The Journal of Social Sciences Research
ISSN(e): 2411-9458, ISSN(p): 2413-6670
Special Issue. 6, pp: 52-58, 2018
URL: https://arpgweb.com/journal/journal/7/special_issue
DOI: https://doi.org/10.32861/jssr.spi6.52.58
Academic Research Publishing Group
Original Research Open Access

The Performance of Robust Multivariate Ewma Control Charts

F. Jamaluddin*
School ofQuantitative Sciences, UniversitiUtara Malaysia, 06010Sintok, Kedah, Malaysia

H. H. Ali
School ofQuantitative Sciences, UniversitiUtara Malaysia, 06010Sintok, Kedah, Malaysia

S. S. Syed Yahaya
School ofQuantitative Sciences, UniversitiUtara Malaysia, 06010Sintok, Kedah, Malaysia

Z. Zain
School ofQuantitative Sciences, UniversitiUtara Malaysia, 06010Sintok, Kedah, Malaysia
Center for Testing Measurement and Appraisal, UniversitiUtara Malaysia, 06010 Sintok, Kedah, Malaysia

Abstract

Multivariate Exponential Weighted Moving Average (MEWMA) control chart is a popular statistical tool for monitoring multivariate process over time. However, this chart is sensitive to the presence of outliers arising from the use of classical mean vector and covariance matrix in estimating the MEWMA statistic. These classical estimators are known to be sensitive to the outliers. To address this problem, robust MEWMA control charts based on modified one-step M-estimator (MOM) and Winsorized modified one-step M-estimator (WM) are proposed. Their performance is then compared with the standard MEWMA control chart in various situations. The findings revealed that the proposed robust MEWMA control charts are more effective in controlling false alarm rates especially for large sample sizes and high percentage of outliers.

Keywords: Multivariate Control Chart; Robust MEWMA control Chart; Outliers; Modified one-step M-estimator; Winsorized modified one-step M-estimator.

1. Introduction

HotellingT^2control chart is the earliest multivariate control chart developed and it is widely adopted in industries due to its simplicity and ease of implementation, which consider only current samples (Aparisi, 1996; Chen, 2007; Faraz *et al.*, 2016; Yeong *et al.*, 2016). This control chart is a multivariate version of the Shewhart control chart. However, the use of Hotelling $T^2$and Shewhart control charts are limited to processes with random pattern. The existence of non-random pattern in the process, cause both control charts beunable to identify out-of-control signal precisely (Mason and Young, 2002). Therefore, we cannot just simply use the control chart designed for random process on non-random process, as this will increase the risk of obtaining misleading result regardless whether the process is in-control or out-of-control. Failing to detect the existence of any non-random patterns in the process, render the Hotelling$T^2$and Shewhart control charts less sensitive to a small process mean shift (Klein, 2000; Koutras *et al.*, 2007; Niaki *et al.*, 2011; Rakitzis and Antzoulakos, 2016).

Exponential weighted moving average (EWMA) control chart which was proposed by Roberts (1959) is one of the exponential smoothing time series methods that is capable of handling non-random pattern that exists in a univariate process. This method is believed to be the most effective way to smooth out the non-random fluctuations which exist in a process (Borror *et al.*, 1999). In addition to that, the EWMA control chart performs better than Shewhart control chart in detecting the shifts in process mean when the size of the shift is small as it considers the information from not only the current sample, but also from the samples taken previously (Saleh *et al.*, 2015; Serel and Moskowitz, 2008; Serel, 2009; Yang, 2013).

In 1992, Lowry *et al.* (1992) proposed multivariate EWMA (hereafter known as MEWMA) control chart as an extension of the original EWMA control chart to be used in multivariate settings. The MEWMA chart proposed by Lowry *et al.* (1992) is based on the MEWMA statistic,E^2. Let$\boldsymbol{X}_i$represent p number of quality characteristics at time i. Then, theE^2statistic for$\boldsymbol{X}_i$is estimated as:

$$E_i^2 = \boldsymbol{Z}_i^t \boldsymbol{\Sigma}_{\boldsymbol{Z}_i}^{-1} \boldsymbol{Z}_i, \quad (1)$$

$$\boldsymbol{Z}_i = r\boldsymbol{X}_i + (1 - r)\boldsymbol{Z}_{i-1}, \quad (2)$$

$$\boldsymbol{\Sigma}_{\boldsymbol{Z}_i} = \frac{r}{(2-r)}\boldsymbol{\Sigma}_0, \quad (3)$$

*Corresponding Author

where $\boldsymbol{Z}_i$ is the MEWMA vectors, Σ_{Z_i} is the covariance matrix of $\boldsymbol{Z}_i$, r is a smoothing constant and $\boldsymbol{Z}_{i-1}$ equal to a process mean vector, $\boldsymbol{\mu}_0$.

This chart gives an out-of-control signal when the E^2 statistic is greater than the upper control limit, h. The h value is the value used to achieve the desired false alarm rate, α. The MEWMA control chart designed by Lowry *et al.* (1992) assumes known process mean vector, $\boldsymbol{\mu}_0$ and covariance matrix, Σ_0, used in estimating the E^2 statistics as in Equation (1). However, these two parameters, $\boldsymbol{\mu}_0$ and Σ_0 are often unknown and are estimated from the sample mean vector, $\overline{\boldsymbol{X}}$ and sample covariance matrix, $\boldsymbol{S}$. When using sample mean vector and covariance matrix, Equation (1) can be rewritten as follows:

$$E_i^2 = (\boldsymbol{Z}_i - \overline{\boldsymbol{X}})^t S_{\boldsymbol{Z}_i}^{-1} (\boldsymbol{Z}_i - \overline{\boldsymbol{X}}), \quad (4)$$

where $\boldsymbol{Z}_i = r\boldsymbol{X}_i + (1-r)\,\boldsymbol{Z}_{(i-1)}$, $\boldsymbol{Z}_0 = \overline{\boldsymbol{X}}$ and $\boldsymbol{S}_{\boldsymbol{Z}i} = r/(2-r)\boldsymbol{S}$.

However, the problem arises when the classical sample mean vector and sample covariance matrix used in Equation (4) are very sensitive to outliers (Alfaro and Ortega, 2009; Ali and Syed Yahaya, 2013; Haddad *et al.*, 2013; Vargas, 2003). Outlier as an observation that appears to deviate markedly from remaining data (Rousseeuw and Van Zomeren, 1990). The statistics based on the classical sample mean vector and covariance matrix are hardly able to detect all the multivariate outliers in a given sample with the consequence that any methods based on classical estimators are not suitable for general use, unless there is certainty that outliers are not present (Filzmoser *et al.*, 2008). Asalternative, past researchers used robust location estimators in control chart and these charts are known as robust control charts. For example, the use of trimmed mean of the sample means, median of the sample means, mean of the sample medians, median of the sample medians and trimmed mean of the sample trimeans in EWMA control chart (Zwetsloot *et al.*, 2014); (Zwetsloot *et al.*, 2016). These EWMA and robust EWMA control charts were investigated under four different contaminated data such as localized shifts, diffuse shifts, structural shifts and random shifts. Their findings indicated that the EWMA control chart using median function or the trimean based estimators yield the best results under localized, structural or random shifts present in Phase I. However, none of the control charts performed well in the present of diffuse shifts.

Alternatively, some researchers used robust scale estimators in EWMA control charts. For instance, Khoo and Sim (2006) compared the performance of EWMA control chart with robust EWMA control chart based on interquartile range. They concluded that the robust control chart is a superior alternative to the EWMA control chart in the presence of outliers. Another research proposed robust EWMA control charts based on six scale estimators such as Gini's mean difference, G, median absolute deviation about the median, MAD, Q_n, S_n, Tau $\hat{\tau}$ and FQ_n. The performance of the proposed robust EWMA control charts are evaluated under several normal and non-normal distributions such as gamma and exponential. Comparing the six robust scale estimators, the EWMA control chart based on Q_n estimator is relatively more sensitive in detecting the out-of-control signal for both normal and non-normal processes (Saeed and Kamal, 2016).

Meanwhile, in multivariate setting, (Midi and Shabbak, 2011) compared the performance of MEWMA control chart with their proposed robust MEWMA based on minimum volume ellipsoid (MVV) and minimum covariance determinant (MCD). They investigated the performance of all control charts under different percentage of outliers, $\boldsymbol{\varepsilon}$ = 5%, 10%, 15% and 20% and found that the robust control charts outperformed the MEWMA control chart, produced high probability of detection regardless of $\boldsymbol{\varepsilon}$ except for $\boldsymbol{\varepsilon}$ = 5%. Although the robust MEWMA control charts outperformed the MEWMA control chart, they produced low probability of detections which are less than 0.4.

Trimmed mean, modified one-step M-estimator (MOM) and Winsorized MOM (WM) are among the best robust estimators adopted by past researchers (Alfaro and Ortega, 2009); (Haddad *et al.*, 2013); (Boente and Vahnovan, 2017); (Nazir *et al.*, 2016). However, the trimmed mean is very problematic, owing to unnecessary or inadequate trimming which is common when adopting this estimator (Wilcox and Keselman, 2003). The ability of WM estimator in controlling the false alarm rates has been confirmed in Hotelling T^2 control chart (Haddad *et al.*, 2013), but not yet tested on MEWMA control chart. Therefore, this paper proposed robust MEWMA control charts using MOM and WM robust estimators. The performance of proposed robust MEWMA control charts are evaluated in term of false alarm rate under various bivariate and multivariate contaminated data. Apart from that, their performance is also compared with the standard MEWMA control chart.

2. Methodology

2.1. Robust MEWMA Control Charts

The sample mean vector and sample covariance matrix used in Equation 4 are sensitive to outliers Ali and Syed Yahaya (2013). To alleviate the problem, in this study, the classical mean vector and covariance matrix are replaced with robust estimators of mean vector and covariance matrix based on modified one-step M-estimator (MOM) and Winsorized MOM (WM).

2.1.1. Robust MEWMA Control Chart based on MOM Estimator, RE^2_{MOM}

The MOM estimator proposed by Wilcox and Keselman (2003) is defined as

$$MOM_j = \left(\sum_{i=i_1+1}^{n_j-i_2} X_{ij} \right) / (n_j - i_1 - i_2) \tag{5}$$

where X_{ij} = i^{th}order statistic in j^{th}quality characteristic variable.

i_1 = Number of X_{ij}that satisfies the criterion ($X_{ij} - \hat{M}_j < -K$*MAD_{nj}) (6)

i_2 = Number of X_{ij}that satisfies the criterion ($X_{ij} - \hat{M}_j > K$*MAD_{nj}) (7)

n_j= Number of observations in each j^{th}quality characteristic variable.

$$\hat{M}_j = \text{med}\ \{X_{1j}, \ldots, X_{nj}\},\ j = 1, \ldots, p$$

$$MAD_{nj} = 1.4826 * \text{med}\ \{|X_{ij} - \hat{M}_j|\} \tag{8}$$

For better efficiency and reasonably smaller standard error under normality, the constant K was adjusted to 2.24 (Wilcox, 2003). The efficiency improved (i.e., 0.9 and 0.88) when the K value is equal to 2.24 for n = 20 and 10 respectively. $MAD_n$is a scale estimator with the best possible breakdown point and bounded influence function (Rousseeuw and Croux, 1993). The simplicity of its formula and fast computation time are among other advantages of MAD_n.

The robust MEWMA, RE^2_{MOM}statistic for $X_i$is estimated as follows:

$$RE^2_{MOMi} = (Z_{MOMi} - MOM)^t S^{-1}_{ZPi} (Z_{MOMi} - MOM), \tag{9}$$

where $Z_{MOMi}$is the robust MEWMA vectors and $S_{ZPi}$is the covariance matrix ofZ_{MOMi}. The estimators Z_{MOMi} and $S_{ZPi}$are defined as in Equation 10 and Equation 11 respectively.

$$Z_{MOMi} = rX_i + (1-r)\ Z_{MOM(i-1)}, \tag{10}$$

and

$$S_{ZPi} = r/(2-r)\ S_P, \tag{11}$$

whereS_P is the corresponding covariance matrix of MOM, estimated using the product of Spearman correlation coefficient, ρ and rescale median absolute deviation, MAD_n as per Equation 12:

$$S_P = \begin{bmatrix} MAD^2_{n1} & \cdots & \rho_{1p} MAD^2_{n1p} \\ \vdots & \ddots & \vdots \\ \rho_{1p} MAD^2_{np1} & \cdots & MAD^2_{np} \end{bmatrix}. \tag{12}$$

2.1.2. Robust MEWMA Control Chart based on WM Estimator, RE^2_{WM1}

The mean vector and covariance matrix of WM estimators given by Haddad *et al.* (2013) are estimated as follows:

$$WM_j = \sum_{i-1}^{m_j} W_{ij} / m_j, \tag{13}$$

and

$$S_{WM}(w_i, w_j) = \frac{1}{m-1} \sum_{k=1}^{m} (w_{ki} - WM_i)(w_{kj} - WM_j), \tag{14}$$

where W_{ij} is Winsorized sample. The Winsorized sample is obtained through two-step process; first is to identify the existence of outlier based on trimming criterion used for MOM estimator as Equation (5) and then the data are Winsorized.

After eliminating the outliers from each sample using criteria (6) and (7), the data are then Winsorized. For each random variable X_{ij}= {X_{1j},...,X_{nj}}, j = 1, …, p, the sample is Winsorized as follows:

$$W_{ij} = \begin{cases} X_{(i_1+1)j}, & if \quad X_{ij} \le X_{(i_1+1)j} \\ X_{ij}, & if \quad X_{(i_1+1)j} < X_{ij} < X_{(n-i_2)j} \\ X_{(n_j-i_2)j}, & if \quad X_{ij} \ge X_{(n-i_2)j} \end{cases}$$

where

i_1: Number of the smallest outliers in the data

i_2: Number of the largest outliers in the data

Thus, the robust MEWMA RE^2_{WM1} statistic for $\boldsymbol{X}_i$ is estimated as follows:

$$RE^2_{WM1i} = (\boldsymbol{Z}_{WMi} - WM)^t \boldsymbol{S}^{-1}_{ZWMi} (\boldsymbol{Z}_{WMi} - WM), \quad (15)$$

where $\boldsymbol{Z}_{WM1}$ is the robust MEWMA vectors and $\boldsymbol{S}_{ZWMi}$ is the covariance matrix of $\boldsymbol{Z}_{WM1}$. The estimators $\boldsymbol{Z}_{WM1}$ and $\boldsymbol{S}_{ZWMi}$ are defined as Equation 16 and Equation 17 respectively:

$$\boldsymbol{Z}_{WMi} = r\boldsymbol{X}_i + (1-r)\, \boldsymbol{Z}_{WM(i-1)}, \quad (16)$$

and

$$\boldsymbol{S}_{ZWMi} = r/(2-r)\, \boldsymbol{S}_{WM}. \quad (17)$$

2.1.3. Robust MEWMA Control Chart based on WM Estimator, RE^2_{WM2}

The RE^2_{WM2} control chart is based on WM estimator as a mean vector and $\boldsymbol{S}_P$ as its corresponding covariance matrix. The WM estimator and $\boldsymbol{S}_P$ are obtained from Equation 13 and Equation 12 respectively. Then, the robust MEWMA RE^2_{WM2} statistic for $\boldsymbol{X}_i$ is estimated as follows:

$$RE^2_{WM2i} = (\boldsymbol{Z}_{WMi} - WM)^t \boldsymbol{S}^{-1}_{ZPi} (\boldsymbol{Z}_{WMi} - WM). \quad (18)$$

2.2. Control Limitsof MEWMA Control Charts

The control limits of the standard and robust MEWMA control charts are estimated using the Monte Carlo simulation method. In this study, Phase I involves the simulation of 5000 data sets from standard multivariate normal distribution $MVN_p\ (0, \boldsymbol{I}_p)$ when false alarm rate, $\alpha = 0.05$. Then, the classical and robust mean vector and covariance matrix for each data set were calculated. Next, in Phase II, we generated an additional observation for each data set and calculated the MEWMA statistics for these observations using the corresponding estimators from Phase I. The control limits for all MEWMA control charts were computed by taking the 95th percentile of 5000 values of MEWMA statistics. Table I reported the upper control limit, h values at $\alpha = 0.05$ of all the investigated MEWMA control charts for different p, n with a given smoothing constant, $r = 0.2$.

2.3. Simulation Design

The MEWMA control charts were investigated and compared in terms of false alarm rate under various conditions to accentuate the strength and weakness of the investigated control charts. Group sizes n = 30, 50, 200 and 400 observations with p = 2, and n = 50, 70, 200 and 400 observations with p = 10 number of dimensions were generated from the mixture normal distribution. The p = 2 and p = 10 number of dimensions represent the bivariate and multivariate data respectively. The mixture of normal distribution suggested in [34]is as follows:

$$(1-\varepsilon)N_p(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0) + \varepsilon N_p(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1) \quad (19)$$

where ε is the proportion of outliers; $\boldsymbol{\mu}_0$ and Σ_0 are the in-control parameters; while $\boldsymbol{\mu}_1$ and Σ_1 are the out-of-control parameters. The covariance matrix, Σ_0 and Σ_1 in Equation 19 represent the identity matrix of p dimensions ($\boldsymbol{I}_p$), as we assume contamination with shift in the mean but no changes in covariance structure. Two different values of percentage of outliers, ε = 5% and 20% and four values of process mean shift, $\boldsymbol{\mu}_1$ = 0.5,1.0,1.5 and 2.0 are considered in creating various conditions. The manipulation of the ε and $\boldsymbol{\mu}_1$ produced 8 types of contaminated data distributions, with estimated control limits shown in Table 1.

The following steps represent the simulation method used in Phase I and Phase II in establishing MEWMA control charts. The in-control parameters which are used together with control limits to develop the control chart are estimated in Phase I control chart. The simulation procedure is as follows:

1. Generate 1000 dataset with n = 30, 50, 200 and 400 and n = 50, 70, 200 and 400 for p = 2 and 10 respectively from the models defined in Equation 18.
2. Then, compute the classical and robust mean vector and covariance matrix for each dataset.

Meanwhile, in Phase II, the false alarm rate based on the estimations in Phase I are determined through the following steps:

1. Randomly generate a new observation from the in-control parameter, $MVN_p\ (0, \boldsymbol{I}_p)$ in Equation 18 and calculate the MEWMA statistics for each new observation using mean vector and covariance matrix obtained in Phase I.
2. Compare the values in step 1 against the control limits obtained in the simulation procedure.
3. The false alarm rate is calculated as the proportion of MEWMA statistics obtained in step 1 that is greater than the control limit in 1000 replications.

3. Results and Discussion

The comparison of empirical results of false alarm rates for E^2, RE^2_{MOM}, RE^2_{WM1} and RE^2_{WM2} control charts under various conditions are presented in Table 2 and 5. The Bradley's liberal criterion of robustness is used to evaluate the robustness of the control charts. According to this criterion, a control chart is considered robust if its empirical false alarm rate is within the robust interval of 0.5α to 1.5α (Bradley, 1978). Therefore, when the nominal value is set at α = 0.05, the control chart is considered robust if its false alarm rate is within robust interval, 0.025 to 0.075. In each table, the false alarm rate values that lie within the robustness interval are bolded. The best control chart is the one that is capable to control the false alarm rates within robust interval and produces the closest false alarm rate to nominal value, 0.05. Thus, in addition to the bolded value, the highlight value in Table 2 and 4 indicated the closest false alarm rates to nominal value.

Table-1. Estimated Control Limits

p	***n***	**Control Limits**			
		E^2	RE^2_{MOM}	RE^2_{WM1}	RE^2_{WM2}
2	30	8.869	11.608	332.376	10.936
	50	7.713	8.809	405.564	8.530
	200	6.392	6.738	1147.980	6.632
	400	6.002	6.183	2152.360	6.121
10	50	30.038	34.700	4786.619	33.426
	70	25.776	28.534	5494.960	27.595
	200	20.542	21.317	11752.120	21.160
	400	19.289	19.619	20534.040	19.541

Table-2. False alarm rates for bivariate case, $p = 2$

n	**% of ε**	μ_1	E^2	RE^2_{MOM}	RE^2_{WM1}	RE^2_{WM2}	***n***	**% of ε**		μ_1	E^2	RE^2_{MOM}	RE^2_{WM1}	RE^2_{WM2}
30	5%	0.5	**0.044**	**0.039**	**0.041**	**0.038**	200	5%		0.5	**0.059**	**0.057**	**0.053**	**0.053**
		1.0	**0.045**	**0.040**	**0.036**	**0.039**				1.0	**0.052**	**0.053**	**0.046**	**0.055**
		1.5	**0.041**	**0.038**	**0.032**	**0.041**				1.5	**0.051**	**0.049**	**0.040**	**0.054**
		2.0	**0.035**	**0.039**	**0.030**	**0.041**				2.0	**0.048**	**0.047**	**0.042**	**0.054**
	20%	0.5	**0.069**	**0.070**	**0.054**	**0.067**		20%		0.5	**0.066**	**0.057**	**0.042**	**0.055**
		1.0	**0.071**	**0.068**	**0.044**	**0.069**				1.0	**0.068**	**0.064**	**0.038**	**0.068**
		1.5	**0.069**	**0.061**	**0.027**	**0.063**				1.5	0.076	**0.064**	**0.026**	0.076
		2.0	**0.063**	**0.050**	0.014	**0.053**				2.0	0.076	**0.056**	0.021	0.084
50	5%	0.5	**0.049**	**0.047**	**0.058**	**0.050**	400	5%		0.5	**0.059**	**0.063**	**0.066**	**0.067**
		1.0	**0.045**	**0.043**	**0.055**	**0.044**				1.0	**0.056**	**0.058**	**0.058**	**0.057**
		1.5	**0.045**	**0.044**	**0.053**	**0.040**				1.5	**0.053**	**0.053**	**0.050**	**0.055**
		2.0	**0.042**	**0.044**	**0.041**	**0.038**				2.0	**0.047**	**0.053**	**0.047**	**0.055**
	20%	0.5	**0.049**	**0.048**	**0.046**	**0.052**		20%		0.5	**0.068**	**0.061**	**0.048**	**0.064**
		1.0	**0.059**	**0.049**	**0.034**	**0.056**				1.0	0.076	**0.072**	**0.043**	0.077
		1.5	**0.055**	**0.042**	**0.026**	**0.046**				1.5	0.085	**0.071**	0.020	0.085
		2.0	**0.054**	**0.044**	0.018	**0.051**				2.0	0.091	**0.070**	0.017	0.095

Table-3. Total bolded and highlight values for bivariate case, $p = 2$

Control Charts	E^2	RE^2_{MOM}	RE^2_{WM1}	RE^2_{WM2}
Total bolded	27	32	27	27
Total highlighted	10	9	10	8

Table 2 and Table 3 represented detail result of the false alarm rates and summary of the total bolded and highlighted values for bivariate case respectively. As reported in Table 2 and Table 3, theRE^2_{MOM} outperformedRE^2_{WM1},RE^2_{WM2} andE^2control charts.TheRE^2_{MOM}control chart is capable in controlling false alarm rate under all simulated conditions which is about 100% (32 out of 32) of the conditions as compared toE^2, RE^2_{WM1} and RE^2_{WM2}which are only effective for 84% (27 out of 32) of the conditions. Even though theE^2 and RE^2_{WM1}control charts produced the highest values of the closest false alarm rates to nominal level (refer Table 3) which are under 10 simulated conditions, their overall performance is greatly affected by large sample size (n = 200 and n = 400) with high percentage of outlier (ε = 20%), as verified by the rates of false alarm far above and below nominal value, 0.05. From the total bolded and highlighted values, we can conclude that the performance of theRE^2_{MOM}control chart is deemed superior than the remaining MEWMAcontrol charts for bivariate case.

Table-4. False alarm rates for multivariate case, p = 10

n	% of ε	μ_1	E^2	RE^2_{MOM}	RE^2_{WM1}	RE^2_{WM2}	n	% of ε	μ_1	E^2	RE^2_{MOM}	RE^2_{WM1}	RE^2_{WM2}
50	5%	0.5	**0.049**	**0.043**	**0.060**	**0.045**	200	5%	0.5	**0.053**	**0.061**	**0.055**	**0.046**
		1	**0.042**	**0.038**	**0.050**	**0.039**			1	**0.045**	**0.055**	**0.049**	**0.046**
		1.5	**0.038**	**0.042**	**0.051**	**0.038**			1.5	**0.041**	**0.052**	**0.039**	**0.044**
		2	**0.038**	**0.041**	**0.054**	**0.039**			2	**0.038**	**0.048**	**0.034**	**0.046**
	20%	0.5	**0.050**	**0.056**	**0.057**	**0.053**		20%	0.5	**0.074**	**0.064**	**0.040**	**0.063**
		1	**0.050**	**0.053**	**0.030**	**0.059**			1	**0.075**	**0.069**	**0.025**	**0.070**
		1.5	**0.050**	**0.059**	0.016	**0.057**			1.5	0.081	**0.067**	0.007	**0.070**
		2	**0.048**	**0.053**	0.006	**0.054**			2	0.076	**0.061**	0.006	**0.075**
70	5%	0.5	**0.050**	**0.048**	**0.057**	**0.053**	400	5%	0.5	**0.053**	**0.053**	**0.061**	**0.052**
		1	**0.051**	**0.049**	**0.042**	**0.051**			1	**0.050**	**0.053**	**0.054**	**0.049**
		1.5	**0.049**	**0.050**	**0.035**	**0.050**			1.5	**0.045**	**0.049**	**0.047**	**0.049**
		2	**0.048**	**0.045**	**0.030**	**0.054**			2	**0.045**	**0.046**	**0.041**	**0.053**
	20%	0.5	**0.063**	**0.052**	**0.058**	**0.057**		20%	0.5	**0.065**	**0.064**	**0.044**	**0.064**
		1	**0.067**	**0.060**	**0.033**	**0.064**			1	**0.074**	**0.071**	**0.032**	**0.073**
		1.5	**0.067**	**0.051**	0.019	**0.058**			1.5	**0.074**	**0.059**	0.016	**0.070**
		2	**0.065**	**0.065**	0.011	**0.062**			2	**0.073**	**0.057**	0.009	**0.073**

Table-5. Total bolded and highlight values for multivariate case, p = 10

Control Charts	E^2	RE^2_{MOM}	RE^2_{WM1}	RE^2_{WM2}
Total bolded	30	32	24	32
Total highlighted	10	13	7	6

When the dimension is increased to p = 10, the RE^2_{MOM} control chart successfully maintains its superior performance even in the case of multivariate data (refer to Table 4 and Table 5). Again, the RE^2_{MOM} control chart is capable to control false alarm rate for all simulated conditions. Besides, it also produced the large values of false alarm rate closest to nominal level which is about 13 conditions as displayed in Table 5. Apart from that, some improvements are observed in the E^2 and RE^2_{WM2} control charts for multivariate data as compared to the bivariate data especially for large sample size, n = 400 and 20% of outliers. However, the increase in the dimension from p = 2 to p = 10 does influence the performance of RE^2_{WM1} control chart. The RE^2_{WM1} control chart is less robust under multivariate case as proven by the reduction in controlling the false alarm rates from 27 conditions (refer Table 3) to 24 conditions (refer Table 5).

4. Conclusion

This paper examined the performance of three robust alternatives to MEWMA control charts in statistical quality control. The robust MEWMA control chart have been developed to protect the MEWMA control chart when outliers are present in Phase I. By means of a simulation study, we have analysed and compared the performance of the E^2, RE^2_{MOM}, RE^2_{WM1} and RE^2_{WM2} control charts under various conditions. Overall, the robust MEWMA control charts are proven capable to improve the performance of the standard MEWMA control chart especially for large sample sizes and high percentage of outliers.

5. Acknowledgement

The authors are gratefully acknowledged Universiti Utara Malaysia and Ministry of Education Malaysia for the opportunity to present this work.

References

Alfaro, J. L. and Ortega, J. F. (2009). A comparison of robust alternatives to Hotelling's T2 control chart. *J. Appl. Stat.*, 36(12): 1385–96.

Ali, H. and Syed Yahaya, S. S. (2013). On robust mahalanobis distance issued from minimum vector variance. *Far East J. Math. Sci.*, 74(2): 249–68.

Aparisi, F. (1996). Hotelling's T2 control chart with adaptive sample sizes. *Int. J. Prod. Res.*, 34(10): 2853–62.

Boente, G. and Vahnovan, A. (2017). Robust estimators in semi-functional partial linear regression models. *J. Multivar. Anal.*, 154: 59–84.

Borror, C. M., Montgomery, D. C. and Runger, G. C. (1999). Robustness of the EWMA control chart to non-normality. *J. Qual. Technol.*, 31(3): 309–16.

Bradley, J. V. (1978). Robustness? *Br. J. Math. Stat. Psychol.*, 31(2): 144–52.

Chen, Y. K. (2007). Adaptive sampling enhancement for Hotelling's T2 charts. *Eur. J. Oper. Res.*, 178(3): 841–57.

Faraz, A., Chalaki, K., Saniga, E. M. and Heuchenne, C. (2016). The robust economic statistical design of the hotelling's T2 chart. *Commun. Stat. Theory Methods*, 45(23): 6989–7001.

Filzmoser, P., Maronna, R. and Werner, M. (2008). Outlier identification in high dimensions. *Comput. Stat. Data Anal.*, 52(3): 1694–711.

Haddad, F. S., Syed Yahaya, S. S. and Alfaro, J. L. (2013). Alternative Hotelling's T2 charts using winsorized modified one-step M-estimator. *Qual. Reliab. Eng. Int.,* 29(4): 583–93.

Khoo, M. B. C. and Sim, S. Y. (2006). A robust exponentially weighted moving average control chart for the process mean. *J. Mod. Appl. Stat. Methods,* 5(2): 464–74.

Klein, M. (2000). Two alternatives to the Shewhart X control chart. *J. Qual. Technol.,* 32(4): 427–31.

Koutras, M. V., Bersimis, S. and Maravelakis, P. E. (2007). Statistical process control using Shewhart control charts with supplementary runs rules. *Methodol. Comput. Appl. Probab.,* 9(2): 207–24.

Lowry, C. A., Woodall, W. H., Champ, C. W. and Ridgon, S. E. (1992). A multivariate exponentially weighted moving average control chart. *Technometrics,* 34(1): 46–53.

Mason, R. and Young, J. C. (2002). Multivariate statistical process control with industrial applications. *SIAM,* 9:

Midi, H. and Shabbak, A. (2011). Robust multivariate control charts to detect small shifts in mean. *Math. Probl. Eng.,* 2011:

Nazir, H. Z., Abbas, N., Riaz, M. and Does, R. J. M. M. (2016). A comparative study of memory-type control charts under normal and contaminated normal environments. *Qual. Reliab. Eng. Int.,* 32(4): 1347–56.

Niaki, S. T. A., Malaki, M. and Ershadi, M. J. W. (2011). A particle swarm optimization approach on economic and economic-statistical designs of MEWMA control charts. *Sci. Iran.,* 18(6): 1529–36.

Rakitzis, A. C. and Antzoulakos, D. L. (2016). Run sum control charts for the monitoring of process variability. *Qual. Technol. Quant. Manag.,* 13(1): 58–77.

Roberts, S. W. (1959). Control chart tests based on geometric moving averages. *Technometrics,* 1(3): 239–50.

Rousseeuw, P. J. and Van Zomeren, B. C. (1990). Unmasking multivariate outliers and leverage points. *J. Am. Stat. Assoc.,* 85(441): 633–39.

Rousseeuw, P. J. and Croux, C. (1993). Alternative to the median absolute deviation. *Am. Stat. Assoc.,* 88(424): 1273–83.

Saeed, N. and Kamal, S. (2016). The EWMA control chart based on robust scale estimators. *Pakistan J. Stat. Oper. Res.,* 12(4): 659–72.

Saleh, N. A., Mahmoud, M. A., Jones-Farmer, A., Zwetsloot, I. M. and Woodall, W. H. (2015). Another look at the EWMA control chart with estimated parameters. *J. Qual. Technol.,* 47(4): 363–82.

Serel, D. A. (2009). Economic design of EWMA control charts based on loss function. *Math. Comput. Model.,* 49(3-4): 745–59.

Serel, D. A. and Moskowitz, H. (2008). Joint economic design of EWMA control charts for mean and variance. *Eur. J. Oper. Res.,* 184(1): 157–68.

Vargas, N. J. (2003). Robust estimation in multivariate control charts for individual observations. *J. Qual. Technol.,* 35(4): 367–76.

Wilcox, R. R. (2003). Multiple comparisons based on a modified one-step M-estimator. *J. Appl. Stat.,* 30(10): 1231–41

Wilcox, R. R. and Keselman, H. J. (2003). Repeated measures one-way ANOVA based on a modified one-step M-estimator. *Br. J. Math. Stat. Psychol.,* 56(1): 15–25.

Yang, S. (2013). Using a new VSI EWMA average loss control chart to monitor changes in the difference between the process mean and target and / or the process variability. *Appl. Math. Model.,* 37(16-17): 7973–82.

Yeong, W. C., Khoo, M. B. C., Lim, S. L. and Castagliola, P. (2016). The economic and economic-statistical designs of the hotelling's T2 chart based on the expected average run length. *Qual. Eng.,* 28(4): 416–28.

Zwetsloot, I. M., Schoonhoven, M. and Does, R. J. M. M. (2014). A robust estimator for location in phase i based on an EWMA chart. *J. Qual. Technol.,* 46(4): 302–16.

Zwetsloot, I. M., Schoonhoven, M. and Does, R. J. M. M. (2016). Robust point location estimators for the EWMA control chart. *Qual. Technol. Quant. Manag.,* 13(1): 29–38.