{i=1}^{R} D_i$, where D_i is the order quantity per year of retailer i. Note that, for each retailer i, the decision variable is D_i but not $p_{r.}$ In a decentralized and competitive environment, a retailer i is not able to determine the retail price. The retail price is the equilibrium value of the competition which satisfies the market clear condition that the total demand is equal to the total supply from the retailers. Each retailer i decide its order quantity and sells all of this quantity in the market, which is denoted by D_i (i.e., this is a Cournot game; the retailers compete on the amount of the product they order from the manufacturer) .

As discussed in chapter 3, the wholesale price, p_w, will affect the profit shares of the members of the supply chain. The manufacturer would prefer a higher p_w, whereas the retailers prefer it to be as low as possible. The manufacturer and the retailers may negotiate with each other to determine the wholesale price, p_w, to decide their individual profit levels. Once p_w is established, each retailer i has to decide the order quantity per year D_i and its economic order size q_i; and the manufacturer needs to determine its production lot size $Q = \sum_{i=1}^{R} (n_i \cdot q_i)$. Note that, each q_i is decided by retailer i. Thus, essentially, the manufacturer decides the number of shipments to each retailer in a production cycle.

The retailer i faces the following maximization problem

$$\begin{aligned}
\underset{q_i,\, D_i}{Max} \quad & D_i \cdot \left(\frac{a - \sum_{j=1}^{R} D_j}{b} - p_w - V_i \right) - \frac{(O_i + F_i) \cdot D_i}{q_i} - \frac{H_{r,i} \cdot q_i}{2} \\
s.t. \quad D_i < \; & P - \sum_{j \neq i} D_j \\
D_i > \; & 0 \\
q_i > \; & 0.
\end{aligned} \tag{4.11}$$

Thus, the optimal q_i for any given D_i can be written as

$$q_i = \sqrt{\frac{2\,(O_i + F_i) \cdot D_i}{H_{r,i}}}. \tag{4.12}$$

Based on equation 4.12, maximization problem 4.11 is equivalent to

$$\begin{aligned}
\underset{D_i}{Max}\; \pi_{r,i}^{d}\,(D_i) \quad = \quad & D_i \cdot \left(\frac{a - \sum_{j=1}^{R} D_j}{b} - p_w - V_i \right) - \sqrt{2H_{r,i} \cdot (O_i + F_i) \cdot D_i} \\
s.t. \quad D_i < \; & P - \sum_{j \neq i} D_j \\
D_i > \; & 0.
\end{aligned} \tag{4.13}$$

The best response of retailer i to the other retailers' decisions is determined by the first order optimality condition. The Nash equilibrium is the solution to the best response of all retailers. Therefore, at the Nash equilibrium,

$$\begin{cases} \left(\frac{a-\sum_{j=1}^{R} D_j}{b} - p_w - v\right) - \frac{2D_i}{b} - \sqrt{\frac{H_{r,i}\cdot(O_i+F_i)}{2D_i}} = 0 & \text{for } i = 1,2,...,R \\ D_i > 0 & \text{for } i = 1,2,...,R \\ \sum_{j=1}^{R} D_j < P & \end{cases}$$

4.3.2 Manufacturer's Maximization Problem

The manufacturer will then make its decision on the production lot size Q, as well as the shipment schedule n_is, in order to maximize its profit. The inventory level of the manufacturer can be expressed as

$$\frac{1}{2}\left[\sum_{j=1}^{R}\left[\left(\frac{2-n_j}{P}\right)D_j - 1\right]\cdot q_j + Q\right].$$

which is derived following the same line of reasoning in SMSR supply chains. Hence, the maximization problem for the manufacturer can be expressed as follows.

$$\begin{aligned} \underset{Q,n_1,n_2,...,n_R}{Max}\ \pi_m^d &= \left(\sum_{j=1}^{R} D_j\right)\cdot\left(p_w - c - \frac{S}{Q}\right) - \frac{H_m}{2}\left[\sum_{j=1}^{R}\left[\left(\frac{2-n_j}{P}\right)D_j - 1\right]\cdot q_j + Q\right] \\ s.t.\quad n_j &\in \{1,2,...\} \\ Q &= \sum_{i=1}^{R} n_j q_j. \end{aligned} \tag{4.14}$$

4.4 Numerical Example

In this section, we use a numerical example to illustrate the proposed algorithm for the centralized supply chain, as well as the solution for the decentralized supply chain. A type of shoes is produced by a shoe factory and distributed to two retailers for sale. The relevant supply chain parameters are given as follows: C=\$10/pair; S=\$800/setup; O_1=\$50/order; F_1=\$100/shipment; V_1=\$1/pair; O_2=\$45/order; F_1=\$95/shipment; V_1=\$0.9/pair; H_m=\$10 per pair per year; $H_{r,1}$=\$11 per pair per year; $H_{r,2}$=\$10.5 per pair per year; P=20,000 pairs per year; a=20,000; b=800; $\alpha_1 = 0.6$; α_2=0.4.

4.4.1 Centralized Supply Chain

We first examine the case where the shoe factory and the two retailers decide to co-operate with each other to maximize the supply chain profits, i.e. the supply chain is centralized. The supply chain parameters are computed as

$$\begin{aligned} \bar{\delta} &= \frac{a}{b} - C - \sum_{i=1}^{R} \alpha_i V_i - 2\sum_{i=1}^{R} \sqrt{\frac{\alpha_i(O_i + F_i) \cdot H_m}{P}} \\ &= 13.281 \end{aligned}$$

$$\begin{aligned} \eta &= \sqrt{1 - \frac{2}{P}\left(\frac{b^2 \cdot S \cdot H_m}{4}\right)^{1/3}} \\ &= \sqrt{1 - \frac{2}{20,000}\left(\frac{800^2 \cdot 800 \cdot 10}{4}\right)^{1/3}} \\ &= 0.9442. \end{aligned}$$

Thus, $\frac{P}{b}\left(1-\eta^{3}\right) < \bar{\delta} < \frac{P}{b}$. According to the algorithm outlined in section 4.2.2, we have following results (the MATLAB code is provided in Appendix 2-2):

$$
\begin{aligned}
n_1* &= 1 \\
n_2* &= 2 \\
T* &= 0.2048 \text{ year} \\
p_r* &= \$18.58 \text{ per pair}
\end{aligned}
$$

Our procedure requires only 5 iterations to yield the optimal solution, which is extremely efficient. The annual demand is 5132 pairs of shoes per year. The maximum profit for the entire supply chain is about $27020 per year.

Compared with the numerical example in the previous chapter, this example has an additional retailer. These two examples illustrate that adding one more retailer actually reduces the profit level of the entire supply chain from $28033 per year to $27020 per year, even though the added retailer (i.e., retailer 2 in this example) has more favorable cost parameters than the existing retailer (i.e., retailer 1 in this example). One of the reasons for this is that distributing a product via multiple retailers requires the manufacturer to coordinate the replenishment schedules of the retailers. This may force some retailers to give up their own individually optimal positions. Another reason is that the transportation cost may be higher for multiple retailers. Products are sent separately to several retailers, thus such separate shipments cannot share the fixed transportation cost. In practice, marketing managers usually believe that it is desirable to incorporate more efficient retailers in the supply chain. Our examples, however, show that, if the supply chain is centralized, this is not necessarily true.

4.4.2 Decentralized Supply Chain

If the manufacturer and retailers do not cooperate with each other, then the profits in the decentralized supply chain are illustrated in following figure.

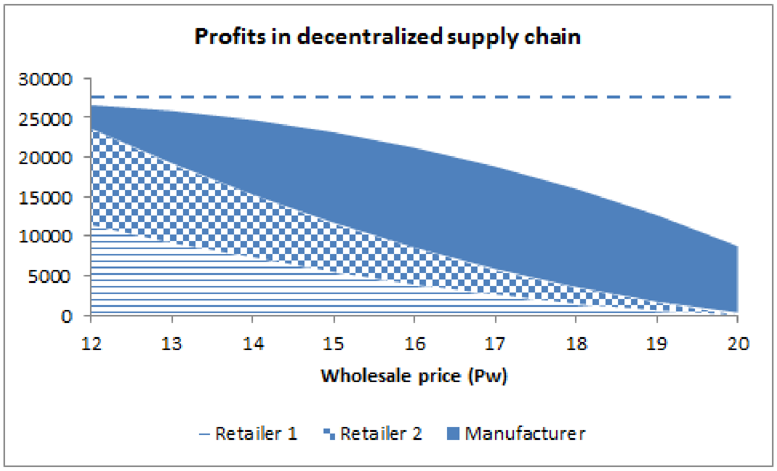

Figure 4.2: Profits in the SMMR decentralized supply chain

As expected, the centralized supply chain yields a higher profit for the entire supply chain than the decentralized case. In a decentralized supply chain, the profit of the entire supply chain decreases with increasing wholesale price. Consequently, when the retailer is the more powerful party and is able to seek a lower wholesale price, the performance of the entire supply chain improves. The equilibrium retail prices are illustrated in Table 2 below.

Table 2: Equilibrium retail price in the SMMR decentralized supply chain

p_w	12	13	14	15	16	17	18	19	20
p_r	19.27	19.78	20.3	20.82	21.24	21.87	22.41	22.96	23.56

Again, consistent with our expectation, the decentralized retail prices are higher than the centralized retail price. As in the single manufacturer single retailer case, the centralization of the supply chain benefit the consumers in the form of a lower retail price.

5 Stochastic Models

Most of existing literature assumes that the market demand of the item in question is deterministic. In inventory management practice, however, market demand usually is stochastic and cannot be predicted with certainty. To study the joint pricing, production and inventory decisions in this situation, we develop a stochastic model under a periodic review policy in this chapter.

Banerjee & Banerjee (1994) is one of the few papers which examines the JELP problem in a stochastic environment. They develop an analytical model for coordinated inventory control between a supplier and multiple buyers. In their model, all the shortages at the buyers and the supplier ends are fully backordered. During each production cycle, the manufacturer produces a production lot based on a produce up-to level of S_m (the amount of product for a new cycle after fulfilling the backorders from the last cycle, if any) and send n shipments to each retailer i with lead time of l_i. The shipment size to retailer i is the difference between a replenish-up-to level S_i and its actual inventory level when the shipment is sent. Banerjee and Banerjee propose an iterative solution algorithm to determine the production cycle length, shipment schedule n, the produce-up-to level S_m and the replenish-up-to level S_i for each retailer. Although due to potential analytical complexities, their solution is approximate and does not guarantee optimality, the numerical examples presented in their paper illustrate the efficiency of their solution algorithm.

Following Banerjee and Banerjee (1994)'s analysis, we develop an analytical model to determine the retail price p_r, production cycle length T, shipment schedule n, and the

(S_m, S_r) policy to maximize the profit of the entire supply chain. To facilitate the model formulation process, we develop a single-manufacturer single-retailer model under an infinite planning horizon. Since incorporating deterministic delivery lead times does not affect the results of our model, for simplicity, we assume that all delivery lead times are zero. We present a model below which may serve as a building block of further extensions in this area of research.

Our model has three important differences from that developed by Banerjee and Banerjee (1994). First, the retail price is not exogenous but a decision variable. market demand is assumed to be stochastic, stationary and dependent on the retail price. Thus, the objective is not to minimize the total relevant costs for the entire supply chain any longer, but to maximize the expected profit of the entire supply chain. Secondly, we use a different cost structure from that adopted by Banerjee and Banerjee (1994)'s model in terms of backorder, ordering and transportation costs. In their paper, Banerjee and Banerjee (1994) assume the backorder cost to be fixed for any stockout incident. We assume that the backorder cost is related to the amount of backordered products. In particular, we describe the backorder cost for the retailer as B_r dollars per backordered unit; whereas the backorder cost for the manufacturer is B_m dollars per backordered unit. Moreover, we incorporate retailer ordering and transportation costs in our model. In particular, the transportation cost consists of a fixed cost per shipment and a variable cost per unit shipped. While these costs are ignored in Banerjee and Banerjee (1994)'s model.

The assumptions and notation used in this chapter are presented in the next section. In section 5.2, we derive an analytical model followed by a proposed solution method in section 5.3. We then illustrate our solution method through some numerical examples in section 5.4, and section 5.5 presents a discussion of our findings.

5.1 Assumptions

Following Banerjee and Banerjee (1994), we make the following major assumptions:

(1) All shortages are backordered. The backorder cost is based on the concept of a fixed cost per unit of shortage, i.e. the total backorder cost is linear in the number of backorders.

(2) Delivery lead times are zero or negligibly small.

(3) "Backorder costs are sufficiently high, such that the average shortage quantities are negligibly small compared to average inventory balance for each party" (an assumption in Banerjee and Banerjee (1994)'s model).

(4) The total annual demand $D = \bar{D} \cdot \epsilon = (a - b \cdot p_r) \cdot \epsilon$; where ϵ is a random variable with expected value of 1 and standard deviation of σ. $g(\epsilon)$ and $G(\epsilon)$ are the probability density function and the cumulative density function of ϵ respectively.

Assumption (1) is widely used in the literature to represent the relationship between backorder cost and number of backorders. We use a linear backorder cost in this thesis, thus the backorder cost is independent of the time of shortage. Our analysis could be easily extended to incorporate backorder cost which depends on both the amount of backordered and the duration of shortage.

Assumptions (2) and (3) are used for presentation and analysis simplicity only. Including lead time in our model does not make any differences in the qualitative results and the model properties. Assuming a small shortage facilitates the derivation of expected inventory cost in the stochastic model. In practice, especially in the competitive environment, companies usually do their best to improve consumer satisfaction and try to fulfill all consumers' demand in time. Therefore, it is reasonable to assume that the unit backorder cost is high and the shortage amount is small.

Assumption (4) indicates that we are using a multiplicative stochastic term in this thesis. The two most widely methods used in existing literature to model stochastic demand are the multiplicative (e.g., Cachon & Kok, 2004; Petruzzi & Dada,1999; Song, Ray & Boyaci, 2006) and the additive forms (e.g., Polatoglu & Sahin, 2000; Chen & Simchi-Levi, 2004a). The multiplicative form assumes that the customer demand is the product of a price-sensitive deterministic component and a non-price-sensitive random variable; whereas the additive form assumes that the customer demand is the sum of these two terms.

An important difference between the additive and the multiplicative forms is the manner in which the random variable affects demand. In the additive demand case, the variance of demand is independent of price, and the coefficient of variation of demand is an increasing function of price. In the multiplicative demand case, the variance of demand is a decreasing function of price, and its coefficient of variation is independent of price. It appears that, it may be important to consider which form is more realistic. Nevertheless, Aiginger (1987) considers this problem and concludes that there is no rational economic basis for choosing between these two forms.

We believe that it may be more reasonable to use the multiplicative form in this thesis. This form is not only widely used but also is supported by empirical evidence (Cachon and Kok 2004, Petruzzi and Dada 1999, Tellis 1988, and references therein). In our setting, the deterministic part of the demand is unknown and variable. Demand may vary significantly depending on the retail price. The additive form assumes that the stochastic demand consists of a price-sensitive deterministic part and a non-price-sensitive stochastic shock. If the magnitude of customer demand is unknown, it is not easy to define an appropriate demand shock under the additive assumption. In addition, it is unlikely that the variance of demand is independent of the retail price (as assumed by the additive form). For example, it is not reasonable to assume that two markets, with expected demands of 1,000 and 10,000 units respectively, have the same demand variance. The multiplicative form does not have these problems and is, thus, more suitable for the setting adopted in this thesis.

5.2 SMSR Stochastic Model

In this section, we develop the model formulation for a supply chain with single manufacturer and single retailer. We use the subscript r to denote the parameters pertaining to a retailer. To formulate the model, we have to examine the revenues and costs associated in the supply chain. We first examine the inventory pattern to derive the inventory cost. Figure 5-1 shows the inventory-time plots for a supply chain consisting of a single manufacturer and a single retailer.

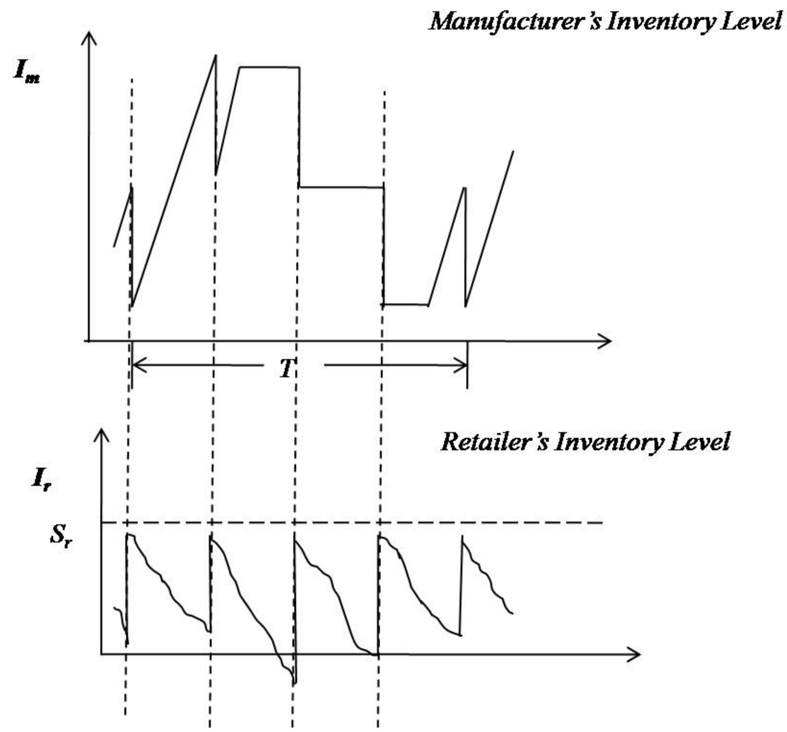

Figure 5.1: Inventory pattern of a supply chain consisting of a manufacturer and a retailer in a stochastic demand environment

Following Banerjee and Banerjee (1994), the average inventory level of the retailer can be expressed as one half of its expected demand each replenishment cycle $(a - b \cdot p_r) \cdot T/n$ plus the safety stock, which is the difference of S_r and the expected demand during each replenishment cycle $D \cdot T/n$. Hence, the average inventory level of the retailer is

$$\begin{aligned} I_r &= \frac{\bar{D} \cdot T/n}{2} + \left[S_r - D \cdot \bar{T}/n\right] \\ &= S_r - \frac{\bar{D} \cdot T}{2n}. \end{aligned} \tag{5.1}$$

Following the same line of reasoning, the average inventory level of the manufacturer can be expressed as its average active inventory plus its safety stock. The average active inventory can be written as (see Chapter 3 of this thesis or Banerjee (1986) for derivation)

$$\frac{\bar{D} \cdot T}{2n} \cdot \left\{ \frac{\bar{D}}{P} (2-n) + n - 1 \right\}.$$

The safety stock of the manufacturer is given by the difference of the produce-up-to level S_m and the expected market demand during the production cycle T. That is, it can be described as $S_m - \bar{D} \cdot T$. Thus, the average inventory level of the manufacturer is

$$I_m = \frac{\bar{D} \cdot T}{2n} \left\{ \frac{\bar{D}}{P} (2-n) + n - 1 \right\} + S_m - \bar{D} \cdot T. \tag{5.2}$$

We then examine the backorder costs for the manufacturer and the retailer respectively. Note that during each cycle, the demand is $D \cdot T$, while the quantity of products available for the cycle is S_m. So, at the manufacturer end, the expected total backorder quantity during time interval T is

$$\int_{\frac{S_m}{\bar{D} \cdot T}}^{\infty} \left(\bar{D} \cdot T \cdot x - S_m \right) \cdot g(x)\, dx.$$

At the retailer's end, the demand during each replenishment cycle is $D \cdot T/n$. While the quantity of products available for this time interval is S_r. So, the expected backorder quantity during the time interval T/n is

$$\int_{\frac{S_r}{\bar{D}T/n}}^{\infty} \left(\frac{\bar{D} \cdot T}{n} \cdot x - S_r \right) g(x)\, dx.$$

The expected total profit is the expected revenue minus the expected total cost, including production cost, inventory cost, backorder cost, transportation cost and ordering cost. Therefore, the expected total profit for the entire supply chain can be expressed as

$$\begin{aligned}
\pi_s &= \bar{D}\cdot(p_r - C) - \left[\frac{n\cdot O + S}{T}\right] - \left[\frac{n\cdot F}{T} + \bar{D}\cdot V\right] \\
&- B_m\cdot\frac{1}{T}\int_{\frac{S_m}{\bar{D}\cdot T}}^{\infty}\left(\bar{D}\cdot T\cdot x - S_m\right)\cdot g\left(x\right)dx \\
&- B_r\cdot\frac{n}{T}\int_{\frac{S_r}{\bar{D}T/n}}^{\infty}\left(\frac{\bar{D}\cdot T}{n}\cdot x - S_r\right)g\left(x\right)dx \\
&- H_m\left\{\frac{\bar{D}\cdot T}{2n}\left[\frac{\bar{D}}{P}\left(2-n\right) - n - 1\right] + S_m\right\} \\
&- H_r\left\{S_r - \frac{\bar{D}\cdot T}{2n}\right\}. \qquad (5.3)
\end{aligned}$$

where the first term represents the revenue less the production cost; the second term is the sum of setup and ordering costs; the third term is the transportation cost; the fourth term is the backorder cost; and the last two term are the holding costs for manufacturer and the retailer, respectively. Therefore, the maximization problem can be formulated as maximizing π_s via manipulating the five decision variables n, T, p_r, S_r, S_m.

It is mathematically intractable to examine the optimality conditions of these five decision variables simultaneously. To solve the maximization problem, we first examine the optimality conditions of S_r and S_m for given $\bar{D}$, T and n. Based on the analysis in Appendix 5, we have the following proposition.

Proposition 8. *For any given $\bar{D}$, T and n, the objective function is jointly concave in*

S_r and S_m. The S_r and S_m to maximize the objective function are given as follows:

$$S_m = \bar{D} \cdot T \cdot G^{-1}\left(1 - \frac{H_m \cdot T}{B_m}\right) \tag{5.4}$$

$$S_r = \frac{\bar{D} \cdot T}{n} \cdot G^{-1}\left(1 - \frac{H_r \cdot T}{B_r \cdot n}\right) \tag{5.5}$$

Proposition 8 enables us to reduce the decision variables of the maximization problem to $\bar{D}$, T and n only. Substituting equation (5.4) and equation (5.5) into the objective function, we have the following equivalent objection function:

$$\begin{aligned}\pi_s &= \bar{D}\left(\frac{a-\bar{D}}{b} - C - V\right) - \frac{n \cdot (O+F) + S}{T} - \frac{\bar{D} \cdot T}{2n}\left\{H_m\left[\frac{\bar{D}}{P}(2-n) - n - 1\right] - H_r\right\} \\ &- \bar{D} \cdot \left\{B_m\left(\int_{G^{-1}\left(1-\frac{H_m \cdot T}{B_m}\right)}^{\infty} x \cdot g(x)\,dx\right) + B_r\left(\int_{G^{-1}\left(1-\frac{H_r \cdot T}{B_r \cdot n}\right)}^{\infty} x \cdot g(x)\,dx\right)\right\}.\end{aligned}$$

5.2.1 A Heuristic Algorithm

It is still mathematically intractable to derive the optimal values of $\bar{D}$, T and n simultaneously. Therefore, we develop a heuristic algorithm by iteratively obtaining the optimal expected demand level and the optimal cycle length as well as the optimal delivery frequencies. To do so, we define following notation:

$$\rho_m = \frac{S_m}{\bar{D} \cdot T}$$

$$\rho_r = \frac{S_r}{\bar{D} \cdot T/n}$$

Thus ρ_m and ρ_r represent the ratio of S_m, S_r and corresponding demands respectively. Note that it is not meaningful to have S_m less than the expected demand in T; similarly, S_r is not less than the expected demand in T/n. Hence, both ρ_m and ρ_r are greater than 1. For notational simplicity, we further denote the expected backorder cost per unit as

$$\begin{aligned}
& L(\rho_m, \rho_r) \\
= \; & \frac{1}{\bar{D} \cdot T} \left\{ B_m \int_{\frac{S_m}{\bar{D} \cdot T}}^{\infty} \left(\bar{D} \cdot T \cdot x - S_m \right) \cdot g(x)\, dx + B_r \cdot n \int_{\frac{S_r}{\bar{D} \cdot T/n}}^{\infty} \left(\frac{\bar{D} \cdot T}{n} \cdot x - S_r \right) g(x)\, dx \right\} \\
= \; & B_m \cdot \int_{\rho_m}^{\infty} (x - \rho_m) \cdot g(x)\, dx + B_r \cdot \int_{\rho_r}^{\infty} (x - \rho_r)\, g(x)\, dx.
\end{aligned}$$

Therefore, the expected backorder cost per unit is defined by ρ_m and ρ_r once $\bar{D}$, T and n are determined. In other words, once ρ_m and ρ_r are known, the expected backorder cost per unit is determined. Thus, the only difference between the maximization problem in this section and the deterministic model in chapter 3 is the determined (fixed) backorder cost per unit. Hence, we are able to use algorithm SMSR in chapter 3 to solve the maximization problem. However, both ρ_m and ρ_r are unknown and variable. A heuristic method to deal with this problem is to initialize ρ_m and ρ_r, and then iteratively update ρ_m and ρ_r until they converge.

Based on the above discussion, a detailed procedure of the proposed heuristic is described below:

Step 1: Initialize ρ_m and ρ_r;

Step 2: Use algorithm SMSR in chapter 3 of this thesis to obtain $\bar{D}$, T and n maximizing π_s;

Step 3: Update ρ_m as $G^{-1}\left(1-\frac{H_m \cdot T}{B_m}\right)$ and ρ_r as $G^{-1}\left(1-\frac{H_r \cdot T}{B_r \cdot n}\right)$;

Step 4: Repeat step 2 until ρ_m and ρ_r converge.

5.2.2 Some Specific Distributions of ϵ

The proposed algorithm above can be used for any distribution of the random variable ϵ when its probability density function and cumulative density function are known. For a specific distribution, $L\left(\rho_m, \rho_r\right)$ can be obtained via numerical analysis or mathematical transformation. Some specific distributions, especially normal distribution and uniform distribution, are frequently discussed in the literature. We discuss how to obtain $L\left(\rho_m, \rho_r\right)$ for some specific distributions of ϵ in this section. Details of the derivation in this section are presented in Appendix 5.

Normal Distribution

When ϵ is normally distributed, then

$$\int_{\rho_m}^{\infty} (x - \rho_m) \cdot g(x)\, dx = \sigma^2 \cdot g(\rho_m) - (\rho_m - 1)\left[1 - G(\rho_m)\right]$$

$$\int_{\rho_r}^{\infty} (x - \rho_r) \cdot g(x)\, dx = \sigma^2 \cdot g(\rho_r) - (\rho_r - 1)\left[1 - G(\rho_r)\right]$$

Thus

$$\begin{aligned} L(\rho_m, \rho_r) &= B_m \cdot T \cdot \int_{\rho_m}^{\infty} (x - \rho_m) \cdot g(x)\, dx + B_r \cdot \frac{T}{n} \cdot \int_{\rho_r}^{\infty} (x - \rho_r)\, g(x)\, dx \\ &= B_m \cdot \left\{\sigma^2 \cdot g(\rho_m) - (\rho_m - 1)\left[1 - G(\rho_m)\right]\right\} \\ &+ B_r \cdot \left\{\sigma^2 \cdot g(\rho_r) - (\rho_r - 1)\left[1 - G(\rho_r)\right]\right\}. \end{aligned}$$

Uniform Distribution

When ϵ is uniformly distributed between $1 - \psi$ and $1 + \psi$ where $\psi \in (0, 1)$, then the standard deviation is $\sigma = \frac{\psi}{\sqrt{3}}$. And

$$\int_{\rho_m}^{\infty} (x - \rho_m) \cdot g(x)\, dx = \frac{1}{4\psi}\left[1 + \psi - \rho_m\right]^2$$

$$\int_{\rho_r}^{\infty} (x - \rho_r) \cdot g(x)\, dx = \frac{1}{4\psi}\left[1 + \psi - \rho_r\right]^2$$

Thus

$$L(\rho_m,\rho_r) = B_m \cdot \int_{\rho_m}^{\infty} (x-\rho_m)\cdot g(x)\,dx + B_r \cdot \int_{\rho_r}^{\infty} (x-\rho_r)\, g(x)\,dx$$
$$= \frac{1}{4\psi}\left\{B_m\left[1+\psi-\rho_m\right]^2 + B_r\left[1+\psi-\rho_r\right]^2\right\}.$$

Gamma Distribution

When ϵ is gamma-distributed with a scale parameter θ and a shape parameter ξ. Since the expected value is 1, $\theta \cdot \xi = 1$. The standard deviation is $\sigma = \theta \cdot \sqrt{\xi}$. Then

$$\int_{\rho_m}^{\infty} (x-\rho_m)\cdot g(x)\,dx = \rho_m \cdot \theta \cdot g(\rho_m) - (\rho_m - 1)\left[1 - G(\rho_m)\right]$$

$$\int_{\rho_r}^{\infty} (x-\rho_r)\cdot g(x)\,dx = \rho_r \cdot \theta \cdot g(\rho_r) - (\rho_r - 1)\left[1 - G(\rho_r)\right].$$

Thus

$$L(\rho_m,\rho_r) = B_m \cdot \int_{\rho_m}^{\infty} (x-\rho_m)\cdot g(x)\,dx + B_r \cdot \int_{\rho_r}^{\infty} (x-\rho_r)\, g(x)\,dx$$
$$= B_m \cdot \left\{\rho_m \cdot \theta \cdot g(\rho_m) - (\rho_m - 1)\left[1 - G(\rho_m)\right]\right\}$$
$$+ \; B_r \cdot \left\{\rho_r \cdot \theta \cdot g(\rho_r) - (\rho_r - 1)\left[1 - G(\rho_r)\right]\right\}.$$

Chi-square Distribution

When ϵ is a chi-square distribution with expected value of 1, the standard deviation is $\sigma = \sqrt{2}$. Then

$$\int_{\rho_m}^{\infty} (x-\rho_m)\cdot g(x)\,dx = \rho_m \cdot g(\rho_m) - (\rho_m - 1)\left[1 - G(\rho_m)\right]$$

$$\int_{\rho_r}^{\infty} (x-\rho_r)\cdot g(x)\,dx = \rho_r \cdot g(\rho_r) - (\rho_r - 1)\left[1 - G(\rho_r)\right]$$

Thus

$$\begin{aligned} L(\rho_m, \rho_r) &= B_m \cdot \int_{\rho_m}^{\infty} (x-\rho_m)\cdot g(x)\,dx + B_r \cdot \int_{\rho_r}^{\infty} (x-\rho_r)\, g(x)\,dx \\ &= B_m \cdot \{\rho_m \cdot g(\rho_m) - (\rho_m - 1)\left[1 - G(\rho_m)\right]\} \\ &+ B_r \cdot \{\rho_r \cdot g(\rho_r) - (\rho_r - 1)\left[1 - G(\rho_r)\right]\}. \end{aligned}$$

5.3 SMMR Stochastic Model

The discussion in the previous section is based on a single manufacturer single retailer supply chain. The model formulation can be extended to supply chains with multiple retailers. In this section, we discuss the extension to the case involving a single manufacturer and multiple retailers.

Following the discussion in chapter 4 of this thesis, we assume that the manufacturer coordinates the deliveries to the retailers by employing a common cycle approach. We further assume that production is organized in such a way that the first shipment for all retailers is made at the same time. We use subscript i to denote the parameters pertaining to retailer i. We also assume that the market will reach a unique equilibrium price p_r, such that, all the retailers will have the same retail price. Furthermore, each retailer i has a known and fixed market share α_i of this item. Thus, $D_i = \alpha_i \cdot D$ and $\sum_{i=1}^{R} \alpha_i = 1$.

Following the same line of reasoning as in the previous section, we first derive the inventory holding cost. The average inventory level of a retailer i can be expressed as one half of its expected demand each replenishment cycle $\bar{D}_i \cdot T/n_i$ plus the safety stock, which is the difference of $S_{r,i}$ and the expected demand during each replenishment cycle $\bar{D}_i \cdot T/n_i$. Hence, the average inventory level of retailer i is

$$\begin{aligned} I_{r,i} &= \frac{\bar{D}_i \cdot T/n_i}{2} + \left[S_{r,i} - D_i \cdot \bar{T}/n_i\right] \\ &= S_{r,i} - \frac{\bar{D}_i \cdot T}{2n_i}. \end{aligned} \tag{5.6}$$

Following the same line of reasoning, the average inventory level of the manufacturer can be expressed as its average active inventory plus its safety stock. According to equation (4.3), the average active inventory can be written as

$$\frac{T \cdot \bar{D}^2}{P} \sum_{i=1}^{R} \frac{\alpha_i}{n_i} + \frac{\bar{D}}{2P} \cdot T \cdot \left(P - \bar{D}\right) - \sum_{i=1}^{R} \left(\frac{\alpha_i}{2} \frac{T \cdot \bar{D}}{n_i}\right).$$

The safety stock of the manufacturer is given by the difference of the produce-up-to level S_m and the expected market demand during the production cycle T. The safety stock, therefore, can be described as $S_m - \bar{D} \cdot T$. It follows that the average inventory level of the manufacturer is

$$
\begin{aligned}
I_m &= \frac{T \cdot \bar{D}^2}{P} \sum_{i=1}^{R} \frac{\alpha_i}{n_i} + \frac{\bar{D}}{2P} \cdot T \cdot \left(P - \bar{D}\right) - \sum_{i=1}^{R} \left(\frac{\alpha_i}{2} \frac{T \cdot \bar{D}}{n_i} \right) + S_m - \bar{D} \cdot T \\
&= \left[T \cdot \bar{D} \left(\frac{\bar{D}}{P} - \frac{1}{2} \right) \right] \sum_{i=1}^{R} \frac{\alpha_i}{n_i} - \frac{\bar{D}}{2P} \cdot T \cdot (P + D) + S_m. \qquad (5.7)
\end{aligned}
$$

We then examine the backorder costs for the manufacturer and the retailer respectively. At the manufacturer's end, the expected total backorder quantity during time interval T is

$$
\int_{\frac{S_m}{\bar{D} \cdot T}}^{\infty} \left(\bar{D} \cdot T \cdot x - S_m \right) \cdot g\left(x\right) dx.
$$

At the retailer's end, the expected backorder quantity during the time interval T/n for retailer i is

$$
\int_{\frac{S_{r,i}}{\bar{D}_i T / n_i}}^{\infty} \left(\frac{\bar{D}_i \cdot T}{n_i} \cdot x - S_{r,i} \right) g\left(x\right) dx.
$$

Therefore, incorporating the revenue, transportation cost, ordering cost, inventory cost and inventory cost, the expected total profit for the entire supply chain can be expressed as

$$
\begin{aligned}
\pi_s &= \bar{D}\cdot(p_r - C) - \bar{D}\cdot\sum_{i=1}^{R}(V_i\cdot\alpha_i) - \frac{S+\sum_{i=1}^{R}[n_i\cdot(O_i+F_i)]}{T} \\
&- B_m\cdot\frac{1}{T}\int_{\frac{S_m}{\bar{D}\cdot T}}^{\infty}\left(\bar{D}\cdot T\cdot x - S_m\right)\cdot g(x)\,dx \\
&- \sum_{i=1}^{R}\left[B_{r,i}\cdot\frac{n_i}{T}\int_{\frac{S_{r,i}}{\bar{D}_i T/n_i}}^{\infty}\left(\frac{\bar{D}_i\cdot T}{n_i}\cdot x - S_{r,i}\right)g(x)\,dx\right] \\
&- H_m\left\{\left[T\cdot\bar{D}\left(\frac{\bar{D}}{P}-\frac{1}{2}\right)\right]\sum_{i=1}^{R}\frac{\alpha_i}{n_i} - \frac{\bar{D}}{2P}\cdot T\cdot(P+D) + S_m\right\} \\
&- \sum_{i=1}^{R}\left[H_{r,i}\left(S_{r,i}-\frac{\bar{D}_i\cdot T}{2n_i}\right)\right]. \qquad (5.8)
\end{aligned}
$$

Consequently, the maximization problem is to maximize π_s, expressed by (5.8), by varying the decision variables $n_i, T, p_r, S_{r,i}, S_m$. As in the case of a single retailer, we have the following propositions.

Proposition 9. *For any given $\bar{D}$, T and n_i, the objective function is jointly concave in $S_{r,i}$ and S_m. The $S_{r,i}$ and S_m that maximize the objective function are obtained as follows:*

$$
S_m = \bar{D}\cdot T\cdot G^{-1}\left(1-\frac{H_m\cdot T}{B_m}\right) \qquad (5.9)
$$

$$
S_{r,i} = \frac{\bar{D}_i\cdot T}{n_i}\cdot G^{-1}\left(1-\frac{H_{r,i}\cdot T}{B_{r,i}\cdot n_i}\right). \qquad (5.10)
$$

As in the last section, we develop a heuristic algorithm to obtain the optimal values of the decision variables. To do so, we adopt the following notation:

$$\begin{aligned}
& \rho_{r,i} \\
= \; & \frac{S_{r,i}}{\bar{D}_i \cdot T/n_i}
\end{aligned}$$

$$\begin{aligned}
& \widetilde{L}\left(\rho_m, \rho_{r,i}\right) \\
= \; & \frac{1}{\bar{D} \cdot T} \left\{ B_m \int_{\frac{S_m}{\bar{D} \cdot T}}^{\infty} \left(\bar{D} \cdot T \cdot x - S_m\right) \cdot g\left(x\right) dx \right\} \\
+ \; & \frac{1}{\bar{D} \cdot T} \left\{ \sum_{i=1}^{R} \left[B_{r,i} \cdot n_i \int_{\frac{S_{r,i}}{\bar{D}_i \cdot T/n_i}}^{\infty} \left(\frac{\bar{D}_i \cdot T}{n_i} \cdot x - S_{r,i} \right) g\left(x\right) dx \right] \right\} \\
= \; & B_m \cdot \int_{\rho_m}^{\infty} \left(x - \rho_m\right) \cdot g\left(x\right) dx + \sum_{i=1}^{R} \left[B_{r,i} \cdot \int_{\rho_{r,i}}^{\infty} \left(x - \rho_{r,i}\right) g\left(x\right) dx \right].
\end{aligned}$$

With the above notation, a detailed procedure of the proposed heuristic is described below:

Step 1: Initialize ρ_m and $\rho_{r,i}$;

Step 2: Use algorithm SMMR in chapter 4 of this thesis to obtain $\bar{D}$, T and n_i maximizing π_s;

Step 3: Update ρ_m as $G^{-1}\left(1 - \frac{H_m \cdot T}{B_m}\right)$ and $\rho_{r,i}$ as $G^{-1}\left(1 - \frac{H_{r,i} \cdot T}{B_{r,i} \cdot n_i}\right)$;

Step 4: Repeat step 2 until ρ_m and $\rho_{r,i}$ converge.

5.4 Numerical Example

We examine a supply chain consisting of a manufacturer and a retailer. The cost and other parameters of the supply chain are the same as those of the numerical example in chapter 3. The demand is not deterministic but has a uniform distribution around its expected value with 20% variation (i.e. ψ=0.2). This means ϵ has a uniform distribution over [0.8, 1.2]. The expected value is 1 and standard deviation σ is $\frac{0.2}{\sqrt{3}}$. The backorder costs at the retailer's and the manufacturer's end are \$5 per unit and \$4 per unit respectively. In this case,

$$\begin{aligned} L\left(\rho_m, \rho_r\right) &= \frac{1}{4\psi}\left\{B_m\left[1+\psi-\rho_m\right]^2 + B_r\left[1+\psi-\rho_r\right]^2\right\} \\ &= \frac{1}{4\times 0.2}\left\{5\times\left[1+0.2-\rho_m\right]^2 + 4\times\left[1+0.2-\rho_r\right]^2\right\} \\ &= 6.25\times\left[1.2-\rho_m\right]^2 + 5\times\left[1.2-\rho_r\right]^2 \end{aligned}$$

Based on the proposed algorithm in section 5.2, we have following results:

$$\begin{aligned} \rho_m &= 1.04 \\ \rho_r &= 1.09 \\ T &= 0.1966 \\ n &= 2 \\ D &= 5095 \end{aligned}$$

Thus, $p_r = 18.63$, $S_m = 1042$, $S_r = 546$. The resulting total supply chain profit is

\$26956 per year, which is \$1077 less than the total supply chain profit under deterministic demand. Thus, the \$1077 additional cost occurs due to the stochastic environment. In this example, ρ_m and ρ_r converge in only 2 iterations, showing the efficiency of the proposed algorithm.

6 Conclusion and Future Research Directions

This dissertation has conducted an extensive study of the optimal decisions that management can make to maximize the expected total profit of a two-echelon supply chain. In particular, the focus is on how a manufacturer and a retailer in such a supply chain can cooperate with each other to make their decisions on production, pricing and inventory control in order to achieve the maximum realizable profit level for the entire supply chain. To the best of our knowledge, this thesis is among the first to systematically address this issue .

We examine supply chains with different structures: (1) a single-manufacturer single-retailer (SMSR) supply chain under deterministic demand, (2) a single-manufacturer multiple-retailer (SMMR) supply chain under deterministic demand, and (3) supply chains under stochastic demand. These three different structures are presented in chapters 3, 4 and 5 of this thesis respectively. In chapter 5, supply chains under stochastic demand, model formulation is developed for single-manufacturer single-retailer supply chains and further extended to single-manufacturer multiple-retailers supply chains. For each supply chain structure, we develop the appropriate mathematical model, outline important propositions on its optimal solution, and propose an algorithm to solve the optimization problem accordingly. Our algorithms are illustrated through numerical examples and shown to be very efficient.

Our research also contributes to the literature of supply chain integration. Existing works in this area either focus on how supply chain members can make integrated decisions on inventory control and pricing to maximize their individual profits, or on how the manufacturer and retailers can make integrated decisions on production and inventory control for maximizing the profit of the entire supply chain. However, due to the analytical difficulties, relatively little attention has been devoted to such integrated decision making. This dissertation fills this gap in the literature, and provides the building blocks for further research extensions.

6.1 Managerial Implications

Our research contributes to management practice by providing some practical implications of supply chain cooperation and integration. One objective of this thesis is to develop some practical insights for making integrated decisions in supply chains. These managerial implications are discussed below.

First, we outline the incentives for supply chain integration, and suggest that, in a single-manufacturer single-retailer supply chain, such integration is always beneficial to the manufacturer, the retailer and the consumer. By integrating supply chains simultaneously in the vertical and horizontal directions, we show how the organizations can cooperate with each other in order to enhance the profitability of the entire supply chain. We find that, in a single-retailer supply chain, cooperation between the manufacturer and the retailer yields greater consumer benefits via lowering of the retail price. In addition, both the manufacturer and the retailer can improve their profits by sharing the surplus.

Our analysis further provides some insights on the effect of the supply chain power structure on the social benefit resulting from such integration. As discussed earlier in this thesis, when the retailer dominates the supply chain, the wholesale price is likely to be relatively low and the decentralized retail price will be closer to a centralized retail price. On the other hand, if the manufacturer is more powerful than the retailer, the wholesale price will be relatively high and the decentralized retail price will be further away from the centralized retail price. Note that the further the decentralized retail price is away from the centralized retail price, the stronger the effect of centralization on extra channel benefit and consumer benefit. Therefore, social benefit of supply chain integration is more evident for supply chains dominated by the manufacturer.

In single-manufacturer multiple-retailer supply chains, it is always advantageous for the manufacturer and the retailers to cooperate and integrate their efforts. It is shown by our numerical analyses that incorporating more retailers is not necessarily desirable for a supply chain even when the newly incorporated retailers are more profitable and cost efficient than the existing ones in the supply chain. In practice, marketing managers usually believe that it is beneficial to incorporate more efficient retailers in the supply chain. Our computational experience indicates that, if the supply chain is centralized, this is not necessarily true.

The results of our numerical examples also show that, in decentralized supply chains, it is possible for the manufacturer to realize more profit by incorporating more retailers in the supply chain. The extra profit gained, however, appears to be limited. Intuitively speaking, on the one hand, increasing the number of retailers would make competition among the retailers more intense, resulting in greater negotiation power on the part of the manufacturer. From this perspective, the manufacturer is likely to achieve higher profit. On the other hand, however, competition among the retailers may also limit the total profit of the supply chain, somewhat negating the profit gained due to the previous phenomenon.

6.2 Future Research Directions

While this dissertation has made contributions in shedding light on many of the key problems involving integrated decision making in supply chains, it is, by no means, an exhaustive study. Much future research can be done based on the findings of this thesis. We examine below some of the possible directions for further research.

6.2.1 Extension to multiple manufacturers

Our analysis in this thesis can be extended to supply chains with multiple manufacturers. It is not unusual that some manufacturers produce same or perfectly substitutable products. These manufacturers may distribute their product via a common retailer or multiple retailers. To extend our analysis to supply chains with multiple-manufacturer and single retailer, an important issue is how the retailer coordinates the production related decisions of the manufacturers. It is quite possible that the total production capacity at the manufacturers' end is much higher than the market demand. If this is the case, the allocation of the market demand to the manufacturers has to be determined.

If the manufactures distribute their product via different retailers, then we may have to consider the competition among different supply chains. For example, two manufacturers A and B produce the some product and distribute it via retailer C and D respectively. In this case, Manufacturer A and retailer C may consist of a supply chain, while manufacturer B and retailer D may consist of another. These two supply chains might compete with each other in the market and thus result in quite different conclusions from the settings in this thesis. An even more complex supply chain structure is that the manufacturers distribute their product via multiple common retailers. Both of these important issues discussed above warrant further study.

6.2.2 Extension to multiple products

We consider only a single product in this study. Our analysis should be extended to multiple products. In managerial practice, a manufacturer usually produces more than one product. For example, a shoe manufacturer might produce different kind of shoes, such as running shoes, slippers, dress shoes and so on. To deal with the manufacturing, distribution and sale of these products, some important issues have to be examined.

One important issue in supply chains with multiple products is whether these products use the same productive facilities. If so, then the scheduling problem become important. In particular, we might assume two different settings. One is that all of these products have the same production rate. This is a simple case, in which the manufacturer has to determine how to allocate its production capacity to these products toward maximizing the total profit of the supply chain. A more complex scenario occurs when the manufacturer has different production rates for these products. In this case, the manufacturer does not have fixed production capacity. Its production capacity depends on product selection and volume decisions.

Another issue to be considered in the multiple-product setting is whether the products share the same transportation facilities. For example, if the products can be shipped via a common truck, then the total transportation cost of these products may be lower than shipping them individually. This can affect the optimal shipping schedule of the products. Sometimes, the products have to be shipped together. For example, a manufacturer producing one kind of a printer and the corresponding ink cartridges usually has to ship both the printer and cartridges to the retailer(s) at the same time. These characteristics will result in a more complex problem deriving the shipping schedules.

Finally, we have to consider the potential substitution and complementarity of multiple products. It is quite common for a manufacturer to produce some substitutable or complementary products. For example, a computer manufacturer, e.g. Dell, usually offers both desktop and laptop computers, which are sometimes substitutable. If the products are substitutable or complementary to each other, then the price of one product may affect the demands of the other products. Hence, the pricing of all these products have to be considered simultaneously, which considerably complicates the profit maximization problem.

6.2.3 Incorporating supply chain coordination

In this thesis, we discuss some of the major issues that result from the cooperation of the manufacturer and the retailer(s) in a centralized supply chain. We also present the bargaining problem between the manufacturer and the retailer(s) about how they can share the extra benefits of centralization. However, due to many practical reasons, the manufacturer and the retailer(s) sometimes are not able to reach an agreeable outcome. In such a situation, some coordination mechanisms are needed to arrive at centralized decisions in decentralized supply chains.

Extensive research has been done to date on the supply chain channel coordination. Various coordination mechanisms have been developed, such as quantity discount (e.g. Gurnani, 2001; Qin, Tang & Guo, 2007), two-part tariff (e.g. Lal, 1990; Moorthy, 1987), returns policy (e.g. Hahn, Hwang & Shinn, 2004; Padmanabhan, 1995), and revenue sharing (e.g. Cachon & Lariviere, 2005; Zhou & Yang, 2008). However, to the best of our knowledge, no coordination mechanism has been developed so far for a supply chain with price-dependent demand when inventory costs are considered.

It is a challenging prospect to develop coordination mechanisms for the supply chain structures considered in this thesis. Most of the existing coordination mechanisms alone are not able to coordinate such structures. For example, it has been shown that, quantity discount alone cannot coordinate a supply chain when inventory costs are considered (Weng, 1995a, 1995b; Viswanathan & Wang, 2003). It has also been shown that a return policy by itself does not coordinate a supply chain under price dependent demand (Emmons & Gilbert, 1998).

Therefore, more innovative coordination mechanisms may be necessary. Actually, some non-traditional mechanisms have been developed in the relevant literature, which provide important insights into the coordination of supply chain structures discussed in this thesis. For example, Chen, Federgruen and Zheng (2001) develop a coordination mechanism in a supply chain with one manufacturer and multiple retailers with consideration of inventory cost and price-dependent demand. They find that coordination is achieved via a combination of periodically charged, fixed fees, and a nontraditional discount pricing scheme under which the discount given to a retailer is the sum of three discount components based on the retailer's (i) annual sales volume, (ii) order quantity, and (iii) order frequency, respectively.

However, Chen, Federgruen and Zheng (2001) do not consider the production process and assume that the production rate is infinite. They also use a simple power-of-two mechanism to coordinate the replenishment of the retailers. The single-manufacturer multiple-retailer (SMMR) supply chain considered in this thesis is more complicated than the one dealt with by Chen, Federgruen and Zheng (2001). Thus, the development of more complex coordination mechanisms appears to be a desirable and necessary direction for future research.

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Appendix 1: Notation

The notation used in this study are listed in alphabetical order:

C: production cost per unit

D: total market demand rate per year

D_i: market demand rate per year on retailer i

F: fixed transportation cost per shipment

F_i: fixed transportation cost per shipment for retailer i

H_i: inventory holding cost per unit per year for retailer i

H_m: inventory holding cost per unit per year for the manufacturer

H_r: inventory holding cost per unit per year for the retailer

I_m: average manufacturer inventory

I_r: average retailer inventory

$I_{r,i}$: average inventory level of retailer i

I_s: average inventory for the entire supply chain

n: number of shipments per production cycle

n_i: number of shipments per production cycle to retailer i

O: retailer ordering cost

O_i: retailer i's ordering cost

p_r: retail price charged to the consumers

p_w: wholesale price charged by the manufacturer to the retailer

P: production rate per year

q: delivery lot size

q_i: delivery lot size to retailer i

Q: production lot size

R: the number of retailers

S: production setup cost

S_m: produce-up-to-level for a manufacturer

S_r: order-up-to-level for a retailer

T: production cycle length

T_P: production time in a manufacturing cycle

V: total variable transportation cost per unit for each shipment

V_i: variable transportation cost per unit for each shipment for retailer i

π_m: manufacturer profit per year

π_r: total retailer profit per year

$\pi_{r,i}$: retailer i's profit per year

π_s: total profit of entire supply chain per year

Appendix 2: MATLAB Codes

A2.1 Code for SMSR Algorithm (section 3.2.2)

```
%% Parameters %%
    C=10;
    S=800;
    O=50;
    F=100;
    v=1;
    Hm=10;
    Hr=11;
    P=20000;
    a=20000;
    b=800;
    %% Get results %%
    [result1 result2 result3]=smsr(C,S,O,F,v,Hm,Hr,P,a,b)
    %% Function smsr %%
```

```
function [nstar,qstar,prstar]=smsr(C,S,O,F,v,Hm,Hr,P,a,b)

e=0.05;

var Dstar;

var prstar;

var nstar;

var qstar;

var pair;

var paim;

var pais;

var L;

var U;

var n;

result=zeros(100,5);

pw=15;

%% delta and eta %%

alpha=Hr/Hm-1;

delta=a/b-C-v-2*sqrt((O+F)*Hm/P);
```

```
eta=sqrt(1-(2*b^2*S*Hm)^(1/3)/P);

%% cases %%

if delta<= P*(1-eta^3)/b disp('Not profitable');

elseif delta>=P*(1+eta^3)/b

Dstar=P;

elseif delta==P/b

Dstar=P/2;

elseif delta>P/b

L=P/2;

U=P*(1+eta)/2;

Dstar=(L+U)/2;

i=1;

while(abs(Y1(Dstar, delta, S, Hm, P)-Y2(Dstar,b))>e)

if Y1(Dstar, delta, S, Hm, P)>Y2(Dstar,b)

L=Dstar;

else

U=Dstar;
```

```
end;
Dstar=(L+U)/2;
result(i,:)=[L, U, Dstar, Y1(Dstar, delta, S, Hm, P), Y2(Dstar,b)];
i=i+1;
end;
else
L=P*(1-eta)/2;
U=P/2;
Dstar=(L+U)/2;
i=1;
while(abs(Y1(Dstar, delta, S, Hm, P)-Y2(Dstar,b))>e)
if Y1(Dstar, delta, S, Hm, P)>Y2(Dstar,b)
L=Dstar;
else
U=Dstar;
end;
Dstar=(L+U)/2;
```

```
result(i,:)=[L, U, Dstar, Y1(Dstar, delta, S, Hm, P), Y2(Dstar,b)];

i=i+1;

end;

end;

%% n and q* %%

n=sqrt(S*(Hr+Hm*(2*Dstar/P-1))/(O+F)/(Hm*(1-Dstar/P)));

qstar=sqrt(2*Dstar*(O+F)/(Hr+Hm*(2*Dstar/P-1)));

prstar=(a-Dstar)/b;

A=floor(n);

paim1=Dstar*(pw-C)-S*Dstar/(A*qstar)

-Hm*(Dstar*qstar/P +(P-Dstar)*qstar*A/P/2-qstar/2);

pair1=Dstar*(prstar-pw-v)-(O+F)*Dstar/qstar-Hr*qstar/2;

pais1=paim1+pair1;

B=A+1;

paim2=Dstar*(pw-C)-S*Dstar/(B*qstar)

-Hm*(Dstar*qstar/P +(P-Dstar)*qstar*B/P/2-qstar/2);

pair2=Dstar*(prstar-pw-v)-(O+F)*Dstar/qstar-Hr*qstar/2;
```

```
pais2=paim2+pair2;

if pais1>pais2

pais=pais1;

paim=paim1;

pair=pair1;

nstar=A;

else

pais=pais2;

paim=paim2;

pair=pair2;

nstar=B;

end;

function y1=Y1(D, delta, S, Hm, P)

y1=delta-sqrt(S*Hm)*(1-2*D/P)/sqrt(2*D*(1-D/P));

function [y2]=Y2(D,b)

y2=2*D/b;
```

A2.2 Code for SMMR Algorithm (section 4.2.2)

```
%% Parameters %%

    C=10;

    S=800;

    Hm=10;

    R=2;

    O=[50 45];

    F=[100 95];

    v=[1 0.9];

    Hr=[11 10.5];

    alpha=[0.6 0.4];

    P=20000;

    a=20000;

    b=800;

    e=0.05;

    pw=15;
```

```
var Dstar;

var prstar;

var Tstar;

var nstar1;

var nstar2;

var pair1;

var pair2;

var paim;

var pais;

var L;

var U;

var n;

result=zeros(100,5);

temp=0;

%% delta and eta %%

for i=1:R

temp=temp+sqrt((O(i)+F(i))*alpha(i)*2*Hm);
```

```
end

delta=a/b-C-sum(alpha*v')-sqrt(2/P)*temp;

eta=sqrt(1-(2*b^2*S*Hm)^(1/3)/P);

%% cases %%

if delta<= P*(1-eta^3)/b

disp('Not profitable');

elseif delta>=P*(1+eta^3)/b

Dstar=P;

elseif delta==P/b

Dstar=P/2;

elseif delta>P/b

L=P/2;

U=P*(1+eta)/2;

Dstar=(L+U)/2;

i=1;

while(abs(Y1(Dstar, delta, S, Hm, P)-Y2(Dstar,b))>e)

if Y1(Dstar, delta, S, Hm, P)>Y2(Dstar,b)
```

```
L=Dstar;

else U=Dstar;

end;

Dstar=(L+U)/2;

result(i,:)=[L, U, Dstar, Y1(Dstar, delta, S, Hm, P), Y2(Dstar,b)];

i=i+1;

end;

else L=P*(1-eta)/2;

U=P/2;

Dstar=(L+U)/2;

i=1;

while(abs(Y1(Dstar, delta, S, Hm, P)-Y2(Dstar,b))>e)

if Y1(Dstar, delta, S, Hm, P)>Y2(Dstar,b)

L=Dstar;

else U=Dstar;

end;

Dstar=(L+U)/2;
```

```
result(i,:)=[L, U, Dstar, Y1(Dstar, delta, S, Hm, P), Y2(Dstar,b)];
i=i+1;
end;
end;
%% n and q* %%
prstar=(a-Dstar)/b;
Tstar=sqrt(2*S*P/(Hm*Dstar*(P-Dstar)));
n=[0 0];
qstar=[0 0];
A=[0 0];
B=[0 0];
temp1=0;
for i=1:R
temp1=temp1+(O(i)+F(i))*alpha(i)*(Hr(i)-Hm)/(2*sqrt(Hr(i)+Hm));
end
for i=1:R
n(i)=Tstar*sqrt(Dstar*alpha(i)*(Hm*Dstar/P+0.5*(Hr(i)-Hm))/(O(i)+F(i)));
```

```
A(i)=floor(n(i));
B(i)=A(i)+1;
end;
maxpais=0;
pais=[0 0 0 0];
pais(1)=Dstar*(prstar-C-sum(alpha*v')) -(S+A(1)*(O(1)+F(1))
+A(2)*(O(2)+F(2)))/Tstar -Hm*(Dstar^2/P*Tstar*(alpha(1)/A(1)
+alpha(2)/A(2)) +Dstar*Tstar*(P-Dstar)/(2*P))
-(Hr(1)-Hm)*alpha(1)*Tstar*Dstar/2/A(1)
-(Hr(2)-Hm)*alpha(2)*Tstar*Dstar/2/A(2);
pais(2)=Dstar*(prstar-C-sum(alpha*v')) -(S+B(1)*(O(1)+F(1))
+A(2)*(O(2)+F(2)))/Tstar -Hm*(Dstar^2/P*Tstar*(alpha(1)/B(1)
+alpha(2)/A(2)) +Dstar*Tstar*(P-Dstar)/(2*P))
-(Hr(1)-Hm)*alpha(1)*Tstar*Dstar/2/B(1)
-(Hr(2)-Hm)*alpha(2)*Tstar*Dstar/2/A(2);
pais(3)=Dstar*(prstar-C-sum(alpha*v')) -(S+A(1)*(O(1)+F(1))
+B(2)*(O(2)+F(2)))/Tstar -Hm*(Dstar^2/P*Tstar*(alpha(1)/A(1)
```

```
+alpha(2)/B(2)) +Dstar*Tstar*(P-Dstar)/(2*P))

-(Hr(1)-Hm)*alpha(1)*Tstar*Dstar/2/A(1)

-(Hr(2)-Hm)*alpha(2)*Tstar*Dstar/2/B(2);

pais(4)=Dstar*(prstar-C-sum(alpha*v')) -(S+B(1)*(O(1)+F(1))

+B(2)*(O(2)+F(2)))/Tstar -Hm*(Dstar^2/P*Tstar*(alpha(1)/B(1)

+alpha(2)/B(2)) +Dstar*Tstar*(P-Dstar)/(2*P))

-(Hr(1)-Hm)*alpha(1)*Tstar*Dstar/2/B(1)

-(Hr(2)-Hm)*alpha(2)*Tstar*Dstar/2/B(2);

maxpais=max(pais);
```

A2.3 Code for Stochastic Model Algorithm (section 5.2.1)

```
%%Parameters%%

    C=10;

    S=800;

    O=50;

    F=100;

    v=1;
```

```
Hm=10;

Hr=11;

P=20000;

a=20000;

b=600;

Bm=5;

Br=4;

T1=0;

LL=0;

rhom=1;

rhor=1;

rhom1=1.1;

rhor1=1.1;

e=0.01;

i=0;

rhomm=zeros(1,100);

rhorr=zeros(1,100);
```

```
lll=zeros(1,100);

while((abs(rhom1-rhom)>e) || (abs(rhor1-rhor)>e))

i=i+1;

rhom=rhom1;

rhor=rhor1;

LL=6.25*(1.2-rhom)^2+5*(1.2-rhor)^2;

C=10+LL;

[nstar1,qstar1,prstar1,Dstar1]=smsr(C,S,O,F,v,Hm,Hr,P,a,b)

T1=nstar1*qstar1/Dstar1;

rhom1=0.8+(1-Hm*T1/Bm)*0.4;

rhor1=0.8+(1-Hr*T1/Br/nstar1)*0.4;

rhomm(1,i)=rhom1;

rhorr(1,i)=rhor1;

lll(1,i)=LL;

end
```

Appendix 3: Mathematics of Proofs for SMSR

A3.1 Proof of Propositions 3-1, 3-2, and 3-3.

Equation (3-14) can be rewritten as

$$\begin{aligned}
\pi_s^c &= (a - b \cdot p_r)\left[p_r - C - V\right] - \sqrt{2(a - b \cdot p_r) \cdot S \cdot H_m \left(1 - \frac{a - b \cdot p_r}{P}\right)} \\
&\quad - \sqrt{2(a - b \cdot p_r) \cdot (O + F) \cdot \left[H_r + H_m \left(2\left(\frac{a - b \cdot p_r}{P}\right) - 1\right)\right]} \\
&= D\left[\frac{a - D}{b} - C - V\right] - \sqrt{2D \cdot S \cdot H_m \left(1 - \frac{D}{P}\right)} \\
&\quad - \sqrt{2D \cdot (O + F) \cdot \left[H_r + H_m \left(2\left(\frac{D}{P}\right) - 1\right)\right]}.
\end{aligned}$$

For notational simplicity, we denote $H_r/H_m - 1$ as α. It is reasonable to assume that H_r is close to H_m, i.e., α is a small number. Thus, we assume that α^2 is negligible. Then the last term of the objective function becomes

$$\begin{aligned}
& \sqrt{2D \cdot (O + F) \cdot \left[H_r + H_m \left(2\left(\frac{D}{P}\right) - 1\right)\right]} \\
&= \sqrt{2D \cdot (O + F) \cdot H_m \cdot \left[\alpha + 2\left(\frac{D}{P}\right)\right]} \\
&= \sqrt{\frac{2 \cdot (O + F) \cdot H_m}{P} \cdot \left[\alpha P D + 2D^2\right]} \\
&= 2\sqrt{\frac{(O + F) \cdot H_m}{P}} \cdot \left[D + \frac{\alpha P}{4}\right].
\end{aligned}$$

Thus, the objective function could be rewritten as

$$\begin{aligned}
\pi_s^c &= D\left[\frac{a-D}{b}-C-V\right]-\sqrt{2D\cdot S\cdot H_m\left(1-\frac{D}{P}\right)} \\
&\quad -\sqrt{2D\cdot(O+F)\cdot\left[H_r+H_m\left(2\left(\frac{D}{P}\right)-1\right)\right]} \\
&= D\left[\frac{a-D}{b}-C-V\right]-\sqrt{2D\cdot S\cdot H_m\left(1-\frac{D}{P}\right)}-2\sqrt{\frac{(O+F)\cdot H_m}{P}}\cdot\left[D+\frac{\alpha P}{4}\right] \\
&= -\frac{D^2}{b}+D\left[\frac{a}{b}-C-V-2\sqrt{\frac{(O+F)\cdot H_m}{P}}\right] \\
&\quad -\sqrt{2D\cdot S\cdot H_m\left(1-\frac{D}{P}\right)}-\sqrt{\frac{(O+F)\cdot H_m}{P}}\cdot\frac{\alpha P}{2}.
\end{aligned}$$

It follows from above that

$$\begin{aligned}
\frac{\partial\pi_s^c}{\partial D} &= -\frac{2D}{b}+\left[\frac{a}{b}-C-V-2\sqrt{\frac{(O+F)\cdot H_m}{P}}\right]-\sqrt{\frac{S\cdot H_m}{P}}\cdot\left[\frac{P-2D}{\sqrt{2D\left(P-D\right)}}\right] \\
\frac{\partial^2\pi_s^c}{\partial D^2} &= -\frac{2}{b}+\sqrt{\frac{S\cdot H_m}{\left[2D\left(1-\frac{D}{P}\right)\right]^3}}.
\end{aligned}$$

Again, for notational simplicity, we denote

$$\begin{aligned}
\delta &= \frac{a}{b}-C-V-2\sqrt{\frac{(O+F)\cdot H_m}{P}} \\
Y_1(D) &= \delta-\sqrt{\frac{S\cdot H_m}{P}}\cdot\left[\frac{P-2D}{\sqrt{2D\left(P-D\right)}}\right] \\
Y_2(D) &= \frac{2D}{b}
\end{aligned}$$

Then the first order optimility condition can be rewritten as

$$Y_1(D) = Y_2(D).$$

Note that

$$\frac{\partial Y_1}{\partial D} = \sqrt{\frac{S \cdot H_m}{\left[2D\left(1 - \frac{D}{P}\right)\right]^3}} > 0$$

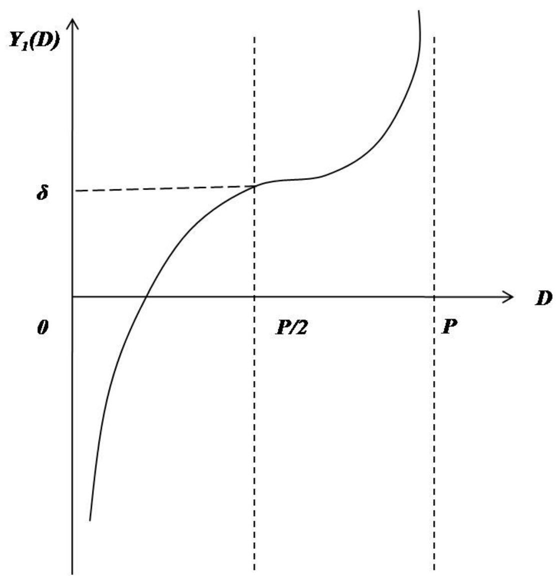
and that

$$\frac{\partial^2 Y_1}{\partial D^2} = 3\sqrt{\frac{S \cdot H_m}{\left[2D\left(1 - \frac{D}{P}\right)\right]^5}} \left[\frac{2D}{P} - 1\right]$$

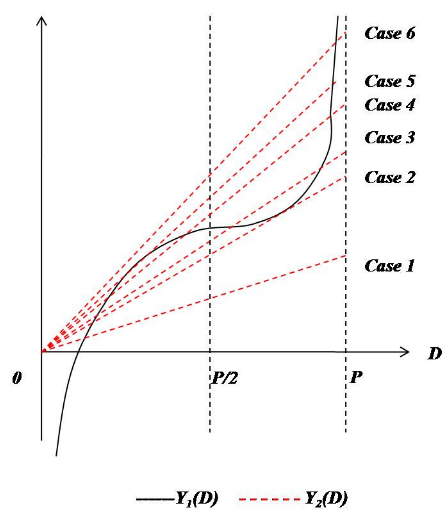
Thus,

$$\begin{cases} \frac{\partial^2 Y_1}{\partial D^2} > 0 \quad if\ D > \frac{P}{2} \\ \frac{\partial^2 Y_1}{\partial D^2} = 0 \quad if\ D = \frac{P}{2} \\ \frac{\partial^2 Y_1}{\partial D^2} < 0 \quad if\ D < \frac{P}{2}. \end{cases}$$

Therefore, Y_1 is increasing and is concave in $(0, P/2]$ and convex in $[P/2, P)$. Moreover, when $D \to 0$, $Y_1 \to -\infty$; when $D \to P$, $Y_1 \to +\infty$. We show Y_1 on the following graph:



Note that Y_2 is a straight line with slope of $2/b$. And the points satisfying the first order conditions are the intersection points of Y_1 and Y_2. The figure below shows the possible cases for Y_1 and Y_2.



Cases 1 and 6: Y_1 and Y_2 have only one intersection point. In these cases, however, the intersection point is a local minimum. Before the intersection, $Y_2 > Y_1$, hence,

$\frac{\partial \pi_s^c}{\partial D} = Y_1 - Y_2 < 0$, so π_s^c is decreasing; while after it, $Y_2 < Y_1$, so π_s^c is increasing after the intersection point. Thus, there is no inner local maximum point in these two cases.

Cases 2 and 5: Y_1 and Y_2 have two intersection points. Following the same lines of reasoning as above, in Case 5, the first intersection point is an inflexion point and the second intersection point is a local minimum; whereas in Case 2, the first intersection point is local minimum and the second is an inflexion point. Thus, there is no inner maximum solution in these two cases.

Cases 3 and 4: Y_1 and Y_2 have three intersection points. In both cases, the first and third intersection points are local minima; while the second intersection point is a local maximum.

To determine the optimality conditions for these different cases, we examine the conditions for inflexion points in Case 2 and Case 5 to happen respectively. In these two cases, $Y_1 = Y_2$ and $\frac{\partial Y_1}{\partial D} = \frac{\partial Y_2}{\partial D}$. That is,

$$\delta - \sqrt{\frac{S \cdot H_m}{P}} \cdot \left[\frac{P - 2D}{\sqrt{2D\,(P-D)}} \right] = \frac{2D}{b} \tag{7.1}$$

$$\sqrt{\frac{S \cdot H_m}{\left[2D\left(1 - \frac{D}{P}\right)\right]^3}} = \frac{2}{b} \tag{7.2}$$

Denote the intersection point as $\hat{D}$. Then, following (A.2), we have

$$\hat{D} = \frac{P + P\sqrt{1 - \frac{2}{P}\left(\frac{b^2 \cdot S \cdot H_m}{4}\right)^{1/3}}}{2} \text{(case 2)}$$

or

$$\hat{D} = \frac{P - P\sqrt{1 - \frac{2}{P}\left(\frac{b^2 \cdot S \cdot H_m}{4}\right)^{1/3}}}{2} (\text{case 5})$$

Denote $\sqrt{1 - \frac{2}{P}\left(\frac{b^2 \cdot S \cdot H_m}{4}\right)^{1/3}}$ by η. Following (A.1), we have

$$\begin{aligned} \delta + \left(\frac{2S \cdot H_m}{b}\right)^{\frac{1}{3}} \eta &= \frac{P}{b}(1+\eta)\,(\text{case 2}) \\ \delta - \left(\frac{2S \cdot H_m}{b}\right)^{\frac{1}{3}} \eta &= \frac{P}{b}(1-\eta)\,(\text{case 5}) \end{aligned}$$

(1) If $\delta - \left(\frac{2S \cdot H_m}{b}\right)^{\frac{1}{3}} \eta \leq \frac{P}{b}(1-\eta)$ (cases 5 & 6), that is, $\delta \leq \frac{P}{b}(1-\eta^3)$, then there is no inner maximum solutions; thus, if there is any maximization solutions, these solutions should either have $D = 0$ or $D = P$. Moreover, under this condition, the system profit at $D = P$ is not greater than 0. That is, it is impossible for the manufacturer and the retailer to gain any profits by running the business. Proposition 3-1 is proved.

(2) If $\delta + \left(\frac{2S \cdot H_m}{b}\right)^{\frac{1}{3}} \eta \geq \frac{P}{b}(1+\eta)$(cases 1 & 2), that is, $\delta \geq \frac{P}{b}(1+\eta^3)$, then there is no inner maximum solutions; thus, we check the boundary solutions. We find that, under this condition, there is a positive system profit when $D = P$. Thus, the optimal market demand is $D \to P$, i.e. $p_r \to \frac{a-P}{b}$. Proposition 3-2 is proved.

(3) If $\delta + \left(\frac{2S \cdot H_m}{b}\right)^{\frac{1}{3}} \eta < \frac{P}{b}(1+\eta)$ and $\delta \geq \frac{P}{b}$ (case 3), that is, $\frac{P}{b} \leq \delta < \frac{P}{b}(1+\eta^3)$, there is a local maximum solution $\frac{P}{2} \leq D < \frac{P}{2}(1+\eta)$. Moreover, $\frac{\partial^2 \pi_s^c}{\partial D^2} = \frac{\partial Y_1}{\partial D} - \frac{\partial Y_2}{\partial D} < 0$. Thus, π_s^c is strictly concave in $\left[\frac{P}{2}, \frac{P}{2}(1+\eta)\right)$. If $\delta - \left(\frac{2S \cdot H_m}{b}\right)^{\frac{1}{3}} \eta > \frac{P}{b}(1-\eta)$ and $\delta \leq \frac{P}{b}$ (case 4), that is, $\frac{P}{b}(1-\eta^3) < \delta \leq \frac{P}{b}$, then there a local maximum solution $\frac{P}{2}(1-\eta) < D \leq \frac{P}{2}$. Moreover, $\frac{\partial^2 \pi_s^c}{\partial D^2} = \frac{\partial Y_1}{\partial D} - \frac{\partial Y_2}{\partial D} > 0$, thus, π_s^c is strictly convex in

$\left(\frac{P}{2}\left(1-\eta\right),\frac{P}{2}\right]$ .Incorporating that $p_r = \frac{a-D}{b}$, proposition 3-3 is proved.

A3.2 For Decentralized Supply Chain

In a decentralized supply chain, for a given wholesale price, the optimization problem of the retailer is written as

$$\begin{aligned} \underset{p_r}{Max}\ \pi_r^d\left(p_r\right) &= (a-b\cdot p_r)\cdot(p_r-p_w-V)-\sqrt{2H_r\cdot(O+F)\cdot(a-b\cdot p_r)} \\ s.t.\quad & p_r > \frac{a-P}{b} \\ & p_r < \frac{a}{b}. \end{aligned}$$

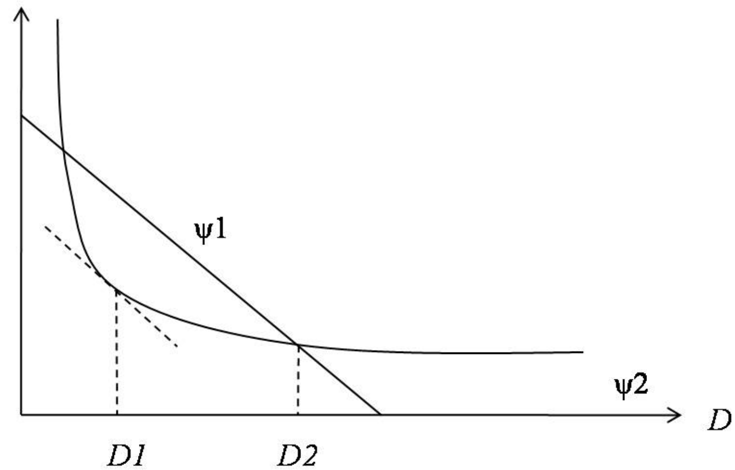
With $D = a - b\cdot p_r$, we can rewrite above problem as

$$\begin{aligned} \underset{D}{Max}\ \pi_r^d\left(D\right) &= D\cdot\left(\frac{a-D}{b}-p_w-V\right)-\sqrt{2H_r\cdot(O+F)\cdot D} \\ s.t.\quad & D > 0 \\ & D < P. \end{aligned}$$

Thus,

$$\frac{\partial \pi_r^d}{\partial D} = \frac{a}{b} - p_w - v - \frac{2D}{b} - \sqrt{\frac{(O+F)\cdot H_r}{2D}}.$$

Let $\psi_1 = \frac{a}{b} - p_w - v - \frac{2D}{b}$ and $\psi_2 = \sqrt{\frac{(O+F)\cdot H_r}{2D}}$, which are illustrated in the following figure:



As indicated by the above figure, for the first order condition to have a local maximum solution, $\psi_1(D_1)$ must be greater than $\psi(D_1)$(D_1 is the demand where and have the same slope). Such that ψ_1 and ψ_2 have two intersection points. In this case, the larger intersection point, D_2, is a local maximum solution. Note that, for D_1, we have $\frac{\partial \psi_1}{\partial D_1} = \frac{\partial \psi_2}{\partial D_1}$. Thus, $D_1 = \left[\frac{b^2 \cdot (O+F) \cdot H_r}{32}\right]^{1/3}$. Therefore, we state the following proposition:

Proposition A2-1. If $p_w \geq \frac{a}{b} - v - 3\left[\frac{b^2 \cdot (O+F) \cdot H_r}{32}\right]^{1/3}$, then optimal demand level $\bar{D} = 0$, i.e. it is not profitable for the retailer to run the business.

If $\psi_1(D_1) \leq \psi_2(D_1)$, i.e. $p_w \geq \frac{a}{b} - v - 3\left[\frac{b^2 \cdot (O+F) \cdot H_r}{32}\right]^{1/3}$, then ψ_1 and ψ_2 do not intersect. In this case, ψ_2 is always not less than ψ_1. Thus, the second term in the objective function is increasing more quickly than the first term, which means the objective function is always decreasing. Therefore, in this case, it is not profitable for the retailer to run the business.

If it is profitable to run the business, then, as discussed above, D_2 is a local maximum. If $D_2 \leq P$, it satisfies the boundary constraint of D. Otherwise, if $D_2 > P$, then the local maximum solution $\widetilde{D} = P$. Note that, $D_2 > P$ is equivalent to $\psi_1(P) > \psi_2(P)$, i.e., $p_w < \frac{a}{b} - v - \frac{2P}{b} - \sqrt{\frac{(O+F)\cdot H_r}{2P}}$. Therefore, we have the following proposition.

Proposition A2-2. If $p_w < \frac{a}{b} - v - 3\left[\frac{b^2\cdot(O+F)\cdot H_r}{32}\right]^{1/3}$, then local maximum solution of demand level $\widetilde{D} = P$. Moreover, the global maximum solution of demand level is $\bar{D} = P$ if $\pi_r(P) > 0$. Otherwise, $\bar{D} = 0$.

Propositions A2-1 and A2-2 enable us to quickly solve the maximization problem if the wholesale price satisfies either one of the conditions in these two propositions. When the wholesale price satisfies neither of them, it is easy to search the local maximum solution of demand level $\widetilde{D} = D_2$ using a binary search method. After that, the global maximum solution of demand level is $\bar{D} = \widetilde{D}$ if $\pi_r\left(\widetilde{D}\right) > 0$. Otherwise, $\bar{D} = 0$.

Appendix 4: Mathematics of Proofs for SMMR

The objective function of the SMMR maximization problem is

$$\begin{aligned}\pi_s &= (a - b \cdot p_r)\left[p_r - C - \sum_{i=1}^{R}(V_i\alpha_i)\right] - 2\sqrt{\frac{H_m \cdot (a - b \cdot p_r) \cdot (P - a + b \cdot p_r) \cdot S}{2P}} \\ &- 2\sum_{i=1}^{R}\sqrt{\left[H_m \cdot \frac{(a - b \cdot p_r)}{P} + \frac{H_{r,i} - H_m}{2}\right] \cdot (a - b \cdot p_r) \cdot \alpha_i \cdot (O_i + F_i)}.\end{aligned}$$

This objective function can be rewritten as

$$\begin{aligned}\pi_s &= D\left[\frac{a - D}{b} - C - \sum_{i=1}^{R}(V_i\alpha_i)\right] - 2\sqrt{\frac{H_m \cdot D \cdot (P - D) \cdot S}{2P}} \\ &- 2\sum_{i=1}^{R}\sqrt{\left[H_m \cdot \frac{D}{P} + \frac{H_{r,i} - H_m}{2}\right] \cdot D_i \cdot (O_i + F_i)}.\end{aligned}$$

For notational simplicity, we denote $H_{r,i}/H_m - 1$ as β_i. It is reasonable to assume that $H_{r,i}$ is close to H_m, i.e., β_i is a small number. Thus, we assume that β_i^2 is negligible. Then the last term of the objective function becomes

$$\begin{aligned}&\sqrt{2D_i \cdot (O_i + F_i) \cdot \left[H_{r,i} + H_m\left(2\left(\frac{D}{P}\right) - 1\right)\right]} \\ &= \sqrt{2D_i \cdot (O_i + F_i) \cdot H_m \cdot \left[\beta_i + 2\left(\frac{D}{P}\right)\right]} \\ &= \sqrt{\frac{2 \cdot \alpha_i(O_i + F_i) \cdot H_m}{P} \cdot [P\beta_i D + 2D^2]} \\ &= 2\sqrt{\frac{\alpha_i(O_i + F_i) \cdot H_m}{P}} \cdot \left[D + \frac{\beta_i P}{4}\right].\end{aligned}$$

Thus, the objective function is rewritten as

$$\begin{aligned}
\pi_s &= D\left[\frac{a-D}{b} - C - \sum_{i=1}^{R}(V_i\alpha_i)\right] - 2\sqrt{\frac{H_m \cdot D \cdot (P-D)\cdot S}{2P}} \\
&\quad -2\sum_{i=1}^{R}\left\{\sqrt{\frac{\alpha_i(O_i+F_i)\cdot H_m}{P}}\cdot\left[D+\frac{\beta_i P}{4}\right]\right\} \\
&= -\frac{D^2}{b} + D\left[\frac{a}{b} - C - \sum_{i=1}^{R}(V_i\alpha_i) - 2\sum_{i=1}^{R}\sqrt{\frac{\alpha_i(O_i+F_i)\cdot H_m}{P}}\right] \\
&\quad -\sqrt{2D\cdot S\cdot H_m\left(1-\frac{D}{P}\right)} - \sum_{i=1}^{R}\left[\sqrt{\frac{\alpha_i(O_i+F_i)\cdot H_m}{P}}\cdot\frac{\beta_i\cdot P}{2}\right].
\end{aligned}$$

Based on the above equation, we have

$$\begin{aligned}
\frac{\partial \pi_s}{\partial D} &= -\frac{2D}{b} + \left[\frac{a}{b} - C - \sum_{i=1}^{R}(V_i\alpha_i) - 2\sum_{i=1}^{R}\sqrt{\frac{\alpha_i(O_i+F_i)\cdot H_m}{P}}\right] \\
&\quad -\sqrt{\frac{S\cdot H_m}{P}}\cdot\left[\frac{P-2D}{\sqrt{2D\,(P-D)}}\right],
\end{aligned}$$

$$\frac{\partial^2 \pi_s}{\partial D^2} = -\frac{2}{b} + \sqrt{\frac{S\cdot H_m}{\left[2D\left(1-\frac{D}{P}\right)\right]^3}}.$$

For notation simplicity, we denote

$$\bar{\delta} = \frac{a}{b} - C - \sum_{i=1}^{R}(V_i\alpha_i) - 2\sum_{i=1}^{R}\sqrt{\frac{\alpha_i(O_i+F_i)\cdot H_m}{P}}.$$

Note that the above maximization problem, including the objective function, the first order derivative and the second order derivative, is essentially the same as the maximization problem for the single-manufacturer single-retailer case in Appendix 3 of this thesis, except that the term δ in Appendix 3 is replaced by $\bar{\delta}$. Therefore, all corresponding analyzes in Appendix 3 can be applied to this maximization problem by replacing δ as $\bar{\delta}$. Propositions 5, 6, and 7 can be proved in the same manner as propositions 1, 2 and 3.

Appendix 5: Mathematics of Proofs for Stochastic Model

A5.1 Proof of Proposition 8

For any given $\bar{D}$, T and n, the first order conditions for S_r and S_m are

$$\begin{aligned}
\frac{\partial \pi_s}{\partial S_m} &= -\frac{\partial}{\partial S_m}\left\{B_m \cdot \frac{1}{T}\int_{\frac{S_m}{\bar{D}\cdot T}}^{\infty}\left(\bar{D}\cdot T\cdot x - S_m\right)\cdot g\left(x\right)dx + H_m \cdot S_m\right\} \\
&= \frac{B_m}{T}\cdot\left[-\frac{S_m}{\bar{D}\cdot T}\cdot g\left(\frac{S_m}{\bar{D}\cdot T}\right) + \left[1 - G\left(\frac{S_m}{\bar{D}\cdot T}\right)\right] + \frac{S_m}{\bar{D}\cdot T}\cdot g\left(\frac{S_m}{\bar{D}\cdot T}\right)\right] - H_m \\
&= \frac{B_m}{T}\cdot\left[1 - G\left(\frac{S_m}{\bar{D}\cdot T}\right)\right] - H_m = 0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \pi_s}{\partial S_r} &= -\frac{\partial}{\partial S_r}\left\{B_r \cdot \frac{n}{T}\int_{\frac{S_r}{\bar{D}T/n}}^{\infty}\left(\frac{\bar{D}\cdot T}{n}\cdot x - S_r\right)g\left(x\right)dx + H_r \cdot S_r\right\} \\
&= \frac{nB_r}{T}\cdot\left[-\frac{nS_r}{\bar{D}\cdot T}\cdot g\left(\frac{nS_r}{\bar{D}\cdot T}\right) + \left[1 - G\left(\frac{nS_r}{\bar{D}\cdot T}\right)\right] + \frac{nS_r}{\bar{D}\cdot T}\cdot g\left(\frac{nS_r}{\bar{D}\cdot T}\right)\right] - H_r \\
&= \frac{nB_r}{T}\cdot\left[1 - G\left(\frac{nS_r}{\bar{D}\cdot T}\right)\right] - H_r = 0.
\end{aligned}$$

Thus,

$$\begin{cases}
\frac{B_m}{T}\cdot\left[1 - G\left(\frac{S_m}{\bar{D}\cdot T}\right)\right] - H_m & = 0 \\
\frac{nB_r}{T}\cdot\left[1 - G\left(\frac{S_r}{\bar{D}\cdot T/n}\right)\right] - H_r & = 0
\end{cases}.$$

Therefore,

$$\begin{aligned} S_m &= \bar{D} \cdot T \cdot G^{-1}\left(1 - \frac{H_m \cdot T}{B_m}\right) \\ S_r &= \frac{\bar{D} \cdot T}{n} \cdot G^{-1}\left(1 - \frac{H_r \cdot T}{B_r \cdot n}\right). \end{aligned}$$

The second order conditions are

$$\begin{aligned} \frac{\partial^2 \pi_s}{\partial \left(S_m\right)^2} &= \frac{\partial}{\partial S_m}\left\{\frac{B_m}{T} \cdot \left[1 - G\left(\frac{S_m}{\bar{D} \cdot T}\right)\right]\right\} \\ &= -\frac{B_m}{T} \cdot \frac{1}{\bar{D} \cdot T} \cdot g\left(\frac{S_m}{\bar{D} \cdot T}\right) < 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \pi_s}{\partial \left(S_r\right)^2} &= \frac{\partial}{\partial S_r}\left\{\frac{nB_r}{T} \cdot \left[1 - G\left(\frac{nS_r}{\bar{D} \cdot T}\right)\right]\right\} \\ &= -\frac{nB_r}{T} \cdot \frac{n}{\bar{D} \cdot T} \cdot g\left(\frac{nS_r}{\bar{D} \cdot T}\right) < 0, \end{aligned}$$

$$\frac{\partial^2 \pi_s}{\partial S_r \partial S_m} = 0.$$

Thus, π_s is jointly concave in S_r and S_m.

A5.2 Derivation for some specific distributions

Normal Distribution

When ϵ is normally distributed, then

$$\begin{aligned}
\int_{\rho_m}^{\infty} (x-\rho_m)\cdot g(x)\,dx &= \int_{\rho_m}^{\infty} (x-1)\cdot g(x)\,dx - \int_{\rho_m}^{\infty} (\rho_m - 1)\cdot g(x)\,dx \\
&= \int_{\rho_m}^{\infty} (x-1)\cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-1)^2}{2\sigma^2}}\,dx - (\rho_m-1)\cdot \int_{\rho_m}^{\infty} g(x)\,dx \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\rho_m - 1}^{\infty} y\cdot e^{-\frac{y^2}{2\sigma^2}}\,dy - (\rho_m-1)\cdot[1-G(\rho_m)] \\
&= \frac{-2\sigma^2}{2\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}|_{\rho_m-1}^{\infty} - (\rho_m-1)\cdot[1-G(\rho_m)] \\
&= \frac{\sigma^2}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\rho_m-1)^2}{2\sigma^2}} - (\rho_m-1)\cdot[1-G(\rho_m)] \\
&= \sigma^2\cdot g(\rho_m) - (\rho_m-1)[1-G(\rho_m)].
\end{aligned}$$

$$\begin{aligned}
\int_{\rho_r}^{\infty} (x-\rho_r)\cdot g(x)\,dx &= \int_{\rho_r}^{\infty} (x-1)\cdot g(x)\,dx - \int_{\rho_r}^{\infty} (\rho_r - 1)\cdot g(x)\,dx \\
&= \int_{\rho_r}^{\infty} (x-1)\cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-1)^2}{2\sigma^2}}\,dx - (\rho_r-1)\cdot \int_{\rho_r}^{\infty} g(x)\,dx \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\rho_r - 1}^{\infty} y\cdot e^{-\frac{y^2}{2\sigma^2}}\,dy - (\rho_r-1)\cdot[1-G(\rho_r)] \\
&= \frac{-2\sigma^2}{2\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}|_{\rho_r-1}^{\infty} - (\rho_r-1)\cdot[1-G(\rho_r)] \\
&= \frac{\sigma^2}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\rho_r-1)^2}{2\sigma^2}} - (\rho_r-1)\cdot[1-G(\rho_r)] \\
&= \sigma^2\cdot g(\rho_r) - (\rho_r-1)[1-G(\rho_r)].
\end{aligned}$$

Uniform Distribution

When ϵ is uniformly distributed between $1-\psi$ and $1+\psi$ where $\psi \in (0,1)$, then the standard deviation is $\sigma = \frac{\psi}{\sqrt{3}}$.

$$\begin{aligned}
\int_{\rho_m}^{1+\psi} (x-\rho_m)\cdot g(x)\,dx &= \frac{1}{2\psi}\int_{\rho_m}^{1+\psi}(x-\rho_m)\,dx \\
&= \frac{1}{2\psi}\int_{0}^{1+\psi-\rho_m} y dy \\
&= \frac{1}{4\psi}\left\{[1+\psi-\rho_m]^2 - 0\right\} \\
&= \frac{1}{4\psi}[1+\psi-\rho_m]^2 .
\end{aligned}$$

$$\begin{aligned}
\int_{\rho_r}^{1+\psi} (x-\rho_r)\cdot g(x)\,dx &= \frac{1}{2\psi}\int_{\rho_r}^{1+\psi}(x-\rho_r)\,dx \\
&= \frac{1}{2\psi}\int_{0}^{1+\psi-\rho_r} y dy \\
&= \frac{1}{4\psi}\left\{[1+\psi-\rho_r]^2 - 0\right\} \\
&= \frac{1}{4\psi}[1+\psi-\rho_r]^2 .
\end{aligned}$$

Gamma Distribution

When ϵ is gamma-distributed with a scale parameter θ and a shape parameter ξ. Since the expected value is 1, $\theta \cdot \xi = 1$. The standard deviation is $\sigma = \theta \cdot \sqrt{\xi}$. Then

$$\begin{aligned}
\int_{\rho_m}^{\infty} (x-\rho_m)\cdot g(x)\,dx &= \int_{\rho_m}^{\infty} x\cdot g(x)\,dx - \int_{\rho_m}^{\infty} \rho_m \cdot g(x)\,dx \\
&= \int_{\rho_m}^{\infty} x\cdot \frac{x^{\xi-1}}{\theta^{\xi}\Gamma(\xi)} e^{-\frac{x}{\theta}}\,dx - \rho_m \cdot \int_{\rho_m}^{\infty} g(x)\,dx \\
&= \frac{1}{\theta^{\xi}\Gamma(\xi)} \int_{\rho_m}^{\infty} x^{\xi}\cdot e^{-\frac{x}{\theta}}\,dx - \rho_m \cdot [1-G(\rho_m)] \\
&= \frac{\theta}{\Gamma(\xi)} \int_{\rho_m}^{\infty} \left(\frac{x}{\theta}\right)^{\xi} \cdot e^{-\frac{x}{\theta}}\,d\left(\frac{x}{\theta}\right) - \rho_m \cdot [1-G(\rho_m)] \\
&= \frac{\theta}{\Gamma(\xi)} \left(\frac{\rho_m}{\theta}\right)^{\xi} e^{-\frac{\rho_m}{\theta}} - (\rho_m - 1)\cdot [1-G(\rho_m)] \\
&= \rho_m \cdot \theta \cdot g(\rho_m) - (\rho_m - 1)[1-G(\rho_m)].
\end{aligned}$$

$$\begin{aligned}
\int_{\rho_r}^{\infty} (x-\rho_r)\cdot g(x)\,dx &= \int_{\rho_r}^{\infty} x\cdot g(x)\,dx - \int_{\rho_r}^{\infty} \rho_r \cdot g(x)\,dx \\
&= \int_{\rho_r}^{\infty} x\cdot \frac{x^{\xi-1}}{\theta^{\xi}\Gamma(\xi)} e^{-\frac{x}{\theta}}\,dx - \rho_r \cdot \int_{\rho_r}^{\infty} g(x)\,dx \\
&= \frac{1}{\theta^{\xi}\Gamma(\xi)} \int_{\rho_r}^{\infty} x^{\xi}\cdot e^{-\frac{x}{\theta}}\,dx - \rho_r \cdot [1-G(\rho_r)] \\
&= \frac{\theta}{\Gamma(\xi)} \int_{\rho_r}^{\infty} \left(\frac{x}{\theta}\right)^{\xi} \cdot e^{-\frac{x}{\theta}}\,d\left(\frac{x}{\theta}\right) - \rho_r \cdot [1-G(\rho_r)] \\
&= \frac{\theta}{\Gamma(\xi)} \left(\frac{\rho_r}{\theta}\right)^{\xi} e^{-\frac{\rho_r}{\theta}} - (\rho_r - 1)\cdot [1-G(\rho_r)] \\
&= \rho_r \cdot \theta \cdot g(\rho_r) - (\rho_r - 1)[1-G(\rho_r)].
\end{aligned}$$

Chi-square Distribution

When ϵ is a chi-square distribution with expected value of 1, then

$$\begin{aligned}
\int_{\rho_m}^{\infty}(x-\rho_m)\cdot g(x)\,dx &= \int_{\rho_m}^{\infty}x\cdot g(x)\,dx-\int_{\rho_m}^{\infty}\rho_m\cdot g(x)\,dx \\
&= \int_{\rho_m}^{\infty}x\cdot\frac{x^{-1/2}}{\sqrt{2}\Gamma(0.5)}e^{-\frac{x}{2}}dx-\rho_m\cdot\int_{\rho_m}^{\infty}g(x)\,dx \\
&= \frac{1}{\sqrt{2}\Gamma(0.5)}\int_{\rho_m}^{\infty}\sqrt{x}\cdot e^{-\frac{x}{2}}dx-\rho_m\cdot[1-G(\rho_m)] \\
&= \frac{-2}{\sqrt{2}\Gamma(0.5)}\sqrt{\rho_m}e^{-\frac{\rho_m}{2}}-(\rho_m-1)\cdot[1-G(\rho_m)] \\
&= \rho_m\cdot g(\rho_m)-(\rho_m-1)[1-G(\rho_m)].
\end{aligned}$$

$$\begin{aligned}
\int_{\rho_r}^{\infty}(x-\rho_r)\cdot g(x)\,dx &= \int_{\rho_r}^{\infty}x\cdot g(x)\,dx-\int_{\rho_r}^{\infty}\rho_r\cdot g(x)\,dx \\
&= \int_{\rho_r}^{\infty}x\cdot\frac{x^{-1/2}}{\sqrt{2}\Gamma(0.5)}e^{-\frac{x}{2}}dx-\rho_r\cdot\int_{\rho_r}^{\infty}g(x)\,dx \\
&= \frac{1}{\sqrt{2}\Gamma(0.5)}\int_{\rho_r}^{\infty}\sqrt{x}\cdot e^{-\frac{x}{2}}dx-\rho_r\cdot[1-G(\rho_r)] \\
&= \frac{-2}{\sqrt{2}\Gamma(0.5)}\sqrt{\rho_r}e^{-\frac{\rho_r}{2}}-(\rho_r-1)\cdot[1-G(\rho_r)] \\
&= \rho_r\cdot g(\rho_r)-(\rho_r-1)[1-G(\rho_r)].
\end{aligned}$$

Vita

Changyuan Yan has published several papers on pricing, scheduling, supply chain management and operations management in academic journals such as International Journal of Production Economics and OMEGA (The International Journal of Management Science). He also presented his research at academic conferences such as INFORMS Annual Meeting. He is a member of Institute for Operations Research and the Management Sciences (INFORMS).

Yan has won several teaching and research awards, including Graduate Student Teaching Excellence Award 2010 (Highly Commended) and Excellence in Research Award, Lebow College of Business, 2010.

Yan did his doctoral study in Business Administration at Drexel University (Philadelphia, PA) with a thesis on Integrated Production, Inventory and Pricing Decisions in Two-Echelon Supply Chains written under the supervision of Dr. Avijit Banerjee. Yan earned his B.S. in Mechanics and in Economics from Peking University (Beijing, China).