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Multiple-Input Single-Output Polynomial Nonlinear State-Space Model of the Li-ion Battery's Short-term Dynamics ⋆

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Abstract: A data-driven methodology is proposed to identify a multi-input single-output (MISO) polynomial nonlinear state-space model (PNLSS) for battery's short-term electrical response, starting from the best linear approximation of the electrical impedance at multiple operating conditions. Furthermore, a transient handling methodology is utilised to minimise the effect of transients arising due to concatenating data from multiple experiments during the identification of the PNLSS model. Experimental validation is done using the data acquired from the operating points lying in the nonlinear regime of an almost depleted battery in terms of the state of charge (SoC) i.e. between $2\%-10\%$ at temperatures between $5°C-40°C$.

Keywords: Nonlinear state-space models, Frequency domain system identification, Battery

1. INTRODUCTION

A good understanding of the short-term dynamics of the battery is essential for desiging good estimators for the state of charge (SoC) of the battery. The short-term voltage response of the battery to the input current load profile at a particular setting of SoC and temperature can be approximately described by the following nonlinear relationship (Relan et al. (2017a)):

$$V(t) \approx f(SoC(t), I(t), T(t)) \qquad (1)$$

where f is a function which maps SoC, current I and temperature T to the terminal voltage V at a particular instant in time. Based on the data-driven nonparametric methodology in (Relan et al. (2016a)), the battery's short-term dynamics between $90\%-20\%$ SoC can be described by a linear model, but this assumption starts to be invalid at a low level of SoC e.g. 10% SoC, because the effect of nonlinear distortion becomes significant.

Modelling short-term dynamics of the battery is a challenging task because the battery's dynamics vary significantly due to the variation in the level of noise and the nonlinear distortions at different operating conditions. In (Relan et al. (2017c,b)) it has been shown how the best linear approximation (BLA) can be developed from the data

acquired at multiple operating conditions. Furthermore, the authors in (Relan et al. (2017a)) showed how a good single-input single-output (SISO) PNLSS model valid at a fixed SoC level and temperature can be developed.

Generally, a change in ambient and the operating temperature impacts the performance of batteries because it limits the power capability, the amount of energy which can be extracted, and the capacity of the battery (Haran et al. (2002)). Experimental results show an important interaction between the electrical and thermal phenomena (Relan et al. (2017c)). An online parameter estimation method based on Lyapunov's direct approach is proposed in (Chaoui and Gualous (2017)) to compensate the estimation inaccuracies introduced by temperature variations by using the surface temperature pre- or the post-compensation scaling methodology. The drawback of the proposed approach is that it is based on the assumption that the battery dynamics at all operating conditions can be represented by a first order equivalent circuit model (ECM) and it requires a priori knowledge about the topology of the ECM elements. The authors in (Jaguemont et al. (2016)) proposed two separate models to capture the electrical and thermal dynamics. The disadvantage of this model is that it cannot capture the dependencies as well as the interaction between the thermal and electrical dynamics. In addition, an identifiability analysis and a procedure to identify the physical parameters of an electro-chemical Doyle-Fuller-Newman (DFN) model from experimental data is reported in (Forman et al. (2012)) but this approach is time consuming for real-world applications.

In addition to the changing physical parameters e.g., temperature, humidity etc. which affect the short-term electrical dynamics of the battery, in practice there are many situations that can lead to a series of sub-records of data of equal (Markovsky and Pintelon (2015)) or unequal lengths (Schoukens et al. (2012)) during the experimentation phase. For example, in a long experiment, due to a sensor failure or due to very large disturbances coming from other processes, some parts in the data can have extremely poor quality. In other experiments, it might be impossible to measure for a very long time without interruption e.g. due to inadequate technical capabilities of data acquisition equipment. In our research investigations, performing a longer experiment was not feasible due to the lack of on-board memory for storing the data. Therefore, it is imperative that, the effect of temperature on the dynamics of battery is accurately accounted in the developed model and the modelling methodology should be able to deal with data records available from multiple experiments.

Hence, to deal with the above mentioned issues, in this paper we propose a methodology to develop a multiple input single output (MISO) PNLSS model starting from the BLA estimated at multiple operating conditions. The PNLSS model not only can accommodate nonlinear effects arising due to any trend, very low SoC levels but it can also deal with the nonlinear effects due to the temperature change. The main idea is to capture the effect of the SoC and the temperature by including them as two extra inputs into the proposed model structure. In addition, the ability to combine data from multiple experiments requires estimation of initial conditions, or in other words handling transients efficiently. Hence, in this paper, we utilise a framework proposed in (Relan et al. (2016b)) to handle transients arising due to concatenation of the data acquired from multiple operating conditions or experiments.

This paper is organized as follows: Section 2 describes the PNLSS model structure. The MISO PNLSS model identification procedure is described in Section 3. Section 4 introduces the experimental set-up and the measurement methodology. Results are presented in Section 5, and finally, the conclusions are given in Section 6.

2. PNLSS MODELS

The PNLSS model structure is very flexible to capture both nonlinear feed-forward and feedback dynamics (Paduart et al. (2010)). The model can be initialised very easily via the BLA. The PNLSS model is mathematically described as:

$$x(t+1) = Ax(t) + Bu(t) + E\zeta(x(t), u(t))$$
$$y(t) = Cx(t) + Du(t) + F\eta(x(t), u(t)) + e(t) \quad (2)$$

The coefficients of the linear terms in $x(t) \in \mathbb{R}^{n_a}$ and $u(t) \in \mathbb{R}^{n_u}$ are given by the matrices $A \in \mathbb{R}^{n_a \times n_a}$ and $B \in \mathbb{R}^{n_a \times n_u}$ in the state equation, $C \in \mathbb{R}^{n_y \times n_a}$ and $D \in \mathbb{R}^{n_y \times n_u}$ in the output equation respectively. The vectors $\zeta(t) \in \mathbb{R}^{n_\zeta}$ and $\eta(t) \in \mathbb{R}^{n_\eta}$ contain nonlinear monomials in $x(t)$ and $u(t)$ of degree two up to a chosen degree P_d. The coefficients of nonlinear terms are given by the matrices $E \in \mathbb{R}^{n_a \times n_\zeta}$ and $F \in \mathbb{R}^{n_y \times n_\eta}$.

3. IDENTIFICATION OF THE PNLSS MODELS

The PNLSS model given in (2) can easily be identified in the frequency domain in four major steps starting from the nonparametric identification of Best Linear Approximation (BLA). The frequency domain identification provides the additional possibility to apply user-defined weighting functions in specific frequency bands of interest.

3.1 The Best Linear Approximation

Definition 1. The Best Linear Approximation: The BLA $G_{BLA}(q)$ of a discrete-time single-input single-output (SISO) model of a nonlinear system, which is excited with signals belonging to the Riemann equivalence class of asymptotically normally distributed excitation signals (Pintelon and Schoukens (2012)), is defined as the model G belonging to the set of linear models $\mathcal{G}$, such that

$$G_{BLA}(q) = \arg\min_{G(q) \in \mathcal{G}} \mathbb{E}_u \left(|y(t) - G(q)u(t)|^2 \right) \quad (3)$$
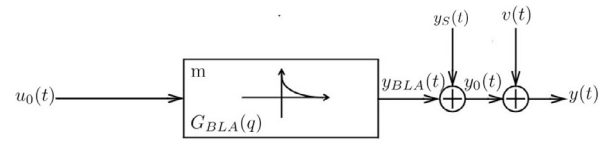

Fig. 1. Time domain representation of the problem

For an infinitely long data record $t = -\infty, \cdots, N-1$, the input-output relation of the nonlinear system is written as:

$$y(t) = G_{BLA}(q)u_0(t) + y_s(t) + H_0(q)e(t). \quad (4)$$

with q^{-1} the backward shift operator ($q^{-1}x(t) = x(t-1)$) where $y_s(t)$ are the stochastic nonlinear contributions. The exact input $u_0(t)$ is assumed to be known, while the output is disturbed with additive noise $v(t)$, then $y(t) = y_0(t) + v(t)$. The noise $v(t)$ is assumed to be filtered white noise, $v(t) = H_0(q)e(t)$, where $H_0(q)$ is the noise model. For a finite record length $t = 0, ..., N-1$, as it is in practical applications, this equation has to be extended with the initial conditions, or in other words, the transient effects of the dynamic plant and noise system t_G, t_H:

$$y(t) = G_{BLA}(q)u_0(t) + y_s(t) + H_0(q)e(t) + t_G(t) + t_H(t). \quad (5)$$

The nonparametric BLA can be calculated using either the Fast or the Robust method explained in Pintelon and Schoukens (2012), or the Local Polynomial Method (LPM)Schoukens et al. (2009).

3.2 BLA from Multiple Experiments

For m independent experiments at different SoC levels and J independent experiments at different temperatures, the common nonparametric BLA termed as the C_{BLA} can be calculated by using any of the two methods discussed in (Relan et al. (2017c,b)). Here we use the method in (Relan et al. (2017c)) to estimate the mean and the variance of the C_{BLA} (calculated at each frequency line k in the set of frequency lines) as detailed below:

$$BLA_{Avg,i}(k) = \frac{1}{m} \sum_{r=1}^{m} BLA_{ir}(k)$$

$$BLA_{Var,i}(k) = \frac{1}{m-1} \sum_{r=1}^{m} |BLA_{ir}(k) - BLA_{Avg,i}(k)|^2$$

$$C_{BLA}(k) = \frac{1}{J} \sum_{i=1}^{J} BLA_{Avg,i}(k)$$

$$\sigma^2{}_{C_{BLA}}(k) = \frac{1}{J} \sum_{i=1}^{J} BLA_{Var,i}(k) \tag{6}$$

3.3 Parametric BLA

In this step, a discrete-time rational transfer function is fitted to the nonparametric estimate of the C_{BLA} and its variance $\sigma^2_{C_{BLA}}$ (which includes both the noise and nonlinear distortion) by minimising the following nonlinear weighted least squares (NLWLS) cost function (Pintelon and Schoukens (2012)):

$$V_{tf}(\theta_{tf}) = \sum_{k=1}^{N_t} \frac{|C_{BLA}(e^{j\omega_k}) - \hat{G}_{C_{BLA}}(e^{j\omega_k}, \theta_{tf})|^2}{\sigma^2_{C_{BLA}}(e^{j\omega_k})}, \tag{7}$$

where $\sigma^2_{C_{BLA}}(j\omega_k)$ includes both noise and nonlinear distortion and N_t is the total number of selected frequencies. The parametric model order can be determined using the minimum description length (MDL) criterion (Pintelon and Schoukens (2012)). Thereafter, a balanced state-space realization $G_{ss} = (\hat{A}_{BLA}, \hat{B}_{BLA}, \hat{C}_{BLA}, \hat{D}_{BLA})$ is calculated, where the subscript ss stands for the state-space. This representation is an equivalent realization for stable systems, for which the controllability and observability Gramians are equal and diagonal (Laub et al. (1987)).

3.4 Estimation of the PNLSS model

Even though only the input current and the output voltage signals were acquired during experiments (see Section 4 for details) but to handle the variations in SoC and temperature, in the last step, simulated profiles of both temperature and SoC are included as two extra inputs to the model. This model structure gives a flexibility to test the model performance at any setting of SoC and temperature.

Assumption 1. The inputs $u(t) \in \mathbb{R}^{n_u}$ are noiseless, where $u(t)$ is a vector containing the measured input current, the simulated SoC and temperature profiles respectively.

Remark 1. The assumption of the input being noiseless is not valid as in any real measurements scenario, the measurements are always corrupted by the measurement noise. Furthermore, to test the robustness of the proposed methodology the simulated inputs of SoC and temperature were also corrupted by additive white Gaussian noise.

Assumption 2. The nonlinearity is assumed to be smooth and can be approximated well using polynomial basis functions.

Remark 2. Note that a uniformly convergent polynomial approximation of a continuous nonlinearity is always possible on a closed interval due to the Weierstrass approximation theorem (Jeffreys and Jeffreys (1999)). The type of

convergence can be relaxed to mean-square convergence, thus allowing for some discontinuous nonlinearities as well. Furthermore, the methodology is not only restricted to polynomial basis functions but is flexible enough to accommodate other set of user-defined basis functions (Marconato et al. (2014)).

For final optimisation, the model structure is initialised with the estimate $\hat{A}_{BLA}, \hat{B}_{BLA}, \hat{C}_{BLA}, \hat{D}_{BLA}$. In addition, all the coefficients corresponding to the temperature and SoC inputs are initialised as 0 vectors in the input B and the feed-through matrix D in (8) respectively. Finally, the coefficients of both the linear and the nonlinear terms in this extended model structure in (8) are identified.

$$x(t+1) = \hat{A}_{BLA}x(t) + [\hat{B}_{BLA} \ \ 0 \ \ 0]u(t) + E\zeta(t)$$
$$y(t) = \hat{C}_{BLA}x(t) + [\hat{D}_{BLA} \ \ 0 \ \ 0]u(t) + F\eta(t) + e(t) \tag{8}$$

where now the dimensions of B and D matrices are $\mathbb{R}^{n_a \times (n_u+2)}$ in the state equation and $\mathbb{R}^{n_y \times (n_u+2)}$ in the output equation respectively. The input $u(t)$ now contains the measured current profile, along with the simulated SoC and temperature profiles. The vectors ζ and η contain the monomials starting from degree 0 (to also estimate explicitly the contribution of any underlying trend in the data) up to a chosen degree P_d, whereas the contribution of the linear terms is already captured in the BLA.

For the identification of the full PNLSS, a weighted least squares (WLS) approach is employed. The WLS cost function that needs to be minimised with respect to (w.r.t.) the parameters $\theta_{NL} = [vec^T(A); vec^T(B); vec^T(C); vec^T(D); vec^T(E); vec^T(F)]^T$ is given by:

$$V_{WLS}(\theta_{NL}) = \sum_{k=1}^{N_t} \frac{|Y_{mod}(j\omega_k, \theta) - Y(j\omega_k)|^2}{W(j\omega_k)} \tag{9}$$

where N_t is the total number of selected frequencies. Y_{mod} and Y are the DFTs of the modelled output and the measured output, respectively. Generally for nonlinear systems, the model errors often dominate the disturbing noise, hence we put the weighting factor $W(j\omega_k) = 1$. Only if the model errors are below the noise level, $W(j\omega_k)$ can be put equal to the noise variance $\sigma_n{}^2(j\omega_k)$. Furthermore, the model error $\epsilon(j\omega_k, \theta_{NL}) \in \mathbb{C}^{n_y}$ is defined as

$$\epsilon(j\omega_k, \theta_{NL}) = Y_{mod}(j\omega_k, \theta_{NL}) - Y(j\omega_k) \tag{10}$$

The minimisation of the non-convex cost function $V_{WLS}(\theta_{NL})$ (9) w.r.t. the model parameters θ_{NL} is tackled via the Levenberg-Marquardt scheme (Moré (1978)). To ensure good initial values, the G_{ss} is used to initialise the PNLSS model. Other ways of estimating the full nonlinear model with different initialisation schemes are proposed in (Van Mulders and Vanbeylen (2013)).

3.5 Transient Handling in Multiple Experiments

The main aspects which need to be carefully handled during the optimisation step while identifying a PNLSS model for the battery's short-term dynamics from the data acquired from multiple experiments are:

(1) the calculation of Jacobian in state update step,
(2) and the handling of the transients due to concatenation of the data.

When computing the state sequence $x(t)$, the initial state x_0 of the model should be carefully taken into account. During the nonlinear optimisation step, the simulated states from the previous Levenberg-Marquardt iteration are used to calculate the Jacobian $J(k, \theta_{NL})$ of the modelled output w.r.t. the model parameters.

$$J(k, \theta_{NL}) = \frac{\partial \epsilon(k, \theta_{NL})}{\partial \theta_{NL}} = \frac{\partial Y_{mod}(k, \theta_{NL})}{\partial \theta_{NL}} \qquad (11)$$

Therefore when calculating the state sequence $x(t)$ of the model from the concatenated data from multiple experiments, we need to clearly define how the Jacobian should be calculated. The concatenated data sequence contains
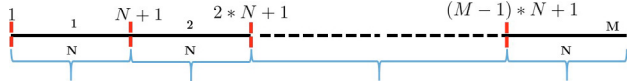

Fig. 2. Schematic representation of the concatenation of the data of M experiments, each containing N samples

blocks of periodic data (see Section 4 for the experimental details) from multiple experiments. Suppose we conduct M independent experiments with N steady state data samples, then the concatenated data sequence looks like: $[[1, ..., N]_1; [1, ..., N]_2; [1, ..., N]_3; \cdots [1, ..., N]_M]$. This concatenation will result in transients at each transition.

To calculate the Jacobian and handle transients efficiently, at the start of each new block in the data sequence i.e. $[1, N + 1, 2 \times N + 1...(M - 1) \times N + 1]$ (see Fig.2), we make use of the second case (for the periodic input-output data) which is detailed in (Paduart et al. (2010); Relan et al. (2016b)). for the calculation and handling of the Jacobian information during nonlinear optimisation. In that case, the model is simulated for multiple periods at each transition (based on the initial guess of the number of transient samples e.g. number of transient samples N_{trans} can be approximated by calculating the impulse response of the linear model estimated in the previous steps) before selecting the Jacobian where the effect of transients is considered negligible. This equally holds when the number of steady state data samples N is different for each individual experiment.
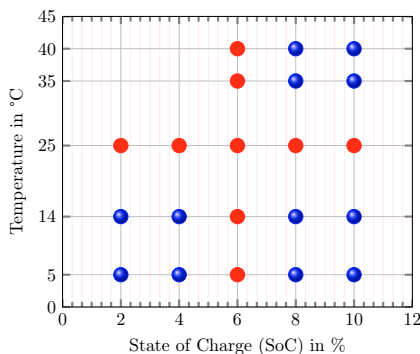
4. EXPERIMENTAL VALIDATION

Fig. 3. Operating points for the data acquisition: Blue balls = estimation and validation datasets, red dots = test dataset.

Experiments are performed on a pre-conditioned battery inside a temperature controlled chamber at different temperatures. A high energy density Li-ion Polymer Battery (EIG-ePLB-C020, Li(NiCoMn)) with the following electrical characteristics: nominal voltage $3.65V$, nominal capacity 20Ah, AC impedance (1 KHz) $< 3m\Omega$ along with the PEC battery tester SBT0550 with 24 channels is used for the data acquisition. Fig. 3 shows the operating points in terms of SoC and temperature at which input-output data were acquired. Blue balls represent the operating points of the estimation dataset and validation datasets whereas the red dots represent the operating points of the test dataset.

An odd random-phase multisine signal is used to excite the battery between 0.1Hz–5Hz. The excitation signal has a period of 5000 samples and the sample frequency f_s is set to 50Hz resulting in a frequency resolution of $f_o = 0.01Hz$. The input is zero mean with a RMS value of 10A. A random realization of the phases of the multisine signal with 7 periods is acquired at different levels of SoC and temperatures. For the test, the battery is first charged using a constant $\frac{C}{3}$ rate, where C is the rated capacity, to the maximum charge voltage of 4.1V using the constant current-constant voltage method. Then, after a relaxation period of 30 minutes, it is discharged to the desired SoC level Ah-based and considering the actual discharge capacity at 25°C until the end of discharge voltage 3.0V of the cell. After each discharge a rest period of 60 minutes is applied before the multisine tests are performed. It is made sure that the synchronisation is maintained between the signal generation and acquisition side.
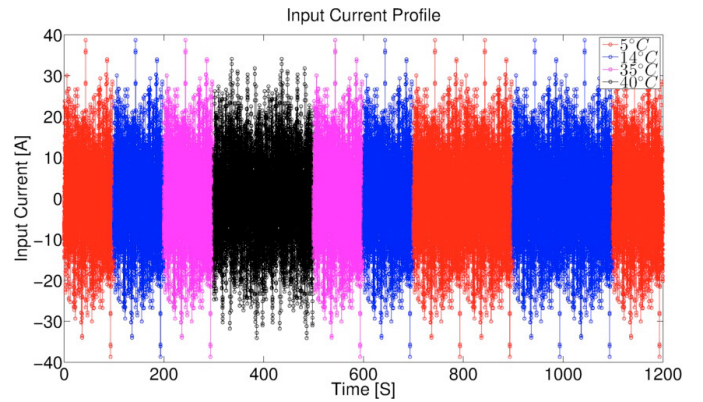
5. RESULTS

Fig. 4. The concatenated input data at different SoC levels and temperatures used as the estimation dataset.

Figures 4 and 5 show the concatenated data records of the input current and the output voltage response of the battery used as the estimation dataset respectively. Different colours represent data acquired at different settings of temperature and SoC. Red, blue, magenta and black colours correspond to 5°C, 14°C, 35°C and 40°C respectively. Within each segment, the data has been arranged according to the simulated input temperature and SoC profiles shown in Figs. 6 and 7 respectively. It can be clearly seen that the output voltage response data of the battery at different levels of SoC and temperatures to a multisine signal of the same RMS value is very different and is very heterogeneous in nature.
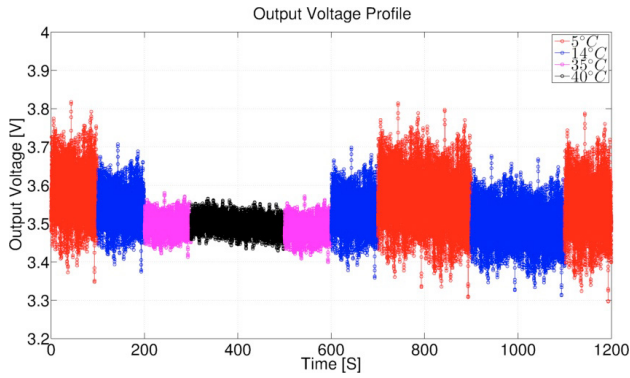
Fig. 5. The concatenated output data at different SoC levels and temperatures used as the estimation dataset.

The additional input profiles represent a realistic discharge in the levels of SoC going from 10% SoC to 2% SoC and a random change in the temperature between 5°C-40°C respectively. It should be noted that, the order of concatenation is irrelevant for this methodology and one is free to adapt any ordering of the data. Although this temperature profile is not realistic in nature as the temperature of a battery will not suddenly jump from 5°C to 14°C and so on in the physical world rather it is a worst case scenario to validate the PNLSS model.
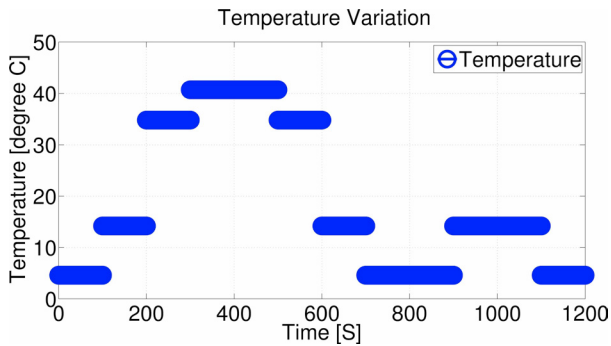

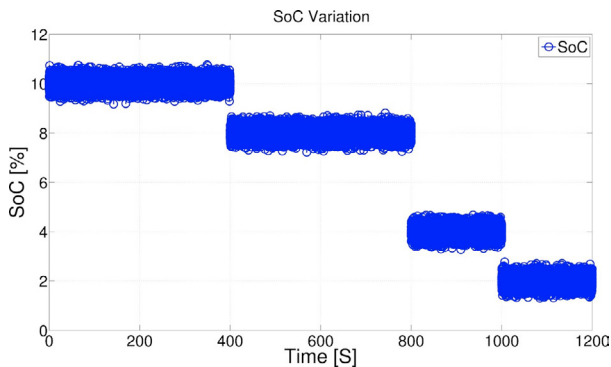Fig. 6. Input temperature profile with added white Gaussian noise with standard deviation of 0.1.

Fig. 7. Input SoC profile with added white Gaussian noise with standard deviation of 0.2.

Fig. 8 shows the performance of the PNLSS model in time domain on the estimation data. It can be clearly observed that the PNLSS model is flexible enough to accommodate the effect of different levels of the noise as well as the nonlinear distortion arising due to variations in

the levels of SoC and temperature. To demonstrate that the PLNSS model is capable of capturing the influence of the SoC, the current level and the temperature in its MIMO settings, the validation of the PNLSS model was performed for different levels of SoC at 5°C, 14°C, 35°C and 40°C respectively to a different (an unseen acquired period of) input load current profile. In addition, the data acquired at 6% SoC at all temperatures ranging between 5°C−40°C and 25°C for all other levels of SoC was kept as an additional test dataset. Figures from 9 to 11 show the
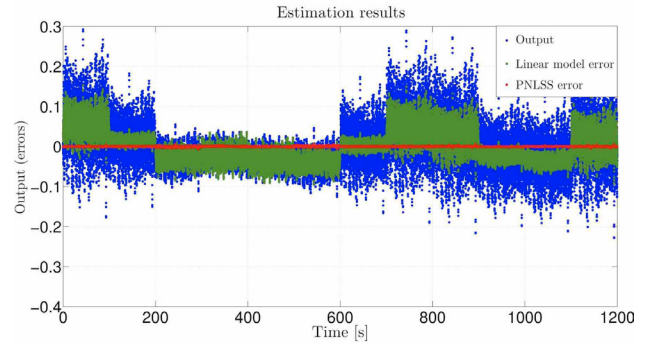

Fig. 8. Estimation result

comparison between the output responses of the common linear model and the PNLSS model at 6% SoC in the frequency domain on a test dataset at 5°C, 40°C and 25°C respectively. It can be clearly observed that the performance of the PNLSS model is approximately 10–15 dB better in the frequency band of interest at 5°C and 40°C. Similar results were obtained at other settings of SoC and temperature. The advantage of estimating the PNLSS
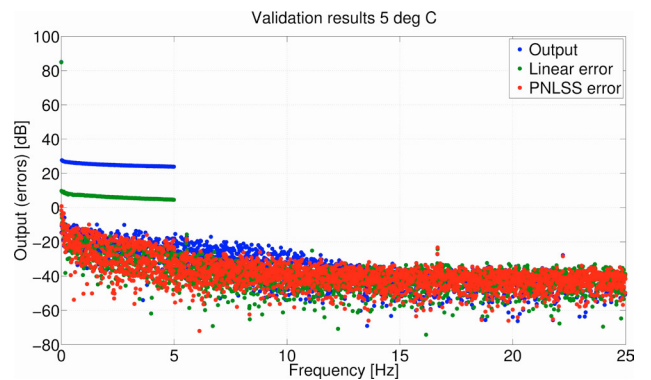

Fig. 9. Model validation at 6% SoC 5°C

model becomes even more evident by looking at Fig. 11, which shows the test results at (6% SoC, 25°C). As stated above the PNLSS model was only estimated using the data acquired at 5°C, 14°C, 35°C and 40°C for different SoC levels. The data acquired at (6% SoC, 25°C) was therefore a completely unseen data record used in the test step. It can be seen that the performance of PNLSS model at (6% SoC, 25°C) is also approximately 8–10 dB better than the commom linear model. This is a significant achievement considering the fact that the battery operates at a very low SoC level and deep inside its nonlinear regime. Similar observations were made at other operating conditions.
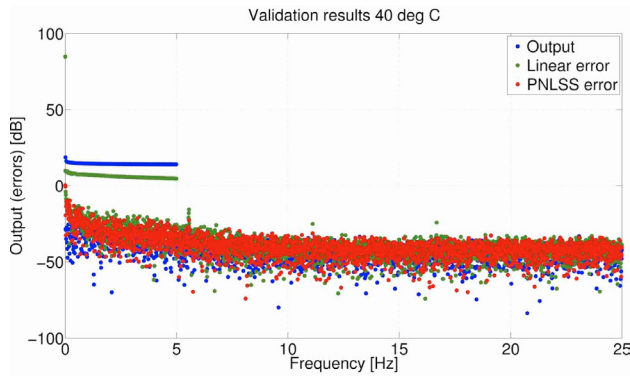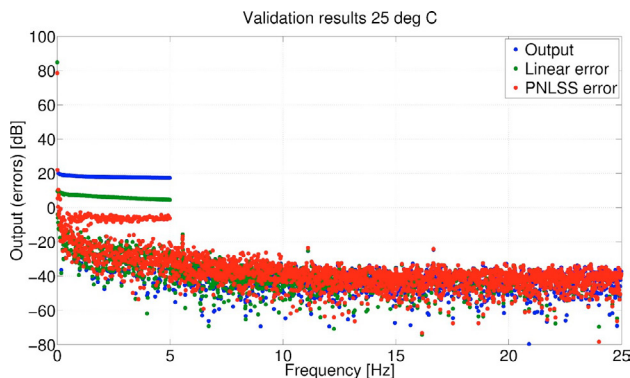
Fig. 10. Model validation at 6% SoC 40°C

Fig. 11. Model validation at 6% SoC 25°C

6. CONCLUSION

In this paper, we proposed a data driven methodology to identify a MISO PNLSS model for the battery's short-term response starting from the BLA over a range of operating conditions. We demonstrated that, it is easy to deal with the practical challenge of estimating a nonlinear model from the data acquired at different operating conditions by efficiently handling the transients arising due to concatenation of data records. This generic approach can easily be extended to include the influence of state of health. The validation of this methodology was performed at extremely low levels of SoC, which is quite a significant achievement as the future aim of the battery manufacturers and consumer industries is to push battery's operation much deeper into its operational regime.

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