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Simple Market Timing with Moving Averages

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Abstract

Consider using the simple moving average (MA) rule of Gartley (1935) to determine when to buy stocks, and when to sell them and switch to the risk-free rate. In comparison, how might the performance be affected if the frequency is changed to the use of MA calculations? The empirical results show that, on average, the lower is the frequency, the higher are average daily returns, even though the volatility is virtually unchanged when the frequency is lower. The volatility from the highest to the lowest frequency is about 30% lower as compared with the buy-and-hold strategy volatility, but the average returns approach the buy-and-hold returns when frequency is lower. The 30% reduction in volatility appears if we invest randomly half the time in stock markets and half in the risk-free rate.

Keywords: Market timing, Moving averages, Risk-free rate, Returns and volatility.

JEL: G32, C58, C22, C41, D23.

1. Introduction

According to the standard investing separation theorem of Tobin (1958), investors allocate investments between risk-free and risky assets. If the risk-free rate is low (high), the investors shift their wealth to (from) the risky assets. Fama (1972) divides forecasters into two categories, namely macro forecasters (or market timers) and micro forecasters (or security analysts), who try to forecast individual stock returns relative to the market returns.

Merton (1981) defines a market timer to forecast when stocks will outperform (underperform) the risk-free asset, indicating that, when $r_t^m > r_t^f$ ($r_t^m < r_t^f$), where r_t^m is average stock market returns, r_t^f is the risk-free asset, $r_t^i = r_t^f + \beta^i(r_t^m - r_t^f) + \varepsilon_t^i$, r_t^i is the return for individual stock i included in the market portfolio m , β^i is a positive parameter, and $E[\varepsilon_t^i | r_t^m] = E[\varepsilon_t^i]$. That is, a market timer only forecasts the statistical properties of $r_t^m - r_t^f$, indicating that their forecasts contain only the differential performance among individual stocks arising from systematic risk in the markets.

Merton (1981) shows theoretically that when investors have heterogeneous beliefs and imperfect information, the value of a random market timing forecast is zero, and if the forecast variable is distributed independently or the forecast is based on public information, its value is zero, too. In fact, Merton shows that the maximum value of skilled market timing is the value of the protective put against buy-and-hold strategy.

Henriksson and Merton (1981) present an empirical procedure whereby correct forecasts can be analyzed statistically. However, if it is assumed that ε_t^i follows an approximate normal distribution, this leads to the CAPM of Sharpe (1964) and Lintner (1965).

We use a simple MA rule for the timing aspect for individual Dow Jones Industrial Average (DJIA) stocks with different frequencies. Zhu and Zhou (2009) show analytically that MA trading rules, as a part of asset allocation rules, can outperform standard allocation rules when stock returns are partly forecastable. The standard rule

means investing a fixed proportion of wealth in risky assets and the rest in risk-free assets, with the ratio determined by the risk tolerance of an investor.

This is the well-known reward/risk (or mean-variance) principle in the spirit of Markowitz (1952), Tobin (1958) and Sharpe (1964). Zhu and Zhou (2009) argue that the fixed allocation rule is not optimal if returns are forecastable by using the MA rule. Therefore, assuming that risk tolerance and the forecast performance of stock market returns are constant, the linear combination rule means that, when the MA rule suggests an uptrend (downtrend), the rule suggests that the total weight should be allocated to stock markets (the risk-free rate).

The empirical findings suggest a low volatility anomaly that might be explained by investors' affection to high volatility, as suggested by Baker et al. (2011) and noted in Ang et al. (2009). On the other hand, the reported predictability of risk premia (see, for example, Cochrane 2008, and Fama 2014) can explain why, for instance, MA rules forecast better than using random highs and lows in the stock market (as noted in Jagannathan and Korajczyk 2017). The topic is important as Friesen and Sapp (2007), among others, report that mutual fund investors had negative outcomes, on average, in their timing to invest and withdraw cash from US mutual funds from 1991 to 2004. Munoz and Vicente (2018) report similar results with more recent data in US markets.

The plan of the remainder of the paper is as follows. Section 2 provides a literature review, and alternative model specifications are presented in Section 3. The empirical analysis is conducted in Section 4, while Section 5 gives some concluding comments.

2. Literature Review

In efficient markets, investors earn above average returns only by taking above average risks (Malkiel 2003). Samuelson (1998) conforms with Fama (1972) by noting that market efficiency can be divided into micro and macro efficiency. The former concerns the relative pricing of individual stocks, and the latter, for markets as a whole. The CAPM by Sharpe (1964), and Lintner (1965) argues that beta is a proper definition for systematic risk for stock i , if unexplained changes in risk adjusted returns for the stock follow approximately normal distribution with zero mean.

Black (1972) states that the slope of the security market line (SML) is flatter if there exists restrictions in borrowing, that is, leverage constraints in the model. Starting from Black et al. (1972), many studies have reported that the security market line is too flat in US stocks compared with the SML suggested by the CAPM version of Sharpe and Lintner.

Ang et al. (2009), Baker et al. (2014), and Frazzini and Pedersen (2014) find that low-beta stocks outperform high-beta stocks statistically significantly. In fact, Frazzini and Pedersen report that significant excess profits in US stocks can be achieved by shorting high-beta stocks and buying low-beta stocks with leverage, but that leverage constraints make them disappear. Using Black (1972), investors often have leverage constraints, thereby making them place too much weight on risky stocks, which results in lower required return for high-beta stocks than would be justified by the Sharpe-Lintner CAPM.

Markowitz (1952) defines portfolio risk simply as the volatility of portfolio returns. Clarke et al. (2010) find that the volatility of stock returns contains potentially an additional risk factor with respect to systematic risk that can be defined in the betas of CAPM by Sharpe and Lintner. Moreover, Ang et al. (2009) report that the total volatility of international stock market returns is highly correlated with US stock returns, thereby suggesting a common risk factor for US stocks.

Baker et al. (2011) suggest that the low-volatility anomaly is due to investor irrational behaviour, mainly because an average fund manager seeks to beat the buy-and hold strategy by overinvesting in high-beta stocks. The explanations include preference for lotteries (Barberis and Huang 2008; Kumar 2009; Bali et al. 2011), overconfidence (Ben-David et al. 2013), and representativeness (Daniel and Titman 2006)), which means that people assess the probability of a state of the world based on how typical of that state the evidence seems to be (Kahneman and Tversky 1974).

Baker and Wurgler (2015) argue that the anomaly is also related to the limits of arbitrage. In fact, the extra costs of shorting prevents to take advantage of overpricing (Hong and Sraer 2016). More importantly, Li et al. (2016) report that the excess returns of low-beta portfolios are due to mispricing in US stocks, indicating that the low-volatility anomaly does not exist because of systematic risk by some rational, stock

specific volatility risk factor. They tested the low-volatility anomaly with monthly data from January 1963 to December 2011 in NYSE, NASDAQ, and AMEX stocks.

Market timing is closely related to technical trading rules. Brown and Jennings (1987) show theoretically that using past prices (like the MA rule in Gartley (1935)) has value for investors, if equilibrium prices are not fully revealing, and signals from past prices have some forecasting qualities. More importantly, Zhu and Zhou (2009) indicate that the MA rules are particularly useful for asset allocation purposes among risk averse investors, when markets are forecastable (quality of signal).

Moskowitz et al. (2012) argue that there are significant time series momentum (TSM) effects in financial markets that are not related to the cross-sectional momentum effect (Jegadeesh and Titman 1993). However, TSM is closely related to MA rules, since it gives a buy (sell) signal according to some historical price reference points, whereas MA rules give a buy (sell) signal, when the current price moves above (below) the historical average of the chosen calculated rolling window measure.

Starting from LeRoy (1973) and Lucas (1978), the literature in financial economics states that financial markets returns in efficient markets are partly forecastable, when investors are risk averse. This leads to the time-varying risk premia of investors, as noted by Fama (2014). For example, Campbell and Cochrane (1999) present a consumption-based model, which indicates that when the markets are in recession (boom), risk averse investors require larger (smaller) risk premium for risky assets. More importantly, Cochrane (2008) notes that the forecastability of excess returns may lead to successful market timing rules.

Brock et al. (1992) test different MA lag rules for US stock markets, and find that they gain profits compared with holding cash. On the other hand, Sullivan et al. (1999) find that MA rules do not outperform the buy-and-hold strategy, if transaction costs are accounted for. Allen and Karjalainen (1999) use a genetic algorithm to develop the best ex-ante technical trading rule model using US data, and find some evidence of outperforming the buy-and-hold strategy. Lo et al. (2000) find that risk averse investors benefit from technical trading rules because they reduce volatility of the portfolio without giving up much returns when compared against the buy-and-hold strategy.

More recently, Neely et al. (2014) use monthly data from January 1951 to December 2011, and report that MA rules forecast the risk premia in US stock markets statistically significantly. Marshall et al. (2017) find that MA rules give an earlier signal than TSM, suggesting better returns for MA rules, but they both work best with large market value stocks.

Moskowitz et al. (2012) use monthly data from January 1965 to December 2009, and report that TSM provides significant positive excess returns in futures markets. However, Kim et al. (2016) report that these positive excess returns produced by TSM are due to the volatility scaling factor used by Moskowitz et. al. (2012).

3. Model Specifications

Consider an overlapping generation economy with a continuum of young and old investors $[0,1]$. A young risk-averse investor j invests their initial wealth, w_t^j , in infinitely lived risky assets $i = 1, 2, \dots, I$, and in risk-free assets that produce the risk-free rate of return, r^f . A risky asset i pays dividend D_t^i , and has x_i^s outstanding. Assuming exogenous processes throughout, the aggregate dividend is D_t .

A young investor j maximizes their utility from old time consumption through optimal allocation of initial resources, w_t^j , between risky and risk-free assets:

$$\max x_t^j \left(\frac{E_t(P_{t+1} + D_{t+1})}{P_t} - (1 + r^f) \right) - \frac{\nu^j}{2} x_t^{j^2} \sigma^2$$

s.t.

$$x_t^j P_t \leq w_t^j$$

where E_t is the expectations operator, P_t is the price of one share of aggregate stock, ν^j is a constant risk-aversion parameter for investor j , σ^2 is the variance of returns for the aggregate stock, and x_t^j is the demand of risky assets for an investor j . The first-order condition is:

$$\frac{E_t(P_{t+1} + D_{t+1})}{P_t} - (1 + r^f) - \nu^j x_t^j \sigma^2 = 0,$$

which results in optimal demand for the risky assets:

$$x_t^j = \frac{E_t((P_{t+1} + D_{t+1}) / P_t) - (1 + r^f)}{\nu^j \sigma_i^2}. \quad (1)$$

Suppose that an investor j is a macro forecaster who allocates their initial wealth, w_t^j , between risky stocks and risk-free assets according to their forecast about the return of the risky alternative. Then, equation (1) says that the investor invests in the risky stocks only if the numerator on the right hand side is positive.

4. Empirical Analysis

This section presents the empirical results from seven frequencies for the (MA) trend-chasing rules. The data consist of 29 companies included in the Dow Jones Industrial Average (DJIA) index in January 2018. The trading data (daily closing prices) cover 30 years from 1 January 1988 to 31 December 2017. Choosing the current DJIA companies for the last 30 years creates a “survivor bias” in the buy-and-hold results. However, this should not be an issue as we intend to compare the performance of the alternative MA frequency rules.

The rolling window is 200 trading days. The first rule is to calculate MA in every trading day; the second frequency takes into account every 5th trading day (thereby providing a proxy for the weekly rule); the third frequency takes into account every 20th trading day (proxy for the monthly rule); the fourth rule is to calculate MA for every 40th trading day (proxy for every other month); the fifth rule takes into account every 60th trading day (proxy for every third month); the sixth rule takes into account every 80th trading day (proxy for every fourth month); and the seventh rule takes into account every 100th trading day (proxy for every fifth month).

For the 29 DJIA companies, 26 of them have daily stock data available from 27 March 1987, thereby giving 4 January 1988 as the first trading day. The data for Cisco are available from 12 February 1990, for Goldman Sachs from 4 May 1999, and for Visa from 19 March 2008. There are 217 569 observations of daily returns from DJIA stocks. Thus, there are $217\ 569 \times 9 = 1\ 958\ 121$ daily returns for the first three frequencies (rules), $217\ 569 \times 4 = 870\ 276$ daily returns for the fourth rule, $217\ 569 \times 3 = 652\ 707$ daily returns for the fifth rule, $217\ 569 \times 2 = 435\ 138$ daily returns for the sixth rule, and 217 569 daily returns for the seventh rule.

The trading rule for all cases is to use a simple crossover rule. When the trend-chasing MA turns lower (higher) than the current daily closing price, we invest the stock (three-month US Treasury Bills) at the closing price of the next trading day. Thus, the trading rule provides a market timing strategy where we invest all wealth either in stocks (separately, every stock included in DJIA), or to the risk-free asset (three-month U.S. Treasury bill), where the moving average rule advises the timing.

At the first frequency (every trading day), we calculate daily returns for MA200, MA180, MA160, MA140, MA120, MA100, MA80, MA60, and MA40. For example, MA200 is calculated as:

$$\left(\frac{P_{t-1} + P_{t-2} + \dots + P_{t-200}}{200} \right) = X_{t-1}.$$

At the lowest frequency, where every 100th daily observation is counted, MAC2 is calculated as:

$$\left(\frac{P_{t-1} + P_{t-100}}{2} \right) = X_{t-1}.$$

If $X_{t-1} < P_{t-1}$, we buy the stock at the closing price, P_t , thereby giving daily returns as

$$R_{t+1} = \ln\left(\frac{P_{t+1}}{P_t}\right).$$

Tables 1-7 in Appendix 1 show that the annualized average log returns of MA200 - MA40 are **+0.053** *after* transaction costs (with 0.1% per change of position). Recall that there are 200 closing day prices in the rolling window MA200, whereas MA40 means that there are 40 closing day prices in the window. The respective log returns for MAW40-MAW8 (weekly) are **+0.063**; for MA10 - MA2 (monthly) **+0.071**; for MAD5 – MAD2 (every other month) **+0.078**; for MAT4 – MAT2 (every third month)**+0.084**, for MAQ3 – MAQ2 (every fourth month) **+0.094**; and for MAC2 (every fifth month) **+0.088** after transaction costs.

Tables 1-7 show that, as the frequency decreases until every fourth month frequency (MAQ3 – MAQ2), average returns tend to increase, and decrease thereafter. In comparison, the biased buy-and-hold strategy produces **+0.117** with equal weights among all DJIA stocks, and with **0.295** annual volatility. A random investment (half the time in the risk-free rate, and half in the equally weighted portfolio from 4 January 1988) produces $(0.117 * 0.5 + 0.022 * 0.5) = +0.070$ annually, on average, with $(1 - \sqrt{0.5} = 0.293) = 29.3\%$ reduction in volatility, indicating **0.209** annual volatility for that portfolio.

The data are dividend excluded, but the average annual dividend yield in DJIA stocks over the last thirty years has been **+0.026**, so that the biased buy and hold strategy produces **+0.143** annually with equal weights among DJIA stocks before taxes. Thus, the random investment strategy produces **+0.083** annually, with survivor bias.

Appendix 1 (that is, the second column of Tables 1-7) also reports the annualized average log returns calculated in the largest sample (full 200 observations) in every category: MA200 **+0.065**; MAW40 **+0.073**; MA10 **+0.079**; MAD5 **+0.083**; MAT4 **+0.089**; MAQ3 **+0.091**; and MAC2 **+0.088** after transaction costs and before dividends. Adding **+0.013** produces after dividends and before taxes: MA200 **+0.078**; MAW40 **+0.086**; MA10 **+0.092**; MAD5 **+0.096**; MAT4 **+0.102**; MAQ3 **+0.104**; and MAC2

+0.101. These results imply that starting from every fifth trading day frequency, a macro forecaster beats the buy and hold strategy **in returns**.

Figure 1 below illustrates the effects of frequency on the returns to volatility ratio (the second column in Tables 1-7).

< Figure 1 goes here >

In Figure 1, the straight line illustrates the return to volatility ratio of portfolios, where wealth is randomly invested in combinations of the three-month Treasury Bill (risk-free rate), with stocks included in the DJIA between 4 January 1988 and 31 December 2017. The red crosses represent the average return/volatility points calculated in the 200-day rolling window with the following frequencies: daily, every five days, every 20 days, every 40 days, every 60 days, every 80 days, and every 100 days (with only the most observations in each frequency giving 200, 40, 10, 5, 4, 3, and 2 observations). The red crosses plot a convex curve that deviates increasingly from the straight return to volatility ratio line, thereby symbolizing superior portfolio efficiency.

Tables 8-14 in Appendix 2 show that the annualized volatility of daily returns read, on average: MA200-MA40 **0.2044**; MAW40-MAW8 **0.205**; MA10-MA2 **0.2091**; MAD5-MAD2 **0.213**; MAT4-MAT2 **0.219**; MAQ3-MAQ2 **0.221**; and MAC2 **0.218**. Thus, there is virtually no difference between the MA frequencies, while the biased buy-and-hold strategy produces **0.295**.

Figure 1 presents the volatilities calculated in the largest sample (full 200 day rolling window in every category, the second column in Tables 8-14). They read MA200 **0.207**; MAW40 **0.208**; MA10 **0.211**; MAD5 **0.213**; MAT4 **0.218**; MAQ3 **0.215**; and MAC2 **0.218** after transaction costs. Investing randomly half of the time in the risk-free rate and the other half in the equally weighted portfolio, produces **0.209**. Thus, the difference between the annual volatilities produced in profitable market timing MA rules (MA10 – MAC2) and random market timing (half and half) ranges from **0.009** to **0.002**.

< Figure 2 goes here >

In Figure 2, the straight line again presents the return to volatility ratio of portfolios with random investment in the risk-free rate and the stocks in DJIA between 4 January 1988 and 31 December 2017. The red crosses plot the average return to volatility ratios, calculated by using a 200 day rolling window, with the following frequencies: daily, every five days, every 20 days, every 40 days, every 60 days, every 80 days, and every 100 days. The averages of every lag are reported in Tables 1-14, Appendices 1 and 2. Thus, all daily returns from Tables 1-14 are included.

Comparing Figures 1 and 2, it is clear that using the whole 200 daily observation windows in the MA rules produces more efficient results in market timing. That is, comparing the products of shorter and longer MA rule rolling windows, say, the last two monthly observations compared with ten monthly observations, average realized returns drop from **+0.079** to **+0.059** before dividends, while volatility remains approximately unchanged (from 0.211 to 0.207). This suggests that, in both cases, about half and half is invested in the equally-weighted DJIA portfolios and in the risk-free rate, and the MA rules advise the timing. More importantly, Tables 8-14 in Appendix 2 show that the range in volatilities with all MA rules varies between 0.202 – 0.227 (with 0.02 difference), whereas Tables 1-7 in Appendix 1 show that realized returns vary between 0.096 – 0.033 before dividends (with 0.063 difference).

These results indicate that a macro market timing with 200 days rolling window produces a reduction in volatility from **0.295** (the buy-and hold) to between 0.207-0.218, but the average annualized returns (dividends included) tend to rise as the MA frequency falls (+0.078 with all 200 observations to +0.104 with every fourth month observations). Thus, the results indicate that MA market timing finds long term stochastic trends more efficiently than short term stochastic trends.

The Sharpe ratio of random market timing (half and half) with dividends is **0.292**; for MA200 **0.271**; for MAW40 **0.308**; for MA10 **0.332**; for the MAD5 **0.347**; for MAT4 **0.370**; for MAQ3 **0.381**; and for MAC2 it is **0.362**.

< Figure 3 goes here >

Figure 3 shows that when the volatility changes 1% in the DJIA stocks , then the average returns change is 0.39%. Figures 1 and 2 suggest that the theoretical change should be such that when the volatility changes 1%, then the average returns change is 0.50%, suggesting a flatter SML line in the data. This suggests strongly that DJIA investors have overweight high-beta stocks in the last 30 years.

It is obvious that transaction costs are crucial in MA performance. In the above calculations, the transaction costs are 0.1% per transaction from current wealth. Tables 15 and 16 in Appendix 3 report the transaction costs for the MA200-MA40 and MA10-MA2 rules. In the MA200-MA40 rules, the average annualized transaction costs are **0.0133**, such that the rules have about 13 changes in positions per year. Meanwhile, for the MA10-MA2 rules, the average annualized transaction costs are **0.0032**, suggesting about 3 changes in positions per year.

Allen and Karjalainen (1999) give reasons for a cost of 0.2% per transaction in their sample, but since technological progress has reduced transaction costs since the mid-nineties, 0.1% per transaction should be fair, on average. Nevertheless, a trial with 0.2% transaction costs shows that, for example, the average annualized daily returns become 0.0403 for the MA200-MA40 rules, and 0.0674 for the MA10-MA2 rules. Note that the returns grow 67%, on average, for the MA10-MA2 rules (with about the same volatility) compared with costs of 0.1% per transaction.

Note that the model prohibits short selling since we have only long positions in stocks or investing in the risk-free rate. Then the limits of arbitrage argument of Baker et al. (2015) are consistent with our results.

5. Concluding Remarks

The analysis suggests that a macro forecaster can obtain higher returns with equal volatility (30 % below that of the buy-and-hold strategy) by reducing the frequency used in MA rules. The return to volatility ratio for risk-averse investors with MA market timing significantly outperforms the random benchmark strategy, when the frequency in

the MA rules is reduced. This indicates that the forecasts are more accurate the longer is the time frame.

The results suggest that a flatter SML in the CAPM can be followed by the irrational preference of investors in high-beta stocks, as suggested by Baker et al. (2011) and Li et al. (2016), since the empirically efficient frontier of portfolios becomes flatter than the theoretically efficient SML (random timing) (see Figure 1). In other words, the empirical results suggests that market timing with the few past obervations (for example, every fourth month) in the past 200 rolling window daily prices, have produced significantly better returns to risk ratio for the portfolio of DJIA equally weighted stocks in the past 30 years than random timing. The finding points to the low-volatility anomaly.

One explanation for the results is that they are due to time-varying risk premiums. This is emphasized by Neely et al. (2014), who claim that MA rules, in effect, forecast changes in the risk premium. If the results are rational products of time-varying risk premiums, the results suggest that investor sensitivity to risk must be extremely high, and their risk premium is larger (smaller) in downs (ups), as suggested by Campbell and Cochrane (1999). As volatility rises (decreases), usually in downs (ups), the results suggest that when volatility is high, investors as a group tolerate significantly more risk (that is, volatility) than in calmer periods.

Consider the following numerical example: Assume that the risk premium is 0.08 in volatile downs, and 0.04 in calm ups, and the variance of returns is 0.03 in downs and 0.09 in ups. Then the risk aversion coefficient must be 0.89 in volatile down periods, and 1.33 in calm up periods. As market timing with MA rules works better in longer periods with few obervations, it seems to be more accurate in longer stochastic (up or down) trends.

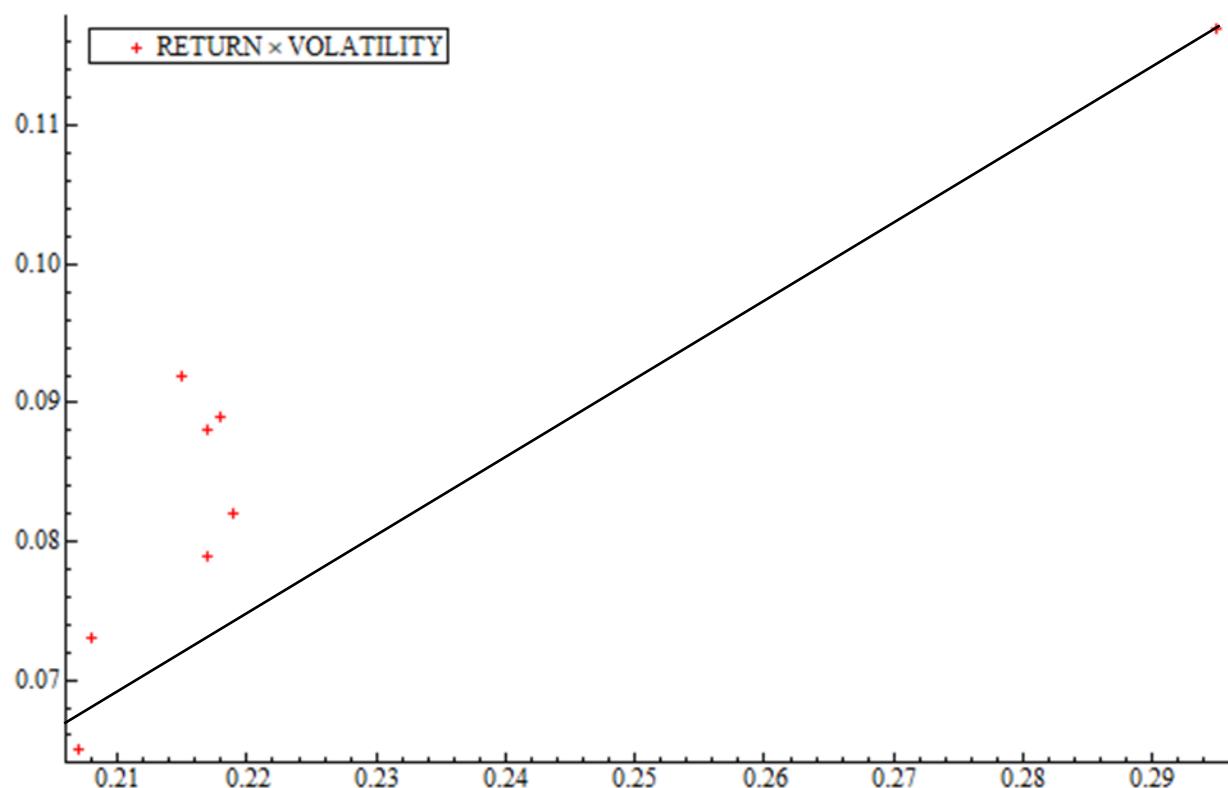


Figure 1: Returns to volatility ratio in MA200, MAW40, MA10, MAD5, MAT4, MAQ3, MAC2, and the theoretical random timing efficient SML

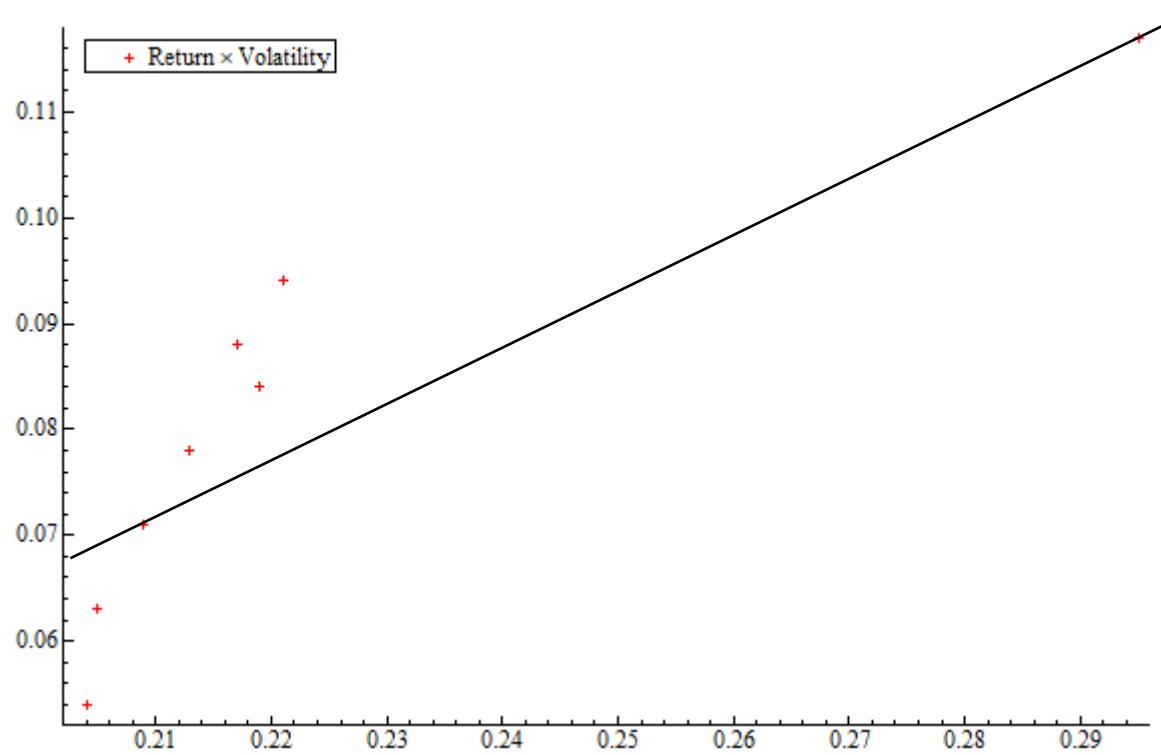
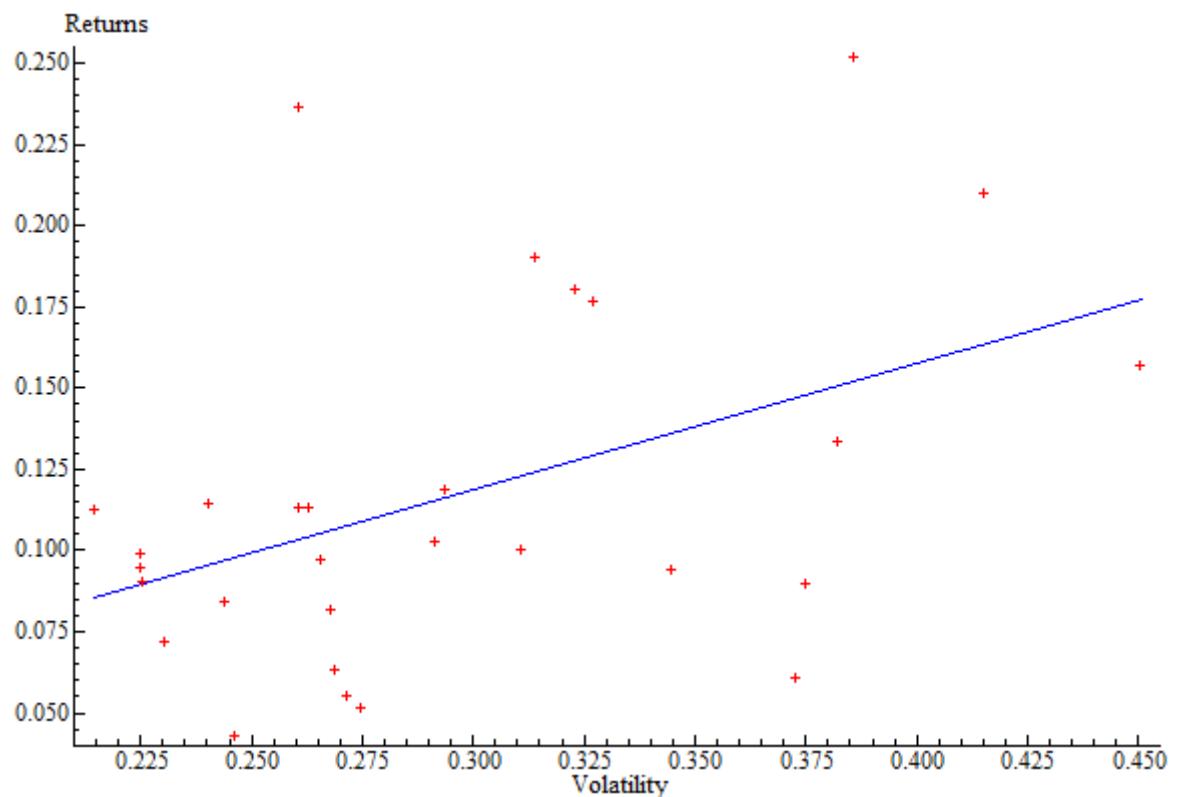


Figure 2: Returns to volatility ratio in MA200-MA40, MAW40-MAW8, MA10-MA2, MAD5-MAD2, MAT4-MAT2, MAQ3-MAQ2, MAC2, and the theoretical random timing efficient SML



**Figure 3: Returns to the volatility ratio in current DJIA stocks in annual averages
from 4 January 1988 to 31 December 2017**

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Appendix 1

Table 1: Annualized daily returns of MA40-MA200, average annualized returns

	Buy &										
	Hold	MA200	MA180	MA160	MA140	MA120	MA100	MA80	MA60	MA40	
3M	0.090	0.042	0.034	0.017	0.015	0.019	0.014	0.006	-0.009	6E-04	
American Express	0.094	0.035	0.037	0.039	0.055	0.039	0.042	0.043	0.041	0.008	
Apple	0.157	0.147	0.145	0.147	0.142	0.156	0.149	0.150	0.146	0.164	
Boeing	0.119	0.088	0.089	0.060	0.055	0.061	0.061	0.058	0.046	0.048	
Caterpillar	0.100	0.075	0.079	0.058	0.058	0.049	0.034	0.028	0.039	0.025	
Chevron	0.084	0.005	0.013	0.002	-0.000	-0.000	0.003	-0.01	-0.025	-0.05	
Coca-Cola	0.099	0.058	0.055	0.030	0.035	0.039	0.027	0.023	0.009	0.003	
Walt Disney	0.103	0.072	0.078	0.079	0.074	0.077	0.074	0.076	0.056	0.048	
Exxon	0.072	-0.011	-0.010	-0.020	-0.030	-0.020	-0.025	-0.01	-0.044	-0.05	
GE	0.052	0.072	0.071	0.058	0.039	0.039	0.033	0.018	0.013	9E-04	
Home Depot	0.190	0.125	0.116	0.102	0.092	0.087	0.076	0.067	0.068	0.058	
IBM	0.055	0.016	0.029	0.033	0.028	0.016	0.021	0.031	0.029	0.048	
Intel	0.134	0.083	0.082	0.083	0.073	0.091	0.082	0.080	0.077	0.078	
Johnson & Johnson	0.113	0.062	0.058	0.053	0.042	0.032	0.044	0.028	0.008	-0.00	
JP Morgan	0.090	0.013	0.014	0.003	0.010	0.017	0.013	0.031	0.038	0.025	
McDonalds	0.114	0.047	0.048	0.040	0.044	0.040	0.035	0.043	0.030	0.018	
Merck	0.063	0.050	0.048	0.044	0.032	0.033	0.029	0.022	0.016	-0.02	
Microsoft	0.180	0.117	0.128	0.105	0.102	0.104	0.095	0.090	0.070	0.062	
Nike	0.177	0.087	0.093	0.085	0.102	0.108	0.107	0.119	0.133	0.112	
Pfizer	0.097	0.059	0.056	0.043	0.042	0.052	0.044	0.040	0.024	0.009	
Procter & Gamble	0.095	0.037	0.045	0.037	0.036	0.037	0.029	0.023	0.004	0.017	
Travellers	0.082	0.036	0.035	0.038	0.029	0.008	-0.004	-9E-04	-0.001	0.006	
United Technologies	0.113	0.051	0.057	0.046	0.059	0.057	0.049	0.049	0.041	0.017	
United Health Group	0.252	0.181	0.182	0.157	0.147	0.136	0.130	0.118	0.125	0.076	
Verizon	0.043	-0.017	-0.020	-0.010	-0.000	-0.020	-0.020	-0.02	-0.029	-0.02	
Wal-Mart	0.113	0.019	0.016	0.010	0.012	0.012	0.016	0.012	0.020	0.024	
Cisco	0.210	0.198	0.194	0.210	0.208	0.198	0.205	0.152	0.096	0.085	
Goldman Sachs	0.061	0.038	0.029	0.033	0.038	0.050	0.057	0.078	0.076	0.063	

Visa	0.236	0.112	0.118	0.129	0.141	0.128	0.132	0.120	0.094	0.085
Average	0.117	0.065	0.066	0.059	0.058	0.057	0.053	0.05	0.041	0.033
										0.054

**Table 2: Annualized daily (every fifth trading day) returns of MAW8-MAW40
(W = number of weeks), average annualized returns**

	Buy&Hold	MAW40	MAW36	MAW32	MAW28	MAW24	MAW20	MAW16	MAW12	MAW8
3M	0.090	0.035	0.033	0.020	0.021	0.019	0.012	0.019	0.032	0.026
American Express	0.094	0.058	0.053	0.062	0.063	0.047	0.046	0.035	0.034	0.015
Apple	0.157	0.130	0.137	0.143	0.131	0.134	0.131	0.188	0.174	0.144
Boeing	0.119	0.089	0.079	0.075	0.074	0.080	0.082	0.066	0.074	0.076
Caterpillar	0.100	0.057	0.062	0.058	0.058	0.061	0.054	0.049	0.043	0.023
Chevron	0.084	0.005	0.015	3E-04	0.004	0.008	0.009	0.004	0.004	-0.03
Coca-Cola	0.099	0.055	0.054	0.054	0.041	0.054	0.047	0.047	0.029	0.011
Walt Disney	0.103	0.071	0.073	0.062	0.080	0.076	0.080	0.078	0.065	0.051
Exxon	0.072	0.018	0.016	0.007	0.008	0.010	0.013	0.020	0.011	0.005
GE	0.052	0.061	0.046	0.047	0.047	0.045	0.023	0.018	0.031	0.023
Home Depot	0.190	0.135	0.133	0.124	0.112	0.110	0.088	0.076	0.096	0.077
IBM	0.055	0.020	0.037	0.044	0.040	0.051	0.027	0.028	0.008	0.016
Intel	0.134	0.088	0.091	0.075	0.061	0.075	0.073	0.070	0.076	0.085
Johnson & Johnson	0.113	0.074	0.079	0.071	0.059	0.050	0.050	0.048	0.042	0.027
JP Morgan	0.090	0.040	0.036	0.027	0.033	0.033	0.048	0.051	0.042	0.020
McDonalds	0.114	0.086	0.068	0.059	0.058	0.052	0.052	0.059	0.058	0.044
Merck	0.063	0.051	0.039	0.029	0.034	0.034	0.030	0.033	0.024	0.029
Microsoft	0.180	0.128	0.125	0.116	0.116	0.116	0.105	0.099	0.062	0.078
Nike	0.177	0.087	0.091	0.098	0.093	0.087	0.094	0.102	0.119	0.091
Pfizer	0.097	0.070	0.061	0.057	0.053	0.063	0.049	0.050	0.044	0.050
Procter & Gamble	0.095	0.050	0.044	0.050	0.051	0.040	0.043	0.042	0.031	0.033
Travellers	0.082	0.020	0.006	0.010	0.014	0.006	0.005	0.008	0.017	0.015
United Technologies	0.113	0.071	0.077	0.062	0.072	0.071	0.056	0.061	0.051	0.053
United Health Group	0.252	0.171	0.133	0.130	0.151	0.124	0.134	0.123	0.113	0.087
Verizon	0.043	-0.00	-0.01	0.002	0.006	-0.01	-0.01	-0.01	-0.009	-0.00
Wal-Mart	0.113	0.050	0.049	0.045	0.038	0.028	0.033	0.026	0.038	0.029
Cisco	0.210	0.209	0.211	0.219	0.222	0.219	0.204	0.164	0.120	0.094
Goldman Sachs	0.061	0.050	0.030	0.031	0.040	0.036	0.071	0.089	0.078	0.077
Visa	0.236	0.143	0.142	0.131	0.171	0.167	0.159	0.113	0.119	0.080
Average	0.117	0.073	0.069	0.066	0.067	0.065	0.062	0.061	0.056	0.046
										0.063

**Table 3: Annualized daily (every 20s trading day) returns of MA2-MA10,
average annualized returns**

	Buy and Hold	MA10	MA9	MA8	MA7	MA6	MA5	MA4	MA3	MA2
3M	0.090	0.033	0.035	0.023	0.023	0.024	0.023	0.038	0.021	0.012
American Express	0.094	0.086	0.087	0.091	0.107	0.088	0.062	0.062	0.036	0.038
Apple	0.157	0.057	0.069	0.056	0.076	0.076	0.094	0.069	0.099	0.071
Boeing	0.119	0.122	0.122	0.102	0.099	0.115	0.110	0.100	0.091	0.077
Caterpillar	0.100	0.065	0.062	0.071	0.083	0.081	0.063	0.057	0.009	0.051
Chevron	0.084	0.022	0.021	0.025	0.026	0.019	0.032	0.032	0.013	0.005
Coca-Cola	0.099	0.083	0.072	0.087	0.071	0.073	0.072	0.069	0.046	0.026
Walt Disney	0.103	0.061	0.066	0.073	0.077	0.071	0.079	0.081	0.073	0.057
Exxon	0.072	0.040	0.038	0.028	0.028	0.034	0.020	0.027	0.025	0.026
GE	0.052	0.079	0.078	0.080	0.072	0.070	0.063	0.018	0.038	0.037
Home Depot	0.190	0.126	0.133	0.134	0.136	0.120	0.14	0.119	0.118	0.110
IBM	0.055	0.029	0.033	0.032	0.038	0.036	0.026	0.033	0.026	0.03
Intel	0.134	0.079	0.080	0.096	0.095	0.085	0.063	0.082	0.110	0.116
Johnson & Johnson	0.113	0.078	0.076	0.071	0.059	0.057	0.058	0.050	0.052	0.031
JP Morgan	0.090	0.057	0.051	0.051	0.063	0.046	0.070	0.079	0.067	0.067
McDonalds	0.114	0.077	0.077	0.057	0.055	0.045	0.056	0.042	0.045	0.033
Merck	0.063	0.069	0.069	0.054	0.059	0.05	0.045	0.027	0.011	3E-04
Microsoft	0.180	0.122	0.127	0.123	0.099	0.112	0.093	0.095	0.090	0.108
Nike	0.177	0.128	0.136	0.130	0.127	0.115	0.111	0.109	0.082	0.089
Pfizer	0.097	0.070	0.069	0.067	0.068	0.066	0.068	0.056	0.040	0.034
Procter & Gamble	0.095	0.057	0.060	0.055	0.042	0.043	0.021	0.024	0.038	0.039
Travellers	0.082	0.045	0.049	0.047	0.041	0.034	0.016	0.009	0.002	0.017
United Technologies	0.113	0.064	0.062	0.074	0.078	0.063	0.046	0.037	0.050	0.050
United Health Group	0.252	0.158	0.162	0.167	0.154	0.168	0.176	0.174	0.180	0.158
Verizon	0.043	0.002	9E-04	0.011	0.017	0.025	-0.00	0.01	-0.00	-0.02
Wal-Mart	0.113	0.046	0.046	0.040	0.044	0.032	0.041	0.037	0.023	0.038
Cisco	0.210	0.228	0.227	0.222	0.221	0.191	0.186	0.184	0.160	0.134
Goldman Sachs	0.061	0.029	0.030	0.020	0.052	0.067	0.065	0.070	0.041	0.068
Visa	0.236	0.171	0.161	0.162	0.149	0.122	0.113	0.115	0.142	0.097
Average	0.117	0.079	0.079	0.078	0.078	0.073	0.069	0.066	0.059	0.055
										0.071

Table 4: Annualized daily (every other month) returns of MAD2-MAD2 (D = every other month, and 5,4,3,2 are the numbers of observations in the rolling window), average annualized returns

	Buy&Hold	MAD5	MAD4	MAD3	MAD2
3M	0.090	0.062	0.063	0.042	0.049
American Express	0.094	0.089	0.098	0.052	0.041
Apple	0.157	0.040	0.042	0.030	0.085
Boeing	0.119	0.112	0.110	0.102	0.110
Caterpillar	0.100	0.079	0.09	0.089	0.084
Chevron	0.084	0.033	0.036	0.026	0.028
Coca-Cola	0.099	0.093	0.102	0.080	0.078
Walt Disney	0.103	0.068	0.074	0.080	0.084
Exxon	0.072	0.022	0.018	0.010	0.009
GE	0.052	0.067	0.066	0.041	0.033
Home Depot	0.190	0.174	0.175	0.156	0.160
IBM	0.055	0.016	0.023	0.017	0.021
Intel	0.134	0.093	0.098	0.089	0.112
Johnson & Johnson	0.113	0.083	0.086	0.048	0.071
JP Morgan	0.090	0.053	0.052	0.048	0.054
McDonalds	0.114	0.094	0.098	0.071	0.070
Merck	0.063	0.084	0.067	0.036	0.031
Microsoft	0.180	0.138	0.136	0.106	0.088
Nike	0.177	0.140	0.144	0.133	0.122
Pfizer	0.097	0.062	0.051	0.061	0.059
Procter & Gamble	0.095	0.048	0.054	0.048	0.034
Travellers	0.082	0.018	0.015	0.018	2E-04
United Technologies	0.113	0.066	0.073	0.096	0.060
United Health Group	0.252	0.181	0.179	0.191	0.207
Verizon	0.043	-0.018	-0.01	-0.02	-0.02
Wal-Mart	0.113	0.067	0.065	0.050	0.061
Cisco	0.210	0.217	0.226	0.207	0.196
Goldman Sachs	0.061	0.041	0.059	0.060	0.039
Visa	0.236	0.174	0.173	0.151	0.120
Average	0.117	0.083	0.085	0.073	0.072
					0.078

Table 5: Annualized daily (every third month) returns of MAT2-MAT4 (T = every third month, and 4,3,2 are the numbers of observations in the rolling window), average annualized returns

	Buy&Hold	MAT4	MAT3	MAT2
3M	0.090	0.061	0.055	0.039
American Express	0.094	0.113	0.091	0.066
Apple	0.157	0.089	0.073	0.096
Boeing	0.119	0.127	0.131	0.114
Caterpillar	0.100	0.070	0.069	0.078
Chevron	0.084	0.047	0.053	0.037
Coca-Cola	0.099	0.077	0.078	0.072
Walt Disney	0.103	0.043	0.042	0.068
Exxon	0.072	0.055	0.049	0.037
GE	0.052	0.084	0.080	0.047
Home Depot	0.190	0.161	0.163	0.128
IBM	0.055	0.054	0.048	0.028
Intel	0.134	0.107	0.115	0.072
Johnson & Johnson	0.113	0.094	0.094	0.074
JP Morgan	0.090	0.058	0.076	0.007
McDonalds	0.114	0.080	0.082	0.069
Merck	0.063	0.062	0.062	0.049
Microsoft	0.180	0.127	0.128	0.080
Nike	0.177	0.146	0.151	0.099
Pfizer	0.097	0.078	0.070	0.056
Procter & Gamble	0.095	0.068	0.072	0.076
Travellers	0.082	0.041	0.043	0.025
United Technologies	0.113	0.077	0.089	0.079
United Health Group	0.252	0.147	0.161	0.178
Verizon	0.043	-0.00	-0.00	-0.02
Wal-Mart	0.113	0.081	0.081	0.083
Cisco	0.210	0.211	0.217	0.213
Goldman Sachs	0.061	0.044	0.026	0.030
Visa	0.236	0.183	0.199	0.177
Average	0.117	0.089	0.089	0.075
				0.084

Table 6: Annualized daily (every fourth month) returns of MAQ2-MAQ3
(Q = every fourth month, and 3 and 2 are the numbers of observations
in the rolling window), average annualized returns

	Buy&Hold	MAQ3	MAQ2
3M	0.090	0.056	0.058
American Express	0.094	0.089	0.094
Apple	0.157	0.094	0.094
Boeing	0.119	0.122	0.128
Caterpillar	0.100	0.064	0.084
Chevron	0.084	0.060	0.054
Coca-Cola	0.099	0.083	0.093
Walt Disney	0.103	0.061	0.062
Exxon	0.072	0.056	0.064
GE	0.052	0.069	0.081
Home Depot	0.190	0.152	0.157
IBM	0.055	0.048	0.031
Intel	0.134	0.064	0.070
Johnson & Johnson	0.113	0.080	0.079
JP Morgan	0.090	0.085	0.091
McDonalds	0.114	0.096	0.112
Merck	0.063	0.056	0.061
Microsoft	0.180	0.143	0.145
Nike	0.177	0.181	0.199
Pfizer	0.097	0.059	0.045
Procter & Gamble	0.095	0.073	0.077
Travellers	0.082	0.051	0.051
United Technologies	0.113	0.080	0.077
United Health Group	0.252	0.185	0.218
Verizon	0.043	0.027	0.023
Wal-Mart	0.113	0.087	0.076
Cisco	0.210	0.195	0.180
Goldman Sachs	0.061	0.042	0.056
Visa	0.236	0.195	0.228
Average	0.117	0.091	0.096
			0.094

Table 7: Annualized daily (every fifth month) returns of MAC2 (C = every fifth month, and 2 = observations accounting in the rolling window), average annualized returns

	Buy & Hold	MAC2
3M	0.090	0.076
American Express	0.094	0.088
Apple	0.157	0.132
Boeing	0.119	0.080
Caterpillar	0.100	0.094
Chevron	0.084	0.047
Coca-Cola	0.099	0.094
Walt Disney	0.103	0.044
Exxon	0.072	0.049
GE	0.052	0.048
Home Depot	0.190	0.143
IBM	0.055	0.032
Intel	0.133	0.057
Johnson & Johnson	0.113	0.081
JP Morgan	0.090	0.045
McDonalds	0.114	0.079
Merck	0.063	0.080
Microsoft	0.180	0.094
Nike	0.177	0.141
Pfizer	0.097	0.099
Procter & Gamble	0.095	0.039
Travellers	0.082	0.068
United Technologies	0.113	0.056
United Health Group	0.252	0.152
Verizon	0.043	0.048
Wal-Mart	0.113	0.093
Cisco	0.210	0.225
Goldman Sachs	0.061	0.053
Visa	0.236	0.217
Average	0.117	0.088

Appendix 2

Table 8: Annualized daily volatility of MA40-MA200, average annualized volatility

	Buy & Hold	MA200	MA180	MA160	MA140	MA120	MA100	MA80	MA60	MA40
3M	0.225	0.164	0.165	0.161	0.161	0.159	0.159	0.158	0.158	0.157
American Express	0.345	0.227	0.228	0.221	0.225	0.224	0.225	0.224	0.228	0.229
Apple	0.451	0.317	0.321	0.315	0.315	0.313	0.315	0.315	0.310	0.305
Boeing	0.294	0.201	0.203	0.199	0.201	0.199	0.198	0.198	0.201	0.204
Caterpillar	0.311	0.216	0.218	0.216	0.216	0.214	0.215	0.214	0.213	0.215
Chevron	0.244	0.167	0.168	0.166	0.166	0.166	0.165	0.164	0.167	0.168
Coca-Cola	0.225	0.164	0.166	0.161	0.160	0.159	0.158	0.158	0.156	0.155
Walt Disney	0.291	0.196	0.201	0.199	0.200	0.199	0.198	0.203	0.204	0.203
Exxon	0.230	0.162	0.163	0.159	0.159	0.159	0.157	0.156	0.155	0.157
GE	0.275	0.174	0.175	0.172	0.173	0.173	0.171	0.168	0.168	0.168
Home Depot	0.314	0.226	0.228	0.223	0.221	0.221	0.219	0.217	0.217	0.214
IBM	0.271	0.187	0.189	0.185	0.184	0.181	0.179	0.177	0.176	0.174
Intel	0.382	0.273	0.275	0.267	0.265	0.263	0.260	0.257	0.256	0.254
Johnson & Johnson	0.215	0.163	0.164	0.161	0.159	0.157	0.155	0.153	0.152	0.149
JP Morgan	0.375	0.223	0.226	0.223	0.224	0.227	0.237	0.242	0.245	0.248
McDonalds	0.240	0.183	0.184	0.18	0.178	0.177	0.176	0.176	0.175	0.174
Merck	0.269	0.177	0.179	0.173	0.173	0.174	0.172	0.174	0.174	0.177
Microsoft	0.323	0.248	0.249	0.243	0.241	0.237	0.236	0.233	0.232	0.231
Nike	0.327	0.243	0.245	0.238	0.236	0.235	0.235	0.232	0.232	0.233
Pfizer	0.266	0.188	0.19	0.187	0.186	0.187	0.186	0.187	0.187	0.187
Procter & Gamble	0.225	0.169	0.169	0.164	0.163	0.161	0.158	0.157	0.156	0.156
Travellers	0.268	0.174	0.175	0.174	0.175	0.178	0.180	0.184	0.182	0.185
United Technologies	0.261	0.179	0.181	0.179	0.178	0.177	0.177	0.177	0.176	0.173
United Health Group	0.386	0.290	0.293	0.290	0.290	0.283	0.282	0.282	0.280	0.273
Verizon	0.246	0.163	0.165	0.164	0.163	0.163	0.163	0.161	0.161	0.163
Wal-Mart	0.263	0.203	0.204	0.200	0.198	0.195	0.191	0.19	0.189	0.191
Cisco	0.415	0.300	0.302	0.297	0.295	0.291	0.290	0.285	0.282	0.275
Goldman Sachs	0.373	0.222	0.226	0.22	0.222	0.223	0.228	0.230	0.227	0.229
Visa	0.260	0.209	0.212	0.209	0.208	0.212	0.208	0.206	0.205	0.197
Average	0.295	0.207	0.209	0.205	0.205	0.204	0.203	0.203	0.202	0.204

Table 9: Annualized daily (every fifth trading day) volatility of MAW8-MAW40
(W = number of weeks), average annualized volatility

	Buy&Hold	MAW40	MAW36	MAW32	MAW28	MAW24	MAW20	MAW16	MAW12	MAW8
3M	0.225	0.165	0.165	0.163	0.163	0.16	0.159	0.157	0.157	0.159
American Express	0.345	0.227	0.224	0.224	0.227	0.225	0.223	0.228	0.232	0.234
Apple	0.451	0.316	0.316	0.313	0.318	0.316	0.343	0.317	0.312	0.309
Boeing	0.294	0.204	0.203	0.204	0.204	0.203	0.203	0.201	0.201	0.206
Caterpillar	0.311	0.216	0.215	0.215	0.217	0.214	0.215	0.215	0.213	0.214
Chevron	0.244	0.169	0.168	0.169	0.168	0.168	0.167	0.166	0.168	0.172
Coca-Cola	0.225	0.165	0.165	0.164	0.162	0.160	0.159	0.159	0.157	0.155
Walt Disney	0.291	0.195	0.198	0.197	0.197	0.199	0.200	0.202	0.203	0.204
Exxon	0.230	0.163	0.161	0.160	0.161	0.160	0.157	0.156	0.153	0.158
GE	0.275	0.174	0.174	0.174	0.175	0.174	0.170	0.169	0.171	0.166
Home Depot	0.314	0.228	0.228	0.226	0.225	0.222	0.224	0.219	0.219	0.214
IBM	0.271	0.190	0.188	0.185	0.184	0.183	0.178	0.177	0.178	0.177
Intel	0.382	0.267	0.267	0.268	0.264	0.263	0.259	0.256	0.259	0.259
Johnson & Johnson	0.215	0.164	0.163	0.162	0.160	0.158	0.156	0.156	0.152	0.15
JP Morgan	0.375	0.222	0.225	0.224	0.230	0.236	0.239	0.243	0.241	0.252
McDonalds	0.240	0.185	0.182	0.181	0.179	0.177	0.177	0.176	0.174	0.171
Merck	0.269	0.179	0.175	0.174	0.173	0.173	0.172	0.175	0.176	0.175
Microsoft	0.323	0.250	0.247	0.245	0.244	0.24	0.236	0.236	0.230	0.232
Nike	0.327	0.244	0.241	0.239	0.240	0.241	0.238	0.235	0.232	0.232
Pfizer	0.266	0.189	0.187	0.186	0.187	0.188	0.190	0.189	0.189	0.184
Procter & Gamble	0.225	0.170	0.168	0.167	0.165	0.164	0.161	0.158	0.160	0.156
Travellers	0.268	0.175	0.175	0.175	0.178	0.177	0.177	0.184	0.184	0.185
United Technologies	0.261	0.181	0.179	0.178	0.177	0.177	0.177	0.177	0.176	0.172
United Health Group	0.386	0.292	0.291	0.292	0.291	0.290	0.289	0.287	0.282	0.278
Verizon	0.246	0.163	0.162	0.162	0.162	0.164	0.162	0.161	0.160	0.159
Wal-Mart	0.263	0.205	0.202	0.201	0.198	0.194	0.191	0.191	0.190	0.192
Cisco	0.415	0.307	0.305	0.300	0.296	0.292	0.293	0.288	0.285	0.281
Goldman Sachs	0.373	0.225	0.223	0.221	0.221	0.220	0.230	0.233	0.241	0.241
Visa	0.260	0.203	0.210	0.209	0.208	0.210	0.208	0.206	0.203	0.195
Average	0.295	0.208	0.207	0.206	0.206	0.205	0.205	0.204	0.203	0.203

Table 10: Annualized daily (rule in every 20s trading day) volatility of MA2-MA10, average annualized volatility

	Buy and									
	Hold	MA10	MA9	MA8	MA7	MA6	MA5	MA4	MA3	MA2
3M	0.225	0.167	0.169	0.162	0.163	0.161	0.161	0.157	0.156	0.156
American Express	0.345	0.232	0.235	0.222	0.218	0.22	0.219	0.22	0.243	0.235
Apple	0.451	0.343	0.347	0.342	0.339	0.339	0.338	0.342	0.335	0.331
Boeing	0.294	0.207	0.210	0.202	0.202	0.199	0.200	0.197	0.207	0.205
Caterpillar	0.311	0.216	0.220	0.217	0.215	0.214	0.217	0.218	0.221	0.224
Chevron	0.244	0.169	0.171	0.172	0.17	0.169	0.169	0.167	0.181	0.171
Coca-Cola	0.225	0.168	0.171	0.169	0.168	0.166	0.161	0.161	0.161	0.156
Walt Disney	0.291	0.203	0.207	0.202	0.203	0.203	0.210	0.212	0.215	0.211
Exxon	0.230	0.166	0.167	0.165	0.164	0.163	0.162	0.157	0.161	0.160
GE	0.275	0.177	0.177	0.175	0.175	0.175	0.172	0.169	0.172	0.180
Home Depot	0.314	0.234	0.235	0.228	0.221	0.230	0.228	0.233	0.225	0.219
IBM	0.271	0.194	0.196	0.196	0.196	0.196	0.19	0.194	0.195	0.190
Intel	0.382	0.273	0.277	0.272	0.272	0.268	0.266	0.266	0.264	0.259
Johnson & Johnson	0.215	0.168	0.169	0.167	0.167	0.162	0.158	0.158	0.154	0.150
JP Morgan	0.375	0.222	0.223	0.217	0.220	0.230	0.233	0.234	0.244	0.234
McDonalds	0.240	0.189	0.189	0.186	0.185	0.185	0.179	0.170	0.171	0.180
Merck	0.269	0.177	0.178	0.173	0.173	0.174	0.173	0.181	0.182	0.192
Microsoft	0.323	0.250	0.251	0.247	0.239	0.233	0.235	0.237	0.233	0.234
Nike	0.327	0.247	0.248	0.244	0.241	0.240	0.235	0.236	0.238	0.248
Pfizer	0.266	0.188	0.190	0.186	0.186	0.186	0.187	0.187	0.191	0.189
Procter & Gamble	0.225	0.173	0.174	0.171	0.167	0.165	0.163	0.164	0.158	0.155
Travellers	0.268	0.171	0.172	0.17	0.169	0.171	0.191	0.186	0.192	0.198
United Technologies	0.261	0.178	0.179	0.178	0.177	0.177	0.175	0.178	0.176	0.173
United Health Group	0.386	0.300	0.302	0.299	0.298	0.294	0.289	0.280	0.283	0.275
Verizon	0.246	0.167	0.167	0.164	0.162	0.160	0.164	0.157	0.160	0.163
Wal-Mart	0.263	0.208	0.210	0.205	0.199	0.196	0.197	0.198	0.198	0.189
Cisco	0.415	0.304	0.307	0.301	0.298	0.300	0.292	0.290	0.281	0.278
Goldman Sachs	0.373	0.230	0.232	0.225	0.232	0.245	0.239	0.253	0.268	0.256
Visa	0.260	0.204	0.203	0.212	0.225	0.221	0.219	0.217	0.217	0.196
Average	0.295	0.211	0.213	0.209	0.208	0.208	0.208	0.208	0.210	0.207
										0.209

Table 11: Annualized daily (every other month) volatility of MAD2-MAD2 (D = every other month, and 5,4,3,2 are the numbers of observations in the rolling window), average annualized volatility

	Buy&Hold	MAD5	MAD4	MAD3	MAD2
3M	0.225	0.168	0.169	0.162	0.159
American Express	0.344	0.222	0.226	0.216	0.211
Apple	0.450	0.351	0.363	0.357	0.338
Boeing	0.294	0.210	0.216	0.211	0.208
Caterpillar	0.311	0.218	0.229	0.215	0.211
Chevron	0.244	0.168	0.175	0.166	0.165
Coca-Cola	0.225	0.168	0.173	0.165	0.158
Walt Disney	0.291	0.197	0.200	0.198	0.203
Exxon	0.230	0.172	0.174	0.159	0.156
GE	0.274	0.175	0.181	0.176	0.182
Home Depot	0.314	0.229	0.230	0.221	0.237
IBM	0.271	0.196	0.199	0.200	0.200
Intel	0.382	0.274	0.286	0.267	0.265
Johnson & Johnson	0.215	0.173	0.175	0.165	0.154
JP Morgan	0.375	0.236	0.241	0.246	0.237
McDonalds	0.240	0.182	0.186	0.178	0.169
Merck	0.269	0.185	0.196	0.188	0.199
Microsoft	0.323	0.245	0.249	0.238	0.250
Nike	0.327	0.252	0.258	0.253	0.253
Pfizer	0.266	0.199	0.203	0.191	0.189
Procter & Gamble	0.225	0.173	0.177	0.169	0.166
Travellers	0.268	0.176	0.178	0.183	0.191
United Technologies	0.261	0.182	0.187	0.178	0.177
United Health Group	0.386	0.313	0.313	0.299	0.305
Verizon	0.246	0.163	0.171	0.165	0.153
Wal-Mart	0.263	0.197	0.199	0.194	0.193
Cisco	0.415	0.312	0.317	0.315	0.285
Goldman Sachs	0.373	0.229	0.245	0.239	0.265
Visa	0.260	0.215	0.215	0.225	0.222
Average	0.295	0.213	0.218	0.212	0.210
					0.213

Table 12: Annualized daily (every third month) volatility of MAT2-MAT4 (T = every third month, and 4,3,2 are the numbers of observations in the rolling window), average annualized volatility

	Buy&Hold	MAT4	MAT3	MAT2
3M	0.225	0.172	0.174	0.171
American Express	0.344	0.230	0.237	0.206
Apple	0.450	0.345	0.357	0.349
Boeing	0.294	0.206	0.219	0.200
Caterpillar	0.311	0.219	0.223	0.214
Chevron	0.244	0.176	0.182	0.170
Coca-Cola	0.225	0.177	0.179	0.181
Walt Disney	0.291	0.220	0.228	0.205
Exxon	0.230	0.168	0.176	0.158
GE	0.274	0.178	0.185	0.177
Home Depot	0.314	0.236	0.251	0.241
IBM	0.271	0.205	0.209	0.193
Intel	0.382	0.285	0.296	0.274
Johnson & Johnson	0.215	0.185	0.188	0.165
JP Morgan	0.375	0.242	0.248	0.240
McDonalds	0.240	0.198	0.204	0.192
Merck	0.269	0.191	0.191	0.180
Microsoft	0.323	0.257	0.267	0.258
Nike	0.327	0.264	0.265	0.258
Pfizer	0.266	0.195	0.206	0.208
Procter & Gamble	0.225	0.177	0.181	0.168
Travellers	0.268	0.187	0.188	0.198
United Technologies	0.261	0.192	0.199	0.187
United Health Group	0.386	0.300	0.308	0.315
Verizon	0.246	0.176	0.176	0.160
Wal-Mart	0.263	0.202	0.208	0.208
Cisco	0.415	0.310	0.311	0.303
Goldman Sachs	0.373	0.226	0.232	0.235
Visa	0.260	0.204	0.215	0.208
Average	0.295	0.218	0.224	0.214
				0.219

Table 13: Annualized daily (every fourth month) volatility of MAQ2-MAQ3 (Q = every fourth month, 3 and 2 are the number of observations in the rolling window), average annualized volatility

	Buy&Hold	MAQ3	MAQ3
3M	0.225	0.168	0.176
American Express	0.344	0.220	0.226
Apple	0.450	0.360	0.373
Boeing	0.294	0.213	0.224
Caterpillar	0.311	0.222	0.239
Chevron	0.244	0.167	0.177
Coca-Cola	0.225	0.173	0.182
Walt Disney	0.291	0.206	0.218
Exxon	0.230	0.160	0.176
GE	0.274	0.180	0.195
Home Depot	0.314	0.237	0.242
IBM	0.271	0.194	0.218
Intel	0.382	0.274	0.293
Johnson & Johnson	0.215	0.181	0.186
JP Morgan	0.375	0.218	0.227
McDonalds	0.240	0.177	0.193
Merck	0.269	0.204	0.212
Microsoft	0.323	0.248	0.260
Nike	0.327	0.258	0.265
Pfizer	0.266	0.198	0.207
Procter & Gamble	0.225	0.173	0.174
Travellers	0.268	0.182	0.192
United Technologies	0.261	0.181	0.188
United Health Group	0.386	0.299	0.314
Verizon	0.246	0.167	0.177
Wal-Mart	0.263	0.194	0.207
Cisco	0.415	0.341	0.349
Goldman Sachs	0.373	0.240	0.260
Visa	0.260	0.212	0.225
Average	0.295	0.215	0.227
			0.221

Table 14: Annualized daily (every fifth month) volatility of MAC2 (C = every fifth month, 2 = observations in rolling window), average annualized volatility

	Buy & Hold	MAC2
3M	0.225	0.176
American Express	0.344	0.226
Apple	0.450	0.323
Boeing	0.294	0.218
Caterpillar	0.311	0.227
Chevron	0.244	0.165
Coca-Cola	0.225	0.168
Walt Disney	0.291	0.206
Exxon	0.230	0.166
GE	0.274	0.187
Home Depot	0.314	0.242
IBM	0.271	0.202
Intel	0.382	0.296
Johnson & Johnson	0.215	0.187
JP Morgan	0.375	0.244
McDonalds	0.240	0.182
Merck	0.269	0.194
Microsoft	0.323	0.250
Nike	0.327	0.249
Pfizer	0.266	0.191
Procter & Gamble	0.225	0.187
Travellers	0.268	0.183
United Technologies	0.261	0.204
United Health Group	0.386	0.298
Verizon	0.246	0.170
Wal-Mart	0.263	0.223
Cisco	0.415	0.333
Goldman Sachs	0.373	0.218
Visa	0.260	0.220
Average	0.295	0.218

Appendix 3

Table 15: Transaction costs per year of MA40-MA200, with one transaction costing 0.1% of total wealth, average annualized transaction costs

	MA200	MA180	MA160	MA140	MA120	MA100	MA80	MA60	MA40
3M	0.010	0.011	0.010	0.011	0.012	0.013	0.016	0.019	0.022
American Express	0.011	0.011	0.011	0.012	0.012	0.013	0.016	0.017	0.023
Apple	0.007	0.008	0.008	0.009	0.010	0.012	0.014	0.015	0.020
Boeing	0.008	0.009	0.010	0.011	0.011	0.012	0.014	0.015	0.020
Caterpillar	0.008	0.009	0.010	0.011	0.012	0.013	0.015	0.015	0.019
Chevron	0.011	0.012	0.012	0.013	0.014	0.016	0.018	0.020	0.024
Coca-Cola	0.009	0.010	0.011	0.011	0.011	0.012	0.015	0.018	0.022
Walt Disney	0.007	0.008	0.009	0.011	0.012	0.012	0.013	0.017	0.021
Exxon	0.011	0.013	0.016	0.017	0.017	0.018	0.019	0.023	0.028
GE	0.007	0.008	0.009	0.010	0.011	0.012	0.014	0.017	0.023
Home Depot	0.008	0.009	0.010	0.011	0.013	0.014	0.016	0.018	0.021
IBM	0.009	0.010	0.010	0.010	0.012	0.012	0.013	0.014	0.019
Intel	0.007	0.009	0.010	0.010	0.012	0.014	0.014	0.016	0.019
Johnson & Johnson	0.009	0.008	0.009	0.010	0.012	0.014	0.016	0.020	0.024
JP Morgan	0.010	0.010	0.011	0.012	0.012	0.014	0.015	0.016	0.020
McDonalds	0.010	0.011	0.011	0.013	0.012	0.014	0.016	0.018	0.023
Merck	0.008	0.009	0.009	0.011	0.011	0.013	0.015	0.017	0.022
Microsoft	0.008	0.009	0.010	0.010	0.011	0.013	0.015	0.015	0.020
Nike	0.009	0.009	0.010	0.010	0.011	0.012	0.013	0.014	0.019
Pfizer	0.008	0.010	0.010	0.011	0.011	0.012	0.014	0.017	0.021
Procter & Gamble	0.010	0.010	0.010	0.011	0.012	0.014	0.016	0.019	0.022
Travellers	0.010	0.011	0.012	0.012	0.013	0.015	0.016	0.018	0.024
United Technologies	0.009	0.010	0.011	0.011	0.012	0.014	0.015	0.018	0.021
United Health Group	0.008	0.008	0.010	0.010	0.011	0.012	0.014	0.017	0.021
Verizon	0.011	0.011	0.011	0.011	0.013	0.014	0.017	0.018	0.023
Wal-Mart	0.010	0.010	0.012	0.013	0.013	0.014	0.015	0.019	0.022
Cisco	0.006	0.006	0.008	0.010	0.009	0.010	0.014	0.017	0.023
Goldman Sachs	0.008	0.010	0.012	0.012	0.014	0.015	0.022	0.026	0.035
Visa	0.008	0.008	0.009	0.009	0.008	0.010	0.011	0.014	0.022
Average	0.009	0.0010	0.010	0.011	0.012	0.013	0.015	0.018	0.022
									0.013

Table 16: Transaction costs per year of MA2-MA10, average annualized transaction costs

	MA10	MA9	MA8	MA7	MA6	MA5	MA4	MA3	MA2
3M	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006
American Express	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.006
Apple	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006
Boeing	0.002	0.002	0.002	0.002	0.002	0.003	0.004	0.004	0.006
Caterpillar	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006
Chevron	0.002	0.003	0.003	0.003	0.003	0.003	0.004	0.005	0.007
Coca-Cola	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.006
Walt Disney	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.006
Exxon	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.005	0.006
GE	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004	0.006
Home Depot	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.006
IBM	0.003	0.002	0.003	0.002	0.003	0.003	0.004	0.004	0.006
Intel	0.002	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.006
Johnson & Johnson	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006
JP Morgan	0.002	0.003	0.003	0.003	0.003	0.003	0.003	0.004	0.006
McDonalds	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.005	0.006
Merck	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.005	0.006
Microsoft	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.006
Nike	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004	0.006
Pfizer	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.006
Procter & Gamble	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.005	0.006
Travellers	0.003	0.002	0.003	0.003	0.003	0.004	0.004	0.005	0.007
United Technologies	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004	0.006
United Health Group	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.006
Verizon	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006
Wal-Mart	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006
Cisco	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.005	0.006
Goldman Sachs	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.005
Visa	0.002	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.005
Average	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.006
									0.003