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*Publication date:*  
2018

*Document Version*  
Peer reviewed version

[Link back to DTU Orbit](#)

*Citation (APA):*

Friis, T., Katsanos, E. I., Tarpø, M., Amador, S., & Brincker, R. (2018). Output-only damping estimation of friction systems in ambient vibrations. Paper presented at International Conference on Noise and Vibration Engineering 2018, Leuven, Belgium.

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# Output-only damping estimation of friction systems in ambient vibrations

T. Friis<sup>1</sup>, E.I. Katsanos<sup>1</sup>, M. Tarpø<sup>2</sup>, S. Amador<sup>1</sup>, R. Brincker<sup>1</sup>

<sup>1</sup> Technical University of Denmark, Department of Civil Engineering, Brovej, Bygning 118, Kongens Lyngby, Denmark  
e-mail: [tofri@byg.dtu.dk](mailto:tofri@byg.dtu.dk)

<sup>2</sup> Aarhus University, Department of Engineering, Inge Lehmanns 10, 8000 Aarhus C, Denmark

## Abstract

Working with dynamics in civil engineering applications, it is experienced that real structures have nonlinear damping. The nonlinearity of the damping can be induced by the hysteresis phenomenon, that occurs due to friction or softening geotechnical boundary conditions. Along these lines, this article focusses on assessing the performance of two output-only methods estimating the linear damping of a friction-induced nonlinearly damped system subjected to random vibrations. In particular, the method that employs auto regressive models and poly-reference, and the poly-reference Least Squares Complex Frequency method are included in the study. The methods are comparatively assessed by comparing their linear damping estimates of friction-induced nonlinear numerical simulations with theoretically derived estimates of equivalent linear damping. It is concluded that the output-only methods underestimate the damping when compared to theoretically derived equivalent linear damping for the present case of Coulomb-type friction-induced nonlinear damping.

## 1 Introduction

It is often experienced in modal analysis of civil engineering structures that they have nonlinear damping. The nonlinearity might have been introduced through the phenomenon of hysteresis, e.g., friction between sliding surfaces in joints, internal friction in materials or from nonlinear material behaviour like yielding. Additionally, the boundary conditions play an important role in civil engineering structures introducing uncertainty as it concerns the dynamic behavior of the structure. For example, there are plenty of cases in which the soil-foundation system experience softening stiffness that may also introduce nonlinear damping through the related hysteretic response. Moreover, the damping can in some cases even be time-varying due to the operational condition, e.g., aerodynamic damping of wind turbines, offshore structures in waves, bridges and bridge cables, high rise buildings and air planes [1, 2]. However, due to simplifications reasons and important mathematical advantages, linear systems are generally assumed when assessing the dynamic behaviour of mechanical systems or when dealing with design of civil engineering structures. Likewise, when identifying the dynamic behaviour of existing structures through the application of operational modal analysis (OMA), damping is assumed to be linear. The topic of this article is not to move towards identification of parameters of nonlinear damping models in operational conditions, but to assess the validity and accuracy of the OMA-based identification of the linear damping for structural systems that experience friction-induced nonlinear damping.

Similar investigations have already been conducted. Zhang et al. [3] investigated the effect of nonlinear stiffness and nonlinear viscous damping on OMA based identification. They concluded that these techniques can extract for nonlinear systems most underlying linear dynamic properties, the latter being related, though, with increased bias, i.e., mean error, and random error, i.e., variance error. Bajric et al [4] evaluated the OMA

damping estimates from simulated response of an aeroelastic model of an 8 MW offshore wind turbine and of a real 8 MW offshore wind turbine under nonoperating conditions. Based on the actual measurements, time-varying damping and natural frequency were detected while a relation between the natural frequency and the amplitude of the response was also established. Friis et al [5] investigated experimentally the performance of OMA methods when applied to friction coupled systems, which consisted of two platforms connected through a bridge loosely placed on top. Based on their findings they concluded that the applied methods were able to identify the underlying linear systems. However, these studies do not deal with the accuracy of OMA damping estimates when applied to friction-induced nonlinearly damped system. Thus, the research question that the present paper is answering is; how does the (best) linear approximation of the OMA based damping estimation fit with theoretical interpretations of equivalent linear damping (ELD) of Coulomb-type friction damped systems in random vibrations? In other words, does the OMA based damping estimation constitute the damping representative of a linear system that will have the same response level as the nonlinear one.

In order to satisfy the objective of the present paper, a numerical simulation study was carried out considering a frictionally damped steel prototype structure. Two different OMA methods were applied to the simulated nonlinear response and the pertinent damping estimates were compared with theoretical interpretations of ELD. The methods for estimating the ELD are based on different assumptions but all employing knowledge of the load, natural frequencies and the mass scaling of the systems, which makes them difficult to apply in practise compared to the OMA methods. Along these lines, this article provides a short description of the ELD and OMA methods and the numerical case study, and finally presents and discusses the findings of the present study.

## 2 Estimation of equivalent linear damping

For the present study of OMA-based damping estimation of friction systems in random vibrations, it is necessary to have a reference for the damping estimate in order to evaluate the accuracy and validity of the respective OMA estimates. For that purpose, three different methods are considered for estimating the equivalent linear damping. The methods and their inherited assumptions are published and described in Friis et al [6] and are therefore only briefly described in the following <sup>1</sup>.

The present study is conducted on the basis of a simple structure, described numerically by a finite element model and, thus, it consists of multiple degrees-of-freedom (DOFs). The OMA-identified damping estimates is assessed with the use of the damping ratio, i.e., the ratio of critical damping, and naturally, also the equivalent linear damping will have to be formulated based on modal ratios of critical damping,  $\zeta_{eq(j)}$ . In the present case, the methods for ELD estimation are based on modal components by a modal decomposition of the response in the time domain by using the pseudo inverse of the mode shape matrix considering more degrees of freedom than the number of modes in order to avoid the issue of overfitting [7].

Regarding the first ELD estimation method used herein, the equivalent linear damping is estimated by balancing the intensity of the response with the integrated effect of the spectral density of the excitation, and hence referred to as the intensity balance method in the following. By assuming a stationary flat broad band excitation around the natural frequency of the mode, the equivalent linear damping can be estimated by:

$$\zeta_{eq(j)} = \pi \frac{S_{0(j)}}{2\sigma_{q_j}^2 \omega_j^3 m_j^2} \quad (1)$$

where  $m_j$  is the mass scaling of the system, i.e., modal mass,  $\omega_j$  is the natural angular frequency,  $S_{0(j)}$  is the flat (or average) spectral input at the natural frequency,  $\sigma_{q_j}^2$  is the variance of the modal response and  $j$  is indicating the mode in question.

The second method, an alteration of a method by Iourtchenko and Dimentberg [8], is based on stochastic averaging of the response in terms of displacement and velocity, by introducing slowly varying amplitude

<sup>1</sup>A journal paper with the derivation of the methods is submitted to Elsevier

and phase, forcing in such a way, the energy to be constant over an oscillation amplitude. With the applied assumption of the method it is referred to as the stochastic averaging method in the following. More details about the stochastic averaging can be found elsewhere [9]. Through this method it is possible to establish the following relation for the equivalent linear damping ratio as described by Eq. (2). The resemblance to the first method in Eq. (1) is notable, however, the second method employs the Rayleigh distribution and the variance the amplitudes of the response,  $\sigma_{Q_j}^2$ , whereas the first method employs the variance of the entire response.

$$\zeta_{eq(j)} = \frac{\pi S_{0(j)}}{(4 - \pi)\sigma_{Q_j}^2 \omega_j^3 m_j^2} \quad (2)$$

The third method modifies the so-called energy-dissipation method by Liang and Feeny [10–12]), and is founded entirely on physics. This method is based on the energy relation of a system in the manner, that the energy of all conservative terms in the nonlinear equation of motion is calculated, and then the estimation of a system's damping is based on the amount of the dissipated energy:

$$W_d = W_e - W_i \quad (3)$$

where  $W_d$  is the dissipated energy,  $W_e$  is the external energy and  $W_i$  is the internal energy. Practically, the amount of dissipated energy is calculated similarly to Eq. (3) in modal components and subsequently, a linear modal damping model is fitted to estimate the equivalent linear damping. The method is referred to as the energy balance method in the following.

### 3 Output-only methods for estimation of linear damping

Two state-of-the-art OMA methods are utilised herein. Especially, the Auto Regressive - Poly-Reference method (AR-PR) is chosen to identify the modal characteristics of both the linear and the friction-induced nonlinear system, while similar identification is conducted mainly in the frequency domain by the use of the Poly-reference Least Squares Complex Frequency method (pLSCF). For the former case, the AR-PR method, the estimates of modal characteristics are conducted in time domain based on correlation functions, while for pLSCF method, the modal characteristics are estimated from half spectral density matrices, thus, also based on correlation functions. A detailed description of the aforementioned OMA methods is considered out of the scope of the current article, and hence, only a short overview is provided next along with relevant references.

#### 3.1 Auto regressive models and poly-reference

The present method is one of the simplest methods within the engineering area of OMA and can be found described in detail in the literature, [7, 13–18]. The outline of the procedure is to compute auto regressive (AR) models in a poly-reference sense based on estimated correlation functions:

$$\mathbf{R}_y(\tau) = \mathbb{E} [\mathbf{y}(t)\mathbf{y}^T(t + \tau)] \quad (4)$$

where  $\mathbb{E}$  is the expectation operator,  $\mathbf{y}(t)$  is the response vector in time,  $t$ , and  $\tau$  is the time lag. By interpreting the correlation functions as free decays,  $\mathbf{Y}(n)$ , and employing the AR models, the measured response can be expressed as:

$$\mathbf{Y}(n) - \mathbf{A}_1\mathbf{Y}(n-1) - \mathbf{A}_2\mathbf{Y}(n-2) - \dots - \mathbf{A}_{na}\mathbf{Y}(n-na) = 0 \quad (5)$$

where  $\mathbf{A} = [\mathbf{A}_{na}, \mathbf{A}_{na-1}, \dots, \mathbf{A}_1]$  is denoted the polynomial coefficient matrices of the AR models,  $\mathbf{Y}(n) = [\mathbf{y}_1(n), \mathbf{y}_2(n), \dots, \mathbf{y}_{nc}(n)] \in \mathbb{R}^{nr \times nc \times np}$  is the free decays at sample  $n$  to the number of samples  $np$  with  $nr$  being the number of outputs and  $nc$  being the number of inputs.

In order to estimate the polynomial coefficient matrices in a poly-reference manner, two block Hankel matrices are formed and a least squared problem is solved. The modal characteristics are then estimated by forming the companion matrix of the polynomial coefficients and computing the eigenvalue decomposition. The main advantage of using the present method, other than its simplicity, is the possibility to exclude noisy time lags of the correlation function. For the present study, the first couple of lags, in accordance with [19], and the so-called noise tail are removed in an automated manner by the method described in detail in Tarpø et al [20]. Finally, the physical poles are also sorted from the mathematical ones in an automated manner by excluding poles with relatively unrealistic poles, i.e., modes with negative or high damping.

### 3.2 Poly-reference Least Squares Complex Frequency

The pLSCF method is a poly-reference generalised version of the Least Squares Complex Frequency method and is well described in the literature (see [21–24]). The main idea of the method is to model the relation between each response and all external excitations in the frequency domain by means of a right matrix-fraction description:

$$\hat{\mathbf{H}}_o(\omega) = \mathbf{N}_o(\omega)\mathbf{D}^{-1}(\omega) \quad (6)$$

where  $\hat{\mathbf{H}}_o(\omega)$  is the estimated  $o$ -th row of the half spectral density matrix,  $\mathbf{N}_o(\omega)$  is the  $o$ -th numerator row-vector polynomial,  $\mathbf{D}(\omega)$  is the common denominator matrix polynomial and  $o$  is indicating the output number from 1 to  $nc$ .

Both the numerator row-vector polynomial and the common denominator matrix polynomial are described in terms of the polynomial basis functions,  $\Omega_j(\omega)$ , and matrix coefficients,  $\mathbf{B}_{oj}$  and  $\mathbf{A}_j$ :

$$\mathbf{N}_o(\omega) = \sum_{j=0}^n \Omega_j(\omega)\mathbf{B}_{oj}, \quad \mathbf{D}(\omega) = \sum_{j=0}^n \Omega_j(\omega)\mathbf{A}_j \quad (7)$$

where  $n$  is the considered model order.

The coefficients of Eq. (7), i.e.,  $\mathbf{B}_{oj}$  and  $\mathbf{A}_j$ , are estimated by either minimising a nonlinear least squared cost function or an approximated linear least squared problem. When the coefficients are estimated, it is possible to compute the poles and modal participation factors, and subsequently the mode shapes, by the eigenvalue decomposition of the companion matrix containing the obtained coefficients. With the estimation of the modal parameters stability diagram can be constructed by varying the model order of the of Eq. (7) in the classical sense. For the present study, the estimation method has been coupled with an agglomerative hierarchical clustering algorithm, running on the stable poles, to collect the physical poles in an automatic manner.

The main advantages of the pLSCF method is its efficiency and the very clear stabilisation diagrams. However, the pLSCF method has a tendency to introduce bias in the damping estimates when statistical errors and/or noise in the tail of the correlation functions are included in the computation of the half spectral matrix. This particular issue is dealt with in the classical sense of applying an exponential window to the correlation functions and subtracting the applied damping from the modal estimates. For more on increasing the accuracy of the estimates see El-Kafafy et al [25].

## 4 Numerical case study

A testbed, for employing the comparative assessment between the ELD and OMA-based damping estimates, was established by creating a finite element model of a simple T-shaped steel structure, see Fig. 1. This respective model was created by using three dimensional beam elements and the system equivalent reduction and expansion process, proposed by O’Callahan [26], was used to reduce the structural system of 156 (initial) degrees-of-freedom to 10 with 10 modes. Furthermore, two types of damping were considered: (i) linear

proportional damping and (ii) friction-induced nonlinear damping. The latter was introduced by including a Coulomb friction element to the top of the structure, being perpendicular to the horizontal top element and in parallel to the third DOF (see Fig. 1). With this particular installation of the friction element only the second mode of the first four modes was effected by the friction-induced nonlinear damping. Thus, the analysis and the consequent discussion of the results are exclusively focused on the second mode. Moreover, by creating the friction system in this particular manner, the amount of included friction-induced nonlinear ramifications of the modal properties were reduced to a minimum. With a sufficiently high ratio of the intensity of the external excitation to the friction force, i.e., insuring the friction element is in the sliding regime for the vast majority of the response, no change of the linear natural frequencies and mode shapes was introduced, i.e., changes of the frequencies and mode shapes without consideration of the friction element. On the contrary, had the employed friction mechanism been of a type that includes a stiffness, e.g., elasto-plastic element (Jenkins element) or any kind of hysteresis-related element, a change in natural frequency would have occurred depending on the amplitude of the response. This particular friction stiffness effect is not included in the study described in the present article, thereby, purely the case of friction-induced nonlinear damping is investigated.

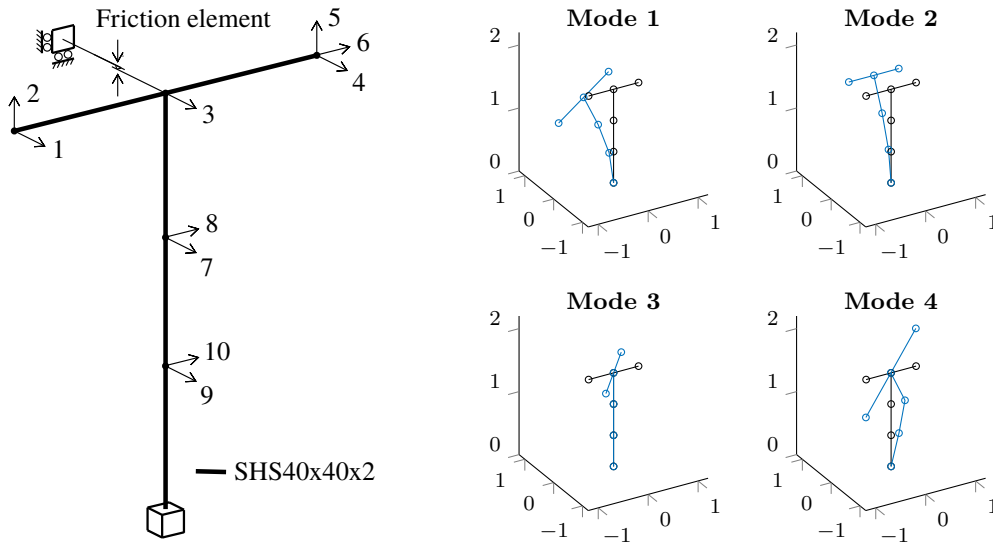


Figure 1: Sketch of the simulated T-shaped steel structure with numbering of DOFs and mode shapes of the first four modes.

To simulate the response of the friction-induced nonlinear structure, the system of second-order differential equations with the classical linear system with the inertia term, viscous damping term, stiffness term and load term is left unchanged. The nonlinear friction force is simply considered through an additional term,  $\mathbf{B} \mathbf{f}_d(\mathbf{y}(t))$ . Thus, the system of equations becomes:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) + \mathbf{P}\mathbf{f}_d(\mathbf{y}(t)) = \mathbf{x}(t) \quad (8)$$

where,  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the system matrices of the linear system,  $\mathbf{P}$  is the damper placement matrix,  $\mathbf{f}_d(\mathbf{y}(t))$  is the nonlinear force from the friction element and  $\mathbf{x}(t)$  is the external excitation.

Nonlinear numerical simulations of the friction damped system is carried out by using the algorithm proposed by Lu et al [27]. The time-stepping method is employing the second-order differential equation (Eq. (8)) in a first order format, i.e., a state-space format. The state-space version of Eq. (8) is subsequently formulated in discrete time format by assuming linear variation of the damper force and external force between time-steps. The nonlinear damping force is then treated by its iteration within each time-step. More details about the aforementioned simulation process can be found elsewhere [27].

In the present case study, the system was subjected to uncorrelated, normally distributed white noise in all 10 DOFs, enabling a comparative study of ELD estimates and OMA damping estimates through 500

Monte Carlo simulations. For each of the 500 simulations, a new excitation series was generated preserving though identical spectral level. Furthermore, for each of the 500 Monte Carlo simulations, both a numerical simulation with only linear damping and simulation with both friction-induced nonlinear damping and linear damping were carried out. For the two cases, referred to as the linear and nonlinear case in the following respectively, the total amount of damping was kept approximately at the same level. Table 1 lists the damping ratios adopted herein for the Monte Carlo simulations along with the vibration frequencies of the first four modes and the friction force.

An important parameter of the simulation study conducted herein is the length (i.e., duration) of the individual simulations that affects substantially the accuracy of the ELD-based damping estimates, the latter being the reference basis to compare with the OMA-identified damping ratios. Additionally, the current study is dealing with random generation of white noise excitation and the estimation of the equivalent linear damping for nonlinear systems subjected to random vibrations. Thus, a rather long simulation length is necessary before the intensity of the load and response is equal between the individual simulations. With these considerations in mind, it was found that an equivalent simulation length of approximately 15,000 periods of the two lowest modes ( $\approx 25$ min) was appropriate to be considered herein.

Mode	1	2	3	4
$f_j$ [Hz]	10.10	10.39	27.58	57.54
$\zeta_j$ [%]	1.5	10 / 1.5	3.0	1.5
$f_c$ [N]		0 / 2.5		

Table 1: Natural frequencies, damping ratios and friction force of the simulated structure. Where two values are listed: linear / nonlinear case.

## 5 Comparison and discussion of linear OMA damping estimates with theoretical equivalent linear damping

The main results of the study will be presented and discussed in this chapter focusing on the ELD and OMA-identified damping estimates of the second mode, the latter being affected by the friction. Especially, two cases are presented and associated with the investigation of: (i) linear damping considered at a level approximately identical to the damping of the nonlinear, friction-related system, (ii) friction-induced nonlinear damping with a relatively small addition of linear damping. By including the case related to the linear damping estimation, the effect of the relatively high damping on the estimation methods can be evaluated without consideration of the friction-induced nonlinearity. In this manner, it is possible to distinguish between the effects of inherited assumptions in the methods and the effects of the friction-induced nonlinear damping.

The first of the two following subsections (section 5.1) presents and discusses the ELD estimates of the two cases. The second subsection (section 5.2) is then presenting and discussing the OMA estimates of the two cases, with comparison to the ELD estimates in the nonlinear case. By including the ELD estimates, it enables the possibility to have a reference of what the OMA damping estimates should be in both cases and not only in the linear case. Thus, it is assumed that the ELD estimates are more accurate in the sense that they constitute the damping representative of a linear system that will have the same response level as the nonlinear one. A small study was conducted to make sure that this is actually the case. From the study, it was concluded that an equivalent linear system based on ELD estimates compared to one with OMA-based estimates is more representative of the nonlinear system. Especially, the response levels, i.e., variances, were identical for the linear system based on ELD estimates and the nonlinear one.

## 5.1 Equivalent linear damping (ELD) estimates

Fig. 2 shows plots of normalised probability distributions of the equivalent linear damping ratios of the second mode for the two cases using the three different methods; (left) purely linear damping, (right) friction-induced nonlinear damping with a relatively small addition of linear damping. All ELD and OMA-based damping estimations have been found to be normal distributed and are therefore plotted as such. Moreover, the mean values are indicated by vertical lines and circles in Fig. 2 and listed in Table 2 together with the coefficient of variation ( $CoV$ ), i.e., the standard deviation divided by the mean value.

As can be seen in Fig. 2, the energy balance method has almost zero variance in the linear case with the line-like shape of the distribution. The latter is not valid for the nonlinear damping case, for which the damping ratio estimates were found to be scattered, however in smaller extent than the damping estimates of the intensity balance and stochastic averaging method respectively. These two methods result in higher dispersion for the damping ratios estimated for the nonlinear case compared to the linear one. For both the linear and the nonlinear case, it is also notable that two of the methods, i.e., the intensity balance and the energy balance method respectively, led to identical mean damping ratio estimates, and for the linear case their mean values are equal to the second mode-related input damping ratio considered for the simulations in the linear case. On the contrary, the mean of the ELD estimates of the method based on stochastic averaging in the linear case are underestimated compared to the input damping ratio of the simulations, while in the nonlinear case, the mean of the ELD estimates is higher compared to the other two methods. These observations are corroborated in Friis et al [6], where it is found that the assumption of constant energy over a cycle of oscillation is introducing bias for higher damping ratios. Along these lines, the superiority (i.e., higher accuracy), found to be associated with ELD estimates from the intensity balance and energy balance method respectively, led the authors to exclude the stochastic average method for the OMA estimates assessment that follows.

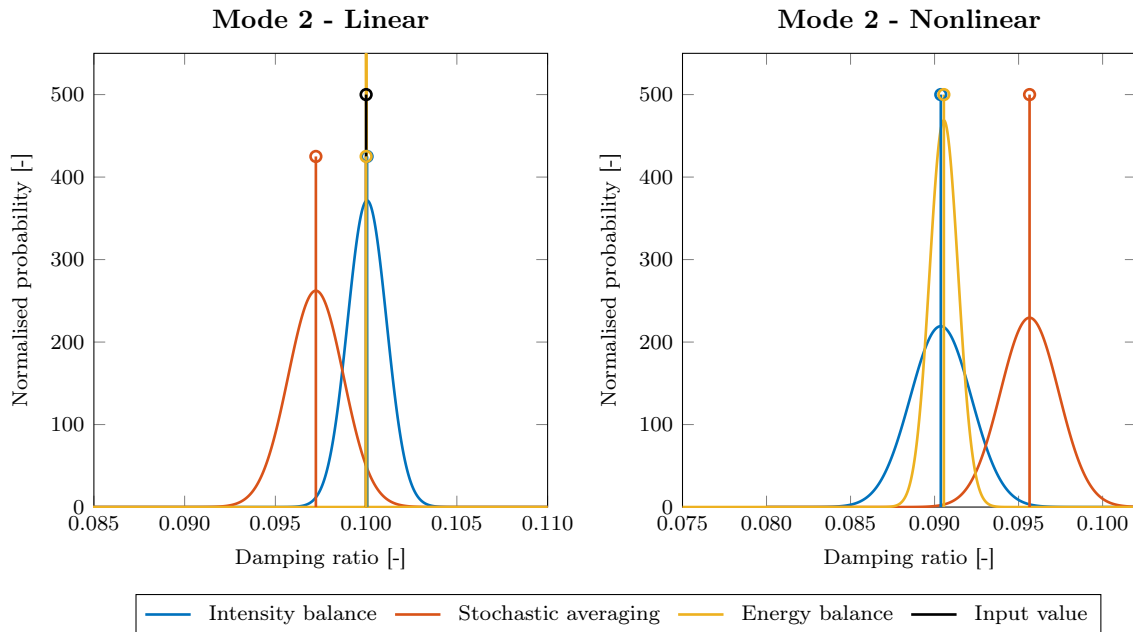


Figure 2: Equivalent linear damping estimates (normalised probability distributions) of mode 2 in the two cases; (left) purely linear damping, (right) friction-induced nonlinear damping with a small addition of linear damping.



## 5.2 Comparison and discussion of OMA estimates with ELD estimates

As for the ELD estimates, Fig. 3 shows plots of normalised probability distributions of the OMA-identified linear damping estimates related to the second mode while considering both the linear and the nonlinear case respectively. Regarding the former case, the mean of the OMA-identified estimates are calculated quite similar to the input value (i.e., the difference was found to be within the range of 0-2 ‰) indicating almost negligible bias for the OMA-identified estimates. Comparing the variance of the OMA-based damping ratios, the two methods, i.e., the pLSCF and the AR-PR, resulted in almost identical *CoV*'s (Table 2). Likewise the linear case, the OMA methods applied also for the nonlinear case led to damping ratios with almost identical mean and variance (i.e., *CoV*) estimates, the latter, is though found higher compared to the linear case. Comparing the *CoV*'s of the two ELD estimation methods, i.e., the methods based on intensity and energy balancing, the same quantifiable increase in variance is found from the linear case to the nonlinear. This observation might indicate a variation of the actual amount of damping between simulations in the nonlinear case, stemming from variation of the external excitation from simulation to simulation generating different responses, although almost with identical intensities, i.e., variances. When the responses are slightly different, the damping might also be slightly different when dealing with amplitude dependent damping. However, it is important to mention that the increase, and the similarity in the increase, might simply come from the fact that nonlinearity is introduced, and that the methods might be effected in the same manner.

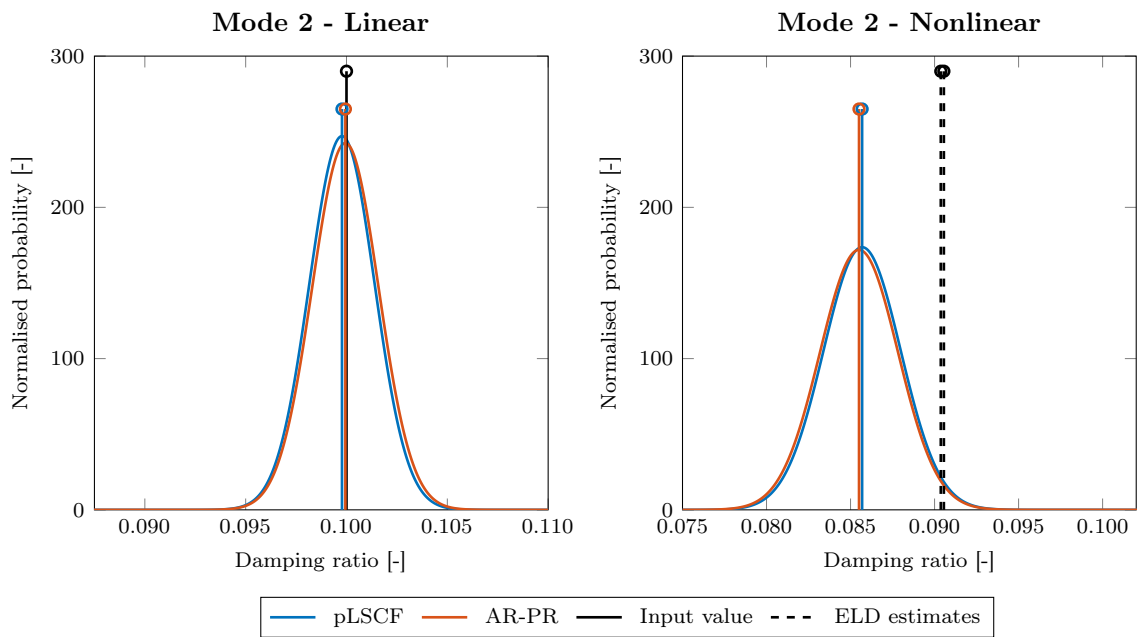


Figure 3: OMA damping estimates of mode 2 in the two cases; (left) purely linear damping, (right) friction-induced nonlinear damping with a small addition of linear damping. Including comparison with ELD estimates in the nonlinear case.

Finally, as can be seen in Table 3, the mean values of OMA-based damping estimates were calculated slightly lower (approximately 5-6%) than the ELD estimates. Such a difference indicates that the OMA methods underestimate the damping of the system with the friction-induced nonlinear damping.

Case	Assessment entity	Intensity balance	Stochastic averaging	Energy balance	pLSCF	AR-PR
Linear	$\bar{\zeta}_2$ [%]	<b>10.0</b>	9.72	<b>10.0</b>	<b>9.98</b>	<b>9.99</b>
	$CoV(\zeta_2)$ [%]	1.07	1.56	0.00	1.62	1.65
Nonlinear	$\bar{\zeta}_{eq(2)}$ [%]	<b>9.04</b>	9.57	<b>9.06</b>	<b>8.57</b>	<b>8.55</b>
	$CoV(\zeta_{eq(2)})$ [%]	2.01	1.82	0.940	2.68	2.71

Table 2: Mean values of equivalent linear damping and OMA linear damping estimates of mode 2 in the two cases; (i) purely linear damping, (ii) friction-induced nonlinear damping with a small addition of linear damping.

## 6 Conclusion

The purpose of this study was to assess the damping estimation performance of OMA-based methods on structural systems in random vibrations experiencing friction-induced nonlinear damping. A Coulomb-type friction mechanism, appropriately introduced at the top of a finite element model of a simple T-shaped prototype steel structure, ensured a numerical testbed for the investigation. Two OMA based methods were comparatively assessed in terms of their related damping estimation potential by comparing their linear damping estimates with theoretically derived estimates of equivalent linear damping. This assessment was based on 500 Monte Carlo friction-induced nonlinear numerical simulations and three different theoretically derived methods were employed for estimation of equivalent linear damping. The study mainly revealed the following.

- (i) From an assessment of the theoretically derived estimates of equivalent linear damping estimates, two methods were found to give similar estimates and to be applicable as a reference for the OMA-based methods, in the particular case with damping ratios of approximately 9%.
- (ii) From employment of the two OMA methods for estimation of linear damping of the T-shaped structure, both methods were found to provide almost identical results. Furthermore, the OMA-based estimates showed no bias when the OMA methods were applied to responses from linear simulations where the damping ratios are known in advance.
- (iii) The two methods for estimation of equivalent linear damping that were found applicable for the comparative study and the two OMA methods showed higher dispersion in their respective estimates when employed to simulated responses of the structure with friction-induced nonlinear damping compared to the case of linear proportional damping.
- (iv) Comparison between the equivalent linear damping estimations with the OMA-based ones revealed a difference of 5-6% indicating that the OMA methods underestimate the system's damping, when the latter is of the friction-induced nonlinear kind.

The underestimation of the damping by the OMA methods together with the effect of including a stiffness component in the friction-induced nonlinearity, which was not included in the present study, calls for additional research to fully comprehend the implications for OMA estimates when friction, and friction like phenomena as hysteresis, influence the dynamic response of a structure.

## Acknowledgements

The authors acknowledge the funding received from Centre for Oil and Gas - DTU/Danish Hydrocarbon Research and Technology Centre (DHRTC).

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