Proposed EN 1992 tension lap strength equation for good bond

Robert Vollum[1]*, Charles Goodchild[2]

1. Department of Civil and Environmental Engineering, Imperial College London, London, SW7 2AZ, UK
2. The Concrete Centre, Gillingham House, 38-44 Gillingham Street, London, SW1V 1HU

* Corresponding author, r.vollum@imperial.ac.uk

Abstract

The paper is concerned with the design of tension laps in reinforced concrete structures. The most recent codified design recommendations for reinforcement laps and anchorages are found in fib Model Code 2010 (MC2010). These recommendations have heavily influenced the draft revision of EN 1992 which is due for publication in 2023. The draft EN 1992 proposal for tension laps is still under development with the main point of discussion being the basic multiplier required to achieve the level of safety prescribed by EN 1990. This is contentious since laps designed to MC2010 can be significantly longer than laps designed to EN 1992 (2004) which many UK designers consider excessive in comparison with previous UK practice. The paper examines the safety of tension laps and proposes a refined design equation for inclusion in the 2023 revision to EN 1992. The proposed design equation achieves the level of safety required by EN 1990 whilst giving lap and anchorage lengths more consistent with current practice than MC2010.

Keywords

Reinforced concrete design, tension laps, Eurocode 2

1 Introduction

EN 1992 (2004) [1] is undergoing a substantial revision which is due to be published in 2023. The draft revision of EN 1992 [2] includes lap and anchorage rules based on the recommendations of fib Bulletin 72 [3] and fib Model Code 2010 (MC2010) [4]. By way of background, MC2010 requires significantly longer laps than EN 1992 (2004) [1], which Cairns and Elighausen [5] showed to have less than the expected margin of safety. Any increase in lap lengths is of concern to UK engineers who already find that the reinforcement detailing requirements of EN 1992 complicate construction and increase project costs compared with previous UK practice [6, 7].

The introduction to the paper describes the design provisions for tension laps in EN 1992, fib Bulletin 72 and MC2010. Subsequently, it describes a reliability analysis carried out by Mancini et al. [8] to determine a suitable safety format for the design of tension laps using equation 3-2 of fib Bulletin 72. The reliability based method of Mancini et al. [8] is used to develop a refined design equation for tension laps which is suggested for inclusion in the 2023 revision to EN 1992. The refined equation proposed in this paper gives full strength lap lengths which are more consistent with current practice than MC2010 and shorter than Mancini et al. [8]. The reader's attention is drawn to Table 6 where the various design proposals are summarised and illustrated with a numerical example.

1.1 EN1992-1-1 [1]

The current EN-1992 [1] lap and anchorage rules are based on guidance given in CEB-FIP Model Code 90 (MC90) [9]. The background to the EN 1992 rules is described by Cairns and Elighausen [5] in their detailed safety assessment of the rules. EN 1992 requires adjacent laps to be staggered by $0.3l_{bd}$ where l_{bd} is the design lap length which is given by:

$$l_{bd} = \alpha_1\alpha_2\alpha_3\alpha_5\alpha_6 l_{b,rqd} \quad [1]$$

where α_1 = 1.0 for straight bars

$\alpha_2 = 1.0 - 0.15(c_d-\phi)/\phi \geq 0.7$ & ≤ 1.0

$\alpha_3 = 1.0 - K\lambda \geq 0.7$ & ≤ 1.0

α_5 = 1.0 for no confining pressure

α_6 = 1.5 for >50% lapped

In which

c_d = min(clear bar spacing/2, side cover, bottom cover)

$\lambda = (\Sigma A_{st}-\Sigma A_{stmin})/A_s$

ΣA_{st} cross-sectional area of the transverse reinforcement along the design anchorage length l_{bd}.

$\Sigma A_{st,min}$ cross-sectional area of the minimum transverse reinforcement $=0.25A_s$ for beams and 0 for slabs.

A_s area of a single anchored bar with maximum bar diameter

K = 0.1 for bar in corner of link, 0.05 if anchored reinforcement is in layer above transverse reinforcement and 0 otherwise.

$$l_{b,rqd} = \frac{\emptyset}{4}\frac{\sigma_{sd}}{f_{bd}} \quad [2]$$

where, ϕ = bar diameter, σ_{sd} = design reinforcement stress and

$f_{bd} = 2.25\eta_1\eta_2 f_{ctk}/\gamma_c$ (design bond strength)

where, η_1= 1.00 for 'good' and 0.70 for 'poor' casting conditions

η_2 = Min (1.0, (132-ϕ)/100)

f_{ctk}	$= 0.21 f_{ck}^{2/3}$ for ≤ C50/60 concrete (f_{ctk} lower characteristic concrete tensile strength, f_{ck} characteristic concrete compressive strength)
γ_C	= 1.5 (partial factor for concrete)

1.2 fib Bulletin 72 [3]

fib Bulletin 72 [3] provides a detailed review of reinforcement anchorages and laps. It proposes equation 3 below [expression 3-2 in the bulletin] for calculating the mean strength of anchorages and laps. These are considered to be the same unlike EN 1992 [1] where lap lengths are obtained by multiplying anchorage lengths by a coefficient $\alpha_6 \geq 1$. According to fib Bulletin 72 the mean lap strength (≡ mean stress in the bar at lap failure) is given by:

$$f_{stm} = 54(f_{cm}/25)^{0.25}(25/\phi)^{0.2}(l_b/\phi)^{0.55}[(c_{min}/\phi)^{0.25}(c_{max}/c_{min})^{0.1} + k_m k_{tr}] \qquad [3]$$

where f_{cm} is the mean concrete strength, l_b is the lap or anchorage length, Ø is the bar diameter, c_{min} and c_{max} are the minimum and maximum of the cover and half the clear bar spacing and $k_m K_{tr}$ is a factor accounting for transverse confinement.

$$k_{tr} = n_l A_{sv}/(s_v \emptyset n_b) \leq 0.05 \qquad [4]$$

in which n_l is the number of legs of a link in each group which cross the potential splitting failure plane, s_v is the spacing between groups of links, A_{sv} is the cross sectional area of each leg of a link and n_b is the number of individual anchored bars or pairs of laps. k_m is an effectiveness factor which is defined in fib Bulletin 72 as follows:

$k_m = 12$ where $a_l \leq 125$ mm or $a_l \leq 5\phi$ in which a_l is the clear spacing between the lap and the nearest vertical leg of a link crossing the splitting plane approximately perpendicularly.

$k_m = 6$ where $a_l > 125$ mm and $a_l > 5\phi$.

$k_m = 0$ where a splitting crack would not intersect transverse reinforcement either because i) the transverse reinforcement is positioned inside the lapped bars or ii) $a_l > 125$ mm and $a_l > 5\phi$ and the clear spacing between pairs of lapped bars is < 4 times the bottom cover to the lapped bars.

In any one lap or anchorage, the minimum k_m should be applied. fib Bulletin 72 [3] limits the ratio $25/\phi$ to a maximum of 2.0 in equation 3 on the basis of "evidence in the database". Expression 3-2 in fib Bulletin 72 provides no upper limit to f_{stm}. However, equation 3 is also presented in MC2010 [4] where f_{stm} is limited to the reinforcement yield strength.

Equation 3-2 of fib Bulletin 72 was derived by curve fitting the fib tension lap database [10] and is valid for: 15 < f_{cm} <110 MPa, 0.5 ≤ c_{min}/ϕ ≤ 3.5, c_{max}/c_{min} ≤ 5.0, k_{tr} ≤ 0.05, l_b ≥ 10ϕ. The term in square brackets accounts for confinement from the concrete cover zone and reinforcement. Consideration of equation 3 shows that the bar stress is not proportional to the lap length as assumed in EN 1992 [1]. Instead, the average bond strength reduces with increasing lap length. Equation (3) can be rearranged as follows to give the required mean lap length in terms of the bar stress to be anchored (σ_s):

$$\frac{l_{b,m}}{\emptyset} = \left(\frac{\sigma_s}{54}\right)^{1.82} \left(\frac{f_{cm}}{25}\right)^{-0.45} \left(\frac{25}{\emptyset}\right)^{-0.36} \left[\left(\frac{c_{min}}{\emptyset}\right)^{0.25} \left(\frac{c_{max}}{c_{min}}\right)^{0.1} + k_m k_{tr}\right]^{-1.82} \quad [5]$$

fib Bulletin 72 [3] uses statistical analysis of test data to show that the characteristic lap strength is 76% of the mean lap strength. With $c_{max} = c_{min} = \phi$ and no confinement from reinforcement, this gives a characteristic basic lap length of:

$$\frac{l_{o,k}}{\phi} = 73.5 \left(\frac{\sigma_s}{435}\right)^{1.82} \left(\frac{f_{cm}}{25}\right)^{-0.45} \left(\frac{25}{\emptyset}\right)^{-0.36} \quad [6]$$

The corresponding average bond stress is given by:

$$f_{bk,o} = \frac{\sigma_s}{4\left(\frac{l_{o,k}}{\phi}\right)} \quad [7]$$

Substitution of $l_{o,k}$ from equation 6 into equation 7 gives the characteristic basic bond strength corresponding to a bar stress of $\sigma_s = f_{yd}$ = 500/(γ_s = 1.15) = 435 MPa as:

$$f_{bk,0} = 1.5 \left(\frac{f_{cm}}{25}\right)^{0.45} \left(\frac{25}{\phi}\right)^{0.36} \quad [8]$$

The characteristic basic bond strength is adjusted for reinforcement grades other that 500 through the multiplier:

$$\eta_4 = \left(\frac{500}{f_{yk}}\right)^{0.82} \quad [9]$$

Consideration of equation 5, shows that the mean basic bond strength should be multiplied by $\alpha'_m = \left[\left(\frac{c_{min}}{\emptyset}\right)^{0.25} \left(\frac{c_{max}}{c_{min}}\right)^{0.1} + k_m k_{tr}\right]^{1.82}$ to account for the effects of cover, bar spacing and confinement from transverse reinforcement. For the calculation of characteristic bond strength, fib Bulletin 72 conservatively replaces α'_m by [α_2+α_3] where:

$$\alpha_2 = \left(\frac{c_{min}}{\phi}\right)^{0.5} \left(\frac{c_{max}}{c_{min}}\right)^{0.15} \quad [10]$$

$$\alpha_3 = k_d k_{tr} \quad [11]$$

with k_{tr} from equation 4 and k_d = 20, 10, 0 for confinement by legs of a link perpendicular to the splitting plane, confinement by straight bars in the cover zone or other circumstances [4].

Finally after some rounding the characteristic bond strength is given as:

$$f_{bk} = 1.6\eta_4 \left(\frac{f_{cm}}{25}\right)^{0.5} \left(\frac{25}{\phi}\right)^{0.3} [\alpha_2 + \alpha_3] \quad [12]$$

The design bond strength is obtained by dividing the characteristic bond strength by γ_c = 1.5. This approach was justified [3] by a more detailed statistical analysis in which the target probability of failure was set such that "*all but 1 in 10^6 instances reinforcement would reach yield before bond failure occurred*" (i.e. β = 4.75). The reported justification is questionable since a target probability of failure of 1 in 10^6 [3] is considerably higher than that required by EN 1990 [11] for moderate (β = 3.8) and high (β = 4.3) consequences of structural failure.

1.3 MC2010 [4]

MC2010 [4] includes equation 3 for f_{stm} as expression 6-1-19 and limits f_{stm} to ≤ f_y where f_y is the reinforcement yield strength. Lap and anchorage lengths are derived via average bond stress. Expression (6.1-25) of MC2010 gives the design lap length (l_{bd}) for bar stress σ_{sd} as:

$$\frac{l_{bd}}{\phi} = \frac{\sigma_{sd}}{4f_{bd}} \geq \frac{l_{bmin}}{\emptyset} \quad [13]$$

where $f_{bd} = (\alpha_2 + \alpha_3) f_{bd,0}$

where α_2 = represents influence of cover

= $(c_{min}/\phi)^{0.5}(c_{max}/c_{min})^{0.15}$ for ribbed bars

α_3 = represents influence of transverse reinforcement

= fn($k_m K_{tr}$) or conservatively = 0.

$f_{bd,0}$ = $\eta_1\eta_2\eta_3\eta_4(f_{ck}/25)^{0.5}/\gamma_c$

where η_1 = 1.75 for ribbed bars

η_2 = 1.0 for good bond conditions

η_3 = Max (1.00, $(25/\phi)^{0.3}$)

η_4 = 1.00 for Grade 500 reinforcement.

$$l_{bmin} = \max\left(\frac{0.3\phi f_{yd}}{4f_{bd}}; 10\phi; 100\ mm\right)$$

Despite appearances, the design equation for bond strength, $f_{bd,0}$, in MC2010 [4] is essentially equation 12 divided by a partial factor of safety for bond of γ_b = 1.5.

1.4 Mancini et al [8]

Mancini et al. [8] undertook a probabilistic analysis of equation 3 using data from the fib tension splice database [10]. The model uncertainty θ was defined as the ratio of measured to calculated lap strength. Mancini et al. [8] considered the effect on θ of variations in i) concrete strength, ii) bar diameter ϕ, iii) normalised lap length l/ϕ, iv) confinement index k_{tr}, v) c_{min}/ϕ and vi) c_{max}/c_{min}. They concluded that "no significant trends of variation are found on the whole database". They also showed that the most likely probabilistic distribution for model uncertainties θ, for both new and existing structures, is lognormal. The calculation methodology used to estimate the fractiles of the resistance random variable for equation 3 followed that proposed by Taerwe [12]. The procedure has previously been used by Koenig and Fischer [13] to calibrate the design equation in EN 1992 [1] for shear in beams without shear reinforcement. The tension lap assessment of Mancini et al. confined uncertainties to the concrete compressive strength and the model uncertainty factor θ. The resistance random variable was defined as R(θ, f_c) where θ is the model uncertainty and f_c is the concrete cylinder compressive strength. The coefficient of variation of the concrete compressive strength V_{fc} was assumed to be 0.15 in accordance with references [4] and [14]. The analysis showed that the general formulation of a fractile R_j, in the function of the characteristic concrete strength, is given by:

$$R_j = f_{stj} = \zeta_j f_{ck}^{0.25} A \quad [14]$$

where

$$\zeta_j = \mu_\theta \exp(a_1 - a_{2j}) \quad [15]$$

$$a_1 = 0.25 \times 1.645\sqrt{\ln(V_{fc}^2 + 1)} \quad [16]$$

$$a_{2j} = h_j\sqrt{\ln\left(V_\theta^2 + 1\right) + 0.0625\ln(V_{fc}^2 + 1)} \quad [17]$$

$$A = 54\left(\frac{1}{25}\right)^{0.25}\left(\frac{l_b}{\phi}\right)^{0.55}\left(\frac{25}{\phi}\right)^{0.2}\left[\left(\frac{c_{min}}{\phi}\right)^{0.25}\left(\frac{c_{max}}{c_{min}}\right)^{0.1} + k_m K_{tr}\right] \quad [18]$$

where ζ_j is the probabilistic coefficient for lap and anchorage strength where j = m, k, d with m = mean value; k = characteristic value (i.e. fractile 5%) and d = design value in the function of a certain reliability index β. The coefficient μ_θ is the mean model uncertainty. The coefficient A equals f_{stm} from equation 3 divided by ${f_{cm}}^{0.25}$.

The coefficients h_j are h_m = 0 for mean value, h_k = 1.645 for characteristic value and $h_d = \alpha_R\beta$ for design value where α_R is the FORM correction factor assumed equal to 0.8 for dominant resistance variables and β is the required reliability index.

Mancini et al. [8] filtered the fib tension splice database into so called new and existing structures defined as follows:

- New structures 20 MPa ≤ f_{cm} ≤ 110 MPa; 0.95 ≤ c_{min}/ϕ ≤ 3.5 and $c_{max}/c_{min} \leq 5$; $l_b \geq 15\phi$; K_{tr} ≤ 0.05
- Existing structures 10 MPa ≤ f_{cm} ≤ 110 MPa; 0.5 ≤ c_{min}/ϕ ≤ 3.5 and $c_{max}/c_{min} \leq 5$; $l_b \geq 10\phi$; K_{tr} ≤ 0.05

Mancini et al. [8] determined the statistical properties of the resulting distributions using Bayesian inference with a non-informative prior distribution on the sample y where y = $\ln\theta_i$. Details of the procedure used are given by Engen et al. [15].The resulting statistical properties as well as probability coefficients ζ_j are summarised in Table 1.

Table 1: Statistical properties of resistance random variable from Mancini et al. [8]

	Number of specimens	Mean μ_θ	Variance σ_θ^2	Covariance V_θ	ζ_k	$\zeta_{d\ \beta = 3.8}$
New Structures	454	0.98	0.016	0.13	0.83	0.69
Existing structures	677	1.02	0.03	0.17	0.80	0.63

Equation 14 can be rearranged to give the required anchorage length. Assuming that the lap strength f_{stj} equals the bar stress σ_s that the lap needs to transfer, Mancini et al. [8] show that for $c_{min} = c_{max} = \phi$, the fractiles of the required basic anchorage length are given by:

$$l_{bj,0} = \phi\left(\frac{25}{f_{ck}}\right)^{0.45}\left(\frac{\sigma_s}{\zeta_j 54}\right)^{1.82}\left(\frac{\phi}{25}\right)^{0.36} \qquad [19]$$

where ζ_j is given by equation 15.

The partial safety factor for bond strength (γ_b) equals the ratio of the design and characteristic basic anchorage lengths corresponding to σ_s. Consequently, γ_b is given by:

$$\gamma_b = \frac{l_{bd,0}}{l_{bk,0}} = \left(\frac{\zeta_k}{\zeta_d}\right)^{1.82} \qquad [20]$$

For the statistics in Table 1, γ_b is calculated to be 1.4 for new structures with β = 3.8. Accounting for the effects of confinement from concrete cover and links, the fractiles of the required lap length for bar stress σ_s are given by:

$$\frac{l_{bj}}{\phi} = C_{anchj}\left(\frac{25}{f_{ck}}\right)^{0.45}\left(\frac{\sigma_s}{435}\right)^{1.82}\left(\frac{\phi}{25}\right)^{0.36}/\alpha'_j \qquad [21]$$

in which the calibration coefficient C_{anchj} is given by:

$$C_{anchj} = (8.06/\zeta_j)^{1.82} \qquad [22]$$

where ζ_j (see equation 15) depends on whether the mean, characteristic or design lap length is sought. For design of new structures with β = 3.8 and ζ_d = 0.69 from Table 1 [8], $C_{anch,d}$ = $(8.06/0.69)^{1.82}$ = 88. Following the approach adopted by fib Bulletin 72 [3], Mancini et al. [8] take the design confinement term in equation 21 as α'_d = $([(c_{min}/\phi)^{0.25}(c_{max}/c_{min})^{0.1}]^{1.82} + \alpha_3)$ with α_3 from equation 11. The term $[(c_{min}/\phi)^{0.25}(c_{max}/c_{min})^{0.1}]^{1.82}$ simplifies to $(c_{min}/\phi)^{0.46}(c_{max}/c_{min})^{0.18}$ which fib Bulletin 72 rounds to $\alpha_2 = (c_{min}/\phi)^{0.5}(c_{max}/c_{min})^{0.15}$ (see equation 10).
Rearranging equation 21, and replacing σ_s with f_{stj}, gives the lap strength:

$$f_{stj} = 54\zeta_j\left(\frac{f_{ck}}{25}\right)^{0.25}\left(\frac{l_b}{\phi}\right)^{0.55}\left(\frac{25}{\phi}\right)^{0.2}\alpha_j'^{0.55} \le f_{yj} \qquad [23]$$

In all the analyses presented in this paper, including Mancini et al. [8], $\alpha'_k = \alpha'_d = \alpha_2 + \alpha_3$ in which α_2 and α_2 are given by equations 10 and 11 respectively. However, $\alpha'_m = \left[\left(\frac{c_{min}}{\emptyset}\right)^{0.25}\left(\frac{c_{max}}{c_{min}}\right)^{0.1} + k_m k_{tr}\right]^{1.82}$ for equivalence with equation 3.

Substituting equation 21 into equation 7 gives the average bond stress:

$$f_{bj} = \frac{108.75}{C_{anchj}}\left(\frac{435}{\sigma_s}\right)^{0.82}\left(\frac{f_{ck}}{25}\right)^{0.45}\left(\frac{25}{\phi}\right)^{0.36}\alpha'_j \qquad [24]$$

in which C_{anchj} is given by equation 22

2 Discussion and Implications

The adoption of the Mancini et al. [8] recommendations would lead to significant increases in full strength lap (and anchorage) lengths over current EN 1992 [1] requirements. This is undesirable since it would make reinforced concrete (rc) construction more expensive through the use of additional reinforcement and more complex working practices. The alternative of using couplers also increases construction costs except for large diameter bars. Furthermore, increasing lap lengths would cause significant uncertainties within the engineering community about the safety of the current stock of rc structures where laps and anchorages, designed to EN 1992 [1] and its precursors, have been performing well and without issue. This was substantiated by a straw poll of European experts conducted by Goodchild which identified no known failures of laps or anchorages (apart from one resulting from gross error). Bearing all these considerations in mind, a group of European experts, convened by Goodchild, deemed it unacceptable to significantly increase lap lengths in the 2023 revision to EN 1992. This paper addresses the disconnect between assessments of lap safety drawn from practice and statistical analysis of databases [3,8].

3. Review of tension lap data

Equation 3 (Exp 3-2 of fib Bulletin 72) was derived by curve fitting a database of around 775 tension lap tests. fib Bulletin 72 gives the limits on Exp 3-2 as 15 MPa < f_{cm}< 110 MPa, 0.5 ϕ ≤c_{min}≤ 3.5 ϕ, c_{max}/c_{min}≤ 5.0 and $l_b \geq 10\phi$. As shown by Mancini et al. [8] changing the limits on equation 3 for new structures to 20≤f_{cm}≤110, 0.95ϕ ≤c_{min}≤ 3.5ϕ, c_{max}/c_{min}≤ 5.0 and and $l_b \geq 15\phi$, significantly reduces the scatter in the ratio of measured to predicted lap strength.

The current authors assessed equation 14 (equation 3 in probabilistic form) for a tension splice database consisting of the fib tension splice database plus 17 specimens tested by Micallef and Vollum [16, 17]. Of these, 516 specimens (see Table 2 for source) remained after filtering as follows:

- 20 MPa ≤ f_{cm} ≤ 90 MPa;
- c_{min}/ϕ ≥ 0.95

These filtering limits are broadly in line with limits applied in EN 1992 [1].

When applying equation 3 to the experimental database, it is necessary to limit the lap strength to a maximum of the reinforcement strength. The lap strength ($f_{st,test}$) exceeded the reported yield strength (f_y) in 72 of 516 tests in the filtered database. In these tests, the peak stress was dependent on the shape of the stress strain curve of the reinforcement which is not generally reported. For example, in tests where the reinforcement has a long yield plateau, like those of Micallef and Vollum [16], extensive yielding can occur under almost constant reinforcement stress prior to flexural compression failure. In tests like these, limiting the reinforcement stress calculated with equation 3 to f_y seems reasonable. On the other hand, for reinforcement without a well-defined yield point the reinforcement stress continually increases after first yield until either lap or flexural failure occurs. Consequently, the limit to apply on the maximum bar stress calculated with equation 3 is unclear. However, it should be noted that the mean lap strength (f_{stm}) is limited to a maximum of the reinforcement yield strength (f_y) in MC2010. In an initial assessment of equation 3, the mean calculated lap strength (f_{stm}) was limited to a maximum of 1.05f_y compared with f_y in MC2010. The results of the analysis are plotted in Figure 1 which shows the influence of measured lap strength ($f_{st,test}$) on $f_{st,test}/f_{stm}$ (where f_{stm} is the mean lap strength calculated with equation 3).

Table 2: Summary of filtered tension specimens considered in analysis

No of tests	fib database reference [10]	Investigators [10]
17		Micallef and Vollum 2017 [16,17]
27	1	Chinn, Ferguson, and Thompson 1955
6	2	Chamberlin 1958
27	4	Ferguson and Breen 1965
12	5	Thompson, Jirsa, Breen, and Meinheit 1975
15	6	Ferguson and Thompson 1965
17	8	Hester, Salamizavaregh, Darwin, and McCabe 1991, 1993
8	9	Choi, Hadje-Ghaffari, Darwin, and McCabe 1990, 1991
32	10	Rezansoff, Konkankar and Fu 1991
12	11	Zekany, Neumann, Jirsa, and Breen 1981
7	12	DeVries, Moehle, and Hester 1991
5	13	Rezansoff, Akanni, and Sparling 1993
57	15	Darwin, Tholen, Idun, and Zuo 1995
39	16	Zuo and Darwin 1998
34	17	Kadoriku 1994
15	18	Hamad, 1999
11	20	Azizinamini, Stark, Roller, Ghosh 1993
13	22	Betzle 1980
16	24	Hamad, Mansour 1996
8	25	Hegger, Burkhardt 1998
8	26	Hwang, Lee, Lee 1994
10	27	Hwang, Leu, Hwang 1996
21	28	Olsen 1990
16	29	Rehm, Eligehausen 1977
1	30	Stöckl, Menne, Kupfer 1977
82	31	Tepfers 1973

Results are shown in Figure 1 for specimens without (no links) and with confining reinforcement (links). The trend lines in Figure 1 show that on average $f_{st,test}/f_{stm}$ increases with measured lap strength ($f_{st,test}$). This implies that equation 3 from fib Bulletin 72 becomes progressively more conservative as the lap strength ($f_{st,test}$) increases. This increasing conservatism of equation 3 with lap strength is not considered in the reliability analysis of Mancini et al. [8].

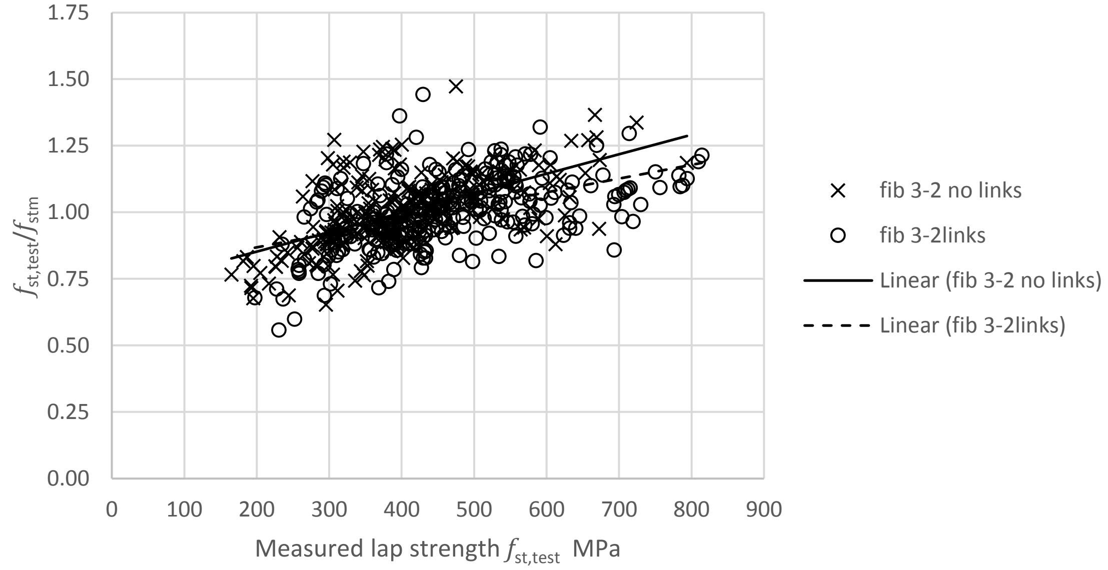


Figure 1: Influence of $f_{st,test}$ on accuracy of equation 3 for mean lap strength f_{stm} (fib Bulletin 72 equation 3-2)

4 Development of improved design method

The conservatism of equation 3 from fib Bulletin 72 [3] with increasing lap strength (see Figure 1) was investigated statistically by separating the tension splice database (see Section 3) into the following four stress bands: $f_{st,test}$ < 300 MPa, 300 MPa ≤ $f_{st,test}$< 400 MPa, 400 MPa ≤ $f_{st,test}$< 500 MPa and $f_{st,test}$ ≥ 500 MPa. The stress $f_{st,test}$ is the measured lap strength. Each stress band was analysed following the procedure of Mancini et al. [8] described in Section 1.4. In line with Mancini et al. [8], the characteristic concrete strength was assumed to have the following lognormal distribution:

$$f_{ck} = f_{cm}\exp(-1.645\sqrt{\ln\left(V_{fc}^2 + 1\right)}) \qquad [25]$$

The coefficient of variation for concrete strength V_{fc} was taken as 0.15 as in [8], giving f_{ck} = $0.782 f_{cm}$ where f_{cm} is the reported concrete strength. Probability coefficients ζ_k, ζ_d and partial bond factor γ_b were calculated separately for each stress band using equations 15 and 20 respectively. In addition to the filters applied in Section 3, the maximum value of $f_{s,test}/f_y$ was limited to 1.10 which left 491 specimens. Furthermore, calculated lap strengths f_{stm} were limited to f_y. The results of the analysis are summarised in Table 3 which shows that significant economies can be achieved on full strength laps if test data are grouped into stress dependent bands as above. Results are also presented for all specimens with calculated lap strength limited to $1.05 f_y$. The latter results are closest to those obtained by Mancini et al. [8] but there is little difference between the results obtained with calculated lap strength f_{stm} limited to f_y and $1.05f_y$.

Table 3: Statistical analysis of banded "good bond" tension splice test data (β = 3.8)

Description	Symbol	Measured lap strength $f_{st,test}$ MPa						Mancini et al. [8]
		<300	300-400	400-500	≥500	All $f_{st,test}$	All $f_{st,test}$	
Number of specimens	N	65	159	153	114	491	491	454
Limit on calculated lap strength	-	f_y	f_y	f_y	f_y	f_y	$1.05f_y$	-
Mean, using eqn 3	μ_θ	0.85	0.98	1.02	1.08	1.00	0.99	0.98
CoV, using eqn 3	V_θ	0.16	0.12	0.09	0.09	0.13	0.13	0.13
Probabilistic coeff., mean (eqn 15)	ζ_m	0.90	1.04	1.08	1.15	1.06	1.05	1.04
Probabilistic coeff., characteristic (eqn 15)	ζ_k	0.70	0.84	0.92	0.97	0.85	0.84	0.83
Probabilistic coeff., design (eqn 15)	ζ_d	0.56	0.70	0.80	0.85	0.70	0.70	0.69
Partial factor for bond (eqn 20)	γ_b	1.50	1.39	1.29	1.29	1.41	1.41	1.40
Calibration coeff., char. (eqn 22)	$C_{anch,k}$	85	61	52	47	60	61	63
Calibration coeff., design (eqn 22)	$C_{anch,d}$	128	85	67	60	85	85	88

Note: $\theta = f_{st,test}/f_{stm}$

Table 3 shows that the coefficients $C_{anch,k}$ and $C_{anch,d}$ (see equation 22) reduce significantly with increasing reinforcement stress. Significantly, for measured lap strengths between 400 and 500 MPa, $C_{anch,k} = 52$ and $C_{anch,d} = 67$ compared with $C_{anch,k} = 60$ and $C_{anchd} = 85$ for the whole database (with $f_{stm} \leq f_y$). If calculated with $\zeta_d = 0.69$ from Mancini et al. [8], $C_{anch,d} = 88$. Reducing $C_{anch,d}$ from 88 to 67 for design lap strength f_{sd} = 435 MPa, gives a 24% reduction in design lap length. Table 3 also shows that Mancini et al. [8] overestimates ζ_j for $f_{st,test} < 300$ MPa. This implies that Mancini et al. [8] has below the level of safety expected by EN 1990 [11] for measured lap strengths $f_{st,test} < 300$ MPa.

4.1 Vollum proposal for good bond

Based on the statistical analysis in Table 3 for f_{st} between 400 – 500 MPa, it is proposed that the lap length for good bond is calculated as follows:

$$\frac{l_{bj}}{\phi} = mC_{anch,j}\left(\frac{25}{f_{ck}}\right)^{0.45}\left(\frac{\phi}{25}\right)^{0.36}/(\alpha_2 + \alpha_3) \qquad [26]$$

in which the probabilistic coefficient $\zeta_{j,Eq\ 26}$ used to calculate $C_{anch,j}$ (see equation 22) is taken from Table 3 for 400 ≤ $f_{st,test}$ < 500 MPa. Hence, $\zeta_{k,Eq\ 26}$ = 0.92 and $\zeta_{d,Eq\ 26}$ = 0.80. Substituting these values into equation 22, gives $C_{anch,k}$ = 52 (characteristic) and $C_{anch,d}$ = 67 (design). The coefficients α_2 and α_3 are calculated with equations 10 and 11 respectively of fib Bulletin 72 [3]. The multiplier m accounts for the bar stress σ_s transferred by the lap and is given by:

$$m = Max\left[\left(\frac{\sigma_s}{435}\right), \left(\frac{\sigma_s}{435}\right)^{1.82}\right] \quad [27]$$

Equation 26 is equivalent to equation 21 for σ_s ≥435 MPa. It assumes a constant average bond strength for reinforcement stress σ_s ≤ 435 MPa. This is a simplification because in reality, the average bond strength is proportional to $\left(\frac{435}{\sigma_s}\right)^{0.82}$ (see equation 24). However, for ease of use, and to compensate for increasing C_{anch} at low σ_s, the Vollum proposal adopts a constant bond strength for σ_s ≤ 435 MPa.

The average bond strength corresponding to equation 26 is found by substituting l_{boj}/ϕ from equation 26 into equation 7. The resulting bond strengths for grade 500 reinforcement (i.e. f_{yd} = 435 MPa) and below are given by:

$$f_{bj} = 2.44\zeta_j^{182}\left(\frac{f_{ck}}{25}\right)^{0.45}\left(\frac{25}{\phi}\right)^{0.36}(\alpha_2 + \alpha_3) \quad [28]$$

For ζ_k = 0.92 and ζ_d = 0.80:

$$f_{bk} = 2.1\left(\frac{f_{ck}}{25}\right)^{0.45}\left(\frac{25}{\phi}\right)^{0.36}(\alpha_2 + \alpha_3) \quad [29]$$

$$f_{bd} = 1.6\left(\frac{1.5}{\gamma_c}\right)^{0.64}\left(\frac{f_{ck}}{25}\right)^{0.45}\left(\frac{25}{\phi}\right)^{0.36}(\alpha_2 + \alpha_3) \quad [30]$$

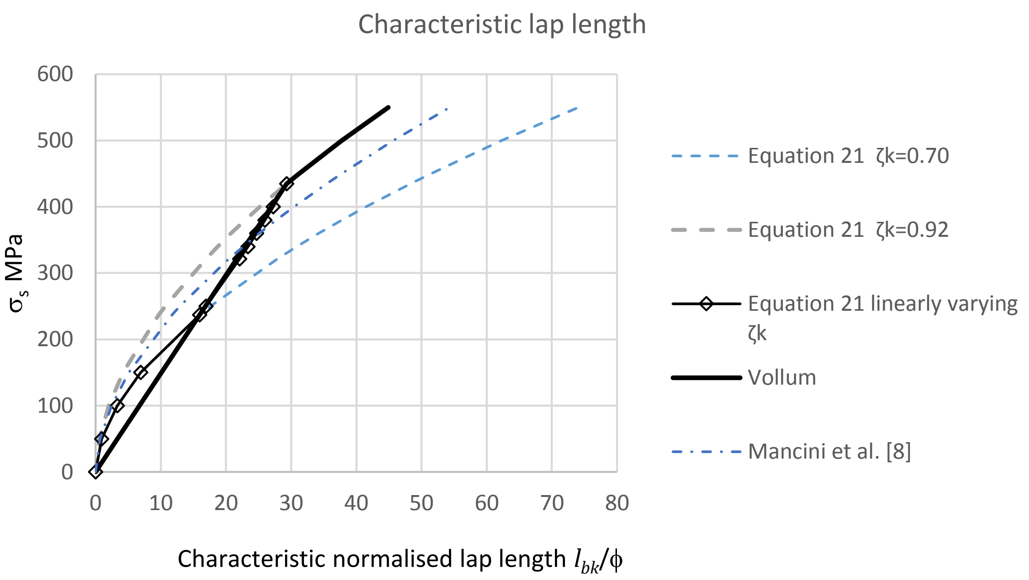
For σ_s > 435 MPa, the bond strength should be multiplied by $\left(\frac{435}{\sigma_s}\right)^{0.82}$.

As explained above, the proposed bond strength is independent of reinforcement stress σ_s for σ_s ≤ 435 MPa. This is unlike the proposal of Mancini et al. [8], where the design bond strength varies with σ_s according to equation 24. In normal design γ_c = 1.5. However, lower γ_c are used in design for fire and accidental actions. The term $\left(\frac{1.5}{\gamma_c}\right)^{0.64}$ is chosen to make the partial bond factor, γ_b = f_{bk}/f_{bd} increase almost linearly with γ_c between γ_c = 1.0 and γ_c = 1.5 and $f_{bd} = f_{bk}$ when γ_c = 1.0. For full strength laps with σ_{sd} = 435 MPa, the bond factor $\gamma_b = f_{bk}/f_{bd}$ is 1.3 for γ_c = 1.5 and 1.14 for γ_c = 1.2 as used for accidental actions.

4.3 Theoretical appraisal of Vollum design proposal (equation 26)

The proposals of Vollum (equation 26) and Mancini et al. [8] (i.e. equation 21 with ζ_k = 0.83, ζ_d = 0.69) are illustrated in Figure 2 respectively for 20 mm bars spaced at 160 mm centres with f_{ck} = 40 MPa and 30 mm cover. Figures 2a and 2b respectively show characteristic and design lap lengths corresponding to bar stress σ_s. Also shown in Figure 2a are characteristic lap lengths calculated using equation 21 with ζ_k = 0.70 (corresponding to $f_{st,test}$ < 300 MPa in Table 3) and ζ_k = 0.92 as used for $C_{anch,k}$ in equation 26. Figure 2b shows that Vollum gives shorter design lap lengths than Mancini et al. [8] for σ_s> 340 MPa.

a)



b)

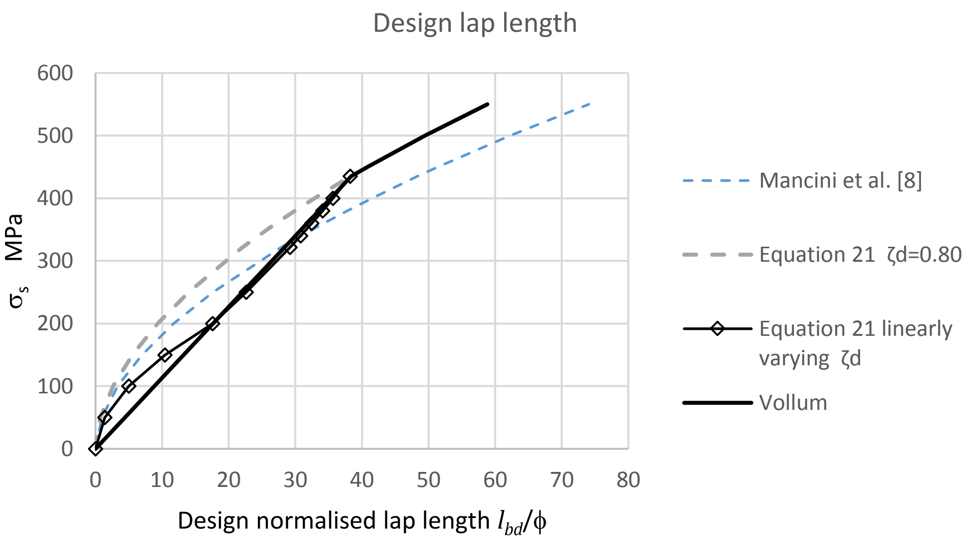


Figure 2: Comparison of design methods for a) characteristic lap length l_{bk}/ϕ and b) design lap length l_{bk}/ϕ

The reinforcement stress at the intersection of the lines in Figure 2 depicted "equation 21 $\zeta_j = \zeta_j^*$" (where j = k in Figure 2a , j = d in Figure 2b and ζ_j^* is the adopted numerical value of ζ_j) and "Vollum" (equation 26) is found by equating lap lengths calculated with equation 21 (with $\zeta_j = \zeta_j^*$) and equation 26 to be:

$$\sigma_s = 435\left(\frac{\zeta_j^*}{\zeta_{j,Eq\ 26}}\right)^{2.22} \qquad [31]$$

in which $\zeta_{j,Eq\ 26}$ is the probability coefficient adopted in equation 26 for calculation of $C_{anch,j}$. In the Vollum proposal, $\zeta_{k,Eq\ 26}$ = 0.92 and $\zeta_{d,Eq\ 26}$ = 0.80. Bar stress σ_s and lap strength f_{stj} are interchangeable in equation 31 dependent on whether lap length or strength, corresponding to given lap length, is being calculated.

For $\zeta_k^* = \zeta_k$ = 0.70 and $\zeta_{k,Eq\ 26}$ = 0.92, equation 31 gives the stress at the intersection of the lines in Figure 2a depicted "Vollum" and "Equation 21 ζ_k = 0.7" as σ_s = 237 MPa. Consideration of Figure 2a, shows that for σ_s < 237 MPa, Vollum is conservative relative to equation 21 with ζ_k = 0.70. For σ_s > 435 MPa, Vollum also provides a safe estimate of characteristic resistance since it is coincident with equation 21 with ζ_k = 0.92. However, for 237 < σ_s < 435 MPa, the variation of ζ_k implicit in the Vollum method requires investigation. Rearranging equation 31 shows that normalised characteristic lap lengths l_{bk}/ϕ calculated with equation 21 (with $\zeta_j = \zeta_j^*$) and the Vollum proposal are equal if:

$$\zeta_j^* = \zeta_{j,Eq\ 26}\left(\frac{\sigma_s}{435}\right)^{0.45} \leq \zeta_{j,Eq\ 26} \qquad [32]$$

For the Vollum proposal to have the level of safety required by EN 1990 [11], ζ_j^* from equation 32 should match ζ_j from Table 3 (depicted "$\zeta_{j,Table\ 3}$") for 237 < σ_s < 435 MPa. The relationship between ζ_j^* from equation 32 (depicted "Equation 32") and $\zeta_{j,Table\ 3}$ (depicted "Table 3") is illustrated in Figures 3a and 3b respectively for characteristic and design lap strengths.

Comparison of ζ_k^* (depicted "Equation 32") and $\zeta_{k,Table\ 3}$ (depicted "Table 3") in Figure 3a suggests that Vollum provides a safe estimate of the characteristic bond strength but statistical verification using test data is required due to the stepped variation of $\zeta_{k,Table\ 3}$. In Figure 3b, probability coefficients from Table 3 are plotted against the following notional design stress bands: f_{std} < 300/γ_b MPa, 300/γ_b MPa ≤ f_{std}< 400/γ_b MPa and f_{std} ≥ 400/γ_b MPa. Probability coefficients from Table 3 are plotted in Figure 3b for γ_b = 1.29 (400 MPa ≤ $f_{st,test}$< 500 MPa) and γ_b = 1.5 ($f_{st,test}$< 300 MPa). By inspection, the "Vollum" (equation 32) probability coefficients in Figure 3b are less than the corresponding Table 3 coefficients. This indicates that the Vollum proposal (equation 26) achieves the required safety level as confirmed with test data in the next section. It is also evident that using γ_b = 1.29 in Figure 3b, rather than γ_b = 1.5, is conservative since it minimises $\zeta_{d,Table\ 3}$ for given f_{std}.

Figures 3a and 3b also show that, if limited to the minimum value in Table 3, ζ_j^* varies almost linearly with lap strength (f_{stj}) as follows:

Characteristic lap length ζ_k^* (depicted "linearly varying ζ_k") [33]

1. ζ_k^* = 0.7 for σ_s ≤ 237 MPa
2. ζ_k^*= 0.7 + (0.92-0.7).(σ_s-237)/(435-237) for 237 MPa ≤ σ_s ≤ 435 MPa
3. ζ_k^*= 0.92 for σ_s ≥ 435 MPa

Design lap length ζ_d^* (depicted "linearly varying ζ_d") [34]

1. ζ_d^*= 0.56 for σ_s ≤ 197 MPa
2. ζ_d^*= 0.56 + (0.80-0.56).(σ_s-197)/(435-197) for 197 MPa ≤ σ_s ≤ 435 MPa
3. ζ_d^*= 0.80 for σ_s ≥ 435 MPa

For assessment of lap strength, σ_s in the above equations for "linearly varying ζ_j" should be replaced by the lap resistance (f_{stj}). By definition, lap lengths calculated using equation 21 with ζ_j^* varying linearly as described above should coincide with the Vollum proposal for f_{stk} ≥ 237 MPa. This is verified in Figures 2a and 2b where the lines depicted 'Equation 21 linearly varying ζ' (calculated assuming ζ varies linearly as described above) coincides with the Vollum proposal for σ_{stk} ≥ 237 MPa (Figure 2a) and σ_{std} ≥ 197 MPa (Figure 2b).

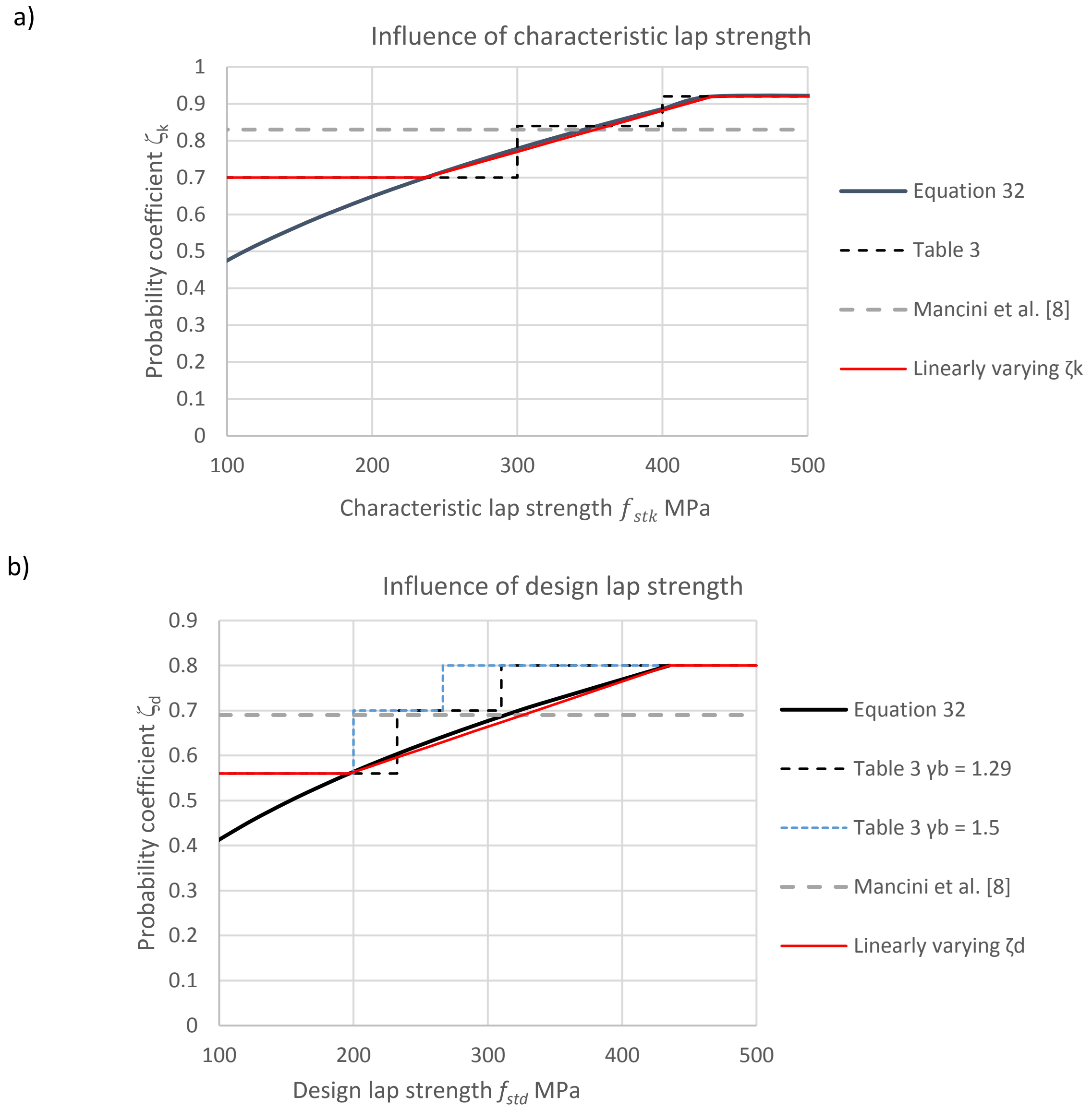


Figure 3: Variation of probability coefficient ζ_j with a) characteristic and b) design lap strength

4.4 Statistical verification of Vollum proposal (equation 26) using test data and comparison with Mancini et al. [8]

As shown in Section 4.3, the Vollum proposal for lap length (equation 26) is equivalent to equation 21 if ζ_j is calculated with equation 32. Equation 23 for lap strength (f_{stj}) is obtained by rearranging equation 21 for lap length (l_{bj}). Consequently, lap strengths calculated using the Vollum proposal for bond strength (equation 28) equal those calculated using equation 23 with ζ_j from equation 32. As shown in Section 4.3, equation 32 can be linearised as described by equation 33 for ζ_k and equation 34 for ζ_d.

Characteristic lap strengths were calculated for the filtered database using equation 23 in which ζ_k was calculated with equation 33. In the discussion below, these strengths are depicted "equation 23 with linearly varying ζ_k". Above around 240 MPa, strengths depicted "equation 23 with linearly varying ζ_k" correspond to the Vollum proposal (equation 26) as shown in Figure 2a. The probability coefficients given by equations 33 and 34 depend on whether the measured or calculated lap strength is used. Probability coefficients are related to the measured lap strength in Table 3 but to the calculated lap strength in the Vollum proposal (equation 26).

For consistency with Table 3, "equation 23 with linearly varying ζ_k" was initially assessed using probability coefficients calculated with equation 33 in terms of the measured lap strength ($f_{st,test}$). Consequently the bar stress (σ_s) was taken as $f_{st,test}$ in equation 33. The calculated characteristic lap strength f_{stk} was limited to a maximum of $1.05f_y$. This limit controlled the strength of 18 test specimens with links and two without links. The results of the analysis are shown in Figure 4 which also shows lap strengths calculated in accordance with Mancini et al. [8] (equation 23 with ζ_k = 0.83) but limited to a maximum of $1.05f_y$. For Mancini et al., the maximum strength of $1.05f_y$ governed for seven test specimens. Data in Figure 4, which assesses the basis of the Vollum method, are grouped into tests without and with confining reinforcement. Strengths calculated using equation 23 with linearly varying ζ_k are depicted "Equation 23" in Figure 4.

Figures 4a to 4c show the influence of a) lap strength ($f_{st,test}$), b) concrete strength (f_c) and c) normalised lap length (l_b/ϕ) on the ratio $f_{st,test}/f_{stk}$. Results are shown for both "equation 23 with linearly varying ζ_k" and Mancini et al. [8]. The influence of each parameter on $f_{st,test}/f_{stk}$ is indicated by the trend lines plotted in Figure 4. As intended, Figure 4a shows that $f_{st,test}/f_{stk}$ is independent of lap strength ($f_{st,test}$) for equation 23 with linearly varying ζ_k (depicted "Equation 23"). This is not the case for Mancini et al. (with ζ_k = 0.83) where $f_{st,test}/f_{stk}$ increases with $f_{st,test}$. Figure 4b shows that $f_{st,test}/f_{stk}$ is independent of concrete strength for both "Equation 23" and Mancini et al. Figure 4c shows that on average $f_{st,test}/f_{stk}$ reduces with increasing l_b/ϕ for "Equation 23". This is not the case for Mancini et al. [8] where $f_{st,test}/f_{stk}$ is independent of l_b/ϕ. The downwards trend of $f_{st,test}/f_{stk}$ with increasing l_b/ϕ for "Equation 23" arises because f_{stk} is frequently limited by reinforcement yield for l_b/ϕ > 40 as observed experimentally.

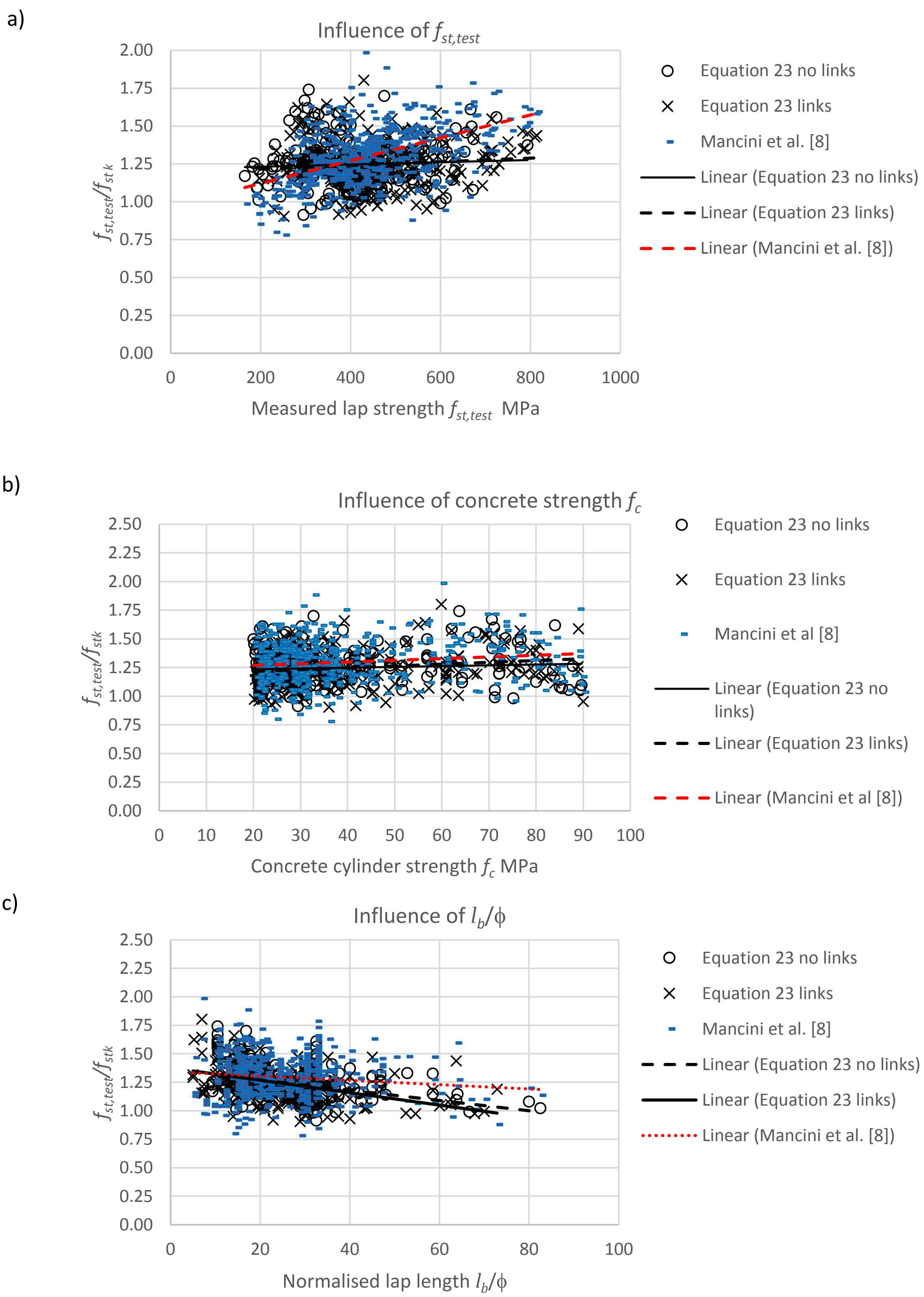


Figure 4: Influence of a) measured lap strength $f_{st,test}$, b) concrete strength f_c and c) l_b/ϕ on $f_{st,test}/f_{stk}$ for equation 23 with linearly varying ζ_k (depicted "Equation 23") and Mancini et al. [8].

Table 4 shows failure rates for the different methods considered. It shows the percentage of specimens with measured strength less than characteristic strength calculated with i) equation 23 with linearly varying ζ_k (basis of Vollum proposal), ii) the Vollum bond strength proposal (equation 29), iii) Mancini et al. [8] (equation 23 with ζ_k = 0.83) and iv) EN1992 [1]. Two sets of analyses were carried out using equation 23 with linearly varying ζ_k. In analyses depicted "equation 23, ζ_k (depicted $\zeta_{k\ fst,\ test}$) was calculated in terms of the measured lap strength ($f_{st,test}$). In analyses depicted "equation 23(a)", ζ_k (depicted $\zeta_{k\ fstk}$) was calculated by iteration in terms of the calculated characteristic lap strength (f_{stk}). Lap strengths calculated with equation 23(a) correspond to lines depicted "Equation 21 linearly varying ζ_k" in Figure 2a. Table 4 also shows the number of specimens in each category. Table 5 shows the mean and standard deviation of $f_{st,test}/f_{stk}$ for each method.

Table 4: Percentage failures for calculated characteristic lap strengths

Measured lap strength $f_{st,test}$ (MPa)	<300	300-400	400-500	>500	All $f_{st,test}$
Number of specimens	65	159	153	114	491
Number of specimens without links	41	96	66	32	235
Number of specimens with links	24	63	87	82	256
Equation 23 (linearly varying $\zeta_{k\ fst,\ test}$)	4.62%	2.52%	3.27%	1.75%	2.85%
Equation 23(a) (linearly varying $\zeta_{k,\ fstk}$)	4.62%	3.14%	4.58%	1.75%	3.24%
Vollum (bond stress: equation 29) reference f_{stk}	4.62%	3.14%	4.58%	1.75%	3.46%
Mancini et al. [8] (equation 23 with ζ_k = 0.83)	21.54%	3.77%	0.65%	0.00%	4.28%
EN 1992 (2004) [1]	9.23%	7.55%	5.23%	7.89%	7.13%
EN 1992 (2004) no links	12.20%	12.50%	6.06%	15.63%	11.06%
EN 1992 (2004) links	4.17%	0.00%	4.60%	4.88%	3.52%

Table 5: Characteristic strength statistics for $f_{st,test}/f_{stk}$

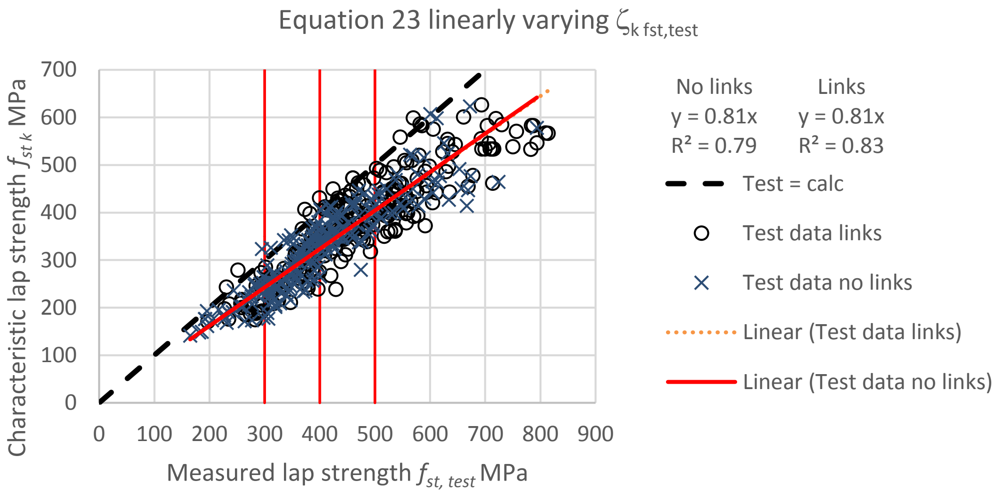
Measured lap strength $f_{st,test}$ (MPa)	Statistic	<300	300-400	400-500	>500	All
Number of specimens	-	65	159	153	114	491
Equation 23 (linearly varying $\zeta_{k\ fst,\ test}$)	μ	1.26	1.27	1.20	1.27	1.25
	σ	0.16	0.16	0.13	0.14	0.15
Equation 23(a) (linearly varying $\zeta_{k\ fstk}$)	μ	1.32	1.45	1.38	1.31	1.38
	σ	0.20	0.24	0.25	0.19	0.23
Vollum (bond stress: equation 29) reference f_{std}	μ	1.56	1.53	1.37	1.31	1.43
	σ	0.35	0.35	0.27	0.18	0.31
Mancini et al. [8] (equation 23; ζ_k = 0.83)	μ	1.11	1.26	1.32	1.40	1.29
	σ	0.17	0.16	0.15	0.16	0.18
EN 1992 (2004) [1]	μ	1.67	1.70	1.65	1.55	1.65
	σ	0.49	0.51	0.43	0.41	0.46

Of all the methods, equation 23, (linearly varying $\zeta_{k\ fst,\ test}$) is most accurate and consistent between stress bands. Equation 23(a), (linearly varying $\zeta_{k\ fstk}$), is satisfactory but less accurate since ζ_k is typically underestimated to a varying degree as a result of the calculated lap strength (f_{stk}) being less than measured ($f_{st,test}$). Table 4 also suggests that Mancini et al. [8] (equation 23 with ζ_k =

0.83) has below the level of safety required by EN 1990 [11] for $f_{st,test}$< 300 MPa but is overly conservative for $f_{st,test}$ > 400 MPa. The current EN 1992 [1] rules appear satisfactory if confining links are provided but have below the expected level of safety without confining reinforcement. Additionally, the mean and standard deviation of each band are greatest for EN 1992 [1] indicating that the method is imperfect and in need of improvement.

Figures 5a and 5b respectively show the influence of measured lap strength $(f_{st,test})$ on the characteristic lap strength calculated using equation 23 (linearly varying $\zeta_{k\ fst,\ test}$) and Mancini et al. [8] (equation 23 with ζ_k = 0.83). Comparison of Figures 5a and 5b shows that the data points in Figure 5a, for equation 23, are grouped more closely around the trend line than in Figure 5b for Mancini et al. [8]. The slope of the trend line is also slightly steeper in Figure 5a. These features are reflected in the statistical data presented for each method in Tables 4 and 5.

a)



b)

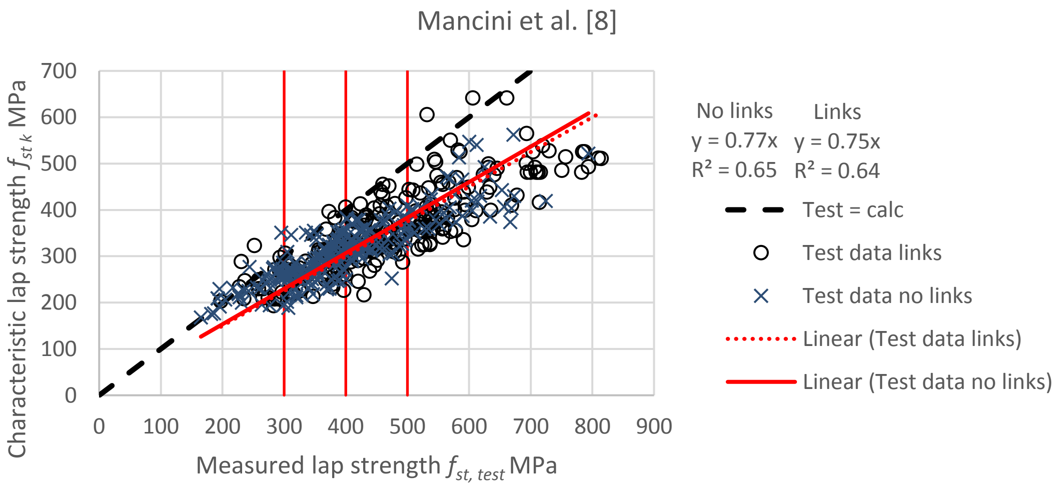


Figure 5: Comparison of measured and characteristic strengths for a) Equation 23 with linearly varying $\zeta_{k\ fst,test}$ and b) Mancini et al. [8] (equation 23 with ζ_k = 0.83)

Figure 6 compares design and measured lap strengths corresponding to i) equation 23 (linearly varying $\zeta_{d\ fst,\ test}$), ii) Mancini et al. [8] (equation 21 with ζ_d = 0.69) and iii) Vollum (equation 26). The maximum lap strength was limited to the measured reinforcement yield strength. Equation 23 with linearly varying $\zeta_{d\ fst,test}$ is seen to be most accurate. The Vollum method is less accurate than equation 23 since the probability coefficient ζ_d implicit in equation 26 depends on the calculated lap strength (f_{std}) rather than the measured strength $(f_{st,test})$. This increases scatter, as shown in Figure 6c, but is conservative since the design strength is always less than measured which reduces the probabilistic coefficient ζ_d calculated with equation 34 for f_{std} < 435 MPa.

4.5 Discussion of safety of Vollum proposal

The Vollum method (equation 26) takes advantage of the increase in probability coefficient ζ_j with lap strength $f_{st,test}$ evident in Table 3. This increase in ζ_j (see equation 15) arises because μ_θ (where μ depicts mean and $\theta = f_{st,test}/f_{stm}$) increases and V_θ (where V depicts coefficient of variation) reduces with increasing lap strength. Tests by Micallef and Vollum [16,17] on “short”, “long” and “very long” laps of the same bar diameter suggest that redistribution of bond stress may explain the observed reduction in V_θ with increasing $f_{st,test}$. “Short” laps failed suddenly due to splitting prior to bar yield. “Long” laps were designed with equation 3 to have mean lap strength equal to the measured reinforcement yield strength. “Long” laps failed due to longitudinal splitting subsequent to extensive bar yield. “Very long” laps varied in length between 1.5 and 2.0 times the length of “long” laps. Specimens with “very long” laps failed in flexure due to concrete crushing at almost the same load as comparable specimens with “long” laps. Despite the difference in failure mode, specimens with “long” and “very long” laps developed very large, and similar, plastic displacements at peak load. In “very long laps” the strain distribution was fairly uniform over the central part of the lap up to peak load. Up to at least 75% of yield, strains were fairly uniform over the central part of “long” laps [17]. However, by first yield, the strain distribution along “long” laps was fairly linear indicating the occurrence of significant bond stress redistribution between 75% of yield and failure. Potential for bond stress redistribution of this type, which increases with lap length, seems a possible explanation for the reduction in V_θ with increasing $f_{st,test}$ evident in Table 3. The proposed approach accounts for the observed behaviour by relating the probability coefficient ζ_j, and hence γ_b, to lap strength.

a)

b)

c)

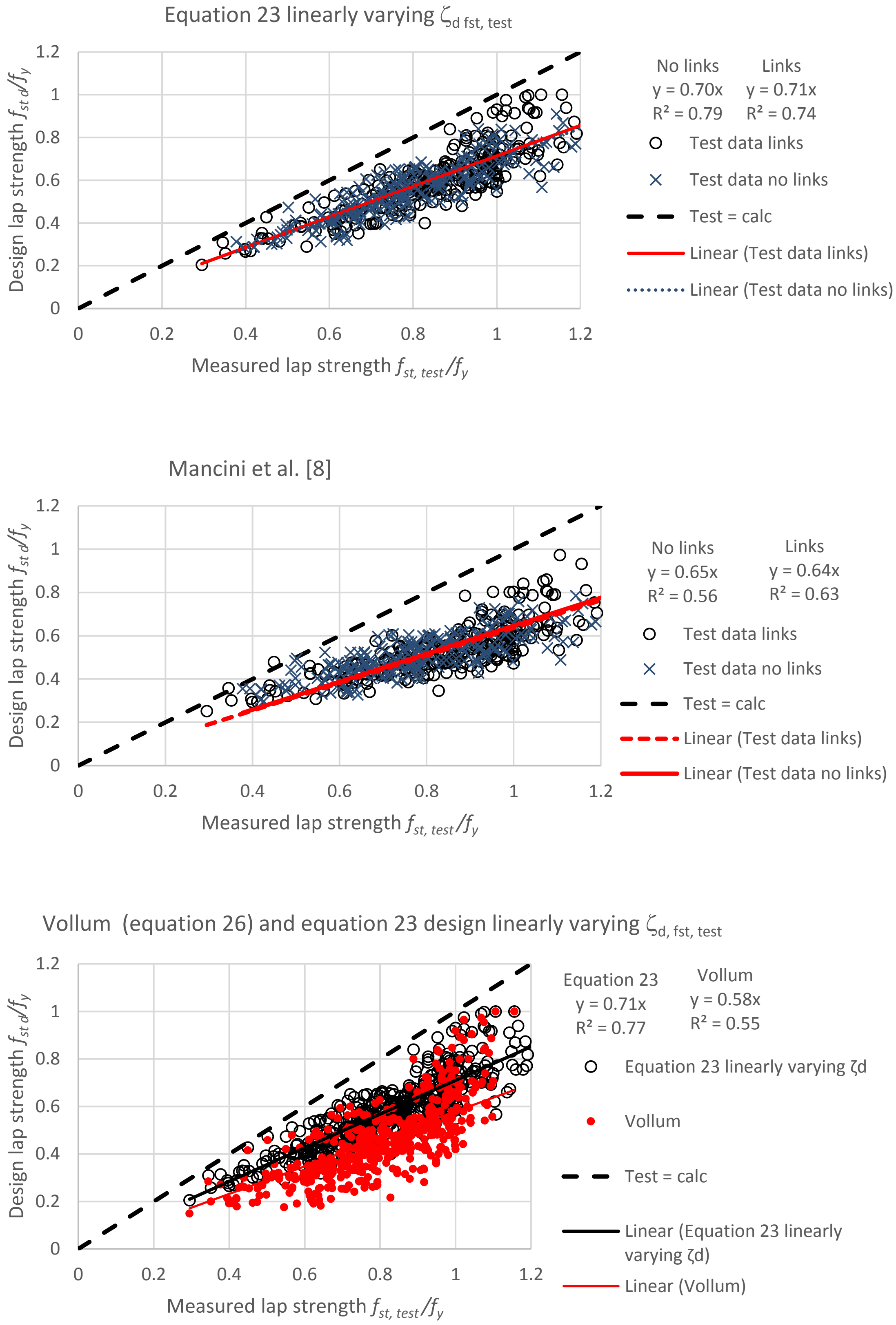


Figure 6: Comparison of measured and design lap strengths for a) equation 23 with linearly varying $\zeta_{d\ fst,\ test}$, b) Mancini et al. [8] (equation 23 with ζ_d = 0.69) and c) Vollum (equation 26) & equation 23 with linearly varying $\zeta_{d\ fst,\ test}$.

4.6 Staggering of laps

EN 1992 [1] requires adjacent laps to be staggered by $0.3l_{b,d}$ where $l_{b,d}$ is the design lap length. Recent research [16,18] shows that staggering laps has a slight detrimental effect on strength whilst having minimal influence on crack width. Therefore, it is proposed that 100% laps be permitted with no requirement to stagger adjacent laps as currently required by EN 1992 [1].

5 Comparison of design methods

Table 6 summarises the methods described in the paper and compares design lap lengths for a 100% lap of 25 mm bars in good bond conditions, bar stress = f_{yd} = 435 MPa, in C30/37 concrete, no confinement from transverse reinforcement or pressure and c_{min} = c_{max} = bar diameter + 10 mm. The proposals in fib Bulletin 72 [3], MC2010 [4], and Mancini et al. [8] are seen to represent a circa 40% increase in full strength (σ_{sd} = 435 MPa) lap lengths over current EN1992 [1] lap lengths for 25 mm bars in 'good' bond conditions in a typical C30/37 concrete. That would equate current good bond conditions to future bad conditions which would be unacceptable to industry. In the UK and elsewhere, the current EN 1992 [1] lap lengths are already regarded as excessive. Contrary to current practice, the anchorage proposals in Mancini et al. [8] relate bond strength (equation 24) to the reinforcement stress. This is advantageous for the design of short tension anchorages but less so for laps which, in UK practice, are typically designed for f_{yd}. This is the case even if laps are not positioned at points of maximum moment since it is not considered practical or economic to design, detail, draw, fix and check each lap for the particular stress in the bars.

A parametric study was carried out to compare the tension lap design recommendations of EN 1992 [1], Mancini et al. [8] (equation 21 with $C_{anch,d}$ = 88) and Vollum (equation 26). Figure 7 compares design bond strengths calculated for 100% laps with bar diameter ϕ = 20 mm, f_{ck} = 30 MPa, cover = 30 mm, clear bar spacing of 110 mm and k_{tr} = 0. The limiting minimum lap strength σ_{sd} shown for each method is the strength calculated for a minimum allowable lap length of 10 bar diameters. For this example, the design bond strengths given by the Vollum method are greater than given by EN 1992 [1]. Design bond stresses corresponding to the Vollum method (equation 30) and equation 24 with ζ_d from equation 34 (depicted Equation 24 linearly varying ζ_d), are equal for $f_{st} \geq 197$ MPa since in this case the two methods are almost identical as explained in Section 4.3. The Vollum proposal is more economic (greater bond stress) than Mancini et al. [8] for design lap strengths above f_{std} = 320 MPa. Below f_{std} = 320 MPa, Mancini et al. [8] is more economic but, according to Table 4, it has below the level of safety expected by EN 1990 [11] for $f_{st,test} \leq 300$ MPa.

Table 6: Comparison of tension lap requirements

In a typical, simple 100% lap for a 25 mm bar in good bond conditions, bar stress = f_{yd} = 435 MPa, in C30/37 concrete, no confinement and c_{max} = c_{min} = bar diameter + 10 mm, the lap length would be:		
Ref	**Calculation**	**$(l_b/\phi)_d$**
EN1992-1-1:2004 [1] Equations 1 and 2	l_{bd}/ϕ $= \alpha_1\ \alpha_2\ \alpha_3\ \alpha_5\ \alpha_6\ (1/4)(\sigma_{sd}/(f_{bd}/\gamma_C)$ $= 1.0x0.94x1.0x1.5(1/4)(435/(2.25x1.0x1.0x0.21f_{ck}^{0.666}/1.5))$ $= 486.8/f_{ck}^{0.666}$	= 50
fib Bulletin 72 [3] Equation 6 and γ_c=1.5	l_{bd}/ϕ $= 73.5\gamma_C\ (25/f_{cm})^{5/11}(\sigma_{sd}/435)^{20/11}(\phi/25)^{4/11}(\phi/c_{min})^{5/11}$ $= 73.5x1.5x(25/38)^{5/11}(435/435)^{20/11}(25/25)^{4/11}(25/35)^{5/11}$ = 110x0.826x1.0x1.0x0.858	= 78
MC2010 [4] Equation 13	l_{bd}/ϕ $= \sigma_{sd}/[4(\alpha_2+\alpha_3)(\eta_1\ \eta_2\ \eta_3\ \eta_4(f_{ck}/25)^{0.5}/\gamma_C]$ $= 435/[4x((35/25)^{0.5}+0)x(1.75x1.0x1.0x1.0x(30/25)^{0.5}/1.5]$ $= 435/[(4x1.183x1.75x1.095/1.5)] \geq l_{b,min}/\phi$ = 435/6.05 =	= 72
Mancini et al. [8] Equation 21 (ζ_d = 0.69)	l_{bd}/ϕ $= C_{anch,d}(25/f_{ck})^{0.45}(\sigma_{sd}/435)^{1.82}(\phi/25)^{0.36}/(\alpha_1+\alpha_2)$ $= 88(25/30)^{0.45}(435/435)^{1.82}(25/25)^{0.36}/((35/25)^{0.5}(35/35)^{0.15}+0)$ = 88x0.921x1.0x1.0/1.18	= 69
Vollum Equation 26	l_{bd}/ϕ $= mC_{anch,d}(25/f_{ck})^{0.45}\ (\phi/25)^{0.36}/(\alpha_2+\alpha_3)$ $= (435/435)x67(25/30)^{0.45}\ (25/25)^{0.36}/((35/25)^{0.5}(35/35)^{0.15}+0)$ = 1.0x67x0.921x1.0 / 1.183	= 52

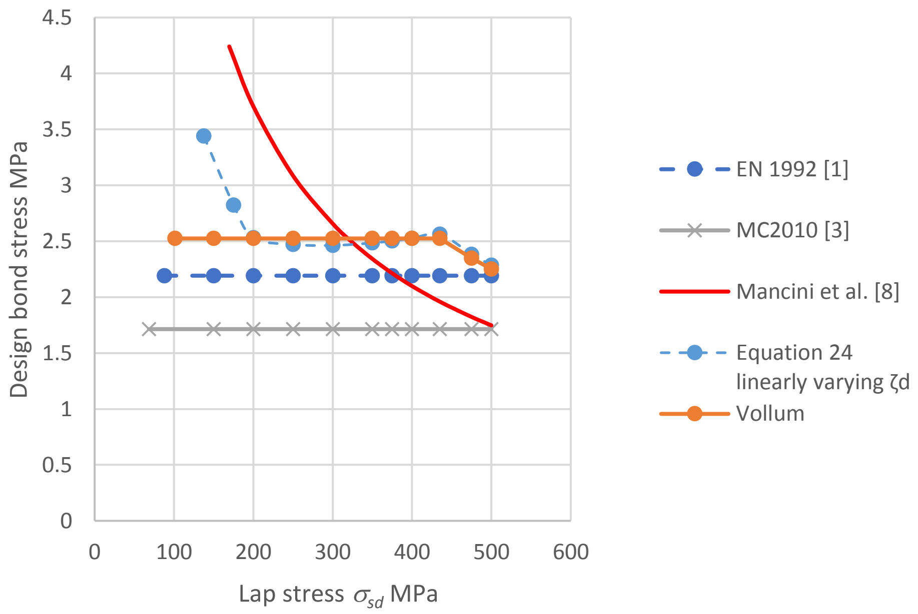


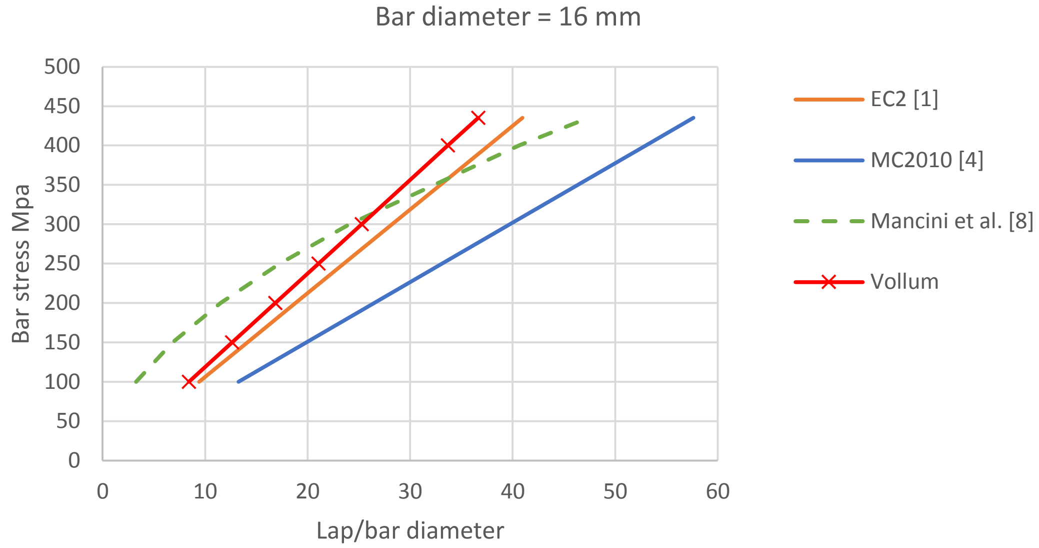
Figure 7: Influence of reinforcement stress on design bond strength

Figures 8a to 8f show the influence of:

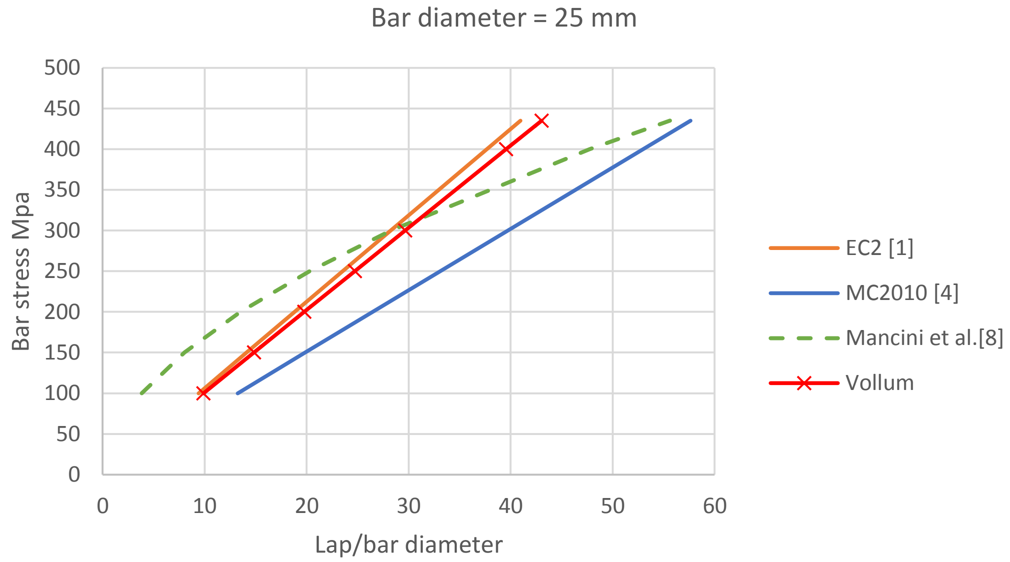
- reinforcement stress on the required design lap length for 16 mm, 25 mm and 32 mm diameter bars (f_{ck} = 40 MPa, cover 1.5ϕ, clear bar spacing 6ϕ, k_{tr} = 0). (see Figures 8a to 8c)
- concrete strength on the full strength design lap strength of 25 mm diameter bars (f_{yd} = 435 MPa, cover 1.5ϕ, clear bar spacing 6ϕ, k_{tr} = 0). (see Figure 8d)
- bar centreline spacing on full strength design lap length of 25 mm diameter bars (f_{yd} = 435 MPa, f_{ck} = 30 MPa, cover 1.5ϕ, k_{tr} = 0). (see Figure 8e)
- Confining reinforcement k_{tr} on full strength design lap length of 25 mm diameter bars ($f_{yd} = 435$ MPa, f_{ck} = 30 MPa, cover 1.5ϕ, clear bar spacing 4ϕ). (see Figure 8f)

The coefficient α_6 was taken as 1.5 in the calculation of EN 1992 [1] laps which corresponds to greater than 50% of bars being lapped at a cross section. Figures 8a to 8f show that the proposed Vollum method tends to give the shortest lap lengths for reinforcement stresses above around 300 MPa. The exceptions are very closely spaced bars and bars of 32 mm or greater diameter where EN 1992 requires slightly shorter laps (~15%) than the proposed Vollum method (equation 26).

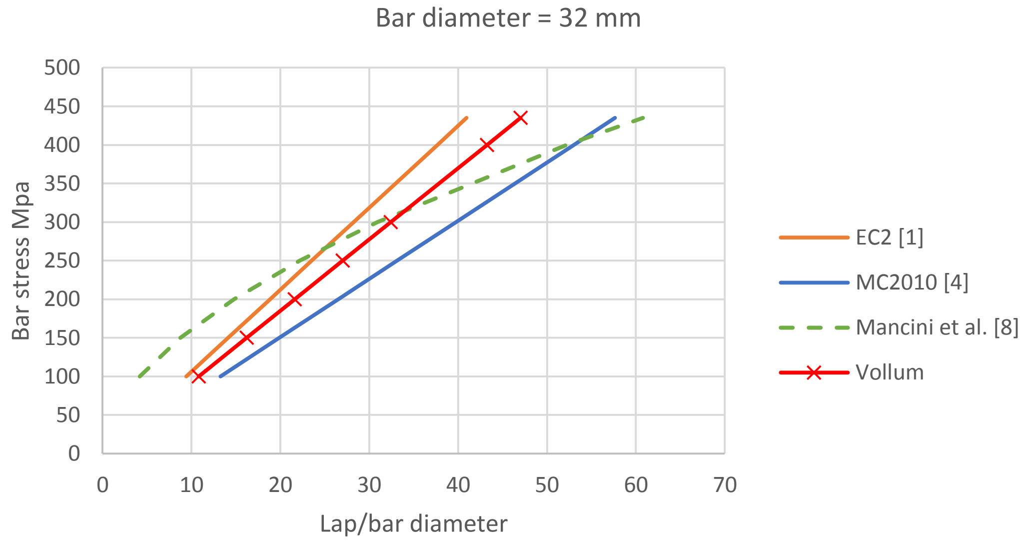
a)


Bar diameter = 16 mm
Bar stress Mpa
500
450
400
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
Lap/bar diameter
EC2 [1]
MC2010 [4]
Mancini et al. [8]
Vollum

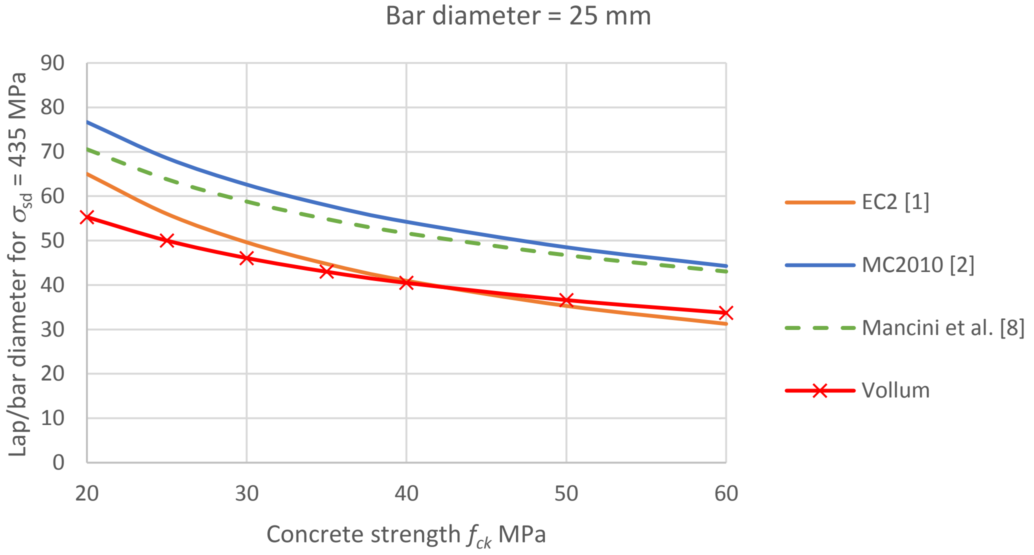
b)


Bar diameter = 25 mm
Bar stress Mpa
500
450
400
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
Lap/bar diameter
EC2 [1]
MC2010 [4]
Mancini et al.[8]
Vollum

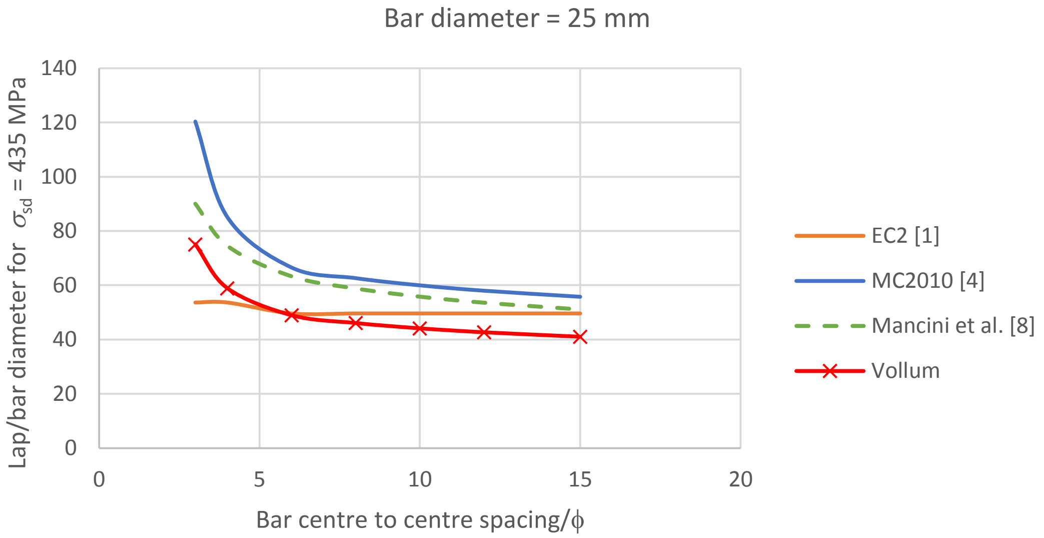
c)


Bar diameter = 32 mm
Bar stress Mpa
500
450
400
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
Lap/bar diameter
EC2 [1]
MC2010 [4]
Mancini et al. [8]
Vollum

d)



e)



f)

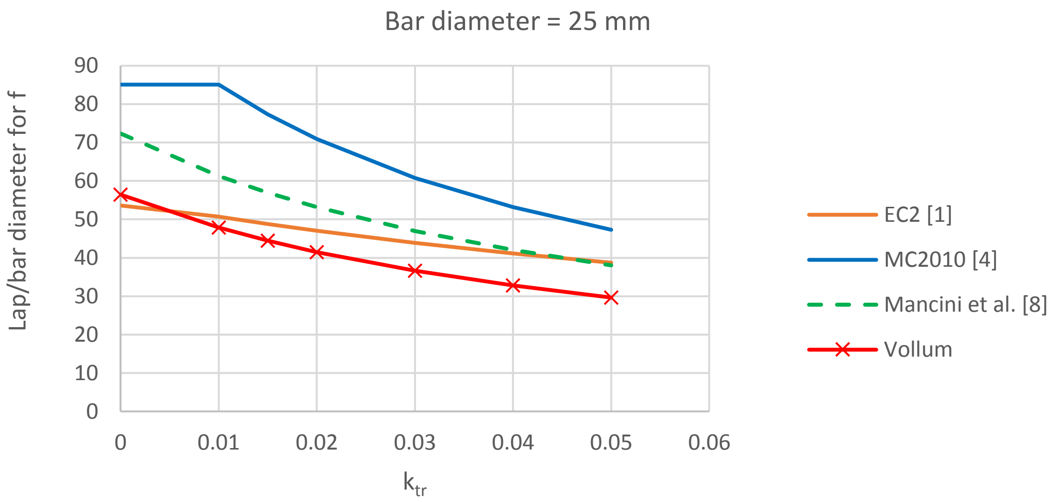


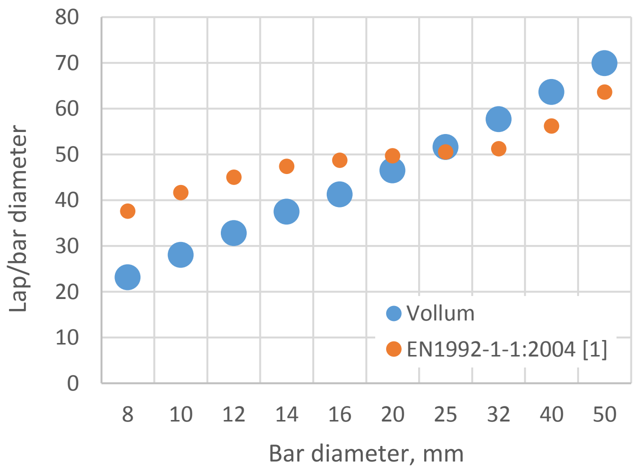
Figure 8: Comparison of design methods: influence of a-c) bar diameter, d) concrete strength, e) bar spacing and f) confining reinforcement index k_{tr}

5.1 Considerations for choice of design method

The choice of design method should ideally be based on accuracy, economy and ease of use. Section 4.4 shows that the Vollum proposal (equation 26) provides the level of safety expected by EN 1990 [11] while Section 5 shows that it scores best in terms of economy for highly stressed laps. For medium sized bars, the Vollum proposal broadly maintains current EN1992 [1] full strength lap lengths for grade 500 reinforcement. Other proposals increase them, sometimes substantially. Furthermore, the Vollum proposal provides conforming and acceptable failure rates across all lap strengths, whereas Mancini et al. [8] (equation 21 with $C_{anch,d}$ = 88) does not for the considered data (see Table 4). For these reasons Vollum (equation 26) is recommended.

The relative economy of the Vollum method compared with EN 1992 is further explored in Figure 9 which shows the influence of bar diameter on full strength lap lengths required by these two methods for f_{ck} = 30 MPa, f_{yd} = 435 MPa, min centres 4ϕ, c_{min} = Max(25,ϕ+10) and no confinement from links. The Vollum proposal is seen to require shorter laps for bar diameters of 20 mm and below but slightly greater laps for bars of 25 mm and above.

Apart from accuracy and economy, ease of use is important. This is somewhat subjective but the Vollum proposal scores well in this regard since the design bond strength is independent of reinforcement stress for grade 500 reinforcement and below. Overall, the Vollum proposal is seen as giving the best outcome on ease of use and economy and is, therefore, advocated.



Note: f_{ck} = 30 MPa, f_{yd} = 435 MPa, min centres 4ϕ, cmin = Max(25,ϕ+10), no confinement

Figure 9: Comparison of full strength lap lengths required by the Vollum proposal and EN1992 [1]

6 Conclusions and recommendations

The paper reviews the background to the MC 2010 design provisions for tension laps as well as the statistical calibration of it by Mancini et al. [8]. These recommendations have been broadly adopted in the draft revision to EN 1992 due for publication in 2023 [2] but the basic multiplier required to achieve the level of safety expected by EN 1990 is still under discussion. The MC2010 design provisions for tension laps are derived from equation 3-2 of fib Bulletin 72 [3] which is shown to become increasingly conservative with increasing lap strength. The authors use a reliability analysis, derived from Mancini et al. [8], to show that the probabilistic coefficient ζ_j used to calibrate equation 3-2 [8] increases with lap strength. On the basis of this analysis, Vollum proposes equation 26 for calculating the required lap length. The Vollum proposal (equation 26) is shown to give reasonable estimates of measured strength for the fib tension splice database [10] as well as 17 specimens tested by Micallef and Vollum [16,17]. It is suggested that the design bond stress for Grade 500 reinforcement and below should be based on the design yield strength of Grade 500 reinforcement which is taken as 435 MPa. The full strength lap lengths given by the Vollum proposal (equation 26) are shown to be significantly shorter than calculated using the recommendations of Mancini et al. [8] (equation 21 with $C_{anch,d} = 88$). For medium sized bars, lap lengths calculated with the Vollum proposal are comparable with those given by EN 1992 [1].

6.1 Recommendations

For design, the Vollum proposal (equation 26) together with values of 67 proven for $C_{anch,d}$ and provision to allow γ_c to vary away from 1.5, is given as equation 33 for anchorage and laps and as equation 38 for bond stress. The following formulae are recommended for inclusion in the c2023 revision of EN1992:

<u>Anchorage and lap length</u>

$$\frac{l_{bd}}{\phi} = 67m\left(\frac{\gamma_c}{1.5}\right)^{0.64}\left(\frac{25}{f_{ck}}\right)^{0.45}\left(\frac{\phi}{25}\right)^{0.36}/(\alpha_2+\alpha_3) \geq 10\phi \quad [33]$$

Where

l_{bd} = design anchorage or lap length

ϕ = bar diameter

m = Max (σ_{sd}/435, $(\sigma_{sd}/435)^{1.82}$) [34]

$$\alpha_2 = \left(\frac{c_{min}}{\phi}\right)^{0.5}\left(\frac{c_{max}}{c_{min}}\right)^{0.15} \quad [10, 35]$$

Where

c_{min}, c_{max} minimum and maximum cover distances according to Figure 6.1-2 of MC2010.

$\alpha_3 = k_d K_{tr}$ [11, 36]

Where

Transverse confinement factor, $K_{tr} = n_l A_{sv}/(s_v \emptyset n_b) \leq 0.05$ [4,37]

Effectiveness factor, k_d = 20 for bars less than 125 mm or 5 bar diameters away from the nearest vertical leg of a link crossing the splitting plane approximately perpendicularly

= 10 for internal bars confined by straight bars within the cover zone with $c_s > 8\phi$ (i.e. spacing > 9ϕ centres)

= 0 for other circumstances [4].

Bond stress

Where bond stress is required, design bond stress may be assumed to be constant for σ_{sd} ≤ 435 MPa and assessed as being:

$$f_{bd} = \min(1, \left(\frac{435}{\sigma_{sd}}\right)^{0.82})1.6\left(\frac{1.5}{\gamma_c}\right)^{0.64}\left(\frac{f_{ck}}{25}\right)^{0.45}\left(\frac{25}{\phi}\right)^{0.36}(\alpha_2 + \alpha_3) \qquad [30, 38]$$

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