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Improved Analysis and Modelling of Leakage Inductance for Planar Transformers

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Abstract

Planar transformers have often been mistaken to essentially have lower leakage inductances. The "radial effect" is a nature characteristic for planar windings due to a higher aspect ratio of conductor width to conductor thickness, which gives a reduction of leakage inductance. Traditional formulas of leakage inductance calculation for traditional transformers where the winding width is much smaller than the winding height, $b<<h$, are not well suitable for planar transformers ($b>>h$). This paper specifically tailors the traditional 1-D solution of leakage inductance calculation by decomposing the leakage flux into longitudinal and transversal flux, and thus the "eddy current effect" and the "radial effect" in leakage inductance can be analyzed individually. The proposed new formula including both ac (high frequency eddy current effect) and dc effects (radial effect) gives an accurate prediction of leakage inductance in planar transformers. Finite Element Analysis (FEA) simulation and measurements are carried out to validate the proposed formula.

1. Introduction

Planar magnetics have gained the popularity in recent years because they have several advantages such as low profile, modularity, good thermal characteristics, ease of manufacture, and predictable parasitics etc. [1]-[9]. In traditional wire-wound components, it is very difficult to control the winding layout, which means variations in leakage inductance and winding capacitance appear in devices manufactured at the same time. With planar magnetics, the windings manufactured by PCB machines are more precise and consistent, yielding magnetic designs with highly controllable and predictable parasitic parameters [5]. Today's new technology including wide bandgap devices (such as GaN SiC semiconductors) and advanced magnetic materials (such as Hitachi ML91S and TDK PC200) pushes the switching frequency into the MHz range, to minimize the size of the inductive components, making the switching loss more severe. PCB windings with enclosed planar magnetic cores have been widely used in such high-frequency and high-power-density [6]-[9]. Therefore, an accurate prediction of leakage inductance for planar transformers is needed in order to implement an optimized in term of high efficiency. Planar transformers have also been studied in resonant converters, such as LLC converters, where the leakage inductance is critical for a well-matched resonant frequency [10]-[14].

Many significant works have been done to understand, characterize, and model the planar transformers [15]-[28]. Most of the efforts are oriented to obtain winding loss based on modifications of the Dowell's equation in [15]. Leakage inductance analytical calculations, nowadays, mainly rely on the works of Dowell, its modified forms [16], which have been since several decades and in the main applied to traditional transformer structures. A new set of formulas for calculating self and mutual impedances of planar coil on homogenous ferromagnetic substrates was proposed in [21]. These formulas are well suitable for planar air thick film and thin film cases. But it is difficult to apply these results to planar PCB transformers. 1-D magnetic component model for planar structures based on transmission lines has been built in [24], [25], and an important characteristic "radial effect" for planar transformers has been reported as well. A general 2-D analytical method which is based on the partial element equivalent circuit (PEEC) method was proposed in [30], [31] to compute the window field, and thus to value leakage inductances of planar transformers. Publications [32] and [33] studied the leakage inductance influenced by the high frequency eddy current effect, and accurate expressions of frequency-dependent leakage inductance were proposed. However, these expressions can only be applied in traditional transformer structures where the "radial effect" of planar transformer was not taken into account.

Planar transformers have often been mistaken to essentially have lower leakage inductances. In leakage inductance is proportional with the mean turn length (MTL) of the conductors. Planar transformers intrinsically have a longer mean turn length than that of the traditional vertical resulting in a higher leakage inductance [1]. However, the important benefit of planar transformers in this regard is the relative ease which primary and secondary windings can be heavily interleaved in a manufacturing or automation environment. Dowell and many of its modified forms for calculating the leakage inductance has been widely applied in traditional wire –wound transformers. However, significant errors have been observed when they are applied to planar transformers. The errors are due to the following reasons: 1) the traditional analysis only express the leakage inductance at low frequency (with frequency independency), and the high frequency eddy current effect was not taken into account; 2) the traditional analytical expressions ignores the insulators' thickness in between layer to layer because they are so small compared to the thickness of conductor layers. However, PCB windings usually have much thicker dielectric layers (>0.2mm) than the copper thickness. For this reason, the error of the traditional analytical expression can reach to ~30% or even more [34], depending on the dimensions of the layout; 3) the "radial effect" was not taken into account. Planar transformer windings situated in a horizontal typically have an extremely high aspect ratio of width to height of a section, causing the currents substantially concentrate the inner edge, and thus impact on the amplitude of magnetic field tangential to the surface of conductors. The contribution of this paper is to propose a new analytical expression of leakage inductance in planar transformers covering all above 3 considerations especially for the "radial effect". The "radial effect" has been reported in planar transformer modelling before, but it is rare to link with frequency-dependent leakage inductance calculation. This paper specifically tailors the traditional 1-D solution of leakage inductance calculation by decomposing the leakage flux into longitudinal and transversal flux, and thus the "eddy current effect" and the "radial effect" in leakage inductance can be analyzed individually. The proposed new formula including both ac frequency eddy current effect) and dc effects (radial effect) gives an accurate prediction of leakage inductance in planar transformers.

2. Improved Calculation of Leakage Inductance for Planar Transformer

As shown in Fig. 1, some flux will leak from the core and return to the air, winding layers and insulator layers, causing imperfect coupling to the secondary winding. If the secondary is short-short-circuited, the main flux in the core which both windings will be negligible because the primary and secondary ampere turns almost cancel. This means only the leakage energy is retained. Assuming that the permeability of core material is infinite ($\mu_r \to \infty$, $\sigma \to 0$), the magnetic field intensity goes to zero inside the core. The magnetic core window height h_w is supposed to much smaller than the core window width b_w, and the conductors are fully filled in the core window width, so that end effects can be taken as negligible. The basic principle to obtain the leakage inductance is to calculate the energy retained in the core window area. Therefore, the magnetic field strength accommodated in the conductors and dielectric insulators become very important.

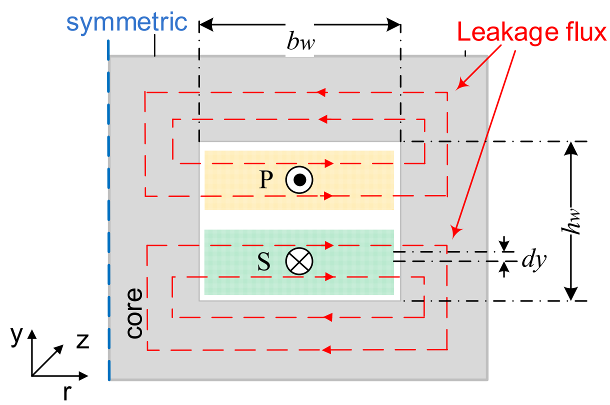

Fig.1 Leakage flux path across half-core window area of a planar transformer.

2.1. Radial Effect

Planar transformer windings situated in a horizontal plane typically have an extremely high aspect ratio of width (b_W) to height (h_w) of the conductor cross-section, with the result of the current is concentrated in the inner path. A typical planar core with rounded center leg is used as an example to calculate the leakage inductance. In any winding conductors, the shorter path on the inside edge (at $r = r_1$ in Fig. 2) of the conducting section means that the resistance to current flow is lower and, therefore, the current density is higher on the inside than on the outside. On the basis of this observation, it is reasonable to assume that there is an inverse relationship between the dc current density $J(r)$ and the radius r [35]:

$$h\int_{r_1}^{r_2} J(r)\cdot dr = h\int_{r_1}^{r_2} \frac{k}{r}\cdot dr = I \qquad (1)$$

where h is the conductor thickness, r_1, r_2 are the distances from the center to the inner edge and the outer edge of the core window respectively. The constant k can be obtained from (1):

$$k = \frac{I}{h\cdot In(\frac{r_2}{r_1})} \qquad (2)$$

Therefore, the dc current density $J(r)$ can be expressed:

$$J(r) = \frac{I}{r\cdot h\cdot In(\frac{r_2}{r_1})} \qquad (3)$$

This relationship has been verified by Finite Element Analysis (FEA) simulation as shown in Fig. 6.

2.2 High Frequency Eddy Current Effect

We often observe from the measurement that the leakage inductance is reduced when the frequency increases. This is mainly due to the that the high frequency effect concentrates the current on the surface of the conductors, and thus equivalently reduces the thickness of the conductors in which the part of leakage energy is stored. As shown in Fig. 3, the magnetic field is longer linearly distributed within the conductors at high frequency because of the eddy current The area of the MMF curve at high frequency is smaller than the one at low frequency, which means the stored leakage energy is actually at high frequency.

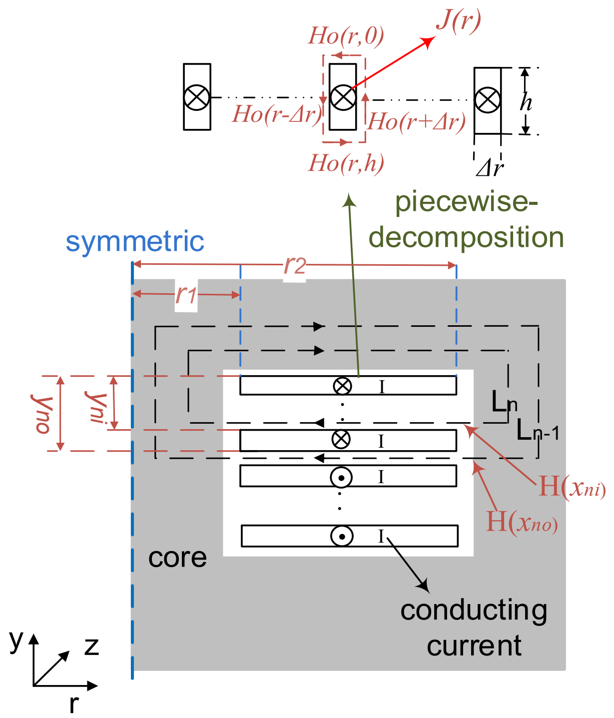

Fig. 2. Inner and outer boundary magnetic field for the n^{th} layer.

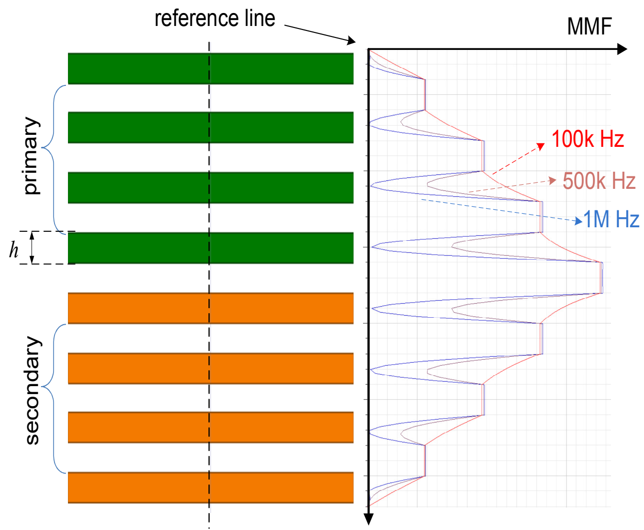

Fig. 3. MMF distribution of planar transformer windings at high frequency (FEA simulation).

Authors' previous work [33] gave a concrete analysis in connection to frequency-dependent leakage inductance particularly for traditional transformer structures. The Maxwell's equations can be rewritten in Cartesian coordinates as a second-order ordinary differential equation, named Helmholtz differential equation:

$$\frac{d^2 H_r}{dy^2} = j\omega\mu_0\sigma H_r \tag{4}$$

The general solution of the Helmholtz equation is given by,

$$H_r(y) = H_1 e^{\gamma y} + H_2 e^{-\gamma y} \tag{5}$$

where H_1 and H_2 are determined by the boundary conditions and complex constants. The complex propagation constant is:

$$\gamma = \sqrt{j\omega\mu_0\sigma} = \frac{1+j}{\delta_w} \tag{6}$$

and δ_w is the skin depth and $\delta_w = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$.

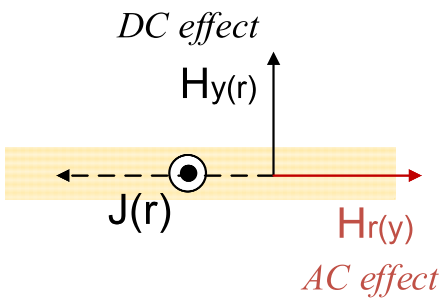

Fig. 4. The magnetic fields due to AC effect and DC effect are decoupled.

However, this analysis only considers the magnetic field along the window height (*y*-component that is parallel to the surface of conductors). This part of flux is the main contribution to the high frequency eddy current (AC) effect. The DC current distribution *J(r)* in (3) due to the radial effect induces a magnetic field only having *r*-component, $H_y(r)$, (changing along with *r* axis). As seen in Fig. 4, the two magnetic fields are orthogonal and decoupled, so that they can be treated individually. How to add the "radial effect" of planar transformers into the previous analysis is the key contribution of this paper. To apply the *r*-component magnetic field $H_y(r)$ into the Helmholtz differential equation, the piecewise decomposition approach is needed to ensure a uniform current distribution in each piece.

2.3 Piecewise Decomposition

The first conduction layer can be decomposed many small segments with the width Δr ($\Delta r \to 0$), as shown in Fig. 2. Taking Ampere's law at radius as shown:

$$J(r) \cdot \Delta r \cdot h = [H_o(r,0) + H_o(r,h)] \cdot \Delta r + [H_o(r-\Delta r) + H_o(r+\Delta r)] \cdot h \tag{7}$$

where the magnetic field intensity in the boundary between the first layer and the high permeability core material $H_o(r,0) = 0$; and $H_o(r-\Delta r) = -H_o(r+\Delta r)$ due to the condition of $\Delta r \to 0$. Thus (7) can be simplified,

$$H_o(r,h) = J(r) \cdot h = \frac{I}{r \cdot In(\frac{r_2}{r_1})} \tag{8}$$

Invoking Ampere's law for the closed loops L_n and L_{n-1} (see in Fig. 2) in a high permeability core, the upper and lower boundary conditions for the n^{th} layer of the winding are obtained,

$$H(r, y_{ni}) = (n-1) \cdot H_o(r,h) \tag{9}$$

$$H(r, y_{no}) = n \cdot H_o(r,h) \tag{10}$$

and y_{ni} and y_{no} are the distance from the upper surface of the first conductor to the upper surface and the lower surface of the n^{th} layer respectively. Applying the boundary conditions (9) and (10) into the Helmholtz equation (5), the magnetic field inside the n^{th} layer is then obtained:

$$H(r,y) = H_o(r,h) \cdot \frac{n\sinh(\gamma y) + (n-1)\sinh(\gamma h - \gamma y)}{\sinh(\gamma h)} \tag{11}$$

2.4 Leakage Energy in Each Element

With the consideration of AC and DC effects described above, the difference to authors' previous works [33], [34] is that the double integrals are used to calculate the leakage energy stored in each element. The differential volume of each element is $2\pi r \cdot dr \cdot dy$, therefore the stored in each layer is:

$$E_i = \frac{1}{2} \cdot \mu_0 \cdot \int_0^h \int_{r_1}^{r_2} H(r,y)^2 \cdot 2\pi r \cdot dr \cdot dy \tag{12}$$

Applying (11) into (12), the energy stored in the primary winding is:

$$E_p = \sum_{i=1}^{n_p} E_i = \frac{\mu_0 \cdot \pi \cdot I_p{}^2 \cdot n_p\left[k_1\left(2n_p{}^2+1\right)+4k_2\left(n_p{}^2-1\right)\right]}{In(\frac{r_2}{r_1}) \cdot 12 \cdot \gamma \sinh^2(\gamma h_p)} \quad (13)$$

where,

$$k_1 = \sinh(2\gamma h_p) - 2\gamma h_p$$
$$k_2 = \gamma h_p \cosh\left(\gamma \cdot h_p\right) - \sinh\left(\gamma \cdot h_p\right)$$

The same approach can be applied to secondary winding. The expression for the leakage energy stored in the secondary winding E_s is similar with equ.(13), with changing all parameters to be secondary windings'.

The dielectric layers are placed between each conducting layer, and the magnetic field inside the n^{th} dielectric layer remains constant and is equal to the magnetic field of the lower surface of the n^{th} conductor layer. The energy stored in the dielectric layer is, therefore, obtained:

$$E_d = \frac{1}{2} \cdot \mu_0 \cdot h_i \cdot \int_{r_1}^{r_2} H(r,h)^2 \cdot 2\pi r \cdot dr = \frac{\mu_0 \cdot \pi \cdot h_i}{In\left(\frac{r_2}{r_1}\right)}\left[I_p{}^2 \sum_{i=1}^{n_p} i^2 + I_s{}^2 \sum_{i=1}^{n_s-1} i^2\right] = \frac{\mu_0 \cdot \pi \cdot h_i}{6In\left(\frac{r_2}{r_1}\right)}\left[I_p{}^2 \cdot n_p\left(n_p+1\right)\left(2n_p+1\right) + I_s{}^2 \cdot n_s(n_s-1)(2n_s-1)\right] \quad (14)$$

where h_p, h_s and h_i are the thickness of primary winding layers, secondary winding layers, and dielectric layer, respectively. The total leakage energy is sum of the energy stored in each elementary layer which can be expressed by:

$$E_{total} = E_p + E_s + E_d \quad (15)$$

Giving an example that has only one turn in each layer, the turns ratio $n = \frac{n_s}{n_p}$ is defined, and the thickness of all winding conductors are kept the same ($h_p = h_s$), then the total leakage inductance is:

$$L_{lk} = \frac{\mu_0 \cdot \pi \cdot n_p}{3In(\frac{r_2}{r_1})}\left\{\frac{n_p{}^2(k_1+2k_2)(n+1)}{\gamma \sinh^2\left(\gamma h_p\right)} + \frac{(k_1-4k_2)(n+1)}{2n\gamma \sinh^2\left(\gamma h_p\right)} + \left[2(1+n) \cdot n_p{}^2 + \frac{1}{n} + 1\right] h_i\right\} \quad (16)$$

The (16) can only be applied in the transformer that has one turn in each layer, but with the same approach, the case with multiple turns in each layer can be also calculated. It is noted that more turns printed in each layer, less effect from the dc current distribution.

3. FEA Simulations and Experimental Verifications

An example of the planar magnetic core ER 51/10/38 was selected to experimentally validate the correctness of the proposed calculation. The parameters of the planar transformer are shown in Table I. The actual PCB windings shown in Fig. 5 are made in consistency with these parameters. Both primary winding and secondary winding have 8 turns with one turn in one layer. The FEA simulation is carried out by ANSYS MAXWELL, and the transformer model is built in 2D plane under the "eddy current" type.

Table I Transformer Parameters

Parameters	Values
Number of turns in primary (n_p)	8
Turns ratio (n)	1
Primary conductor thickness (h_p)	0.15 mm
Secondary conductor thickness (h_s)	0.15 mm
Insulator thickness (h_i)	0.25 mm
Core type	ER 51/10/38
Core material	Ferrite 3C96
Distance from core center to inner edge (r_1)	10 mm
Distance from core center to outer edge (r_2)	20.9 mm

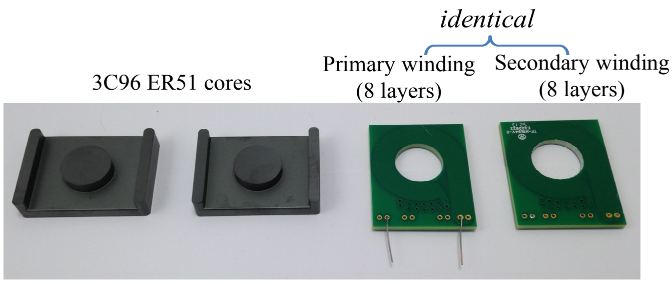

Fig. 5. Photos of planar PCB transformer with 8 primary winding turns and 8 secondary winding turns.

Fig. 6 and Fig. 7 show the current density along with the r axis (horizontal) and the y axis respectively. At low frequency, the current distribution J_r along the r axis well matches the equ.(3) in which the dc current concentrates on inner edge of conductors. At high frequency, the eddy current makes the current concentrating on the surface of conductors. Fig. 8 shows the magnetic field strength $H_o(r,h)$ of the first conductor layer along with the r axis. The FEA simulation for the magnetic strength matches the equ.(8) very well and validates its correctness. 9 shows a comparison on leakage inductance between the Dowell's calculation [15], authors' previous work [33], the proposed calculation, the FEA simulation, and the experimental measurement. The works in [15] and [33] have taken the "radial effect" into account, and the leakage inductances expressed by [15] and [33] are higher than the actual one. The overall agreement between the proposed calculation, the FEA simulation, and the experimental measurement is very good. The measurement was carried out by the impedance analyzer HP-HP-4294A. The leakage inductances were obtained by connecting the primary side to the impedance analyzer and shorting the secondary winding. The measured leakage inductances in Fig. 9 are referred to the primary side. 1.44-μH leakage inductance is obtained at 100 kHz, and 1.22-μH leakage inductance is measured at 1 MHz. The measurement shows the reduction of the leakage inductances with increased frequencies. This is due to the fact of high frequency eddy current effect.

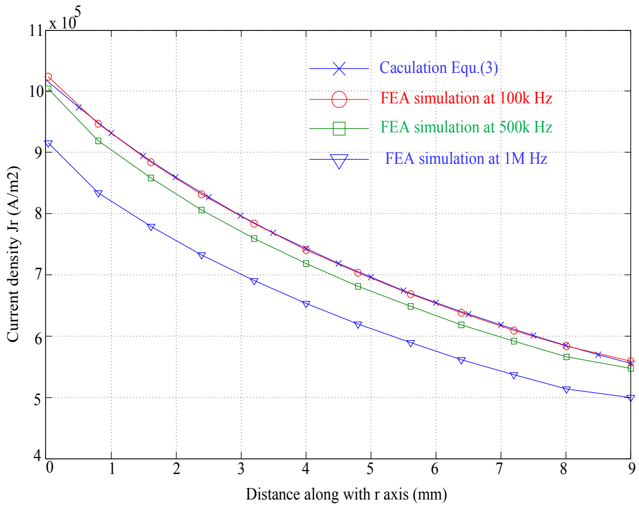

Fig. 6. Current distribution J_r along with the r axis for the first conductor layer

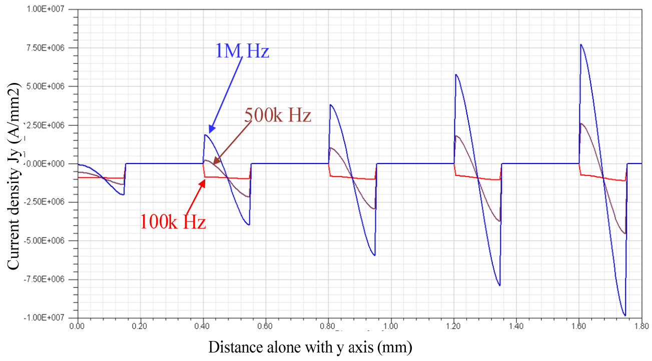

Fig. 7. Current distribution J_y along with the y axis for the first five conductor layers.

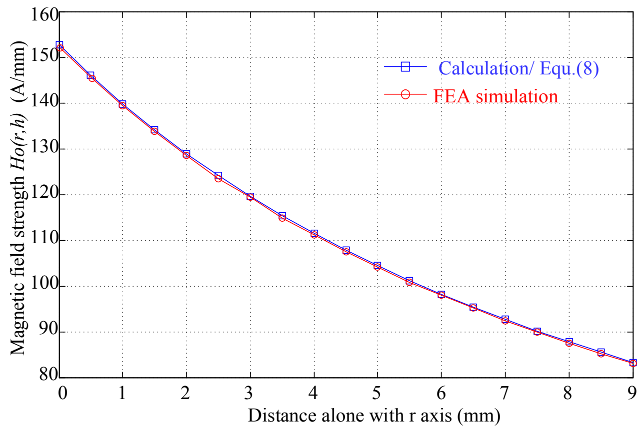

Fig. 8. Magnetic field strength $Ho(r, h)$ along with the r axis for the first conductor layers

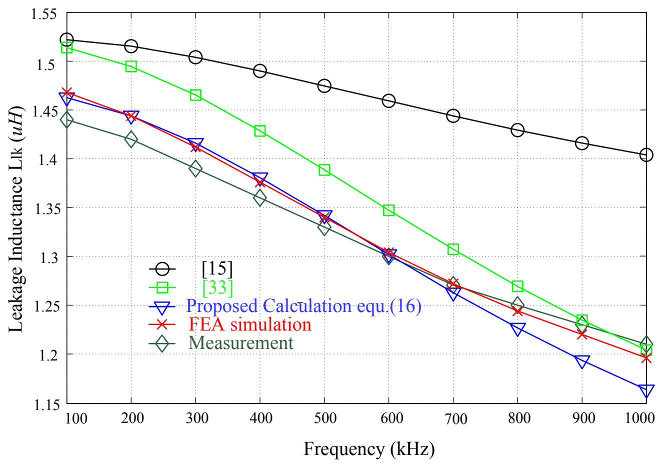

Fig. 9. Comparisons on leakage inductance of the planar transformer.

4. Conclusions

Planar transformer windings situated in a horizontal plane typically have an extremely high aspect ratio, thus concentrating the current in the inner edge of the conductors. This phenomenon gives rise to a lower leakage inductance because the area of the stored leakage energy is reduced. This so-called “radial effect” has been reported in planar transformer modelling before, but it is rare to link with frequency-dependent leakage inductance calculation. This paper gives an accurate prediction of leakage inductance for planar transformers with considerations of the frequency eddy current (AC) effect and the radial (DC) effect. FEA simulations and experimental measurements show good agreements with the prediction. It is noted that the proposed approach is an improved 1-D solution for leakage inductance, and thus only suitable for the cases that ac magnetic flux is substantially parallel to surface of rectangular conductors, meaning that the external ac magnetic flux perpendicular to the surface of conductors (2-D consideration) is negligible. The proposed 1-D solution is to most of planar transformers with PCB windings enclosed by high magnetically permeability cores, and it is also applicable for interleaved windings where each layer is interleaved, but not for complex interleaved cases such as primary secondary windings on the same layer where 2-D consideration may be needed.

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