DTU Library

Fault Detection and Isolation in Linear Heterogeneous Multi-Agent Networks

Nagel, Leon; Galeazzi, Roberto; Wisniewski, Rafal; Blanke, Mogens

Published in:
I F A C Workshop Series

Link to article, DOI:
10.1016/j.ifacol.2018.09.664

Publication date:
2018

Document Version
Publisher's PDF, also known as Version of record

Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 51-24 (2018) 784–789

Fault Detection and Isolation in Linear Heterogeneous Multi-Agent Networks

Leon Nagel * **Roberto Galeazzi** ** **Rafal Wisniewski** *** **Mogens Blanke** **,****

* *DTU Electrical Engineering, Technical University of Denmark, Kgs. Lyngby, Denmark (e-mail: Leon@Nagel.cc)*
** *DTU Electrical Engineering, Technical University of Denmark, Kgs. Lyngby, Denmark (e-mail: {rg,mb}@elektro.dtu.dk)*
*** *Department of Electronic Systems, Aalborg Univerity, Aalborg, Denmark (e-mail: raf@es.aau.dk)*
**** *Centre for Autonomous Marine Operations and Systems, Norwegian University of Science and Technology, Trondheim, Norway*

Abstract: Detection and isolation of faulty behaviours in multi-agent dynamical networks under cooperative control laws is of paramount importance to secure that the network operation is not jeopardized by the failing of one or more of its nodes. The paper investigates under which conditions the fault detectability and isolability (FDI) properties of the individual agent are improved because of the information sharing with other agents in the network. Relying on the parity space method and focusing the analysis on linear heterogeneous dynamical systems two main contributions are achieved: (i) the sharing of information through the cooperative control law does not improve the FDI properties of the individual agents; (ii) the presence of relative measurements among the agents results in improved fault detection and isolation capabilities the multi-agent system under the assumption of a strongly connected network.

Keywords: Multi-agent systems; Detectability; Isolability; Linear heterogeneous systems

1. INTRODUCTION

Cooperative control has received considerable research interest in the last decade, which has lead to great achievements in multi-agent systems. For multi-agent systems to carry out operations over a prolonged period of time, the detection and isolation of faults is of prime importance. Naturally FDI of multi-agent system have received growing attention as the cooperative control have matured. The research have so far been ranging from distributed Kalman filter to fuzzy logic based approaches. For distributed Kalman filters Daigle et al. (2007) Azizi and Khorasani (2009) Azizi and Khorasani (2011) have successfully achieved cooperative diagnosis. In Semsar-Kazerooni and Khorasani (2009) Semsar-Kazerooni and Khorasani (2010) Mehrabian et al. (2012) suppression of faulty agents have been developed through robustness of the control strategies. H_∞ based observers have been explored for fault tolerance in multi-agent system in Léchevin and Rabbath (2007) Meskin and Khorasani (2009b).

Most research done on FDI or FTCC have been centered around a specific methodology for observer design or robustness of control. This have lead to some residual generator designs for mostly homogeneous multi-agent systems with low order dynamics. A few have developed more theoretical results such as Azizi and Khorasani (2009) with an observer based detection framework for discrete event time systems, and Meskin and Khorasani (2009a) with fault signature selection for multi-agent system based on full row canonical decompositions.

The research on FDI for a single linear time invariant dynamical system is very mature. This means that practical analysis tools for such systems are readily available and the design of practical FDI systems can be done for most systems of this kind. In this work we establish some results on what is achievable for a heterogeneous multi-agent LTI system under a synchronizing control law.

1.1 Contribution and Novelty

The paper investigates theoretical conditions that enable the improvement of the fault detectability and isolability properties of a dynamical systems when connected through a network. In particular, by focusing on linear multi-agent systems the paper addresses the issue of *if* and *how* network connectivity can enhance the detection and isolations of faults striking the individual agent. By exploiting the parity space method the paper reaches two major results:

- In absence of relative measurements among the individual agents, the FDI of the single agent is not improved by the sharing of information across the network. This implies that the fault detectability and isolability of the multi-agent system reflects the FDIs of the single agents.
- Under the assumption of a strongly connected network, if two or more agents have relative measurements then the detectability and isolability properties of the single agent are improved by the sharing of information. This implies that the set of faults, which are detectable and possibly isolable at network level is

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2018.09.664

greater than the sets of detectable and isolable faults associated with the individual agents.

1.2 Notation

Throughout the paper the following notation is used for dynamical systems: $\mathbf{x} \in \mathbb{R}^n$ is the state vector; $\mathbf{u} \in \mathbb{R}^m$ is the known input vector; $\mathbf{y} \in \mathbb{R}^p$ is the measured output vector; $\mathbf{d} \in \mathbb{R}^d$ is the unknown disturbance vector; $\mathbf{f} \in \mathbb{R}^l$ is the unknown fault vector. The set of n by m real matrices is denoted by $M(n, m)$. $\text{Col}(\mathbf{x}_1, \dots, \mathbf{x}_n)$ denotes the column vector made by concatenating $\mathbf{x}_1, \dots, \mathbf{x}_n$, $\text{Diag}(\mathbf{M}_1, \dots, \mathbf{M}_n)$ denotes a block diagonal matrix with the matrices $\mathbf{M}_1, \dots, \mathbf{M}_n$ along the diagonal. The field of rational functions is denoted by $\mathbb{R}(s)$.

2. PRELIMINARIES

Let $\mathcal{G} = \mathcal{G}(\mathcal{E}, \mathcal{V})$ be a directed graph, where $\mathcal{V} = \mathcal{V}(\mathcal{G}) = \{1, \dots, N\}$ is the (finite) set of vertices and $\mathcal{E}(\mathcal{G}) = \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. We follow the notation from network control and call the vertices agents and the edges links. We associate with the graph $\mathcal{G}$ the following matrices: the adjacency matrix $\mathbf{E} = [e_{kh}]$ with entries $e_{kh} > 0$ if there is a link $(k, h) \in \mathcal{E}$, otherwise $e_{kh} = 0$; the diagonal in-degree matrix $\mathbf{N} = [n_{kk}]$ with entries $n_{kk} = \sum_{h=1}^{N} e_{kh}$; the Laplacian matrix $\mathbf{L} = \mathbf{N} - \mathbf{E}$.

We regard each agent as a dynamical system whose dynamics in general differs from that of the other agents; however the input-state-output dimensions are the same for all agents. Further the state vector represents the same physical properties for all agents. This leads to the following definition of heterogeneous multi-agent system.

Definition 1. Let $\mathcal{M} : \mathcal{V} \to M(n, n) \times M(n, m) \times M(n, d) \times M(n, l) \times M(p, n) \times M(p, m) \times M(p, d) \times M(p, l)$ be the map

$$k \mapsto \mathcal{M}_k = (\mathbf{A}^k, \mathbf{B}^k, \mathbf{B}_d^k, \mathbf{B}_f^k, \mathbf{C}^k, \mathbf{D}^k, \mathbf{D}_d^k, \mathbf{D}_f^k). \tag{1}$$

As a result each agent k is a linear time invariant (LTI) system of the form

$$\mathcal{M}_k : \begin{cases} \dot{\mathbf{x}}_k = \mathbf{A}^k \mathbf{x}_k + \mathbf{B}^k \mathbf{u}_k + \mathbf{B}_d^k \mathbf{d}_k + \mathbf{B}_f^k \mathbf{f}_k \\ \mathbf{y}_k = \mathbf{C}^k \mathbf{x}_k + \mathbf{D}^k \mathbf{u}_k + \mathbf{D}_d^k \mathbf{d}_k + \mathbf{D}_f^k \mathbf{f}_k \end{cases} \tag{2}$$

The superscript k on the matrices in (2) refers to the specific matrix representation of agent $\mathcal{M}_k$.

Remark 1. Definition 1 is in agreement with Lewis et al. (2013, Chapter 8) and Das and Lewis (2010). However the literature provides other definitions of heterogeneous multi-agent system where each agent may differ also for the dimensionality of the state space, see e.g. Wiedland et al. (2011); Kim et al. (2011).

Remark 2. The fault vector $\mathbf{f}_k$ represents additive faults occurring in actuators and/or sensors, which can be either strongly or weakly detectable.

When the agent $\mathcal{M}_k$ does not share information with other agents in the network ($e_{kh} = 0$, $h = 1, \dots, N$), the set of non-faulty observations ($\mathbf{f}_k = \mathbf{0}$) for that agent is defined according to (Frisk et al., 2009) as

$$\mathcal{O}(NF)_k \triangleq \Bigg\{ (\mathbf{y}_k, \mathbf{u}_k) \in \mathbb{R}(s)^{p+m} \,|\, \exists (\mathbf{x}_k, \mathbf{d}_k) \in \mathbb{R}(s)^{n+d} :$$
$$\begin{bmatrix} \mathbf{A}^k - s\,\mathbf{I}_n & \mathbf{B}_d^k \\ \mathbf{C}^k & \mathbf{D}_d^k \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{d}_k \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{B}_u^k \\ -\mathbf{I}_p & \mathbf{D}_u^k \end{bmatrix} \begin{bmatrix} \mathbf{y}_k \\ \mathbf{u}_k \end{bmatrix} = \mathbf{0} \Bigg\} \tag{3}$$

Given the fault $f_i \in \mathbb{R}(s)$ the set of faulty observations associated with such fault is defined as

$$\mathcal{O}(f_i)_k \triangleq \Bigg\{ (\mathbf{y}_k, \mathbf{u}_k) \in \mathbb{R}(s)^{p+m} \,|\exists (\mathbf{x}_k, \mathbf{d}_k) \in \mathbb{R}(s)^{n+d} :$$
$$\begin{bmatrix} \mathbf{A}^k - s\,\mathbf{I}_n & \mathbf{B}_d^k \\ \mathbf{C}^k & \mathbf{D}_d^k \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{d}_k \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{D}_u^k \\ -\mathbf{I}_p & \mathbf{D}_u^k \end{bmatrix} \begin{bmatrix} \mathbf{y}_k \\ \mathbf{u}_k \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{f,i}^k \\ \mathbf{D}_{f,i}^k \end{bmatrix} f_i = \mathbf{0} \Bigg\} \tag{4}$$

where $\mathbf{B}_{f,i}^k$ and $\mathbf{D}_{f,i}^k$ are the i-th columns of the fault-to-state matrix $\mathbf{B}_f^k$ and the fault-to-output matrix $\mathbf{D}_f^k$.

The set of non-faulty observations $\mathcal{O}(NF)_k$ includes all the input-output pairs $(\mathbf{y}_k, \mathbf{u}_k)$ that are consistent with the dynamics (1) in the absence of faults. Conversely, the set of faulty observations $\mathcal{O}(f_i)_k$ includes the input-output pairs $(\mathbf{y}_k, \mathbf{u}_k)$ that are consistent with the dynamics (1) subject to the fault $\mathbf{f}_i$.

Based on (3) and (4) the fault detectability and isolability properties are defined according to (Frisk et al., 2009).

Definition 2. The fault f_i is detectable in agent $\mathcal{M}_k$ if

$$\mathcal{O}(f_i)_k \not\subseteq \mathcal{O}(NF)_k \tag{5}$$

Definition 3. The fault f_i is isolable from fault f_j in agent $\mathcal{M}_k$ if

$$\mathcal{O}(f_i)_k \not\subseteq \mathcal{O}(f_j)_k \tag{6}$$

Definitions 2 and 3 mean that the set of all faults $\mathcal{F}$ for the system (2) can be seen as the direct sum of detectable and non-detectable faults

$$\mathcal{F} = \mathcal{F}_{det} \oplus \mathcal{F}_{\overline{det}} \tag{7}$$

or the direct sum of isolable and non-isolable faults

$$\mathcal{F} = \mathcal{F}_{iso} \oplus \mathcal{F}_{\overline{iso}} \tag{8}$$

where $\mathcal{F}_{det}$ is the vector space over $\mathbb{R}(s)$ of detectable faults and $\mathcal{F}_{\overline{det}}$ is the is the vector space over $\mathbb{R}(s)$ of non-detectable faults. Similarly, $\mathcal{F}_{iso}$ denotes the is the vector space over $\mathbb{R}(s)$ of isolable fault for the system (2) and $\mathcal{F}_{\overline{iso}}$ the is the vector space over $\mathbb{R}(s)$ of non-isolable faults.

Based on Definitions 2 and 3 criteria for detectability and isolability were derived in (Nyberg, 2002; Frisk et al., 2009), which are here restated and adapted to our notation for the sake of clarity.

Theorem 1. (Nyberg, 2002). A fault f_i is detectable in the agent $\mathcal{M}_k$ if and only if $\forall\, s \in \mathbb{C}$

$$\text{Im}\left(\begin{bmatrix} \mathbf{B}_{f,i}^k \\ \mathbf{D}_{f,i}^k \end{bmatrix} \right) \not\subseteq \text{Im}\left(\begin{bmatrix} \mathbf{A}^k - s\mathbf{I}_n & \mathbf{B}_d^k \\ \mathbf{C}^k & \mathbf{D}_d^k \end{bmatrix} \right) \tag{9}$$

Theorem 2. (Frisk et al., 2009). A fault f_i is isolable from a fault f_j in the agent $\mathcal{M}_k$ if and only if $\forall\, s \in \mathbb{C}$

$$\text{Im}\left(\begin{bmatrix} \mathbf{B}_{f,i}^k \\ \mathbf{D}_{f,i}^k \end{bmatrix} \right) \not\subseteq \text{Im}\left(\left[\begin{bmatrix} \mathbf{A}^k - s\mathbf{I}_n & \mathbf{B}_d^k \\ \mathbf{C}^k & \mathbf{D}_d^k \end{bmatrix} \; \begin{bmatrix} \mathbf{B}_{f,j}^k \\ \mathbf{D}_{f,j}^k \end{bmatrix} \right] \right) \tag{10}$$

Remark 3. Conditions (9) and (10) are independent of the specific control law governing the system (2), i.e. the control system architecture does not influence the FDI properties of the individual agent.

3. FAULT DIAGNOSIS OF MULTI-AGENT SYSTEMS

Given that each agent has its own FDI properties as stand-alone dynamical system, it is of interest to formally determine if becoming part of a multi-agent network enhances the agent's capability of detecting and isolating faults as consequence of the information sharing through the cooperative control law.

Definitions 2 and 3 together with Theorems 1 and 2 clearly formulate the fault detectability and isolability properties for the single agent while disconnected from the network. To analyse if there exist conditions such that the FDI capabilities of the agent $\mathcal{M}_k$ improve when being part of a multi-agent network, equivalent definitions are needed.

Before proceeding we make assumptions about the integrity of the communication network and the control structure the multi-agent system relies on.

Assumption 1. Data communicated though the links described by the graph $\mathcal{G}$ is neither corrupted nor lost.

Assumption 2. The multi-agent system is operating under an output consensus control law, i.e. each agent k is subject to a control input of the form (Lewis et al., 2013)

$$\mathbf{u}_k = c_1^k \mathbf{K}^k \left(\sum_{h=1}^{N} e_{kh}(\hat{\mathbf{x}}_h - \hat{\mathbf{x}}_k) \right) \tag{11}$$

$$\dot{\hat{\mathbf{x}}}_k = \mathbf{A}^k \hat{\mathbf{x}}_k + \mathbf{B}^k \mathbf{u}_k - c_2^k \mathbf{F}^k \left(\sum_{h=1}^{N} e_{kh}(\tilde{\mathbf{y}}_h - \tilde{\mathbf{y}}_k) \right) \tag{12}$$

where $c_1^k > 0$ and $c_2^k > 0$ are the coupling gains for the neighbouring controller and observer, e_{kh} are the coefficients of the adjacency matrix, $\mathbf{K}^k$ is the feedback gain matrix, $\mathbf{F}^k$ is the observer gain matrix, $\hat{\mathbf{x}}_k$ ($\hat{\mathbf{x}}_h$) is the state estimate and $\tilde{\mathbf{y}}_k$ ($\tilde{\mathbf{y}}_h$) is the output estimation error of agent k (h).

Remark 4. Assumption 2 is not restrictive since the cooperative control law (11)-(12) can be replaced with any other cooperative architecture, as e.g. a synchronizing control law, without affecting the subsequent analysis about diagnosis of the multi-agent system.

3.1 Fault Detectability and Isolability on Multi-agent Systems

Let $\mathcal{G}$ be a graph with N nodes, where each node is a dynamical system of the form (2). Let $\bar{\mathbf{x}} = \mathrm{col}(\mathbf{x}_1, \ldots, \mathbf{x}_N) \in \mathbb{R}^{Nn}$, $\bar{\mathbf{u}} = \mathrm{col}(\mathbf{u}_1, \ldots, \mathbf{u}_N) \in \mathbb{R}^{Nm}$, $\bar{\mathbf{y}} = \mathrm{col}(\mathbf{y}_1, \ldots, \mathbf{y}_N) \in \mathbb{R}^{Np}$, $\bar{\mathbf{d}} = \mathrm{col}(\mathbf{d}_1, \ldots, \mathbf{d}_N) \in \mathbb{R}^{Nd}$ and $\bar{\mathbf{f}} = \mathrm{col}(\mathbf{f}_1, \ldots, \mathbf{f}_N) \in \mathbb{R}^{Nl}$. Then the dynamics of the heterogeneous multi-agent system defined over $\mathcal{G}$ is given by

$$\mathcal{M}_\mathcal{G} : \begin{cases} \dot{\bar{\mathbf{x}}} = \mathbf{A}_\mathcal{G}\,\bar{\mathbf{x}} + \mathbf{B}_{u,\mathcal{G}}\,\bar{\mathbf{u}} + \mathbf{B}_{d,\mathcal{G}}\,\bar{\mathbf{d}} + \mathbf{B}_{f,\mathcal{G}}\,\bar{\mathbf{f}} \\ \bar{\mathbf{y}} = \mathbf{C}_\mathcal{G}\,\bar{\mathbf{x}} + \mathbf{D}_{u,\mathcal{G}}\,\bar{\mathbf{u}} + \mathbf{D}_{d,\mathcal{G}}\,\bar{\mathbf{d}} + \mathbf{D}_{f,\mathcal{G}}\,\bar{\mathbf{f}} \end{cases} \tag{13}$$

with $\mathbf{A}_\mathcal{G} = \mathrm{Diag}(\mathbf{A}^1, \ldots, \mathbf{A}^N)$, $\mathbf{B}_{u,\mathcal{G}} = \mathrm{Diag}(\mathbf{B}_u^1, \ldots, \mathbf{B}_u^N)$, $\mathbf{C}_\mathcal{G} = \mathrm{Diag}(\mathbf{C}^1, \ldots, \mathbf{C}^N)$, $\mathbf{D}_{u,\mathcal{G}} = \mathrm{Diag}(\mathbf{D}_u^1, \ldots, \mathbf{D}_u^N)$, $\mathbf{B}_{d,\mathcal{G}} = \mathrm{Diag}(\mathbf{B}_d^1, \ldots, \mathbf{B}_d^N)$, $\mathbf{D}_{d,\mathcal{G}} = \mathrm{Diag}(\mathbf{D}_d^1, \ldots, \mathbf{D}_d^N)$, $\mathbf{B}_{f,\mathcal{G}} = \mathrm{Diag}(\mathbf{B}_f^1, \ldots, \mathbf{B}_f^N)$, $\mathbf{D}_{f,\mathcal{G}} = \mathrm{Diag}(\mathbf{D}_f^1, \ldots, \mathbf{D}_f^N)$. By inserting the consensus control law (11)-(12) into (13) the closed-loop system reads

$$\mathcal{M}_\mathcal{G} : \begin{cases} \dot{\bar{\mathbf{x}}} = \mathbf{A}_\mathcal{G}\,\bar{\mathbf{x}} - \mathbf{B}_{u,\mathcal{G}}\mathbf{K}_\mathcal{G}(\mathbf{L} \otimes \mathbf{I}_N)\,\hat{\bar{\mathbf{x}}} + \mathbf{B}_{d,\mathcal{G}}\,\bar{\mathbf{d}} + \mathbf{B}_{f,\mathcal{G}}\,\bar{\mathbf{f}} \\ \bar{\mathbf{y}} = \mathbf{C}_\mathcal{G}\,\bar{\mathbf{x}} - \mathbf{D}_{u,\mathcal{G}}\mathbf{K}_\mathcal{G}(\mathbf{L} \otimes \mathbf{I}_N)\,\hat{\bar{\mathbf{x}}} + \mathbf{D}_{d,\mathcal{G}}\,\bar{\mathbf{d}} + \mathbf{D}_{f,\mathcal{G}}\,\bar{\mathbf{f}} \\ \dot{\hat{\bar{\mathbf{x}}}} = (\mathbf{A}_\mathcal{G} - \mathbf{B}_{u,\mathcal{G}}\mathbf{K}_\mathcal{G}(\mathbf{L} \otimes \mathbf{I}_N))\,\hat{\bar{\mathbf{x}}} + \mathbf{F}_\mathcal{G}(\mathbf{L} \otimes \mathbf{I}_N)\bar{\mathbf{y}} \end{cases} \tag{14}$$

where $\mathbf{I}_N$ is the N-dimensional identity matrix, $\mathbf{K}_\mathcal{G} = \mathrm{Diag}(c_1^1\mathbf{K}^1, \ldots, c_1^N\mathbf{K}^N)$ and $\mathbf{F}_\mathcal{G} = \mathrm{Diag}(c_1^1\mathbf{F}^1, \ldots, c_1^N\mathbf{F}^N)$.

The set of non-faulty observations associated with the heterogeneous multi-agent system (14) is then defined as

$$\mathcal{O}(NF)_\mathcal{G} \triangleq \left\{ (\bar{\mathbf{y}}, \hat{\bar{\mathbf{x}}}) \in \mathbb{R}(s)^{N(p+n)} \mid \exists (\bar{\mathbf{x}}, \bar{\mathbf{d}}) \in \mathbb{R}(s)^{N(n+d)} : \right.$$
$$\left. \begin{bmatrix} \mathbf{P}^1(s) & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{P}^N(s) \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{d}_1 \\ \vdots \\ \vdots \\ \mathbf{x}_N \\ \mathbf{d}_N \end{bmatrix} + \begin{bmatrix} \mathbf{M}_1^1 & \mathbf{M}_2^1 & \dots & \mathbf{M}_N^1 \\ \mathbf{M}_1^2 & \mathbf{M}_2^2 & \dots & \mathbf{M}_N^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_1^N & \mathbf{M}_2^N & \dots & \mathbf{M}_N^N \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \hat{\mathbf{x}}_1 \\ \vdots \\ \vdots \\ \mathbf{y}_N \\ \hat{\mathbf{x}}_N \end{bmatrix} = \mathbf{0} \right\} \tag{15}$$

where

$$\mathbf{P}^k(s) \triangleq \begin{bmatrix} \mathbf{A}^k - s\mathbf{I}_N & \mathbf{B}_d^k \\ \mathbf{C}^k & \mathbf{D}_d^k \end{bmatrix} \tag{16}$$

is the matrix pencil of the set $(\mathbf{A}^k, \mathbf{B}_d^k, \mathbf{C}^k, \mathbf{D}_d^k)$,

$$\mathbf{M}_k^k \triangleq \begin{bmatrix} \mathbf{0} & -c_1^k\mathbf{B}_u^k\mathbf{K}^k(l_{kk} \otimes \mathbf{I}_N) \\ -\mathbf{I}_p & -c_1^k\mathbf{D}_u^k\mathbf{K}^k(l_{kk} \otimes \mathbf{I}_N) \end{bmatrix} \tag{17}$$

$$\mathbf{M}_h^k \triangleq \begin{bmatrix} \mathbf{0} & -c_1^k\mathbf{B}_u^k\mathbf{K}^k(l_{kh} \otimes \mathbf{I}_N) \\ \mathbf{0} & -c_1^k\mathbf{D}_u^k\mathbf{K}^k(l_{kh} \otimes \mathbf{I}_N) \end{bmatrix}, k \neq h \tag{18}$$

and l_{kh} is the (k,h) entry of the Laplacian matrix describing the connectivity of agent $\mathcal{M}_k$. It is worth noting that in (15) the set of known variables is given by $(\bar{\mathbf{y}}, \hat{\bar{\mathbf{x}}})$ since the control input $\bar{\mathbf{u}}$ is an explicit function of the state estimate, as shown by (11).

Similarly, given the fault $f_{k,i} \in \mathbb{R}(s)$ the set of faulty observations for the multi-agent system is defined as

$$\mathcal{O}(f_{k,i})_\mathcal{G} \triangleq \left\{ (\bar{\mathbf{y}}, \hat{\bar{\mathbf{x}}}) \in \mathbb{R}(s)^{N(p+n)} \mid \exists (\bar{\mathbf{x}}, \bar{\mathbf{d}}) \in \mathbb{R}(s)^{N(n+d)} : \right.$$
$$\begin{bmatrix} \mathbf{P}^1(s) & \dots & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots & & \vdots \\ \mathbf{0} & \dots & \mathbf{P}^k(s) & \dots & \mathbf{0} \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{P}^N(s) \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{d}_1 \\ \vdots \\ \vdots \\ \mathbf{x}_N \\ \mathbf{d}_N \end{bmatrix} +$$
$$\left. \begin{bmatrix} \mathbf{M}_1^1 & \dots & \mathbf{M}_k^1 & \dots & \mathbf{M}_N^1 \\ \vdots & \ddots & \vdots & & \vdots \\ \mathbf{M}_1^k & \dots & \mathbf{M}_k^k & \dots & \mathbf{M}_N^k \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbf{M}_1^N & \dots & \mathbf{M}_k^N & \dots & \mathbf{M}_N^N \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \hat{\mathbf{x}}_1 \\ \vdots \\ \vdots \\ \mathbf{y}_N \\ \hat{\mathbf{x}}_N \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{B}_{f,i}^k \\ \mathbf{D}_{f,i}^k \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} f_{k,i} = \mathbf{0} \right\} \tag{19}$$

where $f_{k,i}$ is the i-th component of the fault vector $\mathbf{f}_k$.

Having defined the set of non-faulty and faulty observations in (15) and (19), it is now possible to state definitions of detectability and isolability for the heterogeneous multi-agent system.

Definition 4. The fault $f_{k,i}$ at agent $\mathcal{M}_k$ is detectable in the multi-agent system described by $\mathcal{G}$ if

$$\mathcal{O}(f_{k,i})_\mathcal{G} \not\subseteq \mathcal{O}(NF)_\mathcal{G} \tag{20}$$

Definition 5. The fault $f_{k,i}$ at agent $\mathcal{M}_k$ is isolable from a fault $f_{h,j}$ at $\mathcal{M}_h$ in the multi-agent system described by $\mathcal{G}$ if

$$\mathcal{O}(f_{k,i})_{\mathcal{G}} \nsubseteq \mathcal{O}(f_{h,j})_{\mathcal{G}} \tag{21}$$

Equations (15)-(21) extend to linear heterogeneous multi-agent networks the definitions of fault detectability and isolability for a single agent in stand-alone configuration.

Now we define the set of detectable faults for the individual agent $\mathcal{M}_k$ as $\mathcal{F}_{k,\text{det}} = \text{span}\{f_{k,l},\dots,f_{k,p}\}$ with $l,p \in \mathcal{I}$. Similarly, the set of isolable faults can be defined as $\mathcal{F}_{k,\text{iso}} = \text{span}\{f_{k,s},\dots,f_{k,t}\}$ with $s,t \in \mathcal{J} \subset \mathcal{I}$. $\mathcal{I}$ and $\mathcal{J}$ are two set of indexes. Following the rationale of Theorems 1 and 2, we state the first two contributions of the paper with Propositions 1 and 2.

Proposition 1. The set of detectable faults $\mathcal{F}_{\text{det},\mathcal{G}}$ in the multi-agent system $\mathcal{M}_{\mathcal{G}}$ is the direct sum of the sets of detectable faults of all individual agents $\mathcal{M}_k$.

$$\mathcal{F}_{\text{det},\mathcal{G}} = \bigoplus_{k\in\mathcal{V}(\mathcal{G})} \mathcal{F}_{k,\text{det}} \tag{22}$$

Proof: By applying Theorem 1 to the system $\mathcal{M}_{\mathcal{G}}$ the set of detectable faults $\mathcal{F}_{\text{det},\mathcal{G}}$ is given by the image condition

$$\text{Im}\left(\begin{bmatrix}\mathbf{B}_{f,\mathcal{G}}\\ \mathbf{D}_{f,\mathcal{G}}\end{bmatrix}\right) \nsubseteq \text{Im}\left(\begin{bmatrix}\mathbf{A}_{\mathcal{G}} - s\,\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}}\\ \mathbf{C}_{\mathcal{G}} & \mathbf{D}_{d,\mathcal{G}}\end{bmatrix}\right) \tag{23}$$

Expanding (23) it is immediate to see that the N agents are fully decoupled from a diagnostic point of view

$$\text{Im}\left(\begin{bmatrix}\mathbf{B}_f^1 & \dots & \mathbf{0}\\ \vdots & \ddots & \vdots\\ \mathbf{0} & \dots & \mathbf{B}_f^n\\ \mathbf{D}_f^1 & \dots & \mathbf{0}\\ \vdots & \ddots & \vdots\\ \mathbf{0} & \dots & \mathbf{D}_f^n\end{bmatrix}\right) \nsubseteq \text{Im}\left(\begin{bmatrix}\mathbf{A}^1 - s\,\mathbf{I}_n & \dots & \mathbf{0} & \mathbf{B}_d^1 & \dots & \mathbf{0}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \dots & \mathbf{A}^N - s\,\mathbf{I}_n & \mathbf{0} & \dots & \mathbf{B}_d^N\\ \mathbf{C}^1 & \dots & \mathbf{0} & \mathbf{D}_d^1 & \dots & \mathbf{0}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \dots & \mathbf{C}^N & \mathbf{0} & \dots & \mathbf{D}_d^N\end{bmatrix}\right) \tag{24}$$

i.e. each pair $[(\mathbf{B}_f^k)^{\mathrm{T}}, (\mathbf{D}_f^k)^{\mathrm{T}}]^{\mathrm{T}}$ defines the set of detectable faults $\mathcal{F}_{\text{det},k}$ for the agent $\mathcal{M}_k$ that is completely disjoint from the sets of the other agents. Hence (22) follows. □

Remark 5. If the multi-agent network consists of agents with homogeneous dynamics then the image condition (24) can be expressed in a compact form using the Kronecker product

$$\text{Im}\left(\mathbf{I}_{\mathcal{N}} \otimes \begin{bmatrix}\mathbf{B}_{f,j}\\ \mathbf{D}_{f,j}\end{bmatrix}\right) \nsubseteq \text{Im}\left(\mathbf{I}_{\mathcal{N}} \otimes \begin{bmatrix}\mathbf{A} - s\,\mathbf{I}_n & \mathbf{B}_d\\ \mathbf{C} & \mathbf{D}_d\end{bmatrix}\right) \tag{25}$$

Proposition 2. The set of isolable faults $\mathcal{F}_{\text{iso},\mathcal{G}}$ in the multi-agent system $\mathcal{M}_{\mathcal{G}}$ is the direct sum of the sets of isolable faults of all individual agents $\mathcal{M}_k$

$$\mathcal{F}_{\text{iso},\mathcal{G}} = \bigoplus_{k\in\mathcal{V}(\mathcal{G})} \mathcal{F}_{k,\text{iso}} \tag{26}$$

Proof: By applying Theorem 2 to the system $\mathcal{M}_{\mathcal{G}}$ the set of isolable faults $\mathcal{F}_{\text{iso},\mathcal{G}}$ is given by the image condition

$$\text{Im}\left(\begin{bmatrix}\mathbf{B}_{f,i,\mathcal{G}}\\ \mathbf{D}_{f,i,\mathcal{G}}\end{bmatrix}\right) \nsubseteq \text{Im}\left(\left[\begin{bmatrix}\mathbf{A}_{\mathcal{G}} - s\,\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}}\\ \mathbf{C}_{\mathcal{G}} & \mathbf{D}_{d,\mathcal{G}}\end{bmatrix} \begin{bmatrix}\mathbf{B}_{f,j,\mathcal{G}}\\ \mathbf{D}_{f,j,\mathcal{G}}\end{bmatrix}\right]\right) \tag{27}$$

where $\mathbf{B}_{f,i,\mathcal{G}}$ and $\mathbf{D}_{f,i,\mathcal{G}}$ are the i-th columns of the fault-to-state and to fault-to-output matrices of the multi-agent system $\mathcal{M}_{\mathcal{G}}$. Expanding (27) and using similar arguments to those adopted in Proposition 1, it is possible to conclude (26). □

Propositions 1 and 2 point out that the information sharing occurring through the cooperative control law does not improve the fault detection and isolation properties of the individual agents, which then give rise to a network whose diagnosability reflects the FDI capabilities of its own agents.

4. FAULT DIAGNOSIS USING RELATIVE MEASUREMENTS IN MULTI-AGENT SYSTEM

In multi-agent systems it is possible to introduce additional measurements that relate the states of two or more agents in the network. Such measurement is called relative and a trivial example is represented by the distance between two agents.

The dynamics of a single agent including relative measurements is given by

$$\mathcal{M}_k : \begin{cases} \dot{\mathbf{x}}_k = \mathbf{A}^k\,\mathbf{x}_k + \mathbf{B}^k\,\mathbf{u}_k + \mathbf{B}_d^k\,\mathbf{d}_k + \mathbf{B}_f^k\,\mathbf{f}_k\\ \mathbf{y}_k = \mathbf{C}^k\,\mathbf{x}_k + \mathbf{D}^k\,\mathbf{u}_k + \mathbf{D}_d^k\,\mathbf{d}_k + \mathbf{D}_f^k\,\mathbf{f}_k\\ \mathbf{y}_{k,r} = \mathbf{C}_r^k\,\mathbf{x}_k + \sum\limits_{\substack{h=1\\h\neq k}}^{N} \mathbf{C}_{r,h}^k\,\mathbf{x}_h + \mathbf{D}_{rd}^k\,\mathbf{d}_k + \mathbf{D}_{rf}^k\,\mathbf{f}_k \end{cases} \tag{28}$$

where the two matrices $\mathbf{C}_r^k$, $\mathbf{C}_{r,h}^k$ denote the coupling of the states between the agents. If $\mathbf{C}_{r,h}^k \neq 0$ then agent $\mathcal{M}_k$ has a relative measurement with agent $\mathcal{M}_h$. For a strongly connected graph $\mathcal{G}$ an assumption on symmetric relative measurements can be introduced.

Assumption 3. Any relative measurement available at agent k that relates agent k to agent $k+1$ is also present at agent $k+1$ and it relates agent $k+1$ to agent k through the same measurement.

The dynamics of the closed-loop heterogeneous multi-agent system over the graph $\mathcal{G}$ with relative measurements and subject to the consensus control law (11)-(12) is

$$\mathcal{M}_{\mathcal{G}_r} : \begin{cases} \dot{\bar{\mathbf{x}}} = \mathbf{A}_{\mathcal{G}}\,\bar{\mathbf{x}} - \mathbf{B}_{u,\mathcal{G}}\mathbf{K}_{\mathcal{G}}(\mathbf{L}\otimes\mathbf{I}_N)\,\hat{\bar{\mathbf{x}}} + \mathbf{B}_{d,\mathcal{G}}\,\bar{\mathbf{d}}\\ \qquad + \mathbf{B}_{f,\mathcal{G}}\,\bar{\mathbf{f}}\\ \bar{\mathbf{y}} = \mathbf{C}_{\mathcal{G}}\,\bar{\mathbf{x}} - \mathbf{D}_{u,\mathcal{G}}\mathbf{K}_{\mathcal{G}}(\mathbf{L}\otimes\mathbf{I}_N)\,\hat{\bar{\mathbf{x}}} + \mathbf{D}_{d,\mathcal{G}}\,\bar{\mathbf{d}}\\ \qquad + \mathbf{D}_{f,\mathcal{G}}\,\bar{\mathbf{f}}\\ \bar{\mathbf{y}}_r = \mathbf{C}_{r,\mathcal{G}}\bar{\mathbf{x}} + \mathbf{D}_{rd,\mathcal{G}}\,\bar{\mathbf{d}} + \mathbf{D}_{rf,\mathcal{G}}\,\bar{\mathbf{f}}\\ \dot{\hat{\bar{\mathbf{x}}}} = (\mathbf{A}_{\mathcal{G}} - \mathbf{B}_{u,\mathcal{G}}\mathbf{K}_{\mathcal{G}}(\mathbf{L}\otimes\mathbf{I}_N))\,\hat{\bar{\mathbf{x}}}\\ \qquad + \mathbf{F}_{\mathcal{G}}(\mathbf{L}\otimes\mathbf{I}_N)\bar{\mathbf{y}} \end{cases} \tag{29}$$

where

$$\mathbf{C}_{r,\mathcal{G}} = \begin{bmatrix}\mathbf{C}_r^1 & \mathbf{C}_{r,2}^1 & \dots & \mathbf{C}_{r,N}^1\\ \vdots & \ddots & \ddots & \vdots\\ \mathbf{C}_{r,1}^N & \mathbf{C}_{r,2}^N & \dots & \mathbf{C}_r^N\end{bmatrix}$$

$\mathbf{D}_{rd,\mathcal{G}} = \text{Diag}(\mathbf{D}_{rd}^1,\dots,\mathbf{D}_{rd}^N)$; $\mathbf{D}_{rf,\mathcal{G}} = \text{Diag}(\mathbf{D}_{rf}^1,\dots,\mathbf{D}_{rf}^N)$.

The set of non-faulty observations for multi-agent system with relative measurements can then be stated analogues to the set (15)

$$
\mathcal{O}(NF)_{\mathcal{G}_r} \triangleq \Bigg\{ (\bar{\mathbf{y}}, \hat{\bar{\mathbf{x}}}) \in \mathbb{R}(s)^{N(p+n)} \,|\exists (\bar{\mathbf{x}}, \bar{\mathbf{d}}) \in \mathbb{R}(s)^{N(n+d)} : \\
\begin{bmatrix} \mathbf{P}^1_{r,1}(s) & \dots & \mathbf{P}^1_{r,k}(s) & \dots & \mathbf{P}^1_{r,N}(s) \\ \vdots & \ddots & \vdots & & \vdots \\ \mathbf{P}^k_{r,1} & \dots & \mathbf{P}^k_{r,k}(s) & \dots & \mathbf{P}^k_{r,N}(s) \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbf{P}^N_{r,1}(s) & \dots & \mathbf{P}^N_{r,k}(s) & \dots & \mathbf{P}^N_{r,N}(s) \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{d}_1 \\ \vdots \\ \vdots \\ \mathbf{x}_N \\ \mathbf{d}_N \end{bmatrix} + \\
\begin{bmatrix} \mathbf{M}^1_1 & \dots & \mathbf{M}^1_k & \dots & \mathbf{M}^1_N \\ \vdots & \ddots & \vdots & & \vdots \\ \mathbf{M}^k_1 & \dots & \mathbf{M}^k_k & \dots & \mathbf{M}^k_N \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbf{M}^N_1 & \dots & \mathbf{M}^N_k & \dots & \mathbf{M}^N_N \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \hat{\mathbf{x}}_1 \\ \vdots \\ \vdots \\ \mathbf{y}_N \\ \hat{\mathbf{x}}_N \end{bmatrix} = \mathbf{0} \Bigg\} \tag{30}
$$

where $\mathbf{P}^k(s)$ is extended as

$$
\mathbf{P}^k_{r,k} = \begin{bmatrix} \mathbf{A}^k - s\mathbf{I}_N & \mathbf{B}^k_d \\ \mathbf{C}^k & \mathbf{D}^k_d \\ \mathbf{C}^k_r & \mathbf{D}^k_{rd} \end{bmatrix} \tag{31}
$$

and

$$
\mathbf{P}^k_{r,h} \triangleq \begin{bmatrix} \mathbf{0}^{(n\times n)} \\ \mathbf{0}^{(n\times p)} \\ \mathbf{C}^k_{r,h} \end{bmatrix} \tag{32}
$$

The set of faulty observations $\mathcal{O}(f_{k,i})_{\mathcal{G}_r}$ can be extended in a similar way.

Definitions 4 and 5 then still holds for the system (29) with the two fault sets $\mathcal{O}(NF)_{\mathcal{G}_r}$ and $\mathcal{O}(f_{k,i})_{\mathcal{G}_r}$. This means that some columns of the matrices $\mathbf{B}_{f,\mathcal{G}}$ and $\mathbf{D}_{f,\mathcal{G}}$ denoted by $\mathbf{B}_{f,N_j,\mathcal{G}}$ and $\mathbf{D}_{f,N_j,\mathcal{G}}$ satisfy the following image condition.

$$
\mathrm{Im}\left(\begin{bmatrix} \mathbf{B}_{f,N_j,\mathcal{G}} \\ \mathbf{B}_{f,N_j,\mathcal{G}} \end{bmatrix}\right) \subseteq \mathrm{Im}\left(\begin{bmatrix} \mathbf{A}_{\mathcal{G}} - s\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}} \\ \mathbf{C}_{\mathcal{G}} & \mathbf{D}_{d,\mathcal{G}} \end{bmatrix}\right) \tag{33}
$$

Detectability and isolability properties can now be stated as follows.

Proposition 3. Consider the heterogeneous multi-agent system (14) for which a set of non detectable faults $\mathcal{F}_{\overline{det},\mathcal{G}}$ exists. If there exists a matrix $\mathbf{C}_{r,\mathcal{G}}$ such that

$$
\mathrm{Im}\left(\begin{bmatrix} \mathbf{B}_{f,\mathcal{G}} \\ \mathbf{D}_{f,\mathcal{G}} \\ \mathbf{D}_{rd,\mathcal{G}} \end{bmatrix}\right) \nsubseteq \mathrm{Im}\left(\begin{bmatrix} \mathbf{A}_{\mathcal{G}} - s\,\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}} \\ \begin{bmatrix} \mathbf{C}_{\mathcal{G}} \\ \mathbf{C}_{r,\mathcal{G}} \end{bmatrix} & \begin{bmatrix} \mathbf{D}_{d,\mathcal{G}} \\ \mathbf{D}_{rd,\mathcal{G}} \end{bmatrix} \end{bmatrix}\right) \tag{34}
$$

then the set of detectable faults of the multi-agent system with relative measurement (29) satisfies the following relation

$$
\mathcal{F}_{det,\mathcal{G}_r} = \mathcal{F}_{det,\mathcal{G}} \oplus \mathcal{F}_{\overline{det},\mathcal{G}} \tag{35}
$$

Proof: If a set of non detectable faults $\mathcal{F}_{\overline{\mathrm{det}},\mathcal{G}}$ exists for the multi-agent system (14) then the image condition

$$
\mathrm{Im}\left(\begin{bmatrix} \mathbf{B}_{f,N_j,\mathcal{G}} \\ \mathbf{B}_{f,N_j,\mathcal{G}} \end{bmatrix}\right) \subseteq \mathrm{Im}\left(\begin{bmatrix} \mathbf{A}_{\mathcal{G}} - s\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}} \\ \mathbf{C}_{\mathcal{G}} & \mathbf{D}_{d,\mathcal{G}} \end{bmatrix}\right) \tag{36}
$$

is satisfied where $\mathbf{B}_{f,N_j,\mathcal{G}}$ denotes the columns of $\mathbf{B}_{f,\mathcal{G}}$ describing the faults in $\mathcal{F}_{\overline{det},\mathcal{G}}$. This is equivalent to

$$
\mathrm{Rank}\left(\left[\begin{bmatrix} \mathbf{A}_{\mathcal{G}} - s\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}} \\ \mathbf{C}_{\mathcal{G}} & \mathbf{D}_{d,\mathcal{G}} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{f,N_j,\mathcal{G}} \\ \mathbf{B}_{f,N_j,\mathcal{G}} \end{bmatrix}\right]\right) \\
= \mathrm{Rank}\left(\begin{bmatrix} \mathbf{A}_{\mathcal{G}} - s\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}} \\ \mathbf{C}_{\mathcal{G}} & \mathbf{D}_{d,\mathcal{G}} \end{bmatrix}\right) \tag{37}
$$

Let assume that the fault $f_{k,i}$ belongs to the set of non detectable faults, i.e. $f_{k,i} \in \mathcal{F}_{\overline{det},\mathcal{G}}$. Then detectability of the fault can be achieved if a relative measure can be found such that the image condition (34) holds, that is

$$
\mathrm{Im}\left(\begin{bmatrix} \mathbf{B}^1_f & \dots & \mathbf{0} \\ \mathbf{D}^1_f & \dots & \mathbf{0} \\ \mathbf{D}^1_{rf} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{B}^N_f \\ \mathbf{0} & \dots & \mathbf{D}^N_f \\ \mathbf{0} & \dots & \mathbf{D}^N_{rf} \end{bmatrix}\right) \nsubseteq \mathrm{Im}\left(\begin{bmatrix} \mathbf{A}^1 - s\mathbf{I}_n & \mathbf{B}^1_d & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}^1 & \mathbf{D}_d & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{C}^1_r & \mathbf{D}_{rd} & \dots & \mathbf{C}^1_{r,N} & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}^N - s\mathbf{I}_n & \mathbf{B}^N_d \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{C}^N & \mathbf{D}^N_d \\ \mathbf{C}^N_{r,1} & \mathbf{0} & \dots & \mathbf{C}^N_r & \mathbf{D}^N_{rd} \end{bmatrix}\right) \tag{38}
$$

where $\mathbf{C}^k_{r,j} \neq 0$ and $\mathbf{D}^k_{rf} \neq 0$. □

Proposition 4. Consider the heterogeneous multi-agent system (14) for which a set of non-isolable faults $\mathcal{F}_{\overline{iso},\mathcal{G}}$ exists. If there exists a matrix $\mathbf{C}_{r,\mathcal{G}}$ such that $\forall f_i \in \mathcal{F}_{\overline{iso},\mathcal{G}}$ and $\forall f_j \in \mathcal{F}_{iso,\mathcal{G}}$

$$
\mathrm{Im}\left(\begin{bmatrix} \mathbf{B}_{f,i,\mathcal{G}} \\ \mathbf{D}_{f,i,,\mathcal{G}} \\ \mathbf{D}_{rd,\mathcal{G}} \end{bmatrix}\right) \nsubseteq \mathrm{Im}\left(\begin{bmatrix} \mathbf{A}_{\mathcal{G}} - s\,\mathbf{I}_{Nn} & \mathbf{B}_{d,\mathcal{G}} \\ \begin{bmatrix} \mathbf{C}_{\mathcal{G}} \\ \mathbf{C}_{r,\mathcal{G}} \end{bmatrix} & \begin{bmatrix} \mathbf{D}_{d,\mathcal{G}} \\ \mathbf{D}_{rd,\mathcal{G}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{f,j,\mathcal{G}} \\ \mathbf{D}_{f,j,,\mathcal{G}} \\ \mathbf{D}_{rd,\mathcal{G}} \end{bmatrix}\right) \tag{39}
$$

then the set of isolable faults of the multi-agent system with relative measurement (29) satisfies the following relation

$$
\mathcal{F}_{iso,\mathcal{G}_r} = \mathcal{F}_{iso,\mathcal{G}} \oplus \mathcal{F}_{\overline{iso},\mathcal{G}} \tag{40}
$$

Proof: Following similar steps to those taken in the proof of Proposition 3 it is possible to conclude that (40) holds true. □

Remark 6. Propositions 3 and 4 provide a result that is much stronger than just adding additional measurements to each of the agents, since the value of the relative measurement $\mathbf{C}^k_{r,h}$ at agent k can be reconstructed from its symmetric measurement $\mathbf{C}^h_{r,k}$ at agent h in case a fault occurs at one of the relative measurements.

5. CONCLUSIONS

The fault detection and isolation properties of a linear heterogeneous multi-agent system on a graph subject to an output consensus control law have been investigated to determine conditions upon which the detectability and isolability properties of the individual agents are enhanced by the network connectivity.

In absence of relative measurements among the individual agents it is demonstrated (Propositions 1-2) that the FDI properties of the multi-agent network reflects the FDI properties of the single agents being part of the network, i.e. information sharing through the consensus control law does not improve detectability and isolability. Propositions 1 and 2 give a boundary on the achievable FDI for a multi-agent system, without any other interconnection than a communication topology and a consensus law, under a parity approach.

By adding a proper relative measurements between the individual agents it is demonstrated (Propositions 3,4) that detection and isolation properties of a multi-agent system on a strongly connect graph are better than those of the agents being part of the network. Propositions 3 and 4 suggest a change to the design of a multi-agent system, which can be used to increase its FDI properties.

REFERENCES

Azizi, S.M. and Khorasani, K. (2009). A distributed Kalman filter for actuator fault estimation of deep space formation flying satellites. *2009 IEEE International Systems Conference Proceedings*, 354–359. doi: 10.1109/SYSTEMS.2009.4815826.

Azizi, S. and Khorasani, K. (2011). Cooperative actuator fault accommodation in formation flight of unmanned vehicles using relative measurements. *International Journal of Control*, 84(5), 876–894. doi: 10.1080/00207179.2011.582157.

Daigle, M.J., Koutsoukos, X.D., and Biswas, G. (2007). Distributed Diangosis in Formations of Mobile Robots. *Electrical Engineering*, 23(2), 1–16.

Das, A. and Lewis, F.L. (2010). Distributed adaptive control for synchronization of unknown nonlinear networked systems. *Automatica*.

Frisk, E., Krysander, M., and Åslund, J. (2009). Sensor placement for fault isolation in linear differential-algebraic systems. *Automatica*, 45(2), 364–371.

Kim, H., Shim, H., and Seo, J.H. (2011). Output consensus of heterogeneous uncertain linear multi-agent systems. *IEEE Transactions on Automatic Control*, 56(1), 200–206.

Léchevin, N. and Rabbath, C.A. (2007). Robust Decentralized Fault Detection in Leader-to-Follower Formations of Uncertain, Linearly Parameterized Systems. *Journal of Guidance, Control, and Dynamics*, 30(5), 1528–1536. doi:10.2514/1.27938.

Lewis, F.L., Zhang, H., Hengster-Movric, K., and Das, A. (2013). *Cooperative control of multi-agent systems: optimal and adaptive design approaches*. Springer Science & Business Media.

Mehrabian, A.R., Tafazoli, S., and Khorasani, K. (2012). Reconfigurable cooperative control of networked lagrangian systems under actuator saturation constraints. *IET Control Theory & Applications*, 6(4), 578. doi: 10.1049/iet-cta.2010.0571.

Meskin, N. and Khorasani, K. (2009a). Actuator Fault Detection and Isolation for a Network of Unmanned Vehicles. *IEEE Transactions on Automatic Control*, 54(4), 835–840. doi:10.1109/TAC.2008.2009675.

Meskin, N. and Khorasani, K. (2009b). Fault Detection and Isolation of discrete-time Markovian jump linear systems with application to a network of multi-agent systems having imperfect communication channels. *Automatica*, 45(9), 2032–2040. doi: 10.1016/j.automatica.2009.04.020.

Nyberg, M. (2002). Criterions for detectability and strong detectability of faults in linear systems. *International Journal of Control*, 75(7), 490–501.

Semsar-Kazerooni, E. and Khorasani, K. (2009). Analysis of actuator faults in a cooperative team consensus of unmanned systems. *Proceedings of the American Control Conference*, 2618–2623. doi: 10.1109/ACC.2009.5160274.

Semsar-Kazerooni, E. and Khorasani, K. (2010). Team consensus for a network of unmanned vehicles in presence of actuator faults. *IEEE Transactions on Control Systems Technology*, 18(5), 1155–1161. doi: 10.1109/TCST.2009.2032921.

Wiedland, P., Sepulchre, R., and Allgwer, F. (2011). An internal model principle is necessary and sufficient for linear output synchronization. *Automatica*, 47, 1068–1074.