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Control of Non-Minimum-Phase Nonlinear Systems through Constrained Input-Output Linearization

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Abstract— **This paper presents a novel control method that provides optimal output-regulation with guaranteed closed-loop asymptotic stability within an assessable domain of attraction. The closed-loop stability is ensured by requiring state variables to satisfy a hard, second-order Lyapunov constraint. Whenever input-output linearization alone cannot ensure asymptotic closed-loop stability, the closed-loop system evolves while being at the hard constraint. Once the closed-loop system enters a state-space region in which input-output linearization can ensure asymptotic stability, the hard constraint becomes inactive. Consequently, the nonlinear control method is applicable to stable and unstable processes, whether non-minimum- or minimum-phase. The control method is implemented on a chemical reactor with multiple steady states, to show its application and performance.**

I. INTRODUCTION

Since the early 1990's, many research efforts have been made to develop nonlinear model-based control methods that can be implemented on nonlinear systems with a non-minimum-phase (NMP) steady state. These efforts have been mainly within the frameworks of model predictive control, differential geometric control, and Lyapunov control.

In model-predictive control (MPC), controller action is the solution to a constrained optimization problem that is solved repeatedly on-line. In MPC, optimality may not imply closed-loop stability. To ensure closed-loop stability in MPC, the addition of Lyapunov stability constraints or penalty terms to the optimization problem has been proposed [1, 2, 3].

Differential geometric control is a direct synthesis method in which the controller is derived by requesting a desired closed-loop output response in the absence of input constraints. A widely used differential geometric control method is input-output linearization, which cannot be used to operate a process at a NMP steady state. Efforts to make input-output linearization applicable to processes with a NMP steady state include the use of equivalent output(s) for the controller design [4], coordinated control [5], controller design by inverting the minimum-phase part [6, 7], and approximate input-output linearization [8, 9].

In Lyapunov-based control, the central focus in the controller design is on stability through Lyapunov's direct method [10, 11, 12, 13].

This paper presents a new approach to addressing a major limitation of input-output linearization; that is, the internal instability of the closed-loop system when the process is operating at a non-minimum-phase steady state. The resulting control method is applicable to stable and unstable processes, whether non-minimum- or minimum-phase. A novel Lyapunov stability constraint that does not suffer from the singularity (at the desired steady state) problem of the standard Lyapunov constraints is presented. The control method is obtained by further exploiting the connections between input-output linearization and model predictive control [14].

This paper is organized as follows. The scope of the study and some mathematical preliminaries are given in Section II. Section III presents the nonlinear control method. The application and performance of the control method are illustrated by numerical simulation of a chemical reactor in Section IV. Finally, concluding remarks are given in section V.

II. SCOPE AND MATHEMATICAL PRELIMINARIES

Consider the general class of multivariable processes with a mathematical model in the form:

$$\begin{aligned} \dot{x} &= f(x,u), \qquad x(0) = x_0 \\ y &= h(x) \end{aligned} \tag{1}$$

with the input constraints

$$u_{l_i} \le u_i \le u_{h_i}, \quad i = 1, \cdots, m$$

where $x \in X \subset R^n$ is the vector of state variables, $u \in U \subset R^m$ is the vector of manipulated inputs, $y \in R^m$ is the vector of controlled outputs, and $f(x,u)$ and $h(x)$ are smooth functions on $X \times U$ and X, respectively. X is an open connected set that contains the nominal steady state value of x. U is a closed connected set that contains the nominal steady state value of u : $U = \{u | \, u_{l_i} \le u_i \le u_{h_i}$, $i = 1, \cdots, m \}$. The relative orders (degrees) of the controlled outputs $y_1, \ldots, y_m$ are denoted by $r_1, \ldots, r_m$, respectively, where r_i is the smallest integer for which

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$\partial\left[d^{r_i} y_i / dt^{r_i}\right]/\partial u \neq 0$. The following notations are used:

$$\begin{aligned} h_i^0(x) &= h_i(x) = y_i \\ h_i^1(x) &= \frac{dy_i}{dt} \\ &\vdots \\ h_i^{r_i-1}(x) &= \frac{d^{r_i-1}y_i}{dt^{r_i-1}} \\ h_i^{r_i}(x,u) &= \frac{d^{r_i}y_i}{dt^{r_i}} \end{aligned} \tag{2}$$

Assumptions:

A1. The relative orders (degrees), $r_1,\ldots,r_m$, are finite and do not change on $X \times U$.

A2. The characteristic (decoupling) matrix of the process is nonsingular on $X \times U$.

A3. The process is controllable and observable on $X \times U$.

The steady state pair(s) (x_{ss}, u_{ss}) corresponding to a given output set-point, y_{sp}, satisfy:

$$\begin{aligned} 0 &= f(x_{ss}, u_{ss}) \\ y_{sp} &= h(x_{ss}) \end{aligned}$$

These relations are used to describe the dependence of a nominal steady state, x_{ss}^N, on the set point: $x_{ss}^N = F(y_{sp})$.

A. MPC Formulation of Input-Output Linearization

This section presents a very short review of the MPC formulation of input-output linearization introduced in [14]. The formulation is used in the main body of this paper.

A linear response of the following form:

$$\begin{aligned} (\varepsilon_1 D+1)^{r_1} y_1(t) &= y_{sp_1}(t) \\ &\vdots \\ (\varepsilon_m D+1)^{r_m} y_m(t) &= y_{sp_m}(t) \end{aligned} \tag{3}$$

where $D = d/dt$, and $\varepsilon_1,\ldots,\varepsilon_m$ are positive constants that set the speed of the closed-loop output responses, can be induced to an *unconstrained* process of the form (1) by implementing the solution to the following unconstrained optimization problem at each time instant [14]:

$$\min_{u(t)} \sum_{i=1}^{m} \left\| \hat{y}_i(\tau) - y_{d_i}(\tau) \right\|^2 \tag{4}$$

where $\|q(\tau)\|$ denotes the 2-norm of a scalar function $q(\tau)$, given by:

$$\|q(\tau)\| \triangleq \sqrt{\int_t^{t+T_h} q(\tau)^2 \, d\tau}$$

$\hat{y}_i(\tau)$ is model-predicted future value of y_i at time instant τ, given by a Taylor series expansion of y_i around the present time, t:

$$\hat{y}_i(\tau) = h_i(x(t)) + \sum_{l=1}^{r_i-1} h_i^l(x(t)) \frac{[\tau-t]^l}{l!} + h_i^{r_i}(x(t),u(t)) \frac{[\tau-t]^{r_i}}{r_i!} + \text{higher order terms (h.o.t.)}$$

and $y_{d_i}(\tau)$ is the predicted future value of the reference trajectory of y_i at time instant τ, given by:

$$\begin{aligned} (\varepsilon_1 D+1)^{r_1} y_{d_1}(\tau) &= y_{sp_1}(t) \\ &\vdots \\ (\varepsilon_m D+1)^{r_m} y_{d_m}(\tau) &= y_{sp_m}(t) \end{aligned} \tag{5}$$

initialized at:

$$\frac{d^l y_{d_i}(t)}{dt^l} = h_i^l(x(t)), \quad l = 0,\cdots,r_i-1, \qquad i = 1,\cdots,m$$

Taylor series expansion of y_{d_i} around the present time, *t,* yields:

$$y_{d_i}(\tau) = h_i(x(t)) + \sum_{l=1}^{r_i-1} h_i^l(x(t)) \frac{[\tau-t]^l}{l!} + \frac{y_{sp_i}(t) - h_i(x(t)) - \sum_{l=1}^{r_i-1} \varepsilon_i^l \binom{r_i}{l} h_i^l(x(t))}{\varepsilon_i^{r_i}} \frac{[\tau-t]^{r_i}}{r_i!} + \text{h.o.t.}$$

After substituting for $\hat{y}_i(\tau)$ and y_{d_i}, for a very small prediction horizon T_h, the minimization problem of (4) takes the simplified form:

$$\min_u \sum_{i=1}^{m} \left[\frac{y_{sp_i} - h_i(x) - \sum_{l=1}^{r_i-1} \varepsilon_i^l \binom{r_i}{l} h_i^l(x) - \varepsilon_i^{r_i} h_i^{r_i}(x,u)}{\varepsilon_i^{r_i}} \right]^2 \tag{6}$$

Equation (6) represents a nonlinear state feedback, which can be denoted by:

$$u = \tilde{\Psi}(x, y_{sp})$$

because of the non-singularity of the characteristic matrix. If this state feedback makes the performance index in (4) zero at each time instant, the linear closed-loop response of (3) is induced; that is, the state feedback of (6) is input-output linearizing. Needless to mention that the closed-loop system under the input-output linearizing state feedback of (6) is stable when the desired steady state is minimum-phase.

III. Nonlinear Control Method

The nonlinear control method and its implementation are presented in this section.

A. Lyapunov Stability Constraint

The idea of designing an input-output linearizing controller with a stability constraint has been inspired by contractive (stability) constraints used in MPC to ensure closed-loop stability [3] and input-output linearization being a shortest prediction-horizon continuous-time model predictive controller [14].

A widely used stability constraint in MPC is:

$$\| \bar{x}(k+1) \| \leq \alpha \| \bar{x}(k) \|, \quad 0<\alpha<1 \tag{9}$$

where $\bar{x} = x - x_{ss}^N$. The preceding stability constraint has the general form:

$$V(\bar{x}(k+1)) - \alpha V(\bar{x}(k)) \leq 0$$

where $V(\bar{x})$ is a positive definite function. The preceding inequality has the continuous-time form:

$$\frac{dV(\bar{x})}{dt} + \gamma V(\bar{x}) \leq 0 \tag{10}$$

where $\gamma = -\ln\alpha / \Delta t$. The manipulated input that makes $V(\bar{x})$ satisfy (10) is the solution for u of:

$$\frac{\partial V(\bar{x})}{\partial \bar{x}} f(x,u) + \gamma V(\bar{x}) \leq 0$$

Since $\partial V(\bar{x}) / \partial \bar{x}\big|_{x=x_{ss}^N} = 0$, as $x \to x_{ss}^N$, (10) becomes ill-conditioned/singular and thus cannot be solved for u. Approaches/approximations to address this singularity problem in Lyapunov-based control include those in [16, 17].

Consider the specific Lyapunov function:

$$V(\bar{x}) = \bar{x}^T P \bar{x} \tag{11}$$

where P is the positive-definite symmetric matrix that satisfies the Riccati equation

$$A^T P + PA - PB^T BP = -Q$$

Q is a positive definite matrix,

$$A = \frac{\partial f(x,u)}{\partial x}\bigg|_{(x_{ss}^N, u_{ss}^N)} \qquad B = \frac{\partial f(x,u)}{\partial u}\bigg|_{(x_{ss}^N, u_{ss}^N)}$$

If the Lyapunov function, $V(\bar{x})$, satisfies

$$\beta^2 \frac{d^2 V}{dt^2} + 2\beta \frac{dV}{dt} + V \leq 0 \tag{12}$$

in closed-loop over *X*, where β is a positive constant that set the rate of decay of $V(\bar{x})$, then the closed-loop system is asymptotically stable over *X*. If $V(\bar{x})$ is required to be governed by (12), then

$$\beta^2 \left[f^T P f + \frac{\partial V}{\partial x} \left\{ \frac{\partial f}{\partial x} f + \frac{\partial f}{\partial u} \frac{du}{dt} \right\} \right] + 2\beta \frac{\partial V}{\partial x} f + V \leq 0 \tag{13}$$

which is not singular at $x = x_{ss}^N$, allowing one to solve for u at every $x \in X$.

B. Nonlinear State Feedback Design

To derive a state feedback that can achieve output regulation with guaranteed closed-loop stability, we solve the following constrained optimization problem at each time instant, t:

$$\min_{u(t)} \sum_{i=1}^{m} \left\| \hat{y}_i(\tau) - y_{d_i}(\tau) \right\|^2 \tag{14}$$

subject to the process model of (1); that is,

$$\dot{x} = f(x,u), \qquad x(0) = x_0$$
$$y = h(x)$$

with $u_{l_i} \leq u_i \leq u_{h_i}$, $i = 1, \cdots, m$, and the stability constraint:

$$\beta^2 \ddot{V} + 2\beta \dot{V} + V \leq 0$$

where $V = (x - x_{ss}^N)^T P (x - x_{ss}^N)$ and $x_{ss}^N = F(y_{sp})$.
Assuming this optimization is feasible (has a solution $u \in U$), let us denote the solution by the state feedback:

$$u = \Psi(x, y_{sp}) \tag{15}$$

The next theorem summarizes the stability properties of the preceding nonlinear state feedback.

Theorem. For a process in the form of (1), the closed-loop system under the state feedback (15) is asymptotically stable in the region in which the state feedback is feasible.

The tunable parameter β is suggested to be chosen such that $\beta > \max(\varepsilon_1, \ldots, \varepsilon_m)$. This choice of β can prevent unnecessary activation of the stability constraint when the solution to the minimization problem of (6) subject to the input constraints can ensure closed-loop stability.

1) Implementation of the Nonlinear State Feedback

The state feedback of (15) is the solution of the constrained optimization problem:

$$\min_u \sum_{i=1}^{m} \left[\frac{y_{sp_i} - h_i(x) - \sum_{l=1}^{r_i - 1} \varepsilon_i^l \binom{r_i}{l} h_i^l(x) - \varepsilon_i^{r_i} h_i^{r_i}(x,u)}{\varepsilon_i^{r_i}} \right]^2 \tag{16}$$

subject to

$$u_{l_i} \leq u_i \leq u_{h_i}, i = 1, \cdots, m$$
$$\beta^2 \ddot{V} + 2\beta \dot{V} + V \leq 0$$

When the process is away from the nominal steady state x_{ss}^N; that is, $V(x - x_{ss}^N) \geq \sigma$, where σ is a very small positive constant that is set by the controller designer, the stability constraint of (12) takes the form of (13), which is denoted by the compact form:

$$S(x, x_{ss}, u, \dot{u}) \leq 0 \tag{17}$$

When the process is close to the nominal steady state x_{ss}^N; that is, $V(x - x_{ss}^N) < \sigma$, the stability constraint of (12) takes the form:

$$\beta^2 f^T P f + V \leq 0 \tag{18}$$

which is denoted by:

$$S_\varepsilon(x, u) \leq 0 \tag{19}$$

A simple way of solving for u in (17) is to approximate the time-derivative of the manipulated input vector, $\dot{u}$, with $\dot{u}(t) = [u(t) - u(t - \Delta t)] / \Delta t$, where Δt is the time-discretization interval. Thus, the implementation form of the state feedback of (15) is:

$$\min_{u(t)} \sum_{i=1}^{m} \left[\frac{y_{sp_i} - h_i(x(t)) - \sum_{l=1}^{r_i - 1} \varepsilon_i^l \binom{r_i}{l} h_i^l(x(t)) - \varepsilon_i^{r_i} h_i^{r_i}(x(t), u(t))}{\varepsilon_i^{r_i}} \right]^2 \tag{20}$$

subject to

$$u_{l_i} \le u_i \le u_{h_i}, i = 1, \cdots, m$$

$$S(x(t), x_{ss}^N, u(t), [u(t) - u(t - \Delta t)] / \Delta t) \le 0, V(x - x_{ss}^N) \ge \sigma$$

$$S_\sigma(x(t), x_{ss}^N, u(t)) \le 0, \quad V(x - x_{ss}^N) < \sigma$$

Remark 1: In the case that $u \in \Re$, the numerical solution to the constrained optimization problem of (20) is obtained using the following algorithm:

1. Solve

$$\min_{u} \sum_{i=1}^{m} \left[\frac{y_{sp_i} - h_i(x) - \sum_{l=1}^{r_i - 1} \varepsilon_i^l \binom{r_i}{l} h_i^l(x) - \varepsilon_i^{r_i} h_i^{r_i}(x, u)}{\varepsilon_i^{r_i}} \right]^2 \tag{21}$$

subject to

$$u_{l_i} \le u_i(t) \le u_{h_i}, \ i = 1, \cdots, m$$

2. If the u calculated in step 1 satisfies

$$\beta^2 \left[f^T P f + \frac{\partial V}{\partial x} \left\{ \frac{\partial f}{\partial x} f + \frac{\partial f}{\partial u} \frac{du}{dt} \right\} \right] + 2\beta \frac{\partial V}{\partial x} f + V \le 0 \tag{22}$$

then implement the u.

3. If the u calculated in step 1 does not satisfy the inequality of (22), calculate u from

$$S(x(t), x_{ss}^N, u(t), [u(t) - u(t - \Delta t)] / \Delta t) \le 0, \quad V(x(t) - x_{ss}^N) \ge \sigma$$

$$S_\sigma(x(t), x_{ss}^N, u(t)) \le 0, \quad V(x(t) - x_{ss}^N) < \sigma$$

and then implement the u.

The algorithm indicates that the state feedback of (15) is a hybrid of an input-output linearizing state feedback and a Lyapunov-based state feedback. Indeed, at any time instant, it is an input-output linearizing or a Lyapunov-based state feedback and may switch from one state feedback to the other.

C. Reduced-Order State Observer

In general, measurements of all state variables are not available. In such cases, estimates of the unmeasured state variables can be obtained from the output measurements. Here, we use the 'closed-loop', reduced-order, nonlinear, state observer in [15] to reconstruct the unmeasured state variables. The observer has the general form:

$$\begin{aligned} \dot{z} &= F_z(z, y, Y, u) \\ \hat{x} &= O(z, y, Y) \end{aligned} \tag{23}$$

The state observer is applicable to processes operating at both stable and unstable steady states.

D. Integral Action

To ensure offset-free response of the closed-loop system in the presence of constant output disturbances and model errors, the control system should have integral action. An estimate of disturbance-free process outputs is first calculated by using the *closed-loop process model*:

$$\begin{aligned} \dot{w} &= f(w, \Psi(w, y_{sp})) \\ \xi &= h(w) \end{aligned} \tag{24}$$

where ξ is the estimate of the disturbance-free controlled output. The difference between this estimate and the measurement of the controlled outputs, y, is then added to the output set-point, as in internal model control, leading to:

$$\begin{aligned} \dot{w} &= f(w, \Psi(w, v)) \\ v &= y_{sp} - y + h(w) \\ u &= \Psi(\hat{x}, v) \end{aligned} \tag{25}$$

An interesting feature of this approach to adding integral action is that the addition of the dynamic system of (25) to the state feedback of (14) [calculation of $x_{ss_N} = v$ according to (25)] adds no additional conditions for closed-loop asymptotic stability. In other words, if the closed-loop system is asymptotically stable under the state feedback of (14) alone, it is also asymptotically stable under (25).

E. Controller System

Combining the state feedback of (14), the reduced-order observer of (23), and the dynamic sub-system of (24), leads to the following controller system that has integral action:

$$\begin{aligned} \dot{z} &= F_z(z, y, Y, u) \\ \hat{x} &= O(z, y, Y) \\ \dot{w} &= f(w, \Psi(w, v)) \\ v &= F\left(y_{sp} - y + h(w)\right) \\ u &= \Psi(\hat{x}, v) \end{aligned} \tag{26}$$

A parameterization of the controller system is shown in Figure 1.

IV. Application To A Chemical Reactor

Consider a constant-volume, non-isothermal, continuous stirred-tank reactor, in which the reaction $A \rightarrow B$ takes place in the liquid phase. The reactor dynamics are represented by the model:

$$\begin{aligned} \dot{C}_A &= -Z \exp\left(\frac{-E_a}{RT}\right) C_A + (C_{A_i} - C_A) \frac{F}{V_0} \\ \dot{T} &= \gamma Z \exp\left(\frac{-E_a}{RT}\right) C_A + (T_i - T) \frac{F}{V_0} + q \\ y &= T \end{aligned}$$

where $Z = 5.0 \times 10^8 \ s^{-1}$, $E_a / R = 8100 \ K$, $\gamma = 3.9$

$m^3 K\ kmol^{-1}$, $q = -2.519\times10^{-2}\ K\ s^{-1}$, C_{A_i} =12 $kmol\ m^{-3}$, T_i = 300 K, V_0 = 0.1 m^3. This reactor has multiple steady states. It is desired to control the reactor temperature by manipulating the feed flow rate, F. All state variables are assumed to be measured.

Here, $x = [C_A \quad T]^T$, $u = [F]$, and $y = [T]$. For this process, the state feedback of (14) takes the form:

$$\min_u \left[\left\{ \gamma Z \exp\left(\frac{-E_a}{Rx_2}\right) x_1 + q + \frac{x2 - y_{sp}}{\varepsilon_1} \right\} + \frac{T_i - x_2}{V_0} u \right]^2$$

subject to the stability constraint of (13). The process is initially at $[x_1(0),\ x_2(0)] = [10, 290]$, which is located in a minimum-phase region.

Initially y_{sp} =293.9, and it then increases to $y_{sp} = 302.0$. The steady state pair corresponding to y_{sp} =293.9 (x_{1ss} =8.39, x_{2ss} = 293.9, u_{ss} =0.45) is stable (eigenvalues of the Jacobian evaluated at the steady state are -4.5 and -0.5, and the one corresponding to y_{sp} =302 (x_{1ss} =6.319, x_{2ss} =302.0, u_{ss} =0.45) is unstable (eigenvalues at -4.5 and 0.309). The Jacobian of the zero dynamics evaluated at x_{1ss} =8.39 has an eigenvalue at -10.91 and at x_{1ss} =6.319 has an eigenvalue at 36.27. Thus, the steady state corresponding to the lower temperature set-point is stable and minimum-phase, and the one corresponding to the higher temperature is unstable and non-minimum-phase.

Figure 2a shows the closed-loop responses of the state variables under the control system of (25) with ε_1 =0.2, β =0.4, and $P = \begin{bmatrix} 0.0755 & 0.116 \\ 0.1116 & 0.1524 \end{bmatrix}$; the control system successfully operates the reactor at the desired steady state, whether stable minimum- or unstable non-minimum-phase. The figure also shows the controller flag. Controller flag of one indicates that the stability constraint is active.
With $y_{sp} = 293.9$, the Lyapunov stability constraint is active until the controller action calculated by the input-output linearizing state feedback satisfies the stability constraint. When the set point changes to $y_{sp} = 302$, the constraint remains continuously active because the output tracking cannot stabilize the closed-loop system at the non-minimum-phase steady state. The corresponding manipulated input and Lyapunov function profiles are shown in Figure 2b.

V. Concluding Remarks

A control method that can be used to operate nonlinear processes at stable and unstable steady states, whether non-minimum- or minimum-phase, was presented. The control method has advantages of both input-output linearization and Lyapunov control. Input-output linearization performs output regulation while the Lyapunov stability constraint ensures asymptotic closed-loop stability when the regulation is incapable of ensuring the stability. The feasibility of the control system of (14) implies asymptotically stability of the closed-loop system, and the feasibility region depends on the choice of matrix Q.

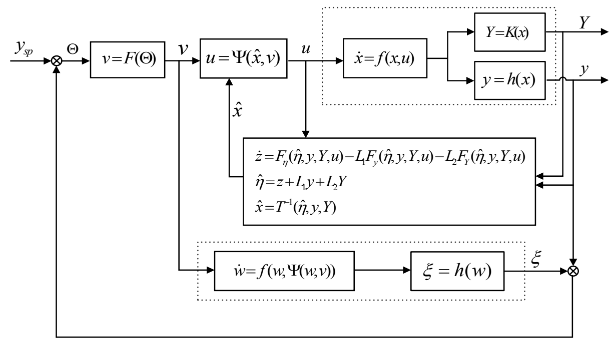

Figure 1 Parameterization of the controller system.

Notation

C_{A_i}	Inlet concentration of reactant A, $kmol\ m^{-3}$
C_A	Outlet concentration of reactant A, $kmol\ m^{-3}$
F	Reactor feed flow rate, $m^3\ h^{-1}$
m	Number of manipulated inputs
n	Number of state variables
r_i	Relative order (degree) of output y_i
t	Time, s
T	Reactor outlet temperature, K
T_i	Reactor inlet temperature, K
u	Vector of manipulated inputs
V	Reactor volume, m^3
x	Vector of state variables
y	Vector of controlled outputs
y_{sp}	Vector of set-points
Z	Reaction rate pre-exponential factor, s^{-1}

Greek

$\varepsilon_1, \ldots, \varepsilon_n$	Adjustable controller parameters
γ	Reactor model parameter, $K\ m^3\ kmol^{-1}$
β	Adjustable controller parameter
σ	Adjustable controller parameter

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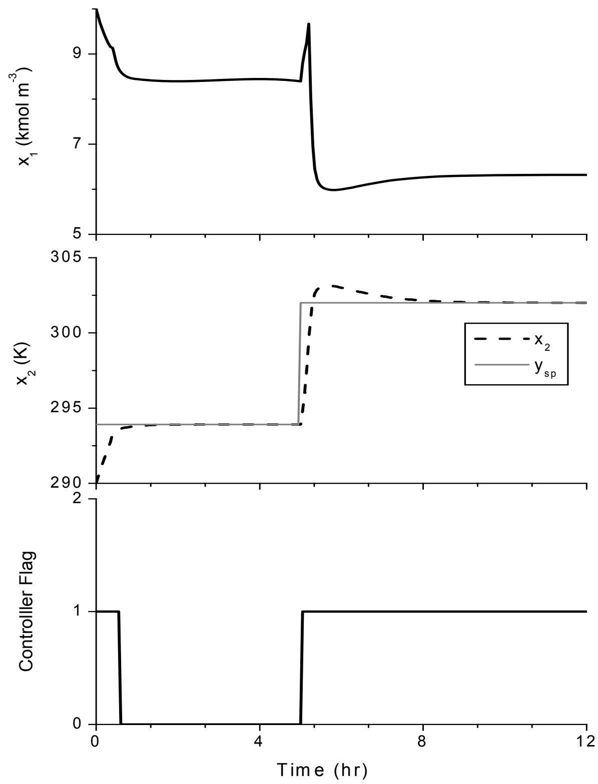

Figure 2a State responses of the chemical reactor under the controller, and the controller flag.

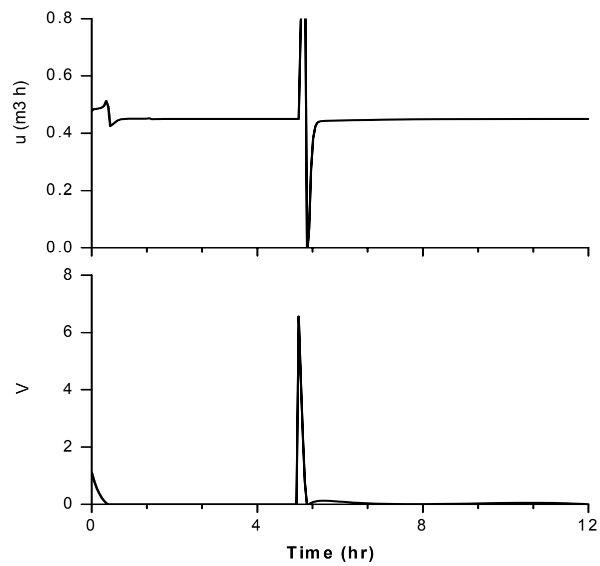

Figure 2b Manipulated input response and the Lyapunov function, corresponding to Figure 3a.