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Revisiting the Effect of Household Size on Consumption Over the Life-Cycle

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Abstract

Although the link between household size and consumption has strong empirical support, there is no consistent way in which demographics are dealt with in standard life-cycle models. We study the relationship between the predictions of the *Single Agent* model (the standard in the literature) versus a simple model extension (the *Demographics* model) where deterministic changes in household size and composition affect optimal consumption decisions. We show theoretically that the *Demographics* model is conceptually preferable to the *Single Agent* model as it captures economic mechanisms ignored by the latter. However, our quantitative analysis demonstrates that differences in predictions for consumption are negligible across models, when using standard calibration strategies. This suggests that it is largely irrelevant which model specification is used.

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1 Introduction

Consumption-savings life-cycle models are one of the workhorse models of modern macroeconomics. Connecting them to the data requires to take a stand on household size and composition effects as these are empirically closely related to household consumption over the life-cycle, as noted by Attanasio and Weber (1995), Attanasio, Banks, Meghir, and Weber (1999), and Gourinchas and Parker (2002). The standard approach in quantitative macroeconomics entails extracting per-adult equivalent consumption facts from household survey data and using them as targets to be replicated by *Single Agent* or *Bachelor* models, which for consistency are calibrated with per-adult equivalent income. Put differently, household effects are controlled for *in the data* but abstracted from in the modeling environment. Some recent papers in this vein include Heathcote, Storesletten, and Violante (2008), who assess the welfare effects of a rise in wage dispersion and the welfare gains of completing markets and eliminating income risk; Low and Pistaferri (2010), who decompose changes in income risk using consumption data based on the predictions of a life-cycle model; Fernández-Villaverde and Krueger (2010), who investigate the role of consumer durables for life-cycle consumption patterns.

There are numerous ways in which household consumption choices might differ from individual ones, e.g. because of two individuals choosing instead of one, the presence of children, uncertainty about household's compositional changes, etc. Probably, the simplest way to take this into account *within the model environment* has been introduced by Attanasio, Banks, Meghir, and Weber (1999) (henceforth labeled as the *Demographics* model): household size and composition change deterministically over the life-cycle and affect consumption/savings choices in a unitary household model. Various specifications of that model have been used to study different questions in the literature: the welfare effects of different bankruptcy laws in Livshits, MacGee, and Tertilt (2007), the effects of German reunification on savings behavior in Fuchs-Schündeln (2008), and the analysis of day care subsidies from an optimal taxation perspective in Domeij and Klein (2013).

The contribution of this paper is to provide a theoretical and quantitative comparison between the *Single Agent* model and the *Demographics* model in its various specifications. We start by studying a simple two period model. We find, not surprisingly, that differences in the way demographic effects are specified across models alter the predictions with respect to the timing of consumption. While both models are obviously reduced-form approaches to more complicated models of the family, the *Demographics* model captures two important channels absent in the *Single Agent* model: first, since tomorrow's assets have to be shared with other household members, the effective interest rate faced by the household varies over time and is lower than in the *Single Agent* model; second, the relative price of consumption across periods varies through changes in the cost of providing consumption to a household of different size (*scale effect*) versus a direct *utility effect* because a different number of household members enjoys utility from consumption.

We then turn to the question wether the different assumptions about household size (changes) matter quantitatively in an off-the-shelf standard model of life-cycle consumption with income uncertainty and incomplete markets as in Storesletten, Telmer, and Yaron (2004). Specifically, we embed both the *Demographics* and *Single Agent* model in this framework and calibrate them using information on income, household composition and other features of the US economy. All models are subject to the same *macroeconomic* restriction, in the sense that we use a common target for the wealth to income ratio. In order to match this ratio, each specification (the *Single Agent* model and the different variations of the *Demographics* model) exhibit different calibrated discount factors, which induce households in our model economies to save as much as their empirical counterparts do in the aggregate.

Using numerical simulations we compare the quantitative predictions for per-adult equivalent consumption (mean and variance) over the life-cycle. We first perform an exercise for the case when the relative price of per-adult equivalent consumption across periods is unchanged by demographics in the *Demographics* model (scale and utility effects cancel each other), and thus only the effective interest rates between the two setups differ. This channel is however quantitatively unimportant: the difference between mean per-adult equivalent consumption is on average 1% (and at most 9% prior to age of retirement) whereas the difference in the log variance of per-adult equivalent consumption never exceeds 3%.

The two specifications of the *Demographics* model featuring the largest differences in the relative price of consumption across periods (relative to each other, due to differences in the strength of the utility effect) generate differences of similar magnitude as above. Mean consumption during working life differs at most by 7% and on average by less than a 1%, while the variance of (log) consumption is virtually the same.

To conclude, our theoretical results show that the *Demographics* model is conceptually preferable to the *Single Agent* model as it captures economic mechanisms ignored by the latter. Our quantitative analysis however demonstrates that differences in predictions of mean and cross-sectional inequality in consumption over the life-cycle are negligible across models as long as each model is restricted to generate the same amount of aggregate savings. The intuition behind this result is simple: when the model is disciplined by an aggregate wealth to income ratio target, the resulting discount factors offset the differences in the effective interest rate or in the relative price effect of consumption stemming from different utility effects.

The structure of the paper is as follows: in Section 2 we discuss the preference structure and optimization problem for the *Demographics* and *Single Agent* model, and present theoretical predictions in a stylized two period framework. In Section 3 we layout the model used to quantify these theoretical predictions. Section 4 presents our quantitative results. We conclude in the last section.

2 A Two Period Model

2.1 Setup

At least since the empirical work by Attanasio and Weber (1993) and Attanasio and Browning (1995) it is well understood that household size changes are important for understanding the patterns of household consumption over the life-cycle. In this section we setup the most simple framework to analyze two popular approaches for dealing with household size changes in the consumption-savings literature.

Households live for two periods. Household size is normalized to one in the first period ($N_1 = 1$, e.g. a young person living alone) and increases deterministically in the second period ($N_2 > 1$, e.g. a child is born). For the theoretical analysis we only need a change of household size between the two periods. The quantitative analysis features a full life-cycle model, which emulates basic facts of the US economy, in terms of earnings processes and family size and composition. Households receive income $\mathbb{Y}_1$ in the first period and $\mathbb{Y}_2$ in the second period. We first consider the case when $\mathbb{Y}_2$ is deterministic and introduce income uncertainty in a second step. Households can borrow up to the natural borrowing constraint at an interest rate r. Unless otherwise noted, we set the interest rate r to zero and the discount factor to one for the ease of exposition. While this obviously affects some of the analytical expressions we derive, it does not alter the qualitative findings regarding the differences between the *Demographics* and *Single Agent* model. Finally, we restrict our attention to utility functions that satisfy the Inada conditions and are strictly concave in consumption.

2.2 Equivalence Scales

Throughout the entire paper we work with equivalence scales which deflate/transform variables measured at the household level into per-adult equivalents. We denote the equivalence scale for consumption by ϕ and for income by κ, both obviously being a function of household demographics. The three mechanisms through which household size affects the intra-temporal rate of transformation between expenditures and consumption services, and that are captured partially through equivalence scales, are family/public goods, economies of scale, and complementarities, see e.g. Lazear and Michael (1980). As a concrete example, consider the widely applied OECD equivalence scale which is given by $\phi_{OECD} = 1 + 0.7(N_{ad} - 1) + 0.5N_{ch}$ with N_{ad} being the number of adults and N_{ch} the number of children in the household. This can be interpreted as follows: it takes \$1.7 of consumption expenditures to generate the same level of welfare out of consumption for a two adult household that \$1 achieves for a single member household. In general, equivalence scales are normalized to one for households of size one and increase by a factor smaller than one, potentially varying with household size and other characteristics as e.g. age, for an additional household member.

Lewbel (1997) provides an extensive survey with a particular emphasis on the main challenges when it comes to equivalence scales: definition, identification and estimation. These three issues are highly controversial and seem to be far from being settled. The equivalence scales used in actual policy making are rather simple weighting schemes as the OECD scale, see Citro and Michaels (1995) for a lengthy discussion. We follow the macroeconomic literature on income and consumption inequality (to whom this paper is the most relevant) where these latter equivalence scales are commonly used.[1]

[1] Two recent macroeconomic papers take a different approach. Salcedo, Schoellman, and Tertilt (2012) develop a theory and quantitative model of household formation in which equivalence scales are an endogenous outcome of the household size, and the expenditure shares on private and public consumption goods. As households get richer they become smaller and spend more on private goods which decreases the 'measured' economies of scale. Hong and Ríos-Rull (2012) estimate household-type specific equivalence scales in the context a life-cycle model with stochastic changes in household size and composition using life-insurance holdings.

2.3 *Demographics* Model

We start with the setup in which household size N_t affects the marginal utility of consumption. In particular, we assume a unitarian framework in which a household decision maker allocates household consumption $\mathbb{C}_t^D$ optimally to the two periods

$$\max_{\mathbb{C}_1^D,\mathbb{C}_2^D} U = u(\mathbb{C}_1^D, N_1) + u(\mathbb{C}_2^D, N_2) \tag{1}$$

subject to

$$\mathbb{C}_1^D + \mathbb{C}_2^D = \mathbb{Y}_1 + \mathbb{Y}_2. \tag{2}$$

Attanasio, Banks, Meghir, and Weber (1999), and Gourinchas and Parker (2002) where the first to use such a framework and specified the dependence of the marginal utility of consumption on demographics via a general taste shifter:

$$u(\mathbb{C}_t, N_t) = \exp(\xi N_t) u\left(\mathbb{C}_t\right). \tag{3}$$

Subsequent papers used a similar specification which we also use in the remainder of the paper:

$$u(\mathbb{C}_t^D, N_t) = \delta(N_t) u\left(\frac{\mathbb{C}_t^D}{\phi(N_t)}\right). \tag{4}$$

Note that there is no private consumption and household consumption is transformed into per-adult equivalent consumption by dividing with the equivalence scale $\phi(N_t)$. $\delta(N_t)$ can be best interpreted as aggregating up the individual utilities from per-adult equivalent consumption $u\left(\frac{\mathbb{C}_t^D}{\phi(N_t)}\right)$ of all household members, or alternatively, as a parameter reflecting altruism towards other household members by the household decision maker. E.g. if the household planner assigns each household member i (including herself) a weight $\bar{\delta}_i$ then $\delta(N_t) = \sum_{i=1}^{N_t} \bar{\delta}_i$. While there are certainly more elaborate models of the household (e.g. Greenwood, Guner, and Knowles (2003), and Mazzocco, Ruiz, and Yamaguchi (2007)), we use this formulation as it nests various specifications used in recent contributions in quantitative macroeconomics: Livshits, MacGee, and Tertilt (2007), Attanasio, Low, and Sanchez-Marcos (2008) set $\delta(N_t) = 1$ such that in each period the household planer maximizes per-capita utility or alternatively the household planer does not have any altruism towards the remaining household members; Fuchs-Schündeln (2008) and Laitner and Silverman (2012) use $\delta(N_t) = \phi(N_t)$, Heathcote, Storesletten, and Violante (2012) use the number of adults in the household and Domeij and Klein (2013) the total number of household members.

2.4 *Single Agent* Model

A more common approach is to assume that households consist only of one single member in any period. To ensure consistency between the model and the data, one popular strategy is to transform total household consumption in the data into a per-adult equivalent consumption by the division with an equivalence scale. An alternative method is to estimate household size/composition effects directly from micro data using least squares regressions, as in Aguiar and Hurst (2013), which then can be trivially converted to an ad-hoc equivalence scale. The predictions of the *Single Agent* model are then compared to the empirical per-adult equivalent consumption.[2] Consequently, income fed into the model is cleaned as well for household size and household composition effects. In particular, in the inequality literature household income is divided by the same equivalence scale used for consumption, see e.g. Cutler and Katz (1992), Krueger and Perri (2006), Blundell, Low, and Preston (2013), Meyer and Sullivan (2013), and the 2010 special issue of the Review of Economic Dynamics (Krueger, Perri, Pistaferri, and Violante (2010)). An alternative is to use only the household heads income as done by Heathcote, Storesletten, and Violante (2008), Low and Pistaferri (2010), and Kaplan (2012). We adjust income with the factor $\kappa(N_t)$ as a stand-in for these different empirical strategies to obtain a per-adult equivalent income. In all considered cases $\kappa(N_t)$ is normalized to one for a household of size one and the household chooses per-adult equivalent consumption c_t^S to solve the following optimization problem

$$\max_{c_1^S, c_2^S} U = u\left(c_1^S\right) + u(c_2^S) \tag{5}$$

subject to

$$c_1^S + c_2^S = \frac{\mathbb{Y}_1}{\kappa_1(N_1)} + \frac{\mathbb{Y}_2}{\kappa_2(N_2)}. \tag{6}$$

The key distinctive feature between the two setups is how demographics affect the optimization

[2]See Blundell, Low, and Preston (2013), Heathcote, Storesletten, and Violante (2008) or Low and Pistaferri (2010).

problem. The *Single Agent* model abstracts from household (changes) inside the model, but is 'calibrated' in a fashion that controls for these effects outside the model. Household effects enter only via the budget constraint. In contrast, in the *Demographics* model household size changes affect utility directly but not the budget constraint.

It is straightforward to make the optimal consumption allocations comparable. The *Single Agent* model directly predicts per-adult equivalent consumption (in our notation the lower case letter c) because the household receives a per-adult equivalent income. In the *Demographics* model, household consumption (in our notation an upper case letter $\mathbb{C}$) is predicted which can be easily transformed into per-adult equivalent consumption by deflating it with the equivalence scale ϕ_t, i.e. $\frac{\mathbb{C}_1^D}{\phi_1} = \frac{\mathbb{C}_1^D}{1}$ and $\frac{\mathbb{C}_2^D}{\phi_2}$.

The objective of the following paragraphs is to compare the predictions of the different specifications of the *Demographics* and *Single Agent* model with each other, and to highlight the economic forces at work which distinguish the different setups.

2.5 Consumption Growth Rate

Proposition 1. *The per-adult equivalent consumption growth rates in the* Demographics *model and* Single Agent *model are the same if and only if $\delta_2 = \phi_2$.*

This result can be immediately read of from the two Euler equations for the *Demographics* model (7) and *Single Agent* model (8):

$$u'(\mathbb{C}_1^D) = \frac{\delta_2}{\phi_2} u'\left(\frac{\mathbb{C}_2^D}{\phi_2}\right) \tag{7}$$

$$u'(c_1^S) = \quad u'(c_2^S). \tag{8}$$

In both first-order conditions only per-adult equivalent consumption appears. Equation (8) implies no per-adult equivalent consumption growth in the *Single Agent* model, i.e. $c_1^S = c_2^S$ which in the *Demographics* model occurs if and only if $\delta_2 = \phi_2$. We specifically want to emphasize here the very different implications for different specifications of the *Demographics* model: if $\delta_2 > \phi_2 \Rightarrow \mathbb{C}_1^D = \frac{\mathbb{C}_1^D}{\phi_1} < \frac{\mathbb{C}_2^D}{\phi_2}$, i.e. per-adult equivalent consumption grows over time, while the opposite is true for $\delta_2 < \phi_2$. The intuition behind this result can be best explained as follows. An additional unit of period two household consumption generates only $\frac{1}{\phi_2}$ units of per-adult equivalent consumption, i.e. each household member does not get the full unit to consume. The implied marginal utility $u'\left(\frac{\mathbb{C}_2^D}{\phi_2}\right)$ accruing to each household member from consuming this additional unit of per-adult equivalent consumption is then aggregated up to the household level by multiplication with the weighting/altruism factor δ_2.

As an example, consider the case of $\delta_2 = 1 < \phi_2$. The larger household size in period two provides an incentive to *decrease* per-adult equivalent consumption in period two. Household consumption needs to be shared with more people but the household does not value the utility from per-adult equivalent consumption enjoyed by these additional members.[3]

The ratio $\frac{\delta_2}{\phi_2}$ has a similar interpretation as $\beta(1+r)$ in standard Euler equations: it changes the effective discount factor in the Euler equation, or alternatively, it changes the relative price of per-adult equivalent consumption between two periods whenever there is a change in household size. This 'demographic' channel is absent in the *Single Agent* model and in the *Demographics* model if and only if $\delta_2 = \phi_2$. Furthermore, this result is completely independent from κ_2, the deflator used to construct per-adult equivalent income for the calibration of the *Single Agent* model.

2.6 Consumption Growth Rates and Income Uncertainty

This section introduces income uncertainty, the empirically more relevant case, in the most simplistic fashion. Period two income can take two values: $\mathbb{Y}_{2,l}$ with probability p_l and $\mathbb{Y}_{2,h}$ with probability $p_h = 1 - p_l$, where $\mathbb{Y}_{2,h} > \mathbb{Y}_{2,l}$. Households are (as before) only allowed to borrow what can be repaid for sure, i.e. at most $\mathbb{Y}_{2,l}$. To highlight the key distinction between the *Demographics* and *Single Agent* model in the presence of income uncertainty, we consider the more general case of a non-zero interest rate but maintain the assumption of $\beta(1+r) = 1$. The implied Euler equations

[3]Note that in the *Demographics* model *household* consumption may nevertheless increase even if $\delta_2 < \phi_2$. In particular with CRRA preferences this is true as long as $\delta_2 > \phi_2^{1-\alpha}$, with α being the coefficient of relative risk aversion.

for the *Demographics* model (9) and *Single Agent* model (10) are given by:

$$u'\left(\mathbb{C}_1^D\right) = \frac{\delta_2}{\phi_2}\sum_{i=l,h} p_i u'\left(\overbrace{\left[\mathbb{Y}_1 - \mathbb{C}_1^D\right]\frac{1+r}{\phi_2} + \frac{\mathbb{Y}_{2,i}}{\phi_2}}^{=\mathbb{C}_2^D/\phi_2}\right) \tag{9}$$

$$u'\left(c_1^S\right) = \sum_{i=l,h} p_i u'\left(\underbrace{\left[\mathbb{Y}_1 - c_1^S\right](1+r) + \frac{\mathbb{Y}_{2,i}}{\kappa_2}}_{=c_2^S}\right) \tag{10}$$

where period two consumption in each Euler equation has been replaced with the respective life-time budget constraint from the *Demographics* (11) and *Single Agent* model (12):

$$\mathbb{C}_1^D + \frac{\mathbb{C}_2^D}{1+r} = \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{1+r} \qquad \forall\ i = l, h \tag{11}$$

$$c_1^S + \frac{c_2^S}{1+r} = \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{(1+r)\kappa_2}\ \forall\ i = l, h \tag{12}$$

While in both models period two per-adult equivalent income (i.e. deflated with the respective equivalence scale) shows up in the marginal utility on the right hand side, period one assets (i.e. period one income less period one consumption) are multiplied with two different effective interest rates. The return to savings is only $\frac{1+r}{\phi_2}$ in the *Demographics* model whereas it is $1+r$ in the *Single Agent* model. It is more expensive (cheaper) to save (borrow) in the *Demographics* model compared to the *Single Agent* model because of the lower effective interest rate payments received (to be paid). This generates differences in the resources required to provide insurance against the low income shock and drives an additional wedge between the two models independent of the ratio of δ and ϕ. Obviously, the difference in the effective interest rates in the two models is also present without income uncertainty but does not affect the choice of per-adult equivalent consumption growth in this case.

In the subsequent analysis, we return to our simplified setting with $r = 0$ and $\beta = 1$. Again, this simplification is only made for the ease of exposition and does not alter the qualitative results regarding the differences between the two model setups.

Proposition 2. *If there is income uncertainty and the optimal per-adult equivalent consumption growth rates in the* Demographics *and* Single Agent *model are the same, then* $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1^D}$.

The detailed proof of Proposition 2 is given in Appendix A.1. It presumes that the Euler equations Equations (9) and (10) are satisfied (same per-adult equivalent consumption growth rate in each state of the world; necessary for optimality) and that the budget constraints hold with equality (ensures feasibility). Therefore Proposition 2 constitutes only a necessary condition, i.e. there exist at most one κ_2 for which the optimal per-adult equivalent consumption profiles in the two models are the same.

Before discussing a sufficient condition, we briefly provide the intuition for the critical value of κ_2 stated in Proposition 2: If the household in the *Demographics* model neither saves nor borrows, κ_2 equals ϕ_2. If the household in the *Demographics* model is a saver, the household receives a lower effective interest rate than the household in the *Single Agent* model. This makes it (as described above) relatively more expensive to provide insurance because of the lower return to savings. To counteract this, κ_2 has to decrease below ϕ_2. Lowering κ_2 has two opposing effects: first, it increases period two per-adult equivalent income in any state with a positive income; second, it increases income uncertainty in period two. Key is that income uncertainty relative to expected life-time per-adult equivalent income increases. This can be seen from the coefficient of variation:

$$CV^S = \frac{S.D.(\mathbb{Y}_1 + \mathbb{Y}_{2,i}/\kappa_2)}{\mathbb{Y}_1 + \sum_{i=l,h} p_i \mathbb{Y}_{2,i}/\kappa_2} = \frac{S.D.(\mathbb{Y}_{2,i})}{\kappa_2 \mathbb{Y}_1 + \sum_{i=l,h} p_i \mathbb{Y}_{2,i}}. \tag{13}$$

Put differently, if the household in the *Demographics* model saves and faces a lower effective return on these savings, the household in the *Single Agent* model has to be endowed with a more risky income process (relative to $\kappa_2 = \phi_2$). Note that for $\mathbb{Y}_1$=0, the critical value for κ in Proposition 2 is not defined. In this case the per-adult equivalent consumption growth rates are never the same. In both models, the household has to borrow but in the *Demographics* model the effective interest rate is smaller. Varying κ does however not change relative income risk any longer if $\mathbb{Y}_1 = 0$, see Equation (13).

The two next propositions, partly building on Proposition 2, are together the counterpart of Proposition 1 for the case of income uncertainty. To obtain analytical solutions, we need to restrict our attention to utility functions whose first derivative u' is homogenous of degree q - a property that the most commonly used utility function in the quantitative macro literature, CRRA preferences, satisfies.

Proposition 3. *If u' is homogenous of degree q and $\delta_2 \neq \phi_2$, then the optimal per-adult equivalent consumption growth rates in the* Demographics *and* Single Agent *model are* **never** *the same.*

Assume to the contrary, the optimal growth rates would be the same, i.e.

$$c_1^S = \eta \mathbb{C}_1^D \text{ and } c_{2,i}^S = \eta \frac{\mathbb{C}_{2,i}^D}{\phi_2} \; \forall \; i = l, h \text{ with } \eta > 0. \tag{14}$$

Plugging allocation (14) in the Euler equation (10) for the *Single Agent* model and using the homogeneity assumption yields

$$\eta^q u' \left(\mathbb{C}_1^D\right) = \eta^q \sum_{i=l,h} p_i u' \left(\frac{\mathbb{C}_{2,i}^D}{\phi_2}\right). \tag{15}$$

Comparing Equation (15) and the Euler equation in the *Demographics* model (9), it is obvious that both cannot hold jointly at the allocation (14) if $\delta_2 \neq \phi_2$. It is important to mention that Proposition 3 is independent of the choice of κ_2. This underpins the claim made above: Proposition 2 constitutes only a necessary condition in the sense that there exists at most one κ_2 for which the per-adult equivalent consumption growth rates in the two models are the same. In fact, Proposition 3 is an example where no such κ_2 exists. Next, we state a sufficient condition under which the per-adult equivalent consumption profiles in the two models are the same.

Proposition 4. *If u' is homogenous of degree q, $\delta_2 = \phi_2$ and $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1^D}$, then the optimal per-adult equivalent consumption growth rates from the* Demographics *model and the* Single Agent *model are the same.*

The detailed proof of Proposition 4 is given in Appendix A.2. Building on Proposition 2, Proposition 4 is a sufficient condition for the per-adult equivalent consumption growth rates in the two models being the same. Propositions 3 and 4 together are the counterpart of Proposition 1 for the case of income uncertainty. However, it takes more than only $\delta_2 = \phi_2$ for the *Single Agent* model to generate the same per-adult equivalent consumption growth rates as in the *Demographics* model: first, homogeneity of u'; second, a specific value of κ_2 which depends on the optimal consumption choice in the *Demographics* model.

3 Quantitative Model

3.1 Basic Setup

We now move to a full life-cycle model, which emulates basic facts of the US economy, in terms of earnings processes and family size and composition to compare the quantitative predictions across models. We set up a standard incomplete markets life-cycle model, which follows closely the one in Storesletten, Telmer, and Yaron (2004). For simplicity, we do however abstract from population growth and prices being determined in general equilibrium.

In the model, households start their economic life in period t_0 with zero assets. During their working life until period t_w they receive a stochastic income in every period. There is no labor supply choice. From period $t_w + 1$ onwards households are retired and have to live from their accumulated savings during working life.[4] Life ends with certainty at age T but there is an age specific survival probability ζ_t. Households have access to a risk-free bond a which pays the interest rate r and can borrow at the same interest rate up to the natural borrowing constraint, i.e. an age specific level $a_{min,t}$ of debt that they can repay for sure. Annuity markets are actuarially fair such that the assets of the dying population are redistributed equally among the surviving members of their cohort. Age T households do not leave bequests and cannot die with debt.

In the *Demographics* model, household size changes over the life-cycle deterministically as in Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002) and is homogenous across all households. The maximization problem is given by

$$\max_{\{a_{t+1}\}_{t=t0}^{T-1}} E_0 \sum_{t=t0}^{T} \left(\Pi_{j=t_0}^{j=t} \zeta_j\right) \beta^{t-t_0} \;\; \delta_t u\left(\frac{c_t}{\phi_t}\right) \tag{16}$$

$$\text{subject to} \tag{17}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + y_t \quad \forall\, t \leq t_w \tag{18}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} \qquad\quad \forall\, t_w < t \leq T \tag{19}$$

$$a_{t+1} \geq a_{min,t} \tag{20}$$

[4] The online appendix shows results with a social security system in place. The qualitative predictions for the differences across models are in line with our benchmark economies and the conclusions are the same.

where δ and ϕ are functions of household size and its composition ($N_{ad,t}$ and $N_{ch,t}$) over the life-cycle. Note that in contrast to Section 2, we now denote all variables with lower case letters. Pre labor tax income y_t is stochastic during the working life, i.e. as long as $t \leq t_w$, and is given by the following process:

$$\ln\, y_t = \varrho_t + \epsilon^F + z_t + \epsilon_t^{Tr} \tag{21}$$

where ϱ_t is an age-dependent, exogenous experience profile common to all individuals, ϵ^F is a fixed effect drawn by households at the beginning of economic life from a normal distribution with mean zero and variance σ_F^2, z_t is a permanent shock to labor income, with

$$z_t = \rho z_{t-1} + \epsilon_t^P \text{ with } \epsilon_t^P \sim N(0, \sigma_P^2) \tag{22}$$

and $\epsilon^{Tr} \sim N(0, \sigma_{Tr}^2)$ is a transitory shock.

The structure of the *Single Agent* problem is very similar. Demographics do not affect the marginal utility of consumption while income y_t is deflated by household size through the equivalence scale κ_t:

$$\max_{\{a_{t+1}\}_{t=t0}^{T-1}} E_0 \sum_{t=t0}^{T} \left(\Pi_{j=t_0}^{j=t} \zeta_j\right) \beta^{t-t_0} \;\; u\left(c_t\right) \quad \text{subject to} \tag{23}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + \frac{y_t}{\kappa_t} \quad \forall\ t \leq t_w \tag{24}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} \qquad\qquad \forall\ t_w < t \leq T \tag{25}$$

$$a_{t+1} \geq a_{\min,t}, \tag{26}$$

with y_t following the same process as for the *Demographics* model given by Equations (21) and (22).

3.2 Quantitative Features of the Model

A model period is one year. Agents start life at age 25, retire when 65 and live until age 95 after which they die with certainty. The common profile for survival probabilities comes from the National Center for Health Statistics (1994). To maintain comparability across models, we keep some parameters fixed: we pick common choices from the literature and set the interest rate at

2.25% (the average of the t-bill rate minus inflation, between 1984 and 2003, IMF Statistics) and work with CRRA preferences ($\alpha = 2$), i.e. for the *Demographics* model we use:

$$u = \delta(N_{ad,t}, N_{ch,t}) \frac{\left(\frac{c_t}{\phi(N_{ad,t}, N_{ch,t})}\right)^{1-\alpha}}{1-\alpha}, \tag{27}$$

and for the *Single Agent* model we use:

$$u = \frac{c_t^{1-\alpha}}{1-\alpha}. \tag{28}$$

We follow Storesletten, Telmer, and Yaron (2004) and choose the discount factor, β, so that the model's aggregate net wealth/income ratio matches that of the lower 99% wealth quantile in the U.S. from Table 6 in Díaz-Giménez, Quadrini, and Ríos-Rull (1997), this ratio is 3.1

We use as the square root scale $\phi_t^{SQR} = \sqrt{N_{ad} + N_{ch}}$ to transform household into per-adult equivalent consumption. This scale is now used by the OECD and almost identical to the equivalence scale preferred by Fernández-Villaverde and Krueger (2007) which is the mean over six representative equivalence scales used in the empirical consumption literature. Furthermore, we use the same equivalence scale for computing per-adult equivalent income and per-adult equivalent consumption ($\kappa_t = \phi_t$) as done in studies investigating jointly income and consumption inequality, see e.g. Cutler and Katz (1992), Krueger and Perri (2006), Meyer and Sullivan (2013), and the 2010 special issue of the Review of Economic Dynamics (Krueger, Perri, Pistaferri, and Violante (2010)).

As for utility weights, we remain agnostic and compare three cases: (i) $\delta_t = 1$ represents the case when there is no altruism towards additional households members (as e.g. in Livshits, MacGee, and Tertilt (2007), Attanasio, Low, and Sanchez-Marcos (2008)); (ii) $\delta_t = N_t = N_{ad,t} + N_{ch,t}$ (as e.g. in Domeij and Klein (2013)) is the opposite with full altruism; and (iii) an intermediate case when $\delta_t = \phi_t$ (as e.g. in Fuchs-Schündeln (2008) and Laitner and Silverman (2012)).

For the deterministic income profiles, we use data from the Current Population Survey, from 1984 to 2003, in particular, the March supplements for years 1985 to 2004, given that questions about income are retrospective. We chose this sample in order to maintain comparability with the Consumer Expenditure Survey sample used to compute consumption facts by related literature (which we refer to below). We use total wage income (deflated by CPI-U, leaving amounts in 2000 US dollars). We construct total household income $W_{i\tau}$ for household i observed in year τ, as the sum of individual incomes in the household for all households with at least one full time/full year worker. The latter is defined as someone who worked more than 40 hours per week and more than 40 weeks per year and earned more than $2 per hour. To get life-cycle experience profiles, we estimate linear regressions on a set of age dummies, controlling for time and cohort effects as described in Deaton and Paxson (1994).[5] The set of estimated coefficients for the age dummies is our object of interest, which we smooth using a quartic polynomial on age, see Figure 4a in Appendix B.

To parameterize the stochastic part of the income process in (21) and (22) we pick the parameter estimates from Storesletten, Telmer, and Yaron (2004) and set $\rho = 0.9989$, $\sigma_F^2 = 0.2105$, $\sigma_P^2 = 0.0166$ and $\sigma_{Tr}^2 = 0.063$. We discretize this calibrated process using the Rouwenhorst method, using 17, 2 and 2 points for the permanent, transitory and fixed effect components respectively. This methodology is specially suited for our case given the high persistence of the process, see the discussion in Kopecky and Suen (2010).

To compute profiles for family size and composition, we use the March supplements of the CPS for the years 1984 to 2003. For each household, we count the number of adults (individuals age 17+) and the number of children: individuals age 16 or less who are identified as being the 'child' of an adult in the household. We compute two separate profiles: one for number of adults and one for number of children. As above, we run dummy regressions to extract life-cycle profiles, where the considered age is that of the head (irrespective of gender) and control for cohort and year effects as described above. After extracting these life-cycle profiles, we smooth them using a cubic polynomial in age, and restrict the number of children to zero after age 60, see Figure 4b in Appendix B.

[5] Deaton and Paxson (1994) impose restrictions on the year dummies to break the collinearity between age, birth cohort and year dummies: the coeffcient estimates of the year dummies have to sum up to one and the sum over the coefficient estimates of year dummies interacted with the respective time trend indicator need to equal zero as well.

4 Results

In the paragraphs below, we relate our theoretical results to two quantitative exercises: first, we compare the *Demographics* model with $\delta = \phi$ to the *Single Agent* model to focus on the role of the different effective interest rate. Second, we compare the different specifications of the *Demographics* model with each other to analyze the role of differences in the relative price of consumption across periods. Similar to the benchmark case in Storesletten, Telmer, and Yaron (2004), our simulated economies do not feature a pension system. The qualitative predictions for the differences across models are in line with our benchmark economies and the conclusions are the same if we introduce a US-style pension system as described in that same paper.[6]

4.1 *Demographics* model with $\delta = \phi$ vs. *Single Agent* model

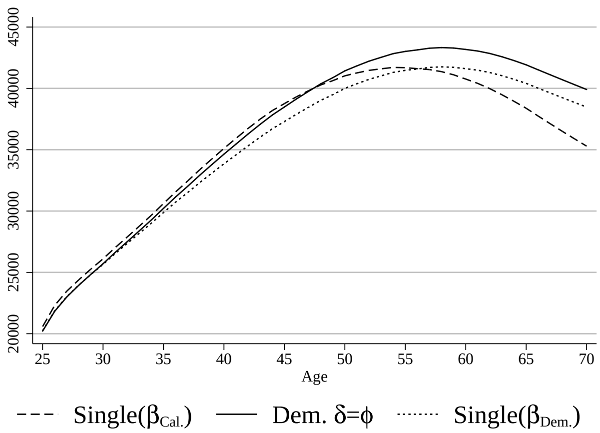
We first focus on the comparison between the *Demographics* model when $\delta = \phi$ and the *Single Agent* model. The key prediction of our theoretical analysis is that per-adult equivalent consumption growth rates are in general different between the two model setups, unless income is certain or the income deflator κ takes on a specific value (which is not guaranteed to exist at all). These results are derived under the assumption that the remaining parameters (preferences, prices) coincide across models. This is also the case in our quantitative exercise except for the discount factor β which is calibrated to match a wealth to income ratio of 3.1 in each economy. The resulting β for the *Demographics* model is 0.959, and 0.946 for the *Single Agent* model respectively. Since the effective return to savings is lower in the *Demographics* model, the discount factor in the *Single Agent* model has to be higher in order to generate the same wealth to income ratio. In order to distinguish more clearly the role of differences in the effective interest rate and differences in the discount factor, we add a further specification to the following analysis: we simulate the *Single Agent* model again with the discount factor from the *Demographics* model, i.e. with $\beta = 0.959$, which increases the wealth to income ratio to 3.65.

Figure 1a shows the mean per-adult equivalent consumption profiles between the three simulations. During retirement, i.e. in the absence of income uncertainty, $[\beta(1 + r)]^{1/\gamma}$ determines the

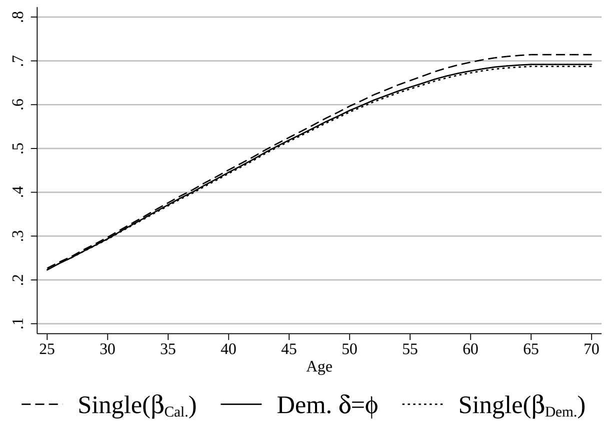
[6] We direct interested readers to the online appendix for that as well as for additional results regarding the implications for insurance coefficients as in Blundell, Pistaferri, and Preston (2008) and Kaplan and Violante (2010), and for empirical estimates of δ based on the work by Attanasio, Banks, Meghir, and Weber (1999).

Figure 1: Per-adult equivalent consumption: *Single Agent* vs. *Demographics* with $\delta = \phi$

(a) Mean



(b) Variance (of Log)



growth rate of per-adult equivalent consumption. Hence, after age 65 per-adult equivalent consumption decreases at a faster rate in the *Single Agent* model with the calibrated discount factor ($\beta = 0.946$) compared to the *Demographics* model and *Single Agent* model with $\beta = 0.959$.

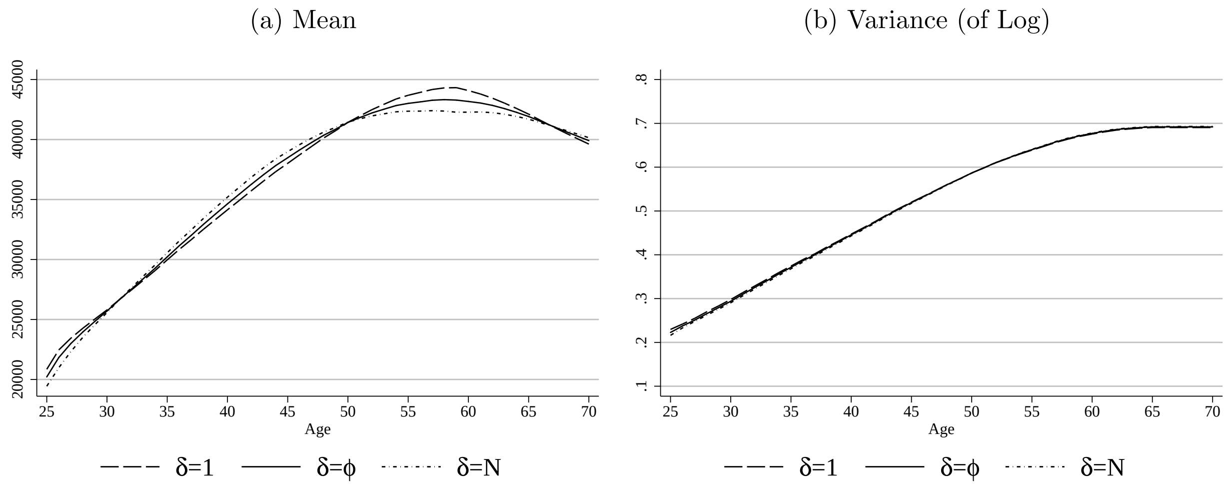
Income uncertainty during working life induces the households to accumulate precautionary savings and generates the upward sloping consumption profile during the first part of the life-cycle in all three simulations despite $\beta(1 + r) < 1$. The consumption profiles for all three simulations are very close to each other during working-life. The difference is on average less than 1%. The lower calibrated discount factor in the *Single Agent* generates a stronger hump-shaped consumption profile and makes mean consumption more different during retirement.[7]

The lower effective return to savings in the *Demographics* model make it more costly to insure against income shocks. This implies less consumption insurance in the *Demographics* model compared to the *Single Agent* model when holding fixed the discount factor between the two models, which results in a slightly higher variance of log (per-adult equivalent) consumption in the former compared to the latter. The lower calibrated discount factor in the *Single Agent* model shifts consumption more to the earlier years (in relative terms) which generates a 3% higher variance during retirement.

Finally, it is worthwhile to mention that per-adult equivalent consumption growth is higher in

[7]There are also level differences in mean per-adult equivalent consumption across models. The online appendix provides a theoretical explanation for this fact.

Figure 2: Per-Adult Equivalent Consumption: Different Specifications of the *Demographics* Model



our benchmark relative to the data as documented in Fernández-Villaverde and Krueger (2007): the peak age of per-adult equivalent consumption is in the early 50s in the data while it is around 57 in our model; at the peak age, per-adult equivalent consumption increases by 30% in the data relative to age 25, whereas in our setup, it more than doubles. On the other hand, the profile for the variance of log per-adult equivalent consumption is roughly in line with the data shown in Storesletten, Telmer, and Yaron (2004), being somewhat higher at the end of life in our simulation.

Overall, although the theoretical predictions of the previous section are validated, the quantitative differences between the three simulations are of second order for the mean consumption profiles and barely perceptible for consumption inequality.

4.2 Different Specifications of the *Demographics* Model

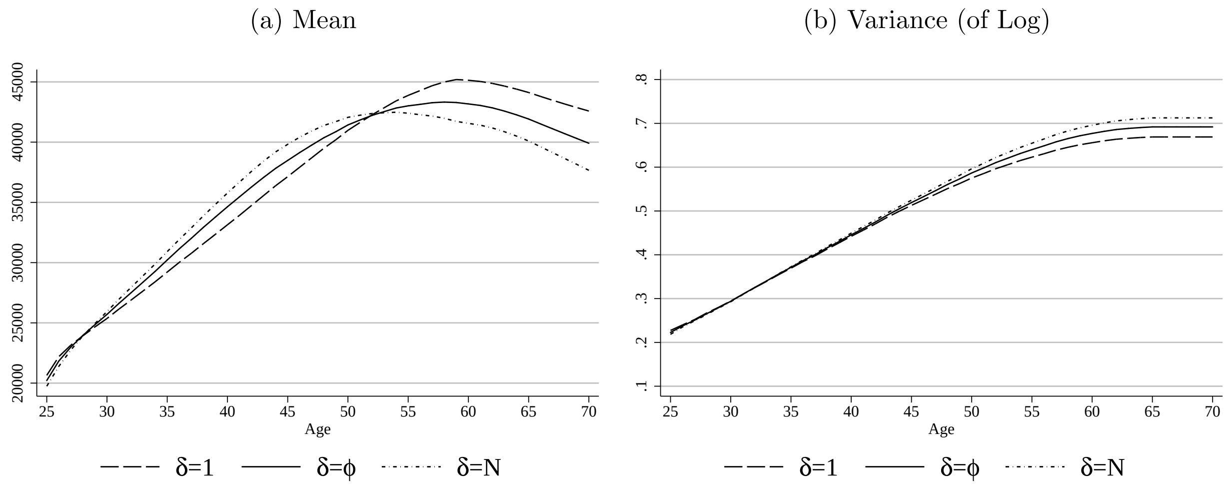
While the discussion of the two period model mainly focussed on the case of $\delta = \phi$, we think that the implications of the Euler equation (7) for different choices of δ deserve more attention in our quantitative analysis here as these vary widely in applied work: from $\delta = 1$ (as e.g. in Livshits, MacGee, and Tertilt (2007) and Attanasio, Low, and Sanchez-Marcos (2008)) to the intermediate cases of $\delta = \phi$ (as e.g. in Fuchs-Schündeln (2008) and Laitner and Silverman (2012)) and number of adults (Heathcote, Storesletten, and Violante (2012)) to household size (Domeij and Klein (2013)).

Recall that the Euler equation (7) implies that the household, ceteris paribus, allocates more per-adult equivalent consumption to periods in which household size is large if $\delta > \phi$ and vice versa. Different choices of the utility weight are associated with different relative prices of per-adult equivalent consumption between two periods whereas the effective interest rates are the same across all specifications. In our quantitative exercise, the discount factors vary across the different choices of δ in order to match for each simulation a wealth-income ratio of 3.1. On average this yields very similar profiles for per-adult equivalent consumption (this is also true for total household consumption) and virtually the same profiles for consumption inequality over the life-cycle, see Figures 2a and 2b respectively.

When $\delta = 1$, the utility that additional household members derive from consumption is not valued by the household decision maker, while at the same time, the scale effect is still in place (consumption has to be shared). When the utility weight equals household size ($\delta = N$) we have the opposite case: households want to allocate more consumption to periods when their size is bigger. In the 20s and around age 60, when household size is small relative to the 30s and 40s, per-adult equivalent consumption in the model with $\delta = 1$ is higher than when δ equals ϕ or 1. In contrast, when $\delta = N$, per-adult equivalent consumption is the highest among the three models when household size is the largest, i.e. during the 30s and 40s. However, the differences in mean per-adult equivalent consumption between these two 'extreme' cases never exceed 7% and are on average less than 1% during working life.

The different utility weights do not exhibit a stronger quantitative impact because of the macroeconomic discipline imposed by the wealth to income ratio target in the calibration stage. The calibrated discount factors are increasing in δ: $\beta = 0.949$ for $\delta = 1$, $\beta = 0.959$ for $\delta = \phi$ and $\beta = 0.969$ for $\delta = N$. In periods when household size is large, households need a stronger preference for current consumption to prevent them from saving too much when $\delta = 1$, whereas when $\delta = N$, households need to be more patient to prevent them from consuming too much. When we do not constrain the models to generate the same wealth to income ratio and hold the discount factor constant across models (we set all βs equal to 0.959), the choice of δ exhibits a stronger impact on the mean profile of per-adult equivalent consumption and also generates differences in consumption inequality, see Figures 3a and 3b. The differences in mean consumption between the setups with $\delta = 1$ and $\delta = N$ during working life are on average above 5% with a maximum of 9%, while the variance of log per-adult equivalent consumption during retirement reaches 6.5%. When $\delta = 1$, households now shift consumption to a larger degree to the later part of the life-cycle (smaller household size) and

Figure 3: Per-Adult Equivalent Cons.: Diff. Specifications of the *Demographics* Model ($\beta = 0.959$)



are more willing to consume comparably little even in the presence of low income realizations. The higher accumulated savings generate the lower consumption inequality relative to $\delta = \phi$ and $\delta = 1$.

To sum up, the choice of *Demographics* model (in terms of different utility weights δ's) can have important qualitative consequences for average and cross-sectional inequality of per-adult equivalent consumption. However, the quantitative importance of this channel is counteracted by the macroeconomic discipline imposed by the wealth to income ratio targeted in the calibration stage, making the theoretical differences seem of second order importance. This finding might explain the results in Livshits, MacGee, and Tertilt (2007), who setup a quantitative life-cycle model, very similar to the one considered here, extended by expenditures shocks (health or divorce) and the option to default on debt. In their baseline setting, they set $\delta = 1$ and show that welfare is lower under an EU-style bankruptcy law compared to the US law. In their robustness analysis they show that this result depends amongst other things on how demographics are modeled. Specifically, they solve their model without an effect of household size on the (marginal) utility of consumption which in fact is nothing else but decreasing the utility weight δ from one to $\phi^{1-\alpha}$ with CRRA preferences, see Equation (27). Under this setting, the EU-style bankruptcy law yields higher ex-ante welfare. This suggest that changes in the slope of the consumption profile induced by differences in δ may have important implications. However, Livshits, MacGee, and Tertilt (2007) do not recalibrate their model to match a certain aggregate moment and thereby potentially overstate the effect of δ on their results.

5 Conclusions

At least since the empirical work by Attanasio and Weber (1993) and Attanasio and Browning (1995) it is well understood that household size changes are important for understanding the patterns of household consumption over the life-cycle. In this paper we compare two widely used versions of the life-cycle model of consumption addressing this issue: the *Single Agent* model, in which household size is constant over the life-cycle and which is calibrated with a per-adult equivalent income; and the *Demographics* model, in which household size changes deterministically over the life-cycle and impacts the marginal utility of consumption, while household income is used in the calibration. Given the widely accepted notion that demographics should be taken into account for applied work, the objective of this paper is to document the implications of using different models for consumption and consumption inequality over the life-cycle.

While both models are obviously reduced-form approaches to more complicated models of the family, the *Demographics* model captures two channels that are absent in the *Single Agent* model: first, since tomorrow's assets have to be shared with other household members, the effective interest rate faced by the household varies over time and is lower than in the *Single Agent* model; second, the relative price of consumption across periods varies through changes in the cost of providing consumption to an additional household member versus the direct preference over an additional household member receiving utility from consumption.

We investigate the quantitative implications of these differences in a standard model of life-cycle consumption with income uncertainty and incomplete markets, similar to the one in Storesletten, Telmer, and Yaron (2004). Specifically, we embed both the *Demographics* and *Single Agent* model in this framework and calibrate them using information on income, household composition and other features of the US economy. All models are subject to the same *macroeconomic* restriction, in the sense of a common target for the wealth to income ratio. In order to match this ratio, each different specification (the *Single Agent* model or the different variations of the *Demographics* model) exhibits a different calibrated discount factor, which makes households in our model economies to save as much as their empirical counterparts do in the aggregate. Under this restriction, the qualitative differences arising from the different setups and assumptions are quantitatively negligible. The intuition behind this result is simple: when the model is disciplined by an aggregate wealth to income ratio target, the resulting discount factors offset the differences in the effective interest rate or in the relative price effect of consumption stemming from different utility effects. Put differently, while our theoretical results show that the *Demographics* model is conceptually preferable to the *Single Agent* model, our quantitative analysis demonstrates that it is largely irrelevant which model (specification) is used.

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Appendix

A Two Period Model

A.1 Proof of Proposition 2

The per-adult equivalent consumption profiles in the two approaches can only be the same if for these allocations the following condition holds

$$c_1^S = \eta \mathbb{C}_1^D \text{ and } c_{2,i}^S = \eta \frac{\mathbb{C}_{2,i}^D}{\phi_2} \;\forall\; i = l, h \text{ with } \eta > 0. \tag{29}$$

Without loss of generality, assume that $\{\mathbb{C}_1^D, \mathbb{C}_{2,l}^D, \mathbb{C}_{2,h}^D\}$ also satisfies the budget constraint (11) in the *Demographics* model, i.e. this allocation is optimal. The allocation (29) in the *Single Agent* model can however also only constitute an optimum if for the low and high income shock the respective budget constraints

$$c_1^S + c_{2,i}^S = \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \;\forall\; i = l, h \tag{30}$$

hold with equality, i.e. the allocation is feasible and no resources are wasted. Replacing condition (29) into the budget constraints of the *Single Agent* model (30) and replacing $\mathbb{C}_{2,i}^D$ with the respective budget constraint from the *Demographics* model yields after some reformulations

$$\mathbb{C}_1^D = \frac{1}{\phi_2 - 1} \frac{1}{\eta} \left[(\phi_2 - \eta)\, \mathbb{Y}_1 + \left(\frac{\phi_2}{\eta \kappa_2} - 1 \right) \mathbb{Y}_{2,i} \right] \;\forall\; i = l, h. \tag{31}$$

Equation (31) has to hold for the low and high period two income realization which can only be the case if

$$\eta = \frac{\phi_2}{\kappa_2}. \tag{32}$$

Using condition (32) we can solve Equation (31) for the κ_2 for which the per-adult equivalent consumption in the two approaches are the same:

$$\kappa_2 = 1 + (\phi_2 - 1) \frac{\mathbb{C}_1^D}{\mathbb{Y}_1}. \tag{33}$$

A.2 Proof of Proposition 4

Recall that our results are derived for $r = 0$ and $\beta = 1$. Without loss of generality, assume that $\{\mathbb{C}_1^D, \mathbb{C}_{2,l}^D, \mathbb{C}_{2,h}^D\}$ satisfies the Euler equation (9) and the budget constraint (11) in the *Demographics* model. Hence, the allocation in the *Demographics* model is optimal. We show that a consumption allocation for the *Single Agent* model exists that implies the same per-adult equivalent consumption profile as the *Demographics* model, satisfies the Euler equation (10) and for $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1}$ the budget constraint in the *Single Agent* model. Since we assume a strictly concave utility function this allocation has to be the unique optimum in the *Single Agent* model.

The profile in the *Single Agent* model is the same as in the *Demographics* model if

$$c_1^S = \eta \mathbb{C}_1^D \text{ and } c_{2,i}^S = \eta \frac{\mathbb{C}_{2,i}^D}{\phi_2} \ \forall \ i = l, h \text{ with } \eta > 0. \tag{34}$$

Plugging allocation (34) in the Euler equation (10) for the *Single Agent* model and using the homogeneity assumption yields

$$\eta^q u'\left(\mathbb{C}_1^D\right) = \eta^q \sum_{i=l,h} p_i u'\left(\frac{\mathbb{C}_{2,i}^D}{\phi_2}\right). \tag{35}$$

Comparing Equation (35) and the Euler equation in the *Demographics* model (9), it is obvious that both hold jointly at the allocation (34) if $\delta_2 = \phi_2$.

The next step is to show that for $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1}$ the budget constraint in the *Single Agent* model holds with equality for the allocation (34) for the low and high income shock. We start with the budget constraint in the *Demographics* model which has to hold for the low ($i = l$) and high ($i = h$) income:

$$\mathbb{Y}_1 + \mathbb{Y}_{2,i} = \mathbb{C}_1^D + \mathbb{C}_2^D \tag{36}$$

Now plug allocation (34) into Equation (36). The derivation of the critical value of κ_2 relied on $\eta = \frac{\phi_2}{\kappa_2}$ which we us as well in this step:

$$\begin{aligned} \mathbb{Y}_1 + \mathbb{Y}_{2,i} &= \frac{1}{\eta} c_1^S + \frac{\phi_2}{\eta} c_{2,i}^S = \frac{\kappa_2}{\phi_2} c_1^S + \kappa_2 c_{2,i}^S \\ \Rightarrow \ \frac{\mathbb{Y}_1}{\kappa_2} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} &= \frac{c_1^S}{\phi_2} + c_{2,i}^S \end{aligned} \tag{37}$$

Now plug allocation (34) in the critical value of κ_2 and solve for κ_2 (again using $\eta = \frac{\phi_2}{\kappa_2}$):

$$\begin{aligned} \kappa_2 &= 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1} = 1 + (\phi_2 - 1)\frac{1}{\eta}\frac{c_1^S}{\mathbb{Y}_1} = 1 + (\phi_2 - 1)\frac{\kappa_2}{\phi_2}\frac{c_1^S}{\mathbb{Y}_1} \\ \Rightarrow \ \kappa_2 &= \frac{1}{1 - \frac{\phi_2 - 1}{\phi_2}\frac{c_1^S}{\mathbb{Y}_1}} \end{aligned} \tag{38}$$

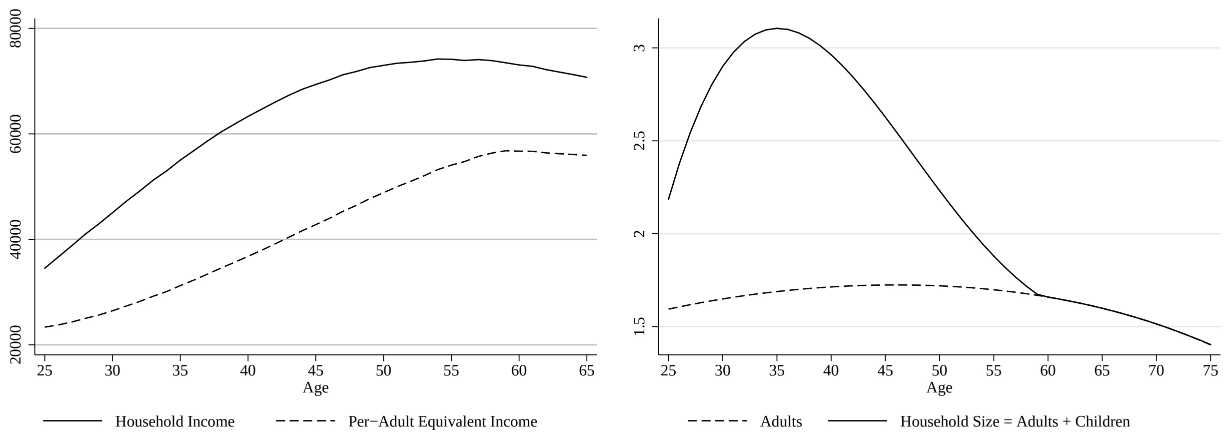
Inserting (38) in (37) for $\frac{\mathbb{Y}_1}{\kappa_2}$ yields after a few reformulations the budget constraint for low ($i = l$) and high ($i = h$) period two income realization in the *Single Agent* model (compare (30)):

$$\begin{aligned} \frac{c_1^S}{\phi_2} + c_{2,i}^S &= \frac{\mathbb{Y}_1}{\kappa_2} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \\ \frac{c_1^S}{\phi_2} + c_{2,i}^S &= \mathbb{Y}_1\left(1 - \frac{\phi_2 - 1}{\phi_2}\frac{c_1^S}{\mathbb{Y}_1}\right) + \frac{\mathbb{Y}_{2,i}}{\kappa_2} = \mathbb{Y}_1 - \mathbb{Y}_1\frac{\phi_2 - 1}{\phi_2}\frac{c_1^S}{\mathbb{Y}_1} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} = \mathbb{Y}_1 - c_1^S + \frac{c_1^S}{\phi_2} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \\ \Rightarrow \ c_1^S + c_{2,i}^S &= \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \end{aligned} \tag{39}$$

B Figures

Figure 4: Model Inputs

(a) Income (b) Household Size and Composition



Note: All profiles are constructed using data from the CPS, 1984-2003.