Efficient Handling of SPARQL OPTIONAL for OBDA (Extended Version)*

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Abstract

`OPTIONAL` is a key feature in SPARQL for dealing with missing information. While this operator is used extensively, it is also known for its complexity, which can make efficient evaluation of queries with `OPTIONAL` challenging. We tackle this problem in the Ontology-Based Data Access (OBDA) setting, where the data is stored in a SQL relational database and exposed as a virtual RDF graph by means of an R2RML mapping. We start with a succinct translation of a SPARQL fragment into SQL. It fully respects bag semantics and three-valued logic and relies on the extensive use of the `LEFT JOIN` operator and `COALESCE` function. We then propose optimisation techniques for reducing the size and improving the structure of generated SQL queries. Our optimisations capture interactions between `JOIN`, `LEFT JOIN`, `COALESCE` and integrity constraints such as attribute nullability, uniqueness and foreign key constraints. Finally, we empirically verify effectiveness of our techniques on the BSBM OBDA benchmark.

1 Introduction

Ontology-Based Data Access (OBDA) aims at easing the access to database content by bridging the semantic gap between *information needs* (what users want to know) and their formulation as executable queries (typically in SQL). This approach hides the complexity of the database structure from users by providing them with a high-level representation of the data as an RDF graph. The RDF graph can be regarded as a view over the database defined by a DB-to-RDF mapping (e.g., following the R2RML specification) and enriched by means of an ontology [4]. Users can then formulate their information needs directly as high-level SPARQL queries over the RDF graph. We focus on the standard OBDA setting, where the RDF graph is not materialised (and is called a *virtual RDF graph*), and the database is relational and supports SQL [18].

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To answer a SPARQL query, an OBDA system reformulates it into a SQL query, to be evaluated by the DBMS. In theory, such a SQL query can be obtained by *(1)* translating the SPARQL query into a relational algebra expression over the ternary relation *triple* of the RDF graph, and then *(2)* replacing the occurrences of *triple* by the matching definitions in the mapping; the latter step is called *unfolding*. We note that, in general, step (1) also includes rewriting the user query with respect to the given (OWL 2 QL) ontology [5, 15]; we, however, assume that the query is already rewritten and, for efficiency reasons, the mapping is saturated; for details, see [15, 24].

SPARQL joins are naturally translated into (`INNER`) `JOIN`s in SQL [9]. However, in contrast to expert-written SQL queries, there typically is a *high margin for optimisation* in naively translated and unfolded queries. Indeed, since SPARQL, unlike SQL, is based on a single ternary relation, queries usually contain many more joins than SQL queries for the same information need; this suggests that many of the `JOIN`s in unfolded queries are redundant and could be eliminated. In fact, the semantic query optimisation techniques such as self-join elimination [6] can reduce the number of `INNER JOIN`s [21, 19].

We are interested in SPARQL queries containing the `OPTIONAL` operator introduced to deal with *missing* information, thus serving a similar purpose [9] to the `LEFT` (`OUTER`) `JOIN` operator in relational databases. The graph pattern P_1 `OPTIONAL` P_2 returns answers to P_1 extended (if possible) by answers to P_2; when an answer to P_1 has no match in P_2 (due to incompatible variable assignments), the variables that occur only in P_2 remain *unbound* (`LEFT JOIN` extends a tuple without a match with `NULL`s). The focus of this work is the efficient handling of queries with `OPTIONAL` in the OBDA setting. This problem is important in practice because *(a)* `OPTIONAL` is very frequent in real SPARQL queries [17, 1]; *(b)* it is a source of computational complexity: query evaluation is PSPACE-hard for the fragment with `OPTIONAL` alone [23] (in contrast, e.g., to basic graph patterns with filters and projection, which are NP-complete); *(c)* unlike expert-written SQL queries, the SQL translations of SPARQL queries (e.g., [8]) tend to have more `LEFT JOIN`s with more complex structure, which DBMSs may fail to optimise well. We now illustrate the difference in the structure with an example.

Example 1. Let `people` be a database relation composed of a primary key attribute `id`, a non-nullable attribute `fullName` and two nullable attributes, `workEmail` and `homeEmail`:

<u>`id`</u>	`fullName`	`workEmail`	`homeEmail`
1	Peter Smith	peter@company.com	peter@perso.org
2	John Lang	`NULL`	joe@perso.org
3	Susan Mayer	susan@company.com	`NULL`

Consider an information need to retrieve the names of people and their e-mail addresses if they are available, with the *preference* given to work over personal e-mails. In standard SQL, the IT expert can express such a preference by means of the `COALESCE` function: e.g., `COALESCE`(v_1, v_2) returns v_1 if it is not `NULL` and v_2 otherwise. The following SQL query retrieves the required names and e-mail addresses:

```
SELECT fullName, COALESCE(workEmail, homeEmail) FROM people.
```

The same information need could naturally be expressed in SPARQL:

```
SELECT ?n ?e { ?p :name ?n  OPTIONAL { ?p :workEmail ?e }
```

```
                          OPTIONAL { ?p :personalEmail ?e } }.
```

Intuitively, for each person ?p, after evaluating the first OPTIONAL operator, variable ?e is bound to the work e-mail if possible, and left unbound otherwise. In the former case, the second OPTIONAL cannot extend the solution mapping further because all its variables are already bound; in the latter case, the second OPTIONAL tries to bind a personal e-mail to ?e. See [9] for a discussion on a similar query, which is weakly well-designed [14].

One can see that the two queries are in fact equivalent: the SQL query gives the same answers on the people relation as the SPARQL query on the RDF graph that encodes the relation by using id to generate IRIs and populating data properties :name, :workEmail and :personalEmail by the non-NULL values of the respective attributes.

However, the unfolding of the translation of the SPARQL query above would produce two LEFT OUTER JOINs, even with known simplifications (see, e.g., Q_2 in [8]):

```
SELECT v3.fullName AS n, COALESCE(v3.workEmail,v4.homeEmail) AS e
FROM (SELECT v1.fullName, v1.id, v2.workEmail FROM people v1
LEFT JOIN people v2 ON v1.id=v2.id AND v2.workEmail IS NOT NULL) v3
LEFT JOIN people v4 ON v3.id=v4.id AND v4.homeEmail IS NOT NULL
        AND (v3.workEmail=v4.homeEmail OR v3.workEmail IS NULL),
```

which is unnecessarily complex (compared to the expert-written SQL query above). Observe that the last bracket is an example of a *compatibility filter* encoding compatibility of SPARQL solution mappings in SQL: it contains disjunction and IS NULL. ❑

Example 1 shows that SQL translations with LEFT JOINs can be simplified drastically. In fact, the problem of optimising LEFT JOINs has been investigated both in relational databases [12, 20] and RDF triplestores [8, 2]. In the database setting, *reordering* of OUTER JOINs has been studied extensively because it is essential for efficient query plans, but also challenging as these operators are neither commutative nor associative (unlike INNER JOINs). To perform a reordering, query planners typically rely on simple joining conditions, in particular, on conditions that reject NULLs and do not use COALESCE [12]. However, the SPARQL-to-SQL translation produces precisely the opposite of what database query planners expect: LEFT JOINs with complex compatibility filters. On the other hand, Chebotko *et al.* [8] proposed some simplifications when an RDBMS stores the *triple* relation and acts as an RDF triplestore. Although these simplifications are undoubtedly useful in the OBDA setting, the presence of mappings brings additional challenges and, more importantly, significant opportunities.

Example 2. Consider Example 1 again and suppose we now want to retrieve people's names, and when available also their work e-mail addresses. We can naturally represent this information need in SPARQL:

```
SELECT ?n ?e { ?p :name ?n OPTIONAL { ?p :workEmail ?e } }.
```

We can also express it very simply in SQL:

```
SELECT fullName, workEmail FROM people.
```

Instead, the straightforward translation and unfolding of the SPARQL query produces

```
SELECT v1.fullName AS n, v2.workEmail AS e
FROM people v1 LEFT JOIN people v2 ON v1.id=v2.id AND
                                  v2.workEmail IS NOT NULL.
```

R2RML mappings filter out NULL values from the database because NULLs cannot appear in RDF triples. Hence, the join condition in the unfolded query contains an IS NOT NULL for the workEmail attribute of v2. On the other hand, the LEFT JOIN of the query assigns a NULL value to workEmail if no tuple from v2 satisfies the join condition for a given tuple from v1. We call an assignment of NULL values by a LEFT JOIN the *padding effect*. A closer inspection of the query reveals, however, that the padding effect only applies when workEmail in v2 is NULL. Thus, the role of the LEFT JOIN in this query boils down to re-introducing NULLs eliminated by the mapping. In fact, this situation is quite typical in OBDA but does not concern RDF triplestores, which do not store NULLs, or classical data integration systems, which can expose NULLs through their mappings. ❑

In this paper we address these issues, and our contribution is summarised as follows.

1. In Sec. 3, we provide a succinct translation of a fragment of SPARQL 1.1 with OPTIONAL and MINUS into relational algebra that relies on the use of LEFT JOIN and COALESCE. Even though the ideas can be traced back to Cyganiak [9] and Chebotko *et al.* [8] for the earlier SPARQL 1.0, our translation fully respects *bag semantics* and the *three-valued logic* of SPARQL 1.1 and SQL [13] (and is formally proven correct).

2. We develop optimisation techniques for SQL queries with complex LEFT JOINs resulting from the translation and unfolding: Compatibility Filter Reduction (CFR, Sec. 4.1), which generalises [8], LEFT JOIN Naturalisation (LJN, Sec. 4.2) to avoid padding, Natural LEFT JOIN Reduction (NLJR, Sec. 4.4), JOIN Transfer (JT, Sec. 4.5) and LEFT JOIN Decomposition (LJD, Sec. 4.6) complementing [12]. By CFR and LJN, compatibility filters and COALESCE are eliminated for well-designed SPARQL (Sec. 4.3).

3. We carried out an evaluation of our optimisation techniques over the well-known OBDA benchmark BSBM [3], where OPTIONALs, LEFT JOINs and NULLs are ubiquitous. Our experiments (Sec. 5) show that the techniques of Sec. 4 lead to a significant improvement in performance of the SQL translations, even for commercial DBMSs.

2 Preliminaries

We first formally define the syntax and semantics of the SPARQL fragment we deal with and then present the relational algebra operators used for the translation from SPARQL.

RDF provides a basic data model. Its vocabulary contains three pairwise disjoint and countably infinite sets of symbols: IRIs I, blank nodes B and RDF literals L. *RDF terms* are elements of $\mathsf{C} = \mathsf{I} \cup \mathsf{B} \cup \mathsf{L}$, *RDF triples* are elements of $\mathsf{C} \times \mathsf{I} \times \mathsf{C}$, and an *RDF graph* is a finite set of RDF triples.

2.1 SPARQL

SPARQL adds a countably infinite set V of *variables*, disjoint from C. A *triple pattern* is an element of $(\mathsf{C} \cup \mathsf{V}) \times (\mathsf{I} \cup \mathsf{V}) \times (\mathsf{C} \cup \mathsf{V})$. A *basic graph pattern* (*BGP*) is a finite set of triple patterns. We consider *graph patterns*, P, defined by the grammar[1]

$$P \ ::= \ B \ \mid \ \textsc{Filter}(P, F) \ \mid \ \textsc{Union}(P_1, P_2) \ \mid \textsc{Join}(P_1, P_2) \ \mid \\ \textsc{Opt}(P_1, P_2, F) \ \mid \ \textsc{Minus}(P_1, P_2) \ \mid \ \textsc{Proj}(P, L),$$

where B is a BGP, $L \subseteq \mathsf{V}$ and F, called a *filter*, is a formula constructed using logical connectives $\wedge$ and $\neg$ from atoms of the form $\mathit{bound}(v)$, $(v = c)$, $(v = v')$, for $v, v' \in \mathsf{V}$ and $c \in \mathsf{C}$. The set of variables in P is denoted by $\mathit{var}(P)$.

Variables in graph patterns are assigned values by *solution mappings*, which are *partial* functions $s\colon \mathsf{V} \to \mathsf{C}$ with (possibly empty) domain $\mathit{dom}(s)$. The *truth-value* $F^s \in \{\top, \bot, \varepsilon\}$ *of a filter F under a solution mapping s* is defined inductively:

- $(\mathit{bound}(v))^s$ is $\top$ if $v \in \mathit{dom}(s)$, and $\bot$ otherwise;
- $(v = c)^s = \varepsilon$ ('error') if $v \notin \mathit{dom}(s)$; otherwise, $(v = c)^s$ is the classical truth-value of the predicate $s(v) = c$; similarly, $(v = v')^s = \varepsilon$ if $\{v, v'\} \not\subseteq \mathit{dom}(s)$; otherwise, $(v = v')^s$ is the classical truth-value of the predicate $s(v) = s(v')$;
- $(\neg F)^s = \begin{cases} \bot, & \text{if } F^s = \top, \\ \top, & \text{if } F^s = \bot, \\ \varepsilon, & \text{if } F^s = \varepsilon, \end{cases}$ and $(F_1 \wedge F_2)^s = \begin{cases} \bot, & \text{if } F_1^s = \bot \text{ or } F_2^s = \bot, \\ \top, & \text{if } F_1^s = F_2^s = \top, \\ \varepsilon, & \text{otherwise.} \end{cases}$

We adopt bag semantics for SPARQL: the answer to a graph pattern over an RDF graph is a multiset (or bag) of solution mappings. Formally, a *bag of solution mappings* is a (total) function Ω from the set of all solution mappings to non-negative integers $\mathbb{N}$: $\Omega(s)$ is called the *multiplicity* of s (we often use $s \in \Omega$ as a shortcut for $\Omega(s) > 0$). Following the grammar of graph patterns, we define respective operations on solution mapping bags. Solution mappings s_1 and s_2 are called *compatible*, written $s_1 \sim s_2$, if $s_1(v) = s_2(v)$, for each $v \in \mathit{dom}(s_1) \cap \mathit{dom}(s_2)$, in which case $s_1 \oplus s_2$ denotes a solution mapping with domain $\mathit{dom}(s_1) \cup \mathit{dom}(s_2)$ and such that $s_1 \oplus s_2\colon v \mapsto s_1(v)$, for $v \in \mathit{dom}(s_1)$, and $s_1 \oplus s_2\colon v \mapsto s_2(v)$, for $v \in \mathit{dom}(s_2)$. We also denote by $s|_L$ the restriction of s on $L \subseteq \mathsf{V}$. Then the SPARQL operations are defined as follows:

- $\textsc{Filter}(\Omega, F) = \Omega'$, where $\Omega'(s) = \Omega(s)$ if $s \in \Omega$ and $F^s = \top$, and 0 otherwise;
- $\textsc{Union}(\Omega_1, \Omega_2) = \Omega$, where $\Omega(s) = \Omega_1(s) + \Omega_2(s)$;
- $\textsc{Join}(\Omega_1, \Omega_2) = \Omega$, where $\Omega(s) = \sum_{\substack{s_1 \in \Omega_1, s_2 \in \Omega_2 \text{ with} \\ s_1 \sim s_2 \text{ and } s_1 \oplus s_2 = s}} \Omega_1(s_1) \times \Omega_2(s_2)$;
- $\textsc{Opt}(\Omega_1, \Omega_2, F) = \textsc{Union}(\textsc{Filter}(\textsc{Join}(\Omega_1, \Omega_2), F), \Omega)$, where $\Omega(s) = \Omega_1(s)$ if $F^{s \oplus s_2} \neq \top$, for all $s_2 \in \Omega_2$ compatible with s, and 0 otherwise;
- $\textsc{Minus}(\Omega_1, \Omega_2) = \Omega$, where $\Omega(s) = \Omega_1(s)$ if $\mathit{dom}(s) \cap \mathit{dom}(s_2) = \emptyset$, for all solution mappings $s_2 \in \Omega_2$ compatible with s, and 0 otherwise;
- $\textsc{Proj}(\Omega, L) = \Omega'$, where $\Omega'(s') = \sum_{s \in \Omega \text{ with } s|_L = s'} \Omega(s)$.

Given an RDF graph G and a graph pattern P, the *answer* $[\![P]\!]_G$ *to P over G* is a bag of solution mappings defined by induction using the operations above and starting from basic graph patterns: $[\![B]\!]_G(s) = 1$ if $\mathit{dom}(s) = \mathit{var}(B)$ and G contains the triple $s(B)$ obtained by replacing each variable v in B by $s(v)$, and 0 otherwise ($[\![B]\!]_G$ is a set).

[1] A slight extension of the grammar and the full translation are given in Appendix A.

2.2 Relational Algebra (RA)

We recap the three-valued and bag semantics of relational algebra [13] and fix the notation. Denote by Δ the underlying domain, which contains a distinguished element *null*. Let U be a finite (possibly empty) set of *attributes*. A *tuple over* U is a (total) map $t\colon U \to \Delta$; there is a unique tuple over $\emptyset$. A *relation* R *over* U is a *bag* of tuples over U, that is, a function from all tuples over U to $\mathbb{N}$. For relations R_1 and R_2 over U, we write $R_1 \subseteq R_2$ ($R_1 \equiv R_2$) if $R_1(t) \leq R_2(t)$ ($R_1(t) = R_2(t)$, resp.), for all t.

A *term* v over U is an attribute $u \in U$, a constant $c \in \Delta$ or an expression *if*(F, v, v'), for terms v and v' over U and a filter F over U. A *filter* F *over* U is a formula constructed from atoms *isNull*(V) and $(v = v')$, for a set V of terms and terms v, v' over U, using connectives $\land$ and $\neg$. Given a tuple t over U, it is extended to terms as follows:

$$t(c) = c, \text{ for constants } c \in \Delta, \qquad \text{and} \qquad t(\textit{if}(F, v, v')) = \begin{cases} t(v), & \text{if } F^t = \top, \\ t(v'), & \text{otherwise,} \end{cases}$$

where the *truth-value* $F^t \in \{\top, \bot, \varepsilon\}$ *of* F *on* t is defined inductively (ε is *unknown*):

- $(\textit{isNull}(V))^t$ is $\top$ if $t(v)$ is *null*, for all $v \in V$, and $\bot$ otherwise;
- $(v = v')^t = \varepsilon$ if $t(v)$ or $t(v')$ is *null*, and the truth-value of $t(v) = t(v')$ otherwise;
- and the standard clauses for $\neg$ and $\land$ in the three-valued logic (see Sec. 2.1).

We use standard abbreviations *coalesce*(v, v') for *if*$(\neg\textit{isNull}(v), v, v')$ and $F_1 \lor F_2$ for $\neg(\neg F_1 \land \neg F_2)$. Unlike Chebotko *et al.* [8], we treat *if* as primitive, even though the renaming operation with an *if* could be defined via standard operations of RA.

For filters in positive contexts, we define a weaker equivalence: filters F_1 and F_2 over U are *p-equivalent*, written $F_1 \equiv^+ F_2$, in case $F_1^t = \top$ iff $F_2^t = \top$, for all t over U.

We use standard relational algebra operations: union $\cup$, difference $\setminus$, projection π, selection σ, renaming ρ, extension ν, natural (inner) join $\bowtie$ and duplicate elimination δ. We say that tuples t_1 over U_1 and t_2 over U_2 are *compatible*[2] if $t_1(u) = t_2(u) \neq \textit{null}$, for all $u \in U_1 \cap U_2$, in which case $t_1 \oplus t_2$ denotes a tuple over $U_1 \cup U_2$ such that $t_1 \oplus t_2 \colon u \mapsto t_1(u)$, for $u \in U_1$, and $t_1 \oplus t_2 \colon u \mapsto t_2(u)$, for $u \in U_2$. For a tuple t_1 over U_1 and $U \subseteq U_1$, we denote by $t_1|_U$ the restriction of t_1 to U. Let R_i be relations over U_i, for $i = 1, 2$. The semantics of the above operations is as follows:

- If $U_1 = U_2$, then $R_1 \cup R_2$ and $R_1 \setminus R_2$ are relations over U_1 satisfying $(R_1 \cup R_2)(t) = R_1(t) + R_2(t)$ and $(R_1 \setminus R_2)(t) = R_1(t)$ if $t \notin R_2$ and 0 otherwise;
- If $U \subseteq U_1$, then $\pi_U R_1$ is a relation over U with $\pi_U R_1(t) = \sum_{t_1 \in R_1 \text{ with } t_1|_U = t} R_1(t_1)$;
- If F is a filter over U_1, then $\sigma_F R_1$ is a relation over U_1 such that $\sigma_F R_1(t)$ is $R_1(t)$ if $t \in R_1$ and $F^t = \top$, and 0 otherwise;
- $R_1 \bowtie R_2$ is a relation R over $U_1 \cup U_2$ such that $R(t) = \sum_{\substack{t_1 \in R_1 \text{ and } t_2 \in R_2 \\ \text{are compatible and } t_1 \oplus t_2 = t}} R_1(t_1) \times R_2(t_2)$;
- If v is a term over U_1 and $u \notin U_1$ an attribute, then the *extension* $\nu_{u \mapsto v} R_1$ is a relation R over $U_1 \cup \{u\}$ with $R(t \oplus \{u \mapsto t(v)\}) = R_1(t)$, for all t. The *extended projection* $\pi_{\{u_1/v_1, \dots, u_k/v_k\}}$ is a shortcut for $\pi_{\{u_1, \dots, u_k\}} \nu_{u_1 \mapsto v_1} \cdots \nu_{u_k \mapsto v_k}$.

[2] Note that, unlike in SPARQL, if u is *null* in either of the tuples, then they are incompatible.

- If $v \in U_1$ and $u \notin U_1$ are distinct attributes, then the *renaming* $\rho_{u/v}R_1$ is a relation over $U_1 \setminus \{v\} \cup \{u\}$ whose tuples t are obtained by replacing v in the domain of t by u. For terms $v_1, \dots, v_k$ over U_1, attributes $u_1, \dots, u_k$ (not necessarily distinct from U_1) and $V \subseteq U_1$, let $u'_1, \dots, u'_k$ be fresh attributes and abbreviate the sequence $\rho_{u_1/u'_1} \cdots \rho_{u_k/u'_k} \pi_{U_1 \cup \{u'_1,\dots,u'_k\} \setminus V} \nu_{u'_1 \mapsto v_1} \cdots \nu_{u'_k \mapsto v_k}$ by $\rho^V_{\{u_1/v_1,\dots,u_k/v_k\}}$.
- δR_1 is a relation over U_1 with $\delta R_1(t) = \min(R_1(t), 1)$.

To bridge the gap between partial functions (solution mappings) of SPARQL and total functions (tuples) of RA, we use a *padding* operation: $\mu_{\{u_1,\dots,u_k\}}R_1$ denotes $\nu_{u_1 \mapsto \mathit{null}} \cdots \nu_{u_k \mapsto \mathit{null}} R_1$, for $u_1, \dots, u_k \notin U_1$. Finally, we define the outer union, the (inner) join and left (outer) join operations by taking

$$R_1 \uplus R_2 \;=\; \mu_{U_2 \setminus U_1} R_1 \,\cup\, \mu_{U_1 \setminus U_2} R_2, \qquad R_1 \bowtie_F R_2 \;=\; \sigma_F(R_1 \bowtie R_2),$$
$$R_1 ⟕_F R_2 \;=\; (R_1 \bowtie_F R_2) \,\uplus\, (R_1 \setminus \pi_{U_1}(R_1 \bowtie_F R_2));$$

note that $\bowtie_F$ and $⟕_F$ are *natural joins*: they are over F as well as shared attributes.

An *RA query* Q is an expression constructed from relation symbols, each with a fixed set of attributes, and filters using the RA operations (and complying with all restrictions). A *data instance* D gives a relation over its set of attributes, for any relation symbol. The *answer to* Q *over* D is a relation $\|Q\|_D$ defined inductively in the obvious way starting from the base case of relation symbols: $\|Q\|_D$ is the relation given by D.

3 Succinct Translation of SPARQL to SQL

We first provide a translation of SPARQL graph patterns to RA queries that improves the worst-case exponential translation of [15] in handling JOIN, OPT and MINUS: it relies on the *coalesce* function (see also [8, 7]) and produces linear-size RA queries.

For any graph pattern P, the RA query $\boldsymbol{\tau}(P)$ returns the same answers as P when solution mappings are represented as relational tuples. For a set V of variables and solution mapping s with $\mathit{dom}(s) \subseteq V$, let $\mathit{ext}_V(s)$ be the tuple over V obtained from s by padding it with *null*s: formally, $\mathit{ext}_V(s) = s \oplus \{v \mapsto \mathit{null} \mid v \in V \setminus \mathit{dom}(s)\}$. The *relational answer* $\|P\|_G$ *to* P *over an RDF graph* G is a bag Ω of tuples over $\mathit{var}(P)$ such that $\Omega(\mathit{ext}_{\mathit{var}(P)}(s)) = [\![P]\!]_G(s)$, for all solution mappings s. Conversely, to evaluate $\boldsymbol{\tau}(P)$, we view an RDF graph G as a data instance $\mathit{triple}(G)$ storing G as a ternary relation *triple* with the attributes *sub*, *pred* and *obj* (note that $\mathit{triple}(G)$ is a set).

The translation of a triple pattern $\langle s, p, o\rangle$ is an RA query of the form $\pi_{\dots}\sigma_F \mathit{triple}$, where the subscript of the extended projection π and filter F are determined by the variables, IRIs and literals in s, p and o; see Appendix A. SPARQL operators UNION, FILTER and PROJ are translated into their RA counterparts: $\uplus$, σ and π, respectively, with SPARQL filters translated into RA by replacing each $\mathit{bound}(v)$ with $\neg \mathit{isNull}(v)$.

The translation of JOIN, OPT and MINUS is more elaborate and requires additional notation. Let P_1 and P_2 be graph patterns with $U_i = \mathit{var}(P_i)$, for $i = 1, 2$, and denote by U their shared variables, $U_1 \cap U_2$. To rename the shared attributes apart, we introduce fresh attributes u^1 and u^2 for each $u \in U$, set $U^i = \{u^i \mid u \in U\}$ and use abbreviations U^i/U and U/U^i for $\{u^i/u \mid u \in U\}$ and $\{u/u^i \mid u \in U\}$, respectively, for $i = 1, 2$. Now we can express the SPARQL solution mapping compatibility:

$$comp_U \;=\; \bigwedge\nolimits_{u \in U} \big[(u^1 = u^2) \lor \mathit{isNull}(u^1) \lor \mathit{isNull}(u^2)\big]$$

(intuitively, the *null* value of an attribute in the context of RA queries represents the fact that the corresponding SPARQL variable is not bound). Next, the renamed apart attributes need to be coalesced to provide the value in the representation of the resulting solution mapping; see $\oplus$ in Sec. 2.1. To this end, given an RA filter F over a set of attributes V, terms $v_1, \ldots, v_k$ over V and attributes $u_1, \ldots, u_k \notin V$, we denote by $F[u_1/v_1, \ldots, u_k/v_k]$ the result of replacing each u_i by v_i in F. We also denote by $\mathit{coalesce}_U$ the substitution of each $u \in U$ with $\mathit{coalesce}(u^1, u^2)$; thus, $F[\mathit{coalesce}_U]$ is the result of replacing each $u \in U$ in F with $\mathit{coalesce}(u^1, u^2)$. We now set

$$\begin{aligned}
\boldsymbol{\tau}(\textsc{Join}(P_1, P_2)) &= \rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}\big[\rho_{U^1/U}\boldsymbol{\tau}(P_1) \bowtie_{comp_U} \rho_{U^2/U}\boldsymbol{\tau}(P_2)\big],\\
\boldsymbol{\tau}(\textsc{Opt}(P_1, P_2, F)) &= \rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}\big[\rho_{U^1/U}\boldsymbol{\tau}(P_1) ⟕_{comp_U \land \boldsymbol{\tau}(F)[\mathit{coalesce}_U]} \rho_{U^2/U}\boldsymbol{\tau}(P_2)\big],\\
\boldsymbol{\tau}(\textsc{Minus}(P_1, P_2)) &= \pi_{U_1}\rho_{U/U^1}\sigma_{\mathit{isNull}(w)}\\
&\qquad \big[\rho_{U^1/U}\boldsymbol{\tau}(P_1) ⟕_{comp_U \land \bigvee_{u \in U}(u^1 = u^2)} \nu_{w \mapsto 1}\rho_{U^2/U}\boldsymbol{\tau}(P_2)\big],
\end{aligned}$$

where $w \notin U_1 \cup U_2$ is an attribute and $1 \in \Delta \setminus \{\mathit{null}\}$ is any domain element. The translation of Join and Opt is straightforward. For Minus, observe that $\nu_{w \mapsto 1}$ extends the relation for P_2 by a fresh attribute w with a non-*null* value. The join condition encodes compatibility of solution mappings whose domains, in addition, share a variable (both u^1 and u^2 are non-*null*). Tuples satisfying the condition are then filtered out by $\sigma_{\mathit{isNull}(w)}$, leaving only representations of solution mappings for P_1 that have no compatible solution mapping in P_2 with a shared variable. Finally, the attributes are renamed back by ρ_{U/U^1} and unnecessary attributes are projected out by π_{U_1}.

Theorem 3. *For any RDF graph G and any graph pattern P,* $\|P\|_G = \|\boldsymbol{\tau}(P)\|_{\mathit{triple}(G)}$.

The complete proof of Theorem 3 can be found in Appendix A.

4 Optimisations of Translated SPARQL Queries

We present optimisations on a series of examples. We begin by revisiting Example 1, which can now be given in algebraic form (for brevity, we ignore projecting away `?p`, which does not affect any of the optimisations discussed):

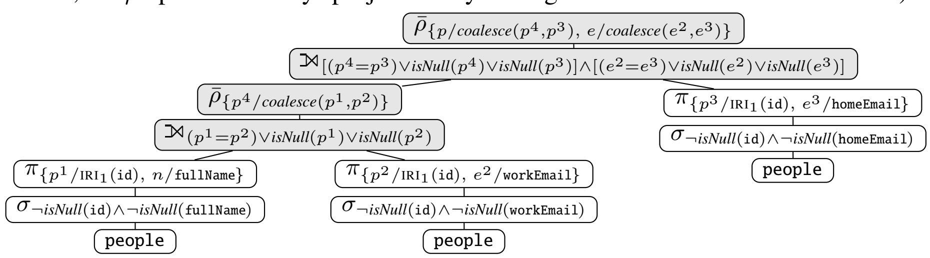
$$\textsc{Opt}(\textsc{Opt}(\texttt{?p :name ?n},\ \texttt{?p :workEmail ?e},\ \top), \texttt{?p :personalEmail ?e},\ \top),$$

where $\top$ denotes the tautological filter (true). Suppose we have the mapping

$$\begin{aligned}
\mathrm{IRI}_1(\texttt{id})\ \texttt{:name fullName} &\leftarrow \sigma_{\neg \mathit{isNull}(\texttt{id}) \land \neg \mathit{isNull}(\texttt{fullName})}\texttt{people},\\
\mathrm{IRI}_1(\texttt{id})\ \texttt{:workEmail workEmail} &\leftarrow \sigma_{\neg \mathit{isNull}(\texttt{id}) \land \neg \mathit{isNull}(\texttt{workEmail})}\texttt{people},\\
\mathrm{IRI}_1(\texttt{id})\ \texttt{:personalEmail homeEmail} &\leftarrow \sigma_{\neg \mathit{isNull}(\texttt{id}) \land \neg \mathit{isNull}(\texttt{homeEmail})}\texttt{people},
\end{aligned}$$

where IRI_1 is a function that constructs the IRI for a person from their ID (an *IRI template*, in R2RML parlance). We assume that the IRI functions are injective and map only *null* to *null*; thus, joins on $\text{IRI}_1(\texttt{id})$ can be reduced to joins on `id`, and $\mathit{isNull}(\texttt{id})$ holds just in case $\mathit{isNull}(\text{IRI}_1(\texttt{id}))$ holds. Interestingly, the IRI functions can encode GLAV mappings, where the target query is a full-fledged CQ (in contrast to GAV mappings, where atoms do not contain existential variables); for more details, see [10].

The translation given in Sec. 3 and unfolding produce the following RA query, where we abbreviate, for example, $\rho^{\{p^1,p^2\}}_{\{p^4/\mathit{coalesce}(p^1,p^2)\}}$ by $\bar{\rho}_{\{p^4/\mathit{coalesce}(p^1,p^2)\}}$ (in other words, the $\bar{\rho}$ operation always projects away the arguments of its *coalesce* functions):



In our diagrams, the white nodes are the contribution of the mapping and the translation of the basic graph patterns: for example, the basic graph pattern `?p :name ?n` produces $\pi_{\{p^1/\text{IRI}_1(\texttt{id}),\ n/\texttt{fullName}\}}\sigma_{\neg \mathit{isNull}(\texttt{id})\wedge\neg \mathit{isNull}(\texttt{fullName})}\texttt{people}$ (we use attributes without superscripts if there is only one occurrence; otherwise, the superscript identifies the relevant subquery). The grey nodes correspond to the translation of the SPARQL operations: for instance, the innermost left join is on $\mathit{comp}_{\{p\}}$ with p renamed apart to p^1 and p^2; the outermost left join is on $\mathit{comp}_{\{p,e\}}$, where p is renamed apart to p^4 and p^3 and e to e^2 and e^3; the two $\bar{\rho}$ are the respective renaming operations with *coalesce*.

4.1 Compatibility Filter Reduction (CFR)

We begin by simplifying the filters in (left) joins and eliminating renaming operations with *coalesce* above them (if possible). First, we can pull up the filters of the mapping through the extended projection and union by means of standard database equivalences: for example, for relations R_1 and R_2 and a filter F over U, we have $\sigma_F(R_1 \cup R_2) \equiv \sigma_F R_1 \cup \sigma_F R_2$, and $\pi_{U'}\sigma_{F'}R_1 \equiv \sigma_{F'}\pi_{U'}R_1$, if F' is a filter over $U' \subseteq U$, and $\rho_{u/v}\sigma_F R_1 \equiv \sigma_{F[u/v]}\rho_{u/v}R_1$, if $v \in U$ and $u \notin U$.

Second, the filters can be moved (in a restricted way) between the arguments of a left join to its join condition: for relations R_1 and R_2 over U_1 and U_2, respectively, and filters F_1, F_2 and F over U_1, U_2 and $U_1 \cup U_2$, respectively, we have

$$\sigma_{F_1}R_1 ⟕_F R_2 \equiv \sigma_{F_1}(R_1 ⟕_F R_2), \quad (1)$$

$$\sigma_{F_1}R_1 ⟕_F R_2 \equiv \sigma_{F_1}R_1 ⟕_{F\wedge F_1} R_2, \quad (2)$$

$$R_1 ⟕_F \sigma_{F_2}R_2 \equiv R_1 ⟕_{F\wedge F_2} R_2; \quad (3)$$

observe that unlike σ_{F_2} in (3), the selection σ_{F_1} cannot be entirely eliminated in (2) but can rather be 'duplicated' above the left join using (1). (We note that (1) and (3) are well-known and can be found, e.g., in [12].) Simpler equivalences hold for inner join: $\sigma_{F_1}R_1 \bowtie_F R_2 \equiv \sigma_{F\wedge F_1}(R_1 \bowtie R_2)$. These equivalences can be, in particular, used to pull up the $\neg$*isNull* filters from mappings to eliminate the *isNull* disjuncts in the compatibility condition $comp_U$ of the (left) joins in the translation by means of the standard p-equivalences of the three-valued logic:

$$(F_1 \lor F_2) \land \neg F_2 \quad \equiv^+ \quad F_1 \land \neg F_2, \tag{4}$$

$$(v = v') \land \neg \mathit{isNull}(v) \quad \equiv^+ \quad (v = v'); \tag{5}$$

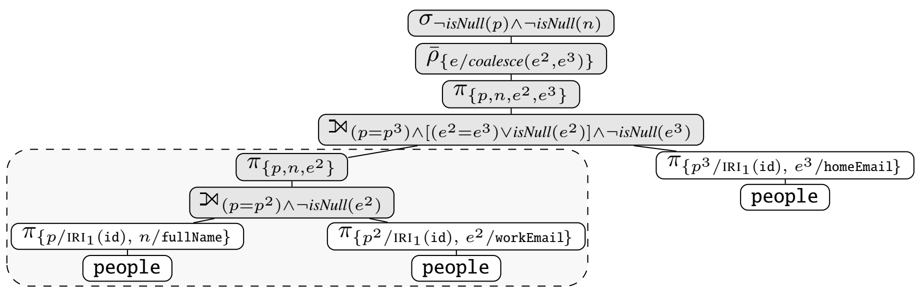
we note in passing that this step refines Simplification 3 of Chebotko *et al.* [8], which relies on the absence of other left joins in the arguments of a (left) join.

Third, the resulting simplified compatibility conditions can eliminate *coalesce* from the renaming operations: for a relation R over U and $u^1, u^2 \in U$, we clearly have

$$\rho^{\{u^1,u^2\}}_{\{u/\mathit{coalesce}(u^1,u^2)\}} \sigma_{\neg \mathit{isNull}(u^1)} R \quad \equiv \quad \sigma_{\neg \mathit{isNull}(u)} \pi_{U \setminus \{u^2\}} R[u/u^1], \tag{6}$$

where $R[u/u^1]$ is the result of replacing each u^1 in R by u. This step generalises Simplification 2 of Chebotko *et al.* [8], which does not eliminate *coalesce* above (left) joins that contain nested left joins.

By applying these three steps to our running example, we obtain (see Appendix C.1)



4.2 Left Join Naturalisation (LJN)

Our next group of optimisations can remove join conditions in left joins (if their arguments satisfy certain properties), thus reducing them to *natural left joins*.

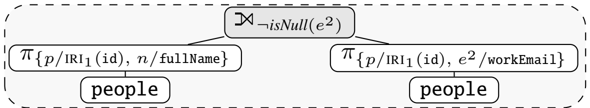
Some equalities in the join conditions of left joins can be removed by means of attribute duplication: for relations R_1 and R_2 over U_1 and U_2, respectively, a filter F over $U_1 \cup U_2$ and attributes $u^1 \in U_1 \setminus U_2$ and $u^2 \in U_2 \setminus U_1$, we have

$$R_1 ⟕_{F \land (u^1 = u^2)} R_2 \quad \equiv \quad R_1 ⟕_F \nu_{u^1 \mapsto u^2} R_2. \tag{7}$$

Now, the duplicated u^2 can be eliminated in case it is actually projected away:

$$\pi_{U_1 \cup U_2 \setminus \{u^2\}} (R_1 ⟕_F \nu_{u^1 \mapsto u^2} R_2) \equiv R_1 ⟕_F R_2[u^1/u^2] \text{ if } F \text{ does not contain } u^2. \tag{8}$$

So, if F is a conjunction of suitable attribute equalities, then by repeated application of (7) and (8), we can turn a left join into a natural left join. In our running example, this procedure simplifies the innermost left join to



Another technique for converting a left join into a natural left join (⟕ is just an abbreviation for $⟕_\top$) is based on the conditional function *if*:

Proposition 4. *For relations R_1 and R_2 over U_1 and U_2, respectively, and a filter F over $U_1 \cup U_2$, we have*

$$R_1 ⟕_F R_2 \;\equiv\; \rho^{\{U_2 \setminus U_1\}}_{\{u/\mathit{if}(F,u,\mathit{null}) \mid u \in U_2 \setminus U_1\}}(R_1 ⟕ R_2) \quad \mathit{if} \quad \pi_{U_1}(R_1 \bowtie R_2) \subseteq R_1. \tag{9}$$

Proof. Denote $R_1 \bowtie R_2$ by S. Then $\pi_{U_1} S \subseteq R_1$ implies that every tuple t_1 in R_1 can have at most one tuple t_2 in R_2 compatible with it, and S consists of all such extensions (with their cardinality determined by R_1). Therefore, $\pi_{U_1}(S \setminus \sigma_F S)$ is precisely the tuples in R_1 that cannot be extended in such a way that the extension satisfies F, whence

$$\pi_{U_1}(S \setminus \sigma_F S) \;\equiv\; \pi_{U_1} S \setminus \pi_{U_1} \sigma_F S. \tag{10}$$

By a similar argument, $R_1 \setminus \pi_{U_1} S$ consists of the tuples in R_1 (with the same cardinality) that cannot be extended by a tuple in R_2, and $\pi_{U_1} S \setminus \pi_{U_1} \sigma_F S$ of those tuples that can be extended but only when F is not satisfied. By taking the union of the two, we obtain

$$(R_1 \setminus \pi_{U_1} S) \;\cup\; (\pi_{U_1} S \setminus \pi_{U_1} \sigma_F S) \;\equiv\; R_1 \setminus \pi_{U_1} \sigma_F S. \tag{11}$$

The claim is then proved by distributivity of ρ and μ over $\cup$; see Appendix B. ❑

Proposition 4 is, in particular, applicable if the attributes shared by R_1 and R_2 uniquely determine tuples of R_2. In our running example, `id` is a primary key in `people`, and so we can eliminate $\neg\mathit{isNull}(e^2)$ from the innermost left join, which becomes a natural left join, and then simplify the term $\mathit{if}(\neg\mathit{isNull}(e^2), e^2, \mathit{null})$ in the renaming to e^2 by using equivalences on complex terms: for a term v and a filter F over U, we have

$$\mathit{if}(F \land \neg\mathit{isNull}(v), v, \mathit{null}) \;\equiv\; \mathit{if}(F, v, \mathit{null}), \tag{12}$$

$$\mathit{if}(\top, v, \mathit{null}) \;\equiv\; v. \tag{13}$$

Thus, we effectively remove the renaming operator introduced by the application of Proposition 4; for full details, see Appendix C.1.

4.3 Translation for Well-Designed SPARQL

We remind the reader that a SPARQL pattern P that uses only JOIN, FILTER and binary OPT (that is, OPT with the tautological filter $\top$) is *well-designed* [16] if every its subpattern P' of the form $\text{OPT}(P_1, P_2, \top)$ satisfies the following condition: every variable u that occurs in P_2 and outside P' also occurs in P_1.

Proposition 5. *If P is well-designed, then its unfolded translation can be equivalently simplified by* (a) *removing all compatibility filters $comp_U$ from joins and left joins and* (b) *eliminating all renamings $u/coalesce(u^1, u^2)$ by replacing both u^1 and u^2 with u.*

Proof. Since P is well-designed, any variable u occurring in the right-hand side argument of any OPT either does not occur elsewhere (and so, can be projected away) or also occurs in the left-hand side argument. The claim then follows from an observation that, if the translation of P_1 or P_2 can be equivalently transformed to contain a selection with $\neg isNull(u)$ at the top, then the translation of $\text{JOIN}(P_1, P_2)$, $\text{OPT}(P_1, P^*, \top)$ and $\text{FILTER}(P_1, F)$ can also be equivalently simplified so that it contains a selection with the $\neg isNull(u^1)$ or, respectively, $\neg isNull(u^2)$ condition at the top. ❑

Rodríguez-Muro & Rezk [22] made a similar observation. Alas, Example 1 shows that Proposition 5 is not directly applicable to *weakly* well-designed SPARQL [14].

4.4 Natural Left Join Reduction (NJR)

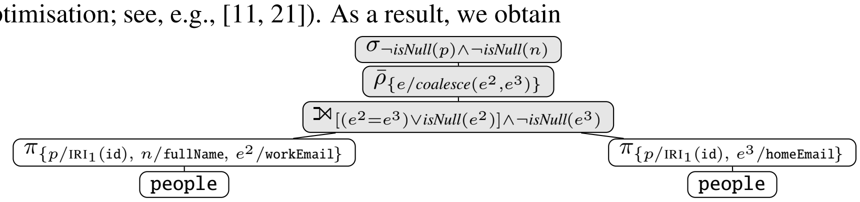
A natural left join can then be replaced by a natural *inner* join if every tuple of its left-hand side argument has a match on the right, which can be formalised as follows.

Proposition 6. *For relations R_1 and R_2 over U_1 and U_2, respectively, we have*

$$\sigma_{\neg isNull(K)} R_1 ⟕ R_2 \;\equiv\; R_1 \bowtie R_2, \quad \textit{if } \delta\pi_K R_1 \subseteq \pi_K R_2, \textit{ for } K = U_1 \cap U_2. \tag{14}$$

Proof. By careful inspection of definitions. Alternatively, one can assume that the left join has an additional selection on top with filters of the form $(u^1 = u^2) \lor isNull(u^2)$, for $u \in K$, where u^1 and u^2 are duplicates of attributes from R_1 and R_2, respectively. Given $\delta\pi_K R_1 \subseteq \pi_K R_2$, one can eliminate the $isNull(u^2)$ because any tuple of R_1 has a match in R_2. The resulting *null*-rejecting filter then effectively turns the left join to an inner join by the outer join simplification of Galindo-Legaria & Rosenthal [12]. ❑

Observe that the inclusion $\delta\pi_K R_1 \subseteq \pi_K R_2$ is satisfied, for example, if R_1 has a foreign key K referencing R_2. It can also be satisfied if both R_1 and R_2 are based on the same relation, that is, $R_i \equiv \sigma_{F_i}\pi_{\dots}R$, for $i = 1, 2$, and F_1 logically implies F_2, where F_1 and/or F_2 can be $\top$ for the vacuous selection. Note that, due to δ, attributes K do not have to uniquely determine tuples in R_1 or R_2. In our running example, trivially, $\delta\pi_{\{p\}}(\pi_{\{p/\text{IRI}_1(\texttt{id}),\ n/\texttt{fullName}\}}\texttt{people}) \subseteq \pi_{\{p\}}(\pi_{\{p/\text{IRI}_1(\texttt{id}),\ e^2/\texttt{workEmail}\}}\texttt{people})$. Therefore, the inner left join can be replaced by a natural inner join, which can then be eliminated altogether because `id` is the primary key in `people` (this is a well-known optimisation; see, e.g., [11, 21]). As a result, we obtain



The running example is wrapped up and discussed in detail in Appendices C.1 and C.2.

4.5 Join Transfer (JT)

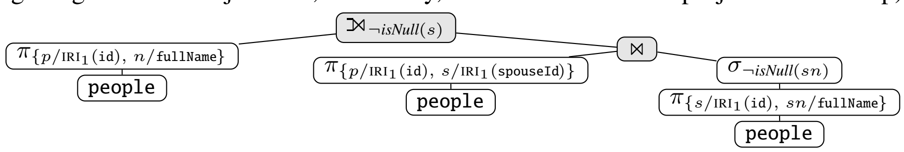
To introduce and explain another optimisation, we need an extension of relation `people` with a nullable attribute `spouseId`, which contains the `id` of the person's spouse if they are married and `NULL` otherwise. The attribute is mapped by an additional assertion:

$$\text{IRI}_1(\texttt{id})\ \texttt{:hasSpouse}\ \text{IRI}_1(\texttt{spouseId}) \quad \leftarrow \quad \sigma_{\neg \mathit{isNull}(\texttt{id}) \land \neg \mathit{isNull}(\texttt{spouseId})}\texttt{people}.$$

Consider now the following query in SPARQL algebra:

$$\text{PROJ}(\text{OPT}(\texttt{?p :name ?n},\ \text{JOIN}(\texttt{?p :hasSpouse ?s},\ \texttt{?s :name ?sn}),\ \top),\ \{\,\texttt{?n}, \texttt{?sn}\,\}),$$

whose translation can be unfolded and simplified with optimisations in Secs. 4.1 and 4.2 into the following RA query (we have also pushed down the filter $\neg \mathit{isNull}(sn)$ to the right argument of the join and, for brevity, omitted selection and projection at the top):

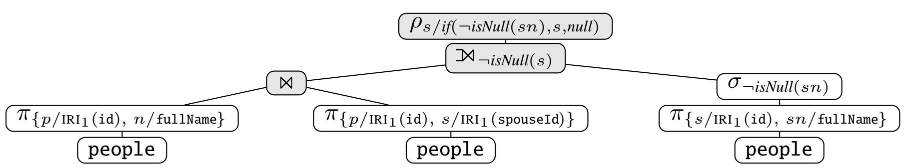


see Appendix C.4 for full details. Observe that the inner join cannot be eliminated using the standard self-join elimination techniques because it is not on a primary (or alternate) key. The next proposition (proved in Appendix B) provides a solution for the issue.

Proposition 7. *Let R_1, R_2 and R_3 be relations over U_1, U_2 and U_3, respectively, F a filter over $U_1 \cup U_2 \cup U_3$ and w an attribute in $U_3 \setminus (U_1 \cup U_2)$. Then*

$$\begin{aligned} R_1 ⟕_F (R_2 \bowtie \sigma_{\neg \mathit{isNull}(w)} R_3) \ \equiv \\ \rho^{\{U_2 \setminus U_1\}}_{\{u/\mathit{if}(\neg \mathit{isNull}(w), u, \mathit{null})\ \mid\ u \in U_2 \setminus U_1\}}((R_1 \bowtie R_2) ⟕_F \sigma_{\neg \mathit{isNull}(w)} R_3), \\ \mathit{if}\ \pi_{U_1}(R_1 \bowtie R_2) \equiv R_1. \end{aligned} \quad (15)$$

By Proposition 7, we take sn as the non-nullable attribute w and get the following:



Now, the inner self-join can be eliminated (as `id` is the primary key of `people`) and the ρ operation removed (as its result is projected away); see Appendix C.4.

4.6 Left Join Decomposition (LJD): Left Join Simplification [12] Revisited

In Sec. 4.4, we have given an example of a reduction of a left join to an inner join. The following equivalence is also helpful (for an example, see Appendix C.3): for relations R_1 and R_2 over U_1 and U_2, respectively, and a filter F over $U_1 \cup U_2$,

$$\pi_{U_1}(R_1 ⟕_F R_2) \ \equiv \ R_1, \quad \text{if} \quad \pi_{U_1}(R_1 \bowtie R_2) \subseteq R_1. \quad (16)$$

Galindo-Legaria & Rosenthal [12] observe that $\sigma_G(R_1 \mathbin{⟕}_F R_2) \equiv R_1 \bowtie_{F\wedge G} R_2$ whenever G rejects *null*s on $U_2 \setminus U_1$. In the context of SPARQL, however, the compatibility condition comp_U does not satisfy the *null*-rejection requirement, and so, this optimisation is often not applicable. In the rest of this section we refine the basic idea.

Let R_1 and R_2 be relations over U_1 and U_2, respectively, and F and G filters over $U_1 \cup U_2$. It can easily be verified that, in general, we can *decompose* the left join:

$$\sigma_G(R_1 \mathbin{⟕}_F R_2) \equiv (R_1 \bowtie_{F\wedge G} R_2) \uplus \sigma_{\mathit{nullify}_{U_2\setminus U_1}(G)} R_1 \setminus \pi_{U_1}(R_1 \bowtie_{F\wedge \mathit{nullify}_{U_2\setminus U_1}(G)} R_2), \quad (17)$$

where $\mathit{nullify}_{U_2\setminus U_1}(G)$ is the result of replacing every occurrence of an attribute from $U_2 \setminus U_1$ in G with *null*. Observe that if G is *null*-rejecting on $U_2 \setminus U_1$, then $\mathit{nullify}_{U_2\setminus U_1}(G) \equiv^+ \bot$, and the second component of the union in (17) is empty. We, however, are interested in a subtler interaction of the filters when the second component of the difference or, respectively, the first component of the union is empty:

$$\sigma_G(R_1 \mathbin{⟕}_F R_2) \equiv R_1 \bowtie_{F\wedge G} R_2 \uplus \sigma_{\mathit{nullify}_{U_2\setminus U_1}(G)} R_1, \quad \text{if } F \wedge \mathit{nullify}_{U_2\setminus U_1}(G) \equiv^+ \bot, \quad (18)$$

$$\sigma_G(R_1 \mathbin{⟕}_F \sigma_{\neg\mathit{isNull}(w)} R_2) \equiv \sigma_{\mathit{isNull}(w)\wedge \mathit{nullify}_{U_2\setminus U_1}(G)}(R_1 \mathbin{⟕}_F \sigma_{\neg\mathit{isNull}(w)} R_2), \quad \text{if } F \wedge G \equiv^+ \bot \text{ and } w \in U_2 \setminus U_1. \quad (19)$$

These cases are of particular relevance for the SPARQL-to-SQL translation of `OPTIONAL` and `MINUS`. We illustrate the technique in Appendix C.5 on the following example:

FILTER(OPT(OPT(`?p a :Product`,
  FILTER(`{ ?p :hasReview ?r .  ?r :hasLang ?l }`, `?l` = `"en"`), $\top$),
  FILTER(`{ ?p :hasReview ?r .  ?r :hasLang ?l }`, `?l` = `"zh"`), $\top$), *bound*(`?r`)).

The technique relies on two properties of *null* propagation from the right-hand side of left joins. Let R_1 and R_2 be relations over U_1 and U_2, respectively. First, if $v = v'$ is a left join condition and v is a term over $U_2 \setminus U_1$, then v is either *null* or v' in the result:

$$R_1 \mathbin{⟕}_{F\wedge(v=v')} R_2 \equiv \sigma_{\mathit{isNull}(v)\vee(v=v')}(R_1 \mathbin{⟕}_{F\wedge(v=v')} R_2). \quad (20)$$

Second, non-nullable terms v, v' over $U_2 \setminus U_1$ are simultaneously either *null* or not *null*:

$$R_1 \mathbin{⟕}_F \sigma_{\neg\mathit{isNull}(v)\wedge\neg\mathit{isNull}(v')} R_2 \equiv \sigma_{[\neg\mathit{isNull}(v)\wedge\neg\mathit{isNull}(v')]\vee[\mathit{isNull}(v)\wedge\mathit{isNull}(v')]}(R_1 \mathbin{⟕}_F \sigma_{\neg\mathit{isNull}(v)\wedge\neg\mathit{isNull}(v')} R_2). \quad (21)$$

The two equivalences introduce *no new* filters apart from *isNull* and their negations. The introduced filters, however, can help simplify the join conditions of the left joins containing the left join under consideration.

5 Experiments

In order to verify effectiveness of our optimisation techniques, we carried out a set of experiments based on the BSBM benchmark [3]; the materials for reproducing the experiments are available online[3]. The BSBM benchmark is built around an e-commerce use case in which vendors offer products that can be reviewed by customers. It comes with a mapping, a data generator and a set of SPARQL and equivalent SQL queries.

Hardware and Software. The experiments were performed on a `t2.xlarge` Amazon EC2 instance with four 64-bit vCPUs, 16G memory and 500G SSD hard disk under Ubuntu 16.04LTS. We used five database engines: free MySQL 5.7 and PostgreSQL 9.6 are run normally, and 3 commercial systems (which we shall call X, Y and Z) in Docker.

Queries. In total, we consider 11 SPARQL queries. Queries Q1–Q4 are based on the original BSBM queries 2, 3, 7 and 8, which contain `OPTIONAL`; we modified them to reduce selectivity: e.g., Q1, Q3 and Q4 retrieve information about 1000 products rather than a single product in the original BSBM queries; we also removed `ORDER BY` and `LIMIT` clauses. Q1–Q4 are well-designed (WD). In addition, we created 7 weakly well-designed (WWD) SPARQL queries: Q5–Q7 are similar to Example 1, Q8–Q10 to the query in Sec. 4.6, and Q11 is along the lines of Sec. 4.5. More information is below:

query	description	SPARQL	optimisations
Q1	2 simple `OPTIONAL`s for the padding effect (derived from BSBM query 2)	WD	LJN, NLJR
Q2	1 `OPTIONAL` with a `!BOUND` filter (encodes `MINUS`) derived from BSBM query 3	WD	JT
Q3	2 outer-level `OPTIONAL`s, the latter with 2 nested `OPTIONAL`s derived from BSBM query 7	WD	LJN, NLJR
Q4	4 `OPTIONAL`s: ratings from attributes of the same relation derived from BSBM query 8	WD	LJN, NLJR
Q5/6/7	2/3/4 `OPTIONAL`s: preference over 2/3/4 ratings of reviews	WWD	LJN, NLJR
Q8/9/10	2/3/4 `OPTIONAL`s: preference of reviews over 2/3/4 languages	WWD	LJN, LJD
Q11	2 `OPTIONAL`s: country-based preference of home pages of reviewed products	WWD	LJN, NLJR, JT

Data. We used the BSBM generator to produce CSV files for 1M products and 10M reviews. The CSV files (20GB) were loaded into DBs, with the required indexes created.

Evaluation. For each SPARQL query, we computed two SQL translations. The *non-optimised* (N/O) translation is obtained by applying to the unfolded query only the standard (previously known and widely adopted) structural and semantic optimisations [4] as well as CFR (Sec. 4.1) to simplify compatibility filters and eliminate unnecessary `COALESCE`. To obtain the *optimised* (O) translations, we further applied the other optimisation techniques presented in Sec. 4 (as described in the table above). We note that the optimised Q1 and Q4 have the same structure as the SQL queries in the original benchmark suite. On the other hand, the optimised Q2 is different from the SQL query in BSBM because the latter uses (`NOT`) `IN`, which is not considered in our optimisations.

[3] `https://github.com/ontop/ontop-examples/tree/master/iswc-2018-optional`

Each query was executed three times with cold runs to avoid any variation due to caching. The size of query answers and their running times (in secs) are as follows:

	#	PostgreSQL		MySQL		X		Y		Z	
query	answers	N/O	O	N/O	O	N/O	O	N/O	O	N/O	O
Q1	19,267	1.79	1.77	0.43	0.38	0.90	0.80	0.56	0.52	29.06	25.09
Q2	6,746	18.75	2.07	19.95	0.36	40.00	16.07	0.44	0.37	27.99	5.97
Q2$_{BSBM}$			3.88		0.37		20.55		0.38		5.91
Q3	1,355	4.20	0.09	4.70	0.11	5.50	1.60	2.04	0.14	5.45	0.65
Q4	1,174	2.14	0.16	0.86	0.04	3.00	0.60	1.78	0.11	4.38	0.53
Q5	2,294	0.56	0.05	0.01	0.01	1.80	0.30	0.30	0.08	0.51	0.53
Q6	2,294	102.35	0.18	>10min	0.04	1.90	0.40	4.50	0.14	0.82	0.54
Q7	2,294	102.00	0.17	>10min	0.04	2.60	0.40	14.57	0.14	1.21	0.53
Q8	1,257	0.07	0.06	0.01	0.01	8.40	1.30	0.08	0.08	295.25	0.40
Q9	1,311	101.20	0.16	>10min	0.04	>10min	2.70	4.30	0.11	>10min	0.43
Q10	1,331	103.30	0.15	>10min	0.05	>10min	4.20	5.20	0.14	>10min	0.43
Q11	3,388	5.26	0.87	3.80	0.21	107.06	2.68	177.95	0.22	7.82	0.13

The main outcomes of our experiments can be summarised as follows.

(a) The running times confirm that the optimisations are effective for all database engines. All optimised translations show better performance in all DB engines, and most of them can be evaluated in less than a second.

(b) Interestingly, our optimised translation is even slightly more efficient than the SQL with (`NOT`) `IN` from the original BSBM suite (see Q2$_{BSBM}$ in the table).

(c) The effects of the optimisations are significant. In particular, for challenging queries (some of which time out after 10 mins), it can be up to three orders of magnitude.

6 Discussion and Conclusions

The optimisation techniques we presented are intrinsic to SQL queries obtained by translating SPARQL in the context of OBDA with mappings, and their novelty is due to the interaction of the components in the OBDA setting. Indeed, the optimisation of `LEFT JOIN`s can be seen as a form of "reasoning" on the structure of the query, the data source and the mapping. For instance, when functional and inclusion dependencies along with attribute nullability are taken into account, one may infer that every tuple from the left argument of a `LEFT JOIN` is guaranteed to match (*i*) at least one or (*ii*) at most one tuple on the right. This information can allow one to replace `LEFT JOIN` by a simpler operator such as an `INNER JOIN`, which can further be optimised by the known techniques.

Observe that, in normal SQL queries, most of the `NULL`s come from the database rather than from operators like `LEFT JOIN`. In contrast, SPARQL triple patterns always bind their variables (no `NULL`s), and only operators like `OPTIONAL` can "unbind" them. In our experiments, we noticed that avoiding the padding effect is probably the most effective outcome of the `LEFT JOIN` optimisation techniques in the OBDA setting.

From the Semantic Web perspective, our optimisations exploit information unavailable in RDF triplestores, namely, database integrity constraints and mappings. From the DB perspective, we believe that such techniques have not been developed because `LEFT JOIN`s and/or complex conditions like compatibility filters are not introduced accidentally in expert-written SQL queries. The results of our evaluation support this hypothesis and show a significant performance improvement, even for commercial DBMSs.

We are working on implementing these techniques in the OBDA system Ontop [4].

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A Full Translation and Proof of Theorem 3

Syntax. We consider *graph patterns*, P, defined by the grammar

$$P ::= B \mid \textsc{Filter}(P, F) \mid \textsc{Bind}(P, v, c) \mid \textsc{Union}(P_1, P_2) \mid \textsc{Join}(P_1, P_2) \mid \textsc{Opt}(P_1, P_2, F) \mid \textsc{Minus}(P_1, P_2) \mid \textsc{Proj}(P, L) \mid \textsc{Dist}(P),$$

where B is a BGP, $v \in \mathsf{V}$ does not occur in P, $c \in \mathsf{C}$ is a constant, $L \subseteq \mathsf{V}$, and F, called *filter*, is a formula constructed using the logical connectives $\wedge$ and $\neg$ from atoms of the form *bound*(v), $(v = c)$, $(v = v')$, for $v, v' \in \mathsf{V}$ and $c \in \mathsf{C}$, and possibly other built-in predicates. The set of variables in P is denoted by *var*(P). We assume (without mentioning it again) that all graph patterns of the form $\textsc{Filter}(P, F)$ are safe in the sense that every variable in F also occurs in P.

We do not consider solution modifiers other than DIST and PROJ; we also define a simplified variant of BIND, where c is a constant rather than an (arithmetic) expression (which are beyond the scope of the paper). Our results, however, can easily be extended to the general form of BIND.

Semantics. The semantics of SPARQL operations is defined as follows:

- $\textsc{Filter}(\Omega, F) = \Omega'$, where $\Omega'(s) = \Omega(s)$ if $s \in \Omega$ and $F^s = \top$, and 0 otherwise;
- $\textsc{Bind}(\Omega, v, c) = \Omega'$, where $\Omega'(s \oplus \{v \mapsto c\}) = \Omega(s)$ if $s \in \Omega$, and 0 otherwise;
- $\textsc{Union}(\Omega_1, \Omega_2) = \Omega$, where $\Omega(s) = \Omega_1(s) + \Omega_2(s)$;
- $\textsc{Join}(\Omega_1, \Omega_2) = \Omega$, where $\Omega(s) = \sum_{\substack{s_1 \in \Omega_1, s_2 \in \Omega_2 \text{ with} \\ s_1 \sim s_2 \text{ and } s_1 \oplus s_2 = s}} \Omega_1(s_1) \times \Omega_2(s_2)$;
- $\textsc{Opt}(\Omega_1, \Omega_2, F) = \textsc{Union}(\textsc{Filter}(\textsc{Join}(\Omega_1, \Omega_2), F), \Omega)$, where $\Omega(s) = \Omega_1(s)$ if $F^{s \oplus s_2} \neq \top$, for all $s_2 \in \Omega_2$ compatible with s, and 0 otherwise;
- $\textsc{Minus}(\Omega_1, \Omega_2) = \Omega$, where $\Omega(s) = \Omega_1(s)$ if $\mathit{dom}(s) \cap \mathit{dom}(s_2) = \emptyset$, for all solution mappings $s_2 \in \Omega_2$ compatible with s, and 0 otherwise;
- $\textsc{Proj}(\Omega, L) = \Omega'$, where $\Omega'(s') = \sum_{s \in \Omega \text{ with } s|_L = s'} \Omega(s)$;
- $\textsc{Dist}(\Omega) = \Omega'$, where $\Omega'(s) = 1$ if $s \in \Omega$, and 0 otherwise.

Translation. The translation of triple patterns depends on their shape:

$$\boldsymbol{\tau}(\langle s, p, o \rangle) = \begin{cases} \pi_{\emptyset} \sigma_{(\mathit{subj}=s) \wedge (\mathit{pred}=p) \wedge (\mathit{obj}=o)}\, \mathit{triple}, & \text{if } s, p, o \in \mathsf{I} \cup \mathsf{L}, \\ \pi_{\{s \mapsto \mathit{subj}\}}\, \sigma_{(\mathit{pred}=p) \wedge (\mathit{obj}=o)}\, \mathit{triple}, & \text{if } s \in \mathsf{V} \text{ and } p, o \in \mathsf{I} \cup \mathsf{L}, \\ \pi_{\{s \mapsto \mathit{subj},\ o \mapsto \mathit{obj}\}}\, \sigma_{\mathit{pred}=p}\, \mathit{triple}, & \text{if } s, o \in \mathsf{V}, s \neq o, p \in \mathsf{I} \cup \mathsf{L}, \\ \pi_{\{s \mapsto \mathit{subj}\}}\, \sigma_{(\mathit{pred}=p) \wedge (\mathit{subj}=\mathit{obj})}\, \mathit{triple}, & \text{if } s, o \in \mathsf{V}, s = o, p \in \mathsf{I} \cup \mathsf{L}, \\ \dots \end{cases}$$

the remaining cases are similar. The translation of SPARQL operators is as follows, where the tp_i are triple patterns

$$
\begin{aligned}
\boldsymbol{\tau}(\{tp_1, tp_2, \dots, tp_k\}) &= \boldsymbol{\tau}(tp_1) \bowtie \boldsymbol{\tau}(tp_2) \bowtie \cdots \bowtie \boldsymbol{\tau}(tp_k),\\
\boldsymbol{\tau}(\textsc{Union}(P_1, P_2)) &= \boldsymbol{\tau}(P_1) \uplus \boldsymbol{\tau}(P_2),\\
\boldsymbol{\tau}(\textsc{Filter}(P_1, F_1)) &= \sigma_{\boldsymbol{\tau}(F_1)} \boldsymbol{\tau}(P_1),\\
\boldsymbol{\tau}(\textsc{Bind}(P_1, v, c)) &= \nu_{v \mapsto c} \boldsymbol{\tau}(P_1),\\
\boldsymbol{\tau}(\textsc{Proj}(P, L)) &= \pi_L \boldsymbol{\tau}(P),\\
\boldsymbol{\tau}(\textsc{Dist}(P)) &= \delta \boldsymbol{\tau}(P),\\
\boldsymbol{\tau}(\textsc{Join}(P_1, P_2)) &= \rho^{U^1 \cup U^2}_{\mathit{coalesce}_U} \big[\rho_{U^1/U} \boldsymbol{\tau}(P_1) \bowtie_{\mathit{comp}_U} \rho_{U^2/U} \boldsymbol{\tau}(P_2)\big],\\
\boldsymbol{\tau}(\textsc{Opt}(P_1, P_2, F)) &= \rho^{U^1 \cup U^2}_{\mathit{coalesce}_U} \big[\rho_{U^1/U} \boldsymbol{\tau}(P_1) ⟕_{\mathit{comp}_U \wedge \boldsymbol{\tau}(F)[\mathit{coalesce}_U]} \rho_{U^2/U} \boldsymbol{\tau}(P_2)\big],\\
\boldsymbol{\tau}(\textsc{Minus}(P_1, P_2)) &= \pi_{U_1} \rho_{U/U^1} \sigma_{\mathit{isNull}(w)}\\
&\qquad \big[\rho_{U^1/U} \boldsymbol{\tau}(P_1) ⟕_{\mathit{comp}_U \wedge \bigvee_{u \in U} (u^1 = u^2)} \nu_{w \mapsto 1} \rho_{U^2/U} \boldsymbol{\tau}(P_2)\big],
\end{aligned}
$$

Theorem 3. *For any RDF graph G and any graph pattern P, $\|P\|_G = \|\boldsymbol{\tau}(P)\|_{\mathit{triple}(G)}$.*

Proof. The proof is by induction on the structure of P. The basis of induction (for basic graph patterns) is straightforward. The cases for $\textsc{Union}(P_1, P_2)$, $\textsc{Filter}(P_1, F)$, $\textsc{Bind}(P_1, v, c)$, $\textsc{Proj}(P_1, L)$ and $\textsc{Dist}(P_1)$ easily follow from the definitions and the induction hypothesis. It remains to consider the induction step for $P = \textsc{Join}(P_1, P_2)$, $P = \textsc{Opt}(P_1, P_2, F)$ and $P = \textsc{Minus}(P_1, P_2)$. Let $U_i = \mathit{var}(P_i)$, $i = 1, 2$, and $U = U_1 \cap U_2$.

If $\|\textsc{Join}(P_1, P_2)\|_G(t) = m > 0$, then there is a unique solution mapping s such that $\mathit{ext}_{U_1 \cup U_2}(s) = t$ and $[\![\textsc{Join}(P_1, P_2)]\!]_G(s) = m$. By definition, m is the sum of all $m_1 \cdot m_2$ such that $m_i = [\![P_i]\!]_G(s_i)$ for compatible s_1 and s_2 with $s_1 \oplus s_2 = s$. Consider any compatible s_1 and s_2 with $s_1 \oplus s_2 = s$. By IH, we have $\|P_i\|_G(\mathit{ext}_{U_i}(s_i)) = \|\boldsymbol{\tau}(P_i)\|_{\mathit{triple}(G)}(\mathit{ext}_{U_i}(s_i))$. Since s_1 and s_2 are compatible, the structure of the filter and renaming operations in $\boldsymbol{\tau}(\textsc{Join}(P_1, P_2))$ guarantee that $\mathit{ext}_{U_1 \cup U_2}(s_1 \oplus s_2)$ belongs to $\|\boldsymbol{\tau}(\textsc{Join}(P_1, P_2))\|_{\mathit{triple}(G)}$ with multiplicity $\geq m_1 \cdot m_2$. It remains to observe that any such $\mathit{ext}_{U_1 \cup U_2}(s_1 \oplus s_2)$ coincides with t, and so $m \leq \|\boldsymbol{\tau}(\textsc{Join}(P_1, P_2))\|_{\mathit{triple}(G)}(t)$.

Conversely, $\|\boldsymbol{\tau}(\textsc{Join}(P_1, P_2))\|_{\mathit{triple}(G)}(t) = m > 0$, then, for $i = 1, 2$, there are t_i with $\|\boldsymbol{\tau}(P_i)\|_{\mathit{triple}(G)}(t_i) = m_i > 0$ and unique solution mappings s_i such that $t_i = \mathit{ext}_{U_i}(s_i)$, for $i = 1, 2$, s_1 and s_2 are compatible (due to the filter in $\bowtie$) and $t = \mathit{ext}_{U_1 \cup U_2}(s_1 \oplus s_2)$ (due to the renaming operations in $\boldsymbol{\tau}(\textsc{Join}(P_1, P_2))$). By IH, $\|P_i\|_G(\mathit{ext}_{U_i}(s_i)) = m_i$ and so, $[\![P_i]\!]_G(s_i) = m_i$. Thus, $[\![\textsc{Join}(P_1, P_2)]\!]_G(s_1 \oplus s_2) \geq m_1 \cdot m_2$ and $\|\textsc{Join}(P_1, P_2)\|_G(t) \geq m$.

If $\|\textsc{Opt}(P_1, P_2, F)\|_G(t) = m > 0$, then there is a unique s with $\mathit{ext}_{U_1 \cup U_2}(s) = t$ and $[\![\textsc{Opt}(P_1, P_2, F)]\!]_G(s) = m$. Then, by definition, m is the sum of (*a*) all $m_1 \cdot m_2$ such that there are compatible s_1 and s_2 with $s_1 \oplus s_2 = s$, $F^s = \top$ and $[\![P_i]\!]_G(s_i) = m_i > 0$, and (*b*) $m' = [\![P_1]\!]_G(s) > 0$ in case there is no $s_2 \in [\![P_2]\!]_G$ compatible with s such that $F^{s \oplus s_2} = \top$. Item (*a*) is as the case of Join (with an additional filter), so we consider only item (*b*). By IH, $\|\boldsymbol{\tau}(P_1)\|_{\mathit{triple}(G)}(\mathit{ext}_{U_1}(s)) = m'$ and there is no s_2 such that $\mathit{ext}_{U_2}(s_2) \in \|\boldsymbol{\tau}(P_2)\|_{\mathit{triple}(G)}$, s and s_2 are compatible and $F^{s\oplus s_2} = \top$. Due to the shape of the condition, $\theta = \mathit{comp}_U \wedge \boldsymbol{\tau}(F)[\mathit{coalesce}_U]$, in the $⟕$ operation, the tuple $\mathit{ext}_{U_1}(s)$ belongs to $\|\rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}(\rho_{U^1/U}\boldsymbol{\tau}(P_1) \setminus \pi_{U_1^1}(\rho_{U^1/U}\boldsymbol{\tau}(P_1) \bowtie_\theta \rho_{U^2/U}\boldsymbol{\tau}(P_2)))\|_{\mathit{triple}(G)}$ with multiplicity $\geq m'$, where $U_1^1 = (U_1 \setminus U) \cup U^1$; note that $\rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}$ simply renames all u^1 into u, for $u \in U$, because all u^2 are removed by the projection. Therefore, $\mathit{ext}_{U_1 \cup U_2}(s)$ belongs to $\|\boldsymbol{\tau}(\textsc{Opt}(P_1, P_2, F))\|_{\mathit{triple}(G)}$ with multiplicity $\geq m'$. It remains to observe that the tuple $\mathit{ext}_{U_1 \cup U_2}(s)$ coincides with t, and so, $\|\boldsymbol{\tau}(\textsc{Opt}(P_1, P_2, F))\|_{\mathit{triple}(G)}(t) \geq m = \|\textsc{Opt}(P_1, P_2, F)\|_G(t)$.

Conversely, if $\|\boldsymbol{\tau}(\textsc{Opt}(P_1, P_2, F))\|_{\mathit{triple}(G)}(t) = m > 0$, then $m = m'' + m'$, where

(*a*) $m'' = \|\rho^{U^1 \cup U^2}_{\mathit{coalesce}(U)}(\rho_{U^1/U}\boldsymbol{\tau}(P_1) \bowtie_\theta \rho_{U^2/U}(P_2))\|_{\mathit{triple}(G)}(t)$,

(*b*) $m' = \|\rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}(\rho_{U^1/U}\boldsymbol{\tau}(P_1) \setminus \pi_{U_1^1}(\rho_{U^1/U}\boldsymbol{\tau}(P_1) \bowtie_\theta \rho_{U^2/U}\boldsymbol{\tau}(P_2)))\|_{\mathit{triple}(G)}(t)$,

U_1^1 denotes $(U_1 \setminus U) \cup U^1$ and $\theta = \mathit{comp}_U \wedge \boldsymbol{\tau}(F)[\mathit{coalesce}_U]$. Again, item (*a*) is identical to JOIN with FILTER (we just point out that the $\boldsymbol{\tau}(F)[\mathit{coalesce}_U]$ component of the filter in $\bowtie$ can be pulled outside $\rho_{\mathit{coalesce}_U}$ as $\sigma_{\boldsymbol{\tau}(F)}$); so, we focus on item (*b*) only. By construction, there is a unique solution mapping s such that $\mathit{ext}_{U_1}(s) = t$, $\|\rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}\rho_{U^1/U}\boldsymbol{\tau}(P_1)\|_{\mathit{triple}(G)}(s) = m'$ but $\mathit{ext}_{U_1 \cup U_2}(s) \notin \|\rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}\pi_{U_1^1}(\rho_{U^1/U}\boldsymbol{\tau}(P_1) \bowtie_\theta \rho_{U^2/U}\boldsymbol{\tau}(P_2))\|_{\mathit{triple}(G)}$. The latter implies that there is no s_2 compatible with s such that $F^{s\oplus s_2} = \top$ and $\mathit{ext}_{U_2}(s_2) \in \|\rho^{U^1 \cup U^2}_{\mathit{coalesce}_U}\rho_{U^2/U}\boldsymbol{\tau}(P_2)\|_{\mathit{triple}(G)}$. It follows that $\|\boldsymbol{\tau}(P_1)\|_{\mathit{triple}(G)}(\mathit{ext}_{U_1}(s)) \geq m'$ but there is no s_2 compatible with s such that $F^{s\oplus s_2} = \top$ and $\mathit{ext}_{U_2}(s_2) \in \|\boldsymbol{\tau}(P_2)\|_{\mathit{triple}(G)}$. By IH and the definition of OPT, $[\![\textsc{Opt}(P_1, P_2, F)]\!]_G(s) \geq m'$. It follows that $\|\textsc{Opt}(P_1, P_2, F)\|_G(t) \geq m = \|\boldsymbol{\tau}(\textsc{Opt}(P_1, P_2, F))\|_{\mathit{triple}(G)}(t)$.

If $\|\textsc{Minus}(P_1, P_2)\|_G(t) = m > 0$, then there is a unique s with $\mathit{ext}_{U_1 \cup U_2}(s) = t$ and $[\![\textsc{Minus}(P_1, P_2)]\!]_G(s) = m$. Then, by definition, $[\![P_1]\!]_G(s) = m > 0$ and there is no $s_2 \in [\![P_2]\!]_G$ compatible with s and such that $\mathit{dom}(s) \cap \mathit{dom}(s_2) \neq \emptyset$. By IH, $\|\boldsymbol{\tau}(P_1)\|_{\mathit{triple}(G)}(\mathit{ext}_{U_1}(s)) = m$ and there is no s_2 compatible with s such that $\mathit{ext}_{U_2}(s_2) \in \|\boldsymbol{\tau}(P_2)\|_{\mathit{triple}(G)}$ and $\mathit{dom}(s) \cap \mathit{dom}(s_2) \neq \emptyset$. Due to the shape of the filter, $\theta = \mathit{comp}_U \wedge \bigvee_{u \in U}(u^1 = u^2)$, in the $⟕$ operation and the extension $\nu_{w \mapsto 1}$ with the filter $\mathit{isNull}(w)$, the tuple $\mathit{ext}_{U_1}(s)$ belongs to $\|\pi_{U_1}\rho_{U/U^1}\sigma_{\mathit{isNull}(w)}\big[\rho_{U^1/U}\boldsymbol{\tau}(P_1) ⟕_\theta \nu_{w \mapsto 1}\rho_{U^2/U}\boldsymbol{\tau}(P_2)\big]\|_{\mathit{triple}(G)}$ with multiplicity m. Therefore, $\mathit{ext}_{U_1}(s)$ belongs to $\|\boldsymbol{\tau}(\textsc{Minus}(P_1, P_2))\|_{\mathit{triple}(G)}$ with multiplicity m. It remains to observe that the tuple $\mathit{ext}_{U_1}(s)$ coincides with t, and so, $m = \|\boldsymbol{\tau}(\textsc{Minus}(P_1, P_2))\|_{\mathit{triple}(G)}(t)$.

Conversely, if $\|\boldsymbol{\tau}(\textsc{Minus}(P_1, P_2))\|_{\mathit{triple}(G)}(t) = m > 0$, by construction, there is a unique solution mapping s such that $\mathit{ext}_{U_1}(s) = t$, $\|\boldsymbol{\tau}(P_1)\|_{\mathit{triple}(G)}(s) = m$ but $\mathit{ext}_{U_1 \cup U_2 \cup U^1 \cup U^2 \cup \{w\} \setminus U}(s) \notin \|\rho_{U^1/U}\boldsymbol{\tau}(P_1) \bowtie_\theta \nu_{w \mapsto 1}\rho_{U^2/U}\boldsymbol{\tau}(P_2))\|_{\mathit{triple}(G)}$, where $\theta = \mathit{comp}_U \wedge \bigvee_{u \in U}(u^1 = u^2)$. It follows that there is no s_2 such that $\mathit{ext}_{U_2}(s_2) \in \|\boldsymbol{\tau}(P_2)\|_{\mathit{triple}(G)}$ such that s and s_2 are compatible and $\mathit{dom}(s) \cap \mathit{dom}(s_2) \neq \emptyset$. By IH and the definition of MINUS, $[\![\textsc{Minus}(P_1, P_2)]\!]_G(s) = m$.

This completes the proof of Theorem 3. ❑

B Section 4 Proofs

Proposition 4. *For relations R_1 and R_2 over U_1 and U_2, respectively, and a filter F over $U_1 \cup U_2$, we have*

$$R_1 ⟕_F R_2 \;\equiv\; \rho^{\{U_2\setminus U_1\}}_{\{u/\mathit{if}(F,u,\mathit{null})\,\mid\, u\in U_2\setminus U_1\}}(R_1 ⟕ R_2) \qquad \mathit{if}\;\; \pi_{U_1}(R_1 \bowtie R_2) \subseteq R_1. \tag{22}$$

Proof. Denote $R_1 \bowtie R_2$ by S. Then $\pi_{U_1} S \subseteq R_1$ implies that every tuple t_1 in R_1 can have at most one tuple t_2 in R_2 compatible with it, and S consists of all such extensions (with their cardinality determined by R_1). Therefore, $\pi_{U_1}(S \setminus \sigma_F S)$ is precisely the tuples in R_1 that cannot be extended in such a way that the extension satisfies F, whence

$$\pi_{U_1}(S \setminus \sigma_F S) \;\;\equiv\;\; \pi_{U_1} S \setminus \pi_{U_1}\sigma_F S. \tag{10}$$

By a similar argument, $R_1 \setminus \pi_{U_1} S$ consists of the tuples in R_1 (with the same cardinality) that cannot be extended by a tuple in R_2, and $\pi_{U_1} S \setminus \pi_{U_1}\sigma_F S$ of those tuples that can be extended but only when F is not satisfied. By taking the union of the two, we obtain

$$(R_1 \setminus \pi_{U_1} S) \cup (\pi_{U_1} S \setminus \pi_{U_1}\sigma_F S) \;\;\equiv\;\; R_1 \setminus \pi_{U_1}\sigma_F S. \tag{11}$$

Having these two equivalences at hand, we can now prove the claim:

$$\begin{aligned}
&\rho^{\{U_2\setminus U_1\}}_{\{\mathit{if}(F,u,\mathit{null})/u\,\mid\, u\in U_2\setminus U_1\}}(R_1 ⟕ R_2)\\
\text{(definition of LJ)} \equiv\;& \rho^{\{U_2\setminus U_1\}}_{\{\mathit{if}(F,u,\mathit{null})/u\,\mid\, u\in U_2\setminus U_1\}}\big[S \;\uplus\; (R_1 \setminus \pi_{U_1} S)\big]\\
(\rho \text{ distributes over } \cup \text{ and the definiton of } \mathit{if}) \equiv\;& \rho^{\{U_2\setminus U_1\}}_{\{\mathit{if}(F,u,\mathit{null})/u\,\mid\, u\in U_2\setminus U_1\}} S \;\;\uplus\;\; (R_1 \setminus \pi_{U_1} S)\\
(\text{definitions of } \rho \text{ and } \mathit{if}) \equiv\;& \sigma_F S \;\;\uplus\;\; \pi_{U_1}(S \setminus \sigma_F S) \;\;\uplus\;\; (R_1 \setminus \pi_{U_1} S)\\
(10) \equiv\;& \sigma_F S \;\;\uplus\;\; (\pi_{U_1} S \setminus \pi_{U_1}\sigma_F S) \;\uplus\; (R_1 \setminus \pi_{U_1} S)\\
(\mu \text{ distributes over } \cup) \equiv\;& \sigma_F S \;\;\uplus\;\; \big[(\pi_{U_1} S \setminus \pi_{U_1}\sigma_F S) \;\cup\; (R_1 \setminus \pi_{U_1} S)\big]\\
(11) \equiv\;& \sigma_F S \;\;\uplus\;\; (R_1 \setminus \pi_{U_1}\sigma_F S)\\
\text{(definition of LJ)} \equiv\;& R_1 ⟕_F R_2.
\end{aligned}$$

This completes the proof of Proposition 4. ❑

By combining Propositions 4 and 6, we obtain

Corollary 8. *For relations R_1 and R_2 over U_1 and U_2, respectively, and a filter F over $U_1 \cup U_2$, we have*

$$\sigma_{\neg \mathit{isNull}(U_1\cap U_2)} R_1 ⟕_F R_2 \;\equiv\; \rho^{\{U_2\setminus U_1\}}_{\{u/\mathit{if}(F,u,\mathit{null})\,\mid\, u\in U_2\setminus U_1\}}(R_1 \bowtie R_2), \qquad \mathit{if}\;\; \pi_{U_1}(R_1 \bowtie R_2) = R_1. \tag{23}$$

Note that the condition $\pi_{U_1}(R_1 \bowtie R_2) = R_1$ is satisfied, in particular, when R_1 and R_2 are both (extended) projections of the same relation (with a primary key) or when R_1 has a foreign key referencing (a primary or alternate key of) R_2.

Proposition 7. *Let R_1, R_2 and R_3 be relations over U_1, U_2 and U_3, respectively, F a filter over $U_1 \cup U_2 \cup U_3$ and w an attribute in $U_3 \setminus (U_1 \cup U_2)$. Then*

$$\begin{aligned} R_1 ⟕_F (R_2 \bowtie \sigma_{\neg \mathit{isNull}(w)} R_3) \;\; &\equiv \\ &\rho^{\{U_2 \setminus U_1\}}_{\{u/\mathit{if}(\neg \mathit{isNull}(w), u, \mathit{null}) \;|\; u \in U_2 \setminus U_1\}}((R_1 \bowtie R_2) ⟕_F \sigma_{\neg \mathit{isNull}(w)} R_3), \\ &\qquad\qquad \text{if } \pi_{U_1}(R_1 \bowtie R_2) = R_1. \end{aligned} \tag{24}$$

Proof. Denote $R_1 \bowtie R_2$ by S and $\sigma_{\neg \mathit{isNull}(w)} R_3$ by R'_3. First, we can easily establish the following analogue of (10):

$$\pi_{U_1}(S \setminus \pi_{U_1 \cup U_2}(S \bowtie_F R'_3)) \;\; \equiv \;\; \pi_{U_1} S \setminus \pi_{U_1}(S \bowtie_F R'_3), \tag{25}$$

where $\pi_{U_1 \cup U_2}(S \bowtie_F R'_3)$ plays the same role as $\sigma_F S$ in (10). Then the proof of the proposition is immediate from the following sequence of equivalences:

$$\begin{aligned} & R_1 ⟕_F (R_2 \bowtie R'_3) \\ \text{(def. of LJ)} \equiv \;\; & \sigma_F(S \bowtie R'_3) \;\uplus\; (R_1 \setminus \pi_{U_1} \sigma_F(S \bowtie R'_3)) \\ \text{(assumption)} \equiv \;\; & \sigma_F(S \bowtie R'_3) \;\uplus\; (\pi_{U_1} S \setminus \pi_{U_1} \sigma_F(S \bowtie R'_3)) \\ \text{(def. of join)} \equiv \;\; & (S \bowtie_F R'_3) \;\uplus\; (\pi_{U_1} S \setminus \pi_{U_1}(S \bowtie_F R'_3)) \\ \text{(25)} \equiv \;\; & (S \bowtie_F R'_3) \;\uplus\; \pi_{U_1}(S \setminus \pi_{U_1 \cup U_2}(S \bowtie_F R'_3)) \\ \text{(def. of } \mu) \equiv \;\; & (S \bowtie_F R'_3) \;\uplus\; \mu_{U_2 \setminus U_1} \pi_{U_1}(S \setminus \pi_{U_1 \cup U_2}(S \bowtie_F R'_3)) \\ \text{(def. of } \mathit{if}) \equiv \;\; & \rho^{\{U_2 \setminus U_1\}}_{\{u/\mathit{if}(\neg \mathit{isNull}(w), u, \mathit{null}) \;|\; u \in U_2 \setminus U_1\}}[(S \bowtie_F R'_3) \uplus (S \setminus \pi_{U_1 \cup U_2}(S \bowtie_F R'_3))] \\ \text{(def. of LJ)} \equiv \;\; & \rho^{\{U_2 \setminus U_1\}}_{\{u/\mathit{if}(\neg \mathit{isNull}(w), u, \mathit{null}) \;|\; u \in U_2 \setminus U_1\}}(S ⟕_F R'_3). \end{aligned}$$

❑

C Complete Examples in Section 4

C.1 Example 1

We begin by revisiting Example 1, which can now be given in algebraic form (for brevity, we ignore projecting away `?p`, which does not affect the optimisations):

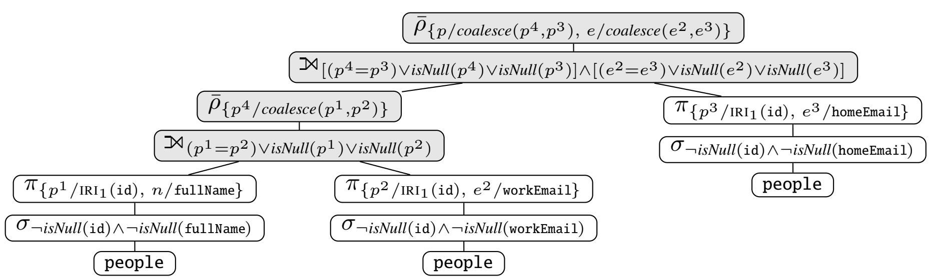
$$\textsc{Opt}(\textsc{Opt}(\texttt{?p :name ?n},\ \texttt{?p :workEmail ?e},\ \top), \texttt{?p :personalEmail ?e},\ \top),$$

where $\top$ denotes the tautological filter (true). Suppose we have the following mapping:

$$\begin{aligned}
\textsc{iri}_1(\texttt{id})\ \texttt{:name fullName} &\leftarrow \sigma_{\neg isNull(\texttt{id}) \land \neg isNull(\texttt{fullName})}\texttt{people},\\
\textsc{iri}_1(\texttt{id})\ \texttt{:workEmail workEmail} &\leftarrow \sigma_{\neg isNull(\texttt{id}) \land \neg isNull(\texttt{workEmail})}\,\texttt{people},\\
\textsc{iri}_1(\texttt{id})\ \texttt{:personalEmail homeEmail} &\leftarrow \sigma_{\neg isNull(\texttt{id}) \land \neg isNull(\texttt{homeEmail})}\,\texttt{people},
\end{aligned}$$

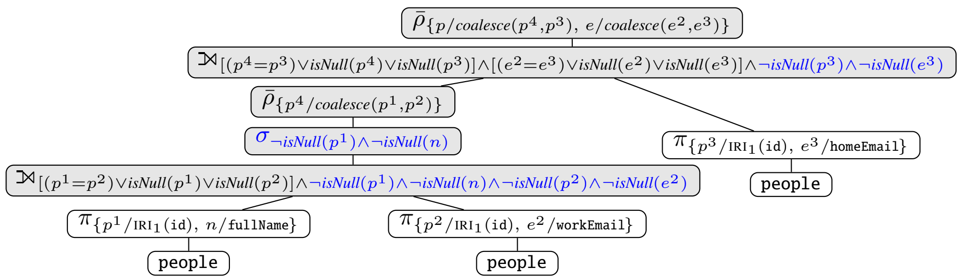
where $\textsc{iri}_1$ is a function that constructs the IRI for a person from their ID (an IRI template, in R2RML parlance).

The translation given in Section 3 and unfolding produce the following RA query, where we abbreviate, for example, $\rho^{\{p^1,p^2\}}_{\{p^4/coalesce(p^1,p^2)\}}$ by $\bar{\rho}_{\{p^4/coalesce(p^1,p^2)\}}$ (in other words, the $\bar{\rho}$ operation always projects away the arguments of its *coalesce* functions):



In our diagrams, the white nodes are the contribution of the mapping and the translation of the basic graph patterns: for example, the basic graph pattern `?p :name ?n` produces $\pi_{\{p^1/\textsc{iri}_1(\texttt{id}),\ n/\texttt{fullName}\}}\sigma_{\neg isNull(\texttt{id}) \land \neg isNull(\texttt{fullName})}\texttt{people}$ (we use attributes without superscripts if there is only one occurrence; otherwise, the superscript identifies the relevant subquery). The grey nodes correspond to the translation of the SPARQL operations: for instance, the innermost left join is on $comp_{\{p\}}$ with p renamed apart to p^1 and p^2; the outermost left join is on $comp_{\{p,e\}}$, where p is renamed apart to p^4 and p^3 and e to e^2 and e^3; the two $\bar{\rho}$ are the respective renaming operations with *coalesce*.

By pulling the filters up for the first time using the standard database equivalences and (1)–(3), we obtain (the additions are shown in blue)

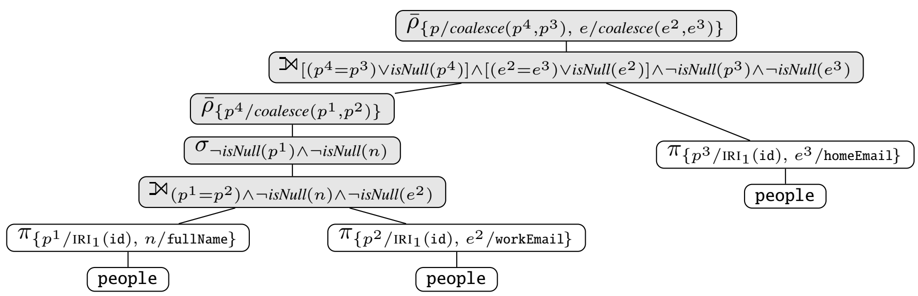


We then remove disjuncts from the join condition of the left joins by using (4), with the following results (the eliminated disjuncts are in gray):

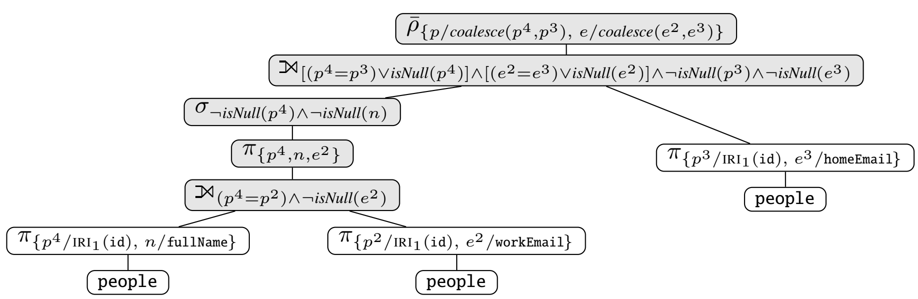
$$[(p^1 = p^2) \vee \mathit{isNull}(p^1) \vee \mathit{isNull}(p^2)] \wedge \neg\mathit{isNull}(p^1) \wedge \neg\mathit{isNull}(n) \wedge \neg\mathit{isNull}(p^2) \wedge \neg\mathit{isNull}(e^2),$$

$$[(p^4 = p^3) \vee \mathit{isNull}(p^4) \vee \mathit{isNull}(p^3)] \wedge [(e^2 = e^3) \vee \mathit{isNull}(e^2) \vee \mathit{isNull}(e^3)] \wedge \neg\mathit{isNull}(p^3) \wedge \neg\mathit{isNull}(e^3),$$

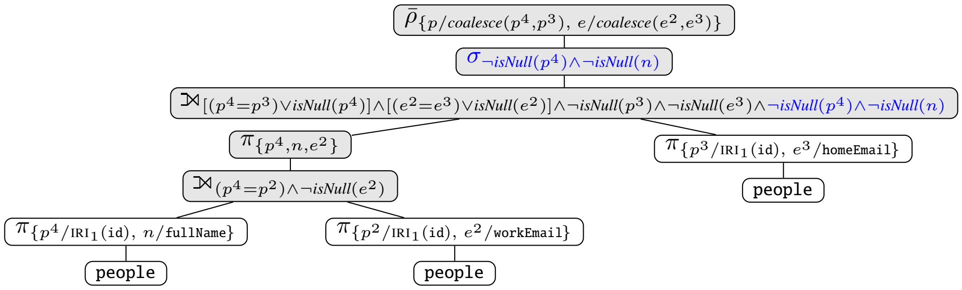
which are then simplified by (5), thus obtaining



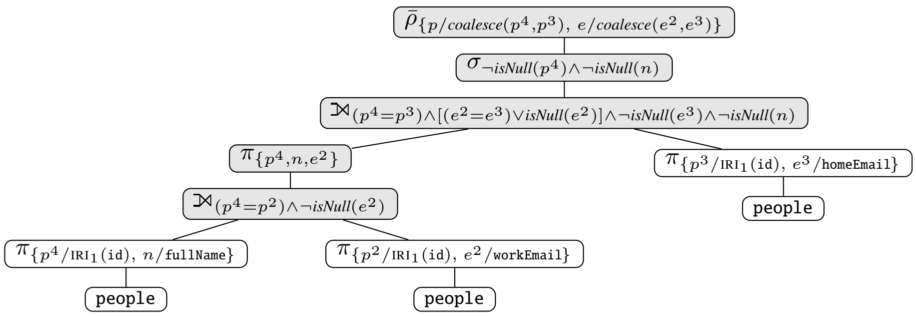
Then we apply (6) to eliminate $\mathit{coalesce}(p^1, p^2)$ with the following result (note that, by (1) and (2), we also remove $\neg\mathit{isNull}(n)$ from the join condition):



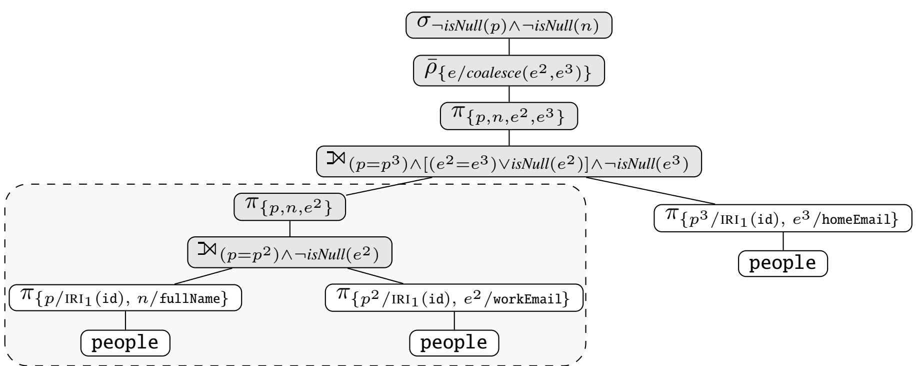
We can now repeat the procedure of pulling up the filter with (1)–(3) to get:



Then we simplify the joining condition by (4) and (5) with the following result:

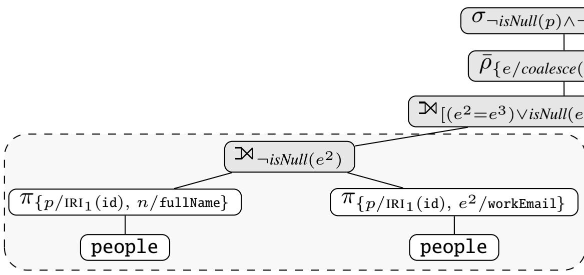


Now we apply (6) to eliminate $coalesce(p^4, p^3)$ (and, by (1) and (2), remove $\neg isNull(n)$ from the join condition) and obtain:

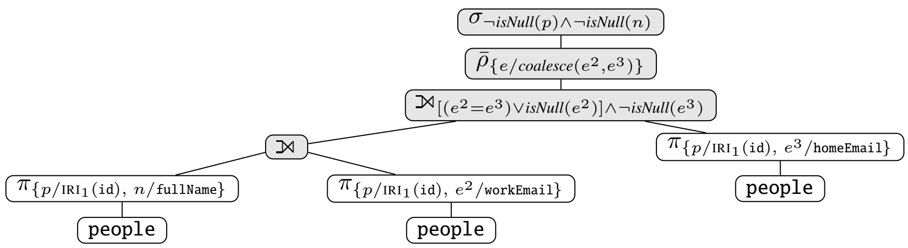


This completes application of the Compatibility Filters Reduction optimisation described in Section 4.1.

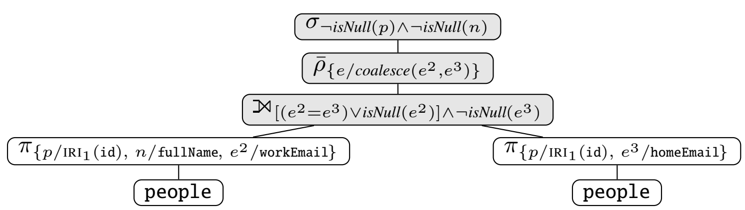
By removing equalities with the help of (7) and (8), we simplify the innermost left join (Left Join Naturalisation, Section 4.2) to obtain



Proposition 4 is, in particular, applicable if the attributes shared by R_1 and R_2 uniquely determine tuples of R_2. In our running example, `id` is a primary key in `people`, and so we can eliminate $\neg isNull(e^2)$ from the innermost left join, which becomes a natural left join, and then simplify the term $if(\neg isNull(e^2), e^2, null)$ in the renaming to e^2 by using equivalences (12) and (13) on complex terms. Thus, we effectively remove the renaming operator we have introduced by the application of Proposition 4:



Next, by applying Proposition 6 (Natural Left Join Reduction, Section 4.4), we can replace the natural left join by an inner join and then eliminate it (because it is on the primary key `id`). So, we arrive at



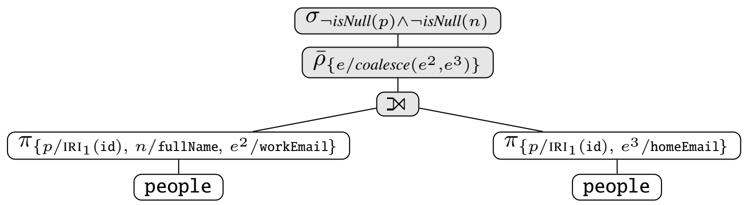
To complete the running example, observe that now, by Proposition 4, the remaining left join can be replaced with a natural left join at the expense of introducing a renaming for e^3 with $if([(e^2 = e^3) \vee isNull(e^2)] \wedge \neg isNull(e^3), e^3, null)$, which, by (12), is equivalent to $if((e^2 = e^3) \vee isNull(e^2), e^3, null)$. This renaming operation is then combined with $\bar{\rho}_{\{e/coalesce(e^2,e^3)\}}$ to produce $if(\neg isNull(e^2), e^2, if((e^2 = e^3) \vee isNull(e^2), e^3, null))$. Next, we use equivalences

$$if(F_1 \vee F_2, v_1, v_2)) \equiv if(F_1, v_1, if(F_2, v_1, v_2)), \tag{26}$$

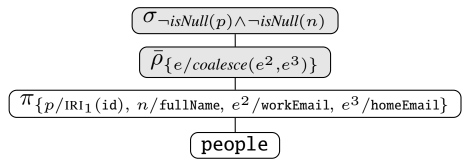
$$if(F_1, if(F_2, v_1, v_2), v_3) \equiv if(F_1 \wedge F_2, v_1, if(F_1, v_2, v_3)), \tag{27}$$

$$if(F_0, v_1, if(F, v_2, v_3)) \equiv if(F_0, v_1, if(F \wedge \neg F_0, v_2, v_3)), \quad \text{if } F_0 \text{ is 2-valued}, \tag{28}$$

to obtain $coalesce(e^2, e^3) = if(\neg isNull(e^2), e^2, e^3)$; by a *2-valued filter* we understand any filter that does not produce ε, for example, $\neg isNull(e^3)$. So, we obtain



Finally, by applying Proposition 6 again, we can replace the natural left join by an inner join and then eliminate it:



which can be simplified to $\pi_{\{p/\text{IRI}_1(\texttt{id}),\ n/\texttt{fullName},\ e/\textit{coalesce}(\texttt{workEmail},\ \texttt{homeEmail})\}}$`people` because `id` and `fullName` are non-nullable in `people`, obtaining the following SQL query:

```
SELECT fullName AS n, COALESCE(workEmail, homeEmail) AS e
FROM people
```

(In this SQL, as well as in the subsequent variations of the example, we ignore variable `?p` because it requires IRI construction, and we do not specify the IRI template.)

C.2 Variation 1 on Example 1

Consider now an extension of Example 1 with another mapping for property `personalEmail`:

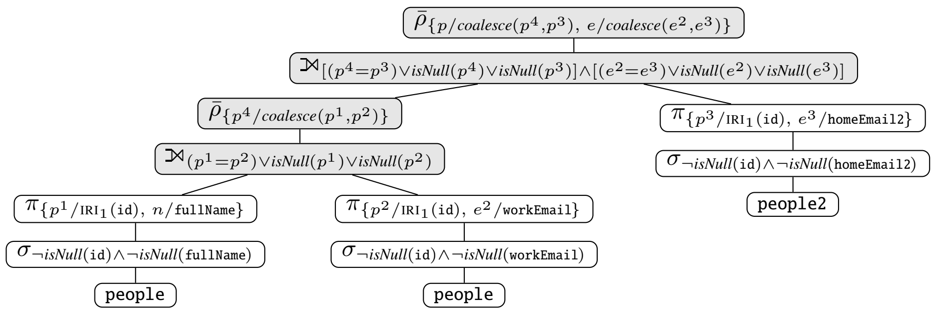
$$\text{IRI}_1(\texttt{id})\ \texttt{:personalEmail homeEmail2} \quad \leftarrow \quad \sigma_{\neg \textit{isNull}(\texttt{id}) \land \neg \textit{isNull}(\texttt{homeEmail2})}\,\texttt{people2},$$

which uses another relation `people2` with attributes `id` and `homeEmail2` (in a possibly different datasource). The additional mapping creates a union in the right-hand side of the outermost left join operation, which blocks an application of Proposition 4 to this left join (a person can now have two personal e-mail addresses, and so, we cannot make the left join natural). The resulting SQL query is

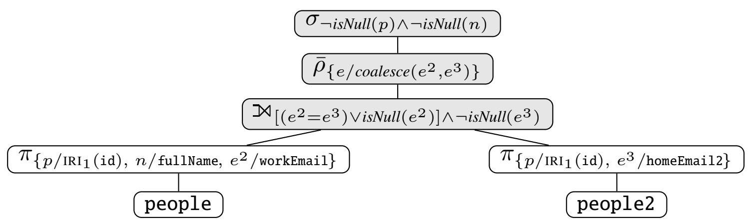
```
SELECT t1.fullName AS n, COALESCE(t1.workEmail, t2.e3) AS e
FROM people t1 LEFT JOIN
   (SELECT id, homeEmail AS e3 FROM people
    UNION
    SELECT id, homeEmail2 AS e3 FROM people2) t2
ON (t1.id = t2.id) AND
   ((t1.workEmail = t2.e2) OR IS NULL(t1.workEmail)) AND
   IS NOT NULL(t2.e3)
```

C.3 Variation 2 on Example 1

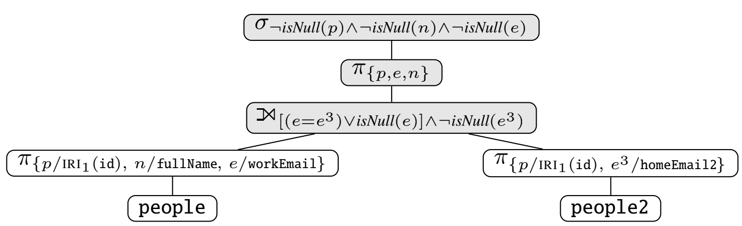
As another variant, consider again the setting of Example 1, where `workEmail` is a non-nullable attribute in `people` and there is a single mapping assertion given in Section C.2 for `:personalEmail`. Then the translated and unfolded query is as follows:



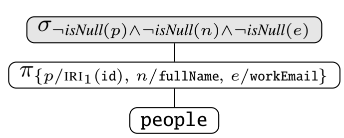
By following the same steps as in Section C.1, we arrive at:



Now, since `workEmail` is a non-nullable attribute in `people`, we can insert selection $\sigma_{\neg isNull(\texttt{workEmail})}$ above `people` and then pull the filter up to the renaming operation $\bar{\rho}$, which, by (6) simplifies to a projection:



We now apply (16) to eliminate the left join together with its right-hand side argument:



Thus, we obtain the following SQL query:

```
SELECT fullName AS n, workEmail AS e FROM people
```

C.4 Example of Join Transfer

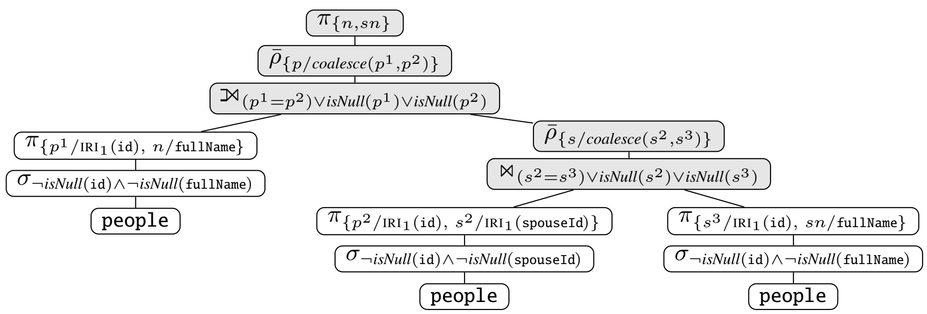
To illustrate an application of Proposition 7, we need an extension of table `people` with a nullable attribute `spouseId`, which contains the `id` of the person's spouse if they are married and `NULL` otherwise. The attribute is mapped by the following additional assertion:

$$\text{IRI}_1(\texttt{id})\ \texttt{:hasSpouse}\ \text{IRI}_1(\texttt{spouseId}) \quad \leftarrow \quad \sigma_{\neg \mathit{isNull}(\texttt{id}) \wedge \neg \mathit{isNull}(\texttt{spouseId})} \texttt{people}.$$

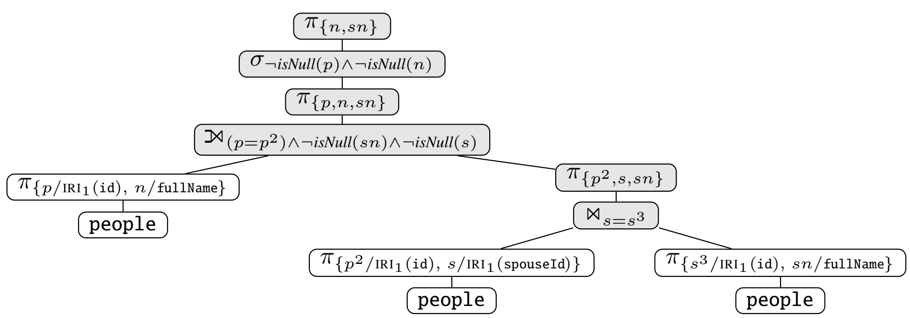
Consider now the following query in SPARQL algebra:

$$\text{PROJ}(\text{OPT}(\texttt{?p :name ?n},\ \text{JOIN}(\texttt{?p :hasSpouse ?s},\ \texttt{?s :name ?sn}),\ \top),\ \{\,\texttt{?n}, \texttt{?sn}\,\}),$$

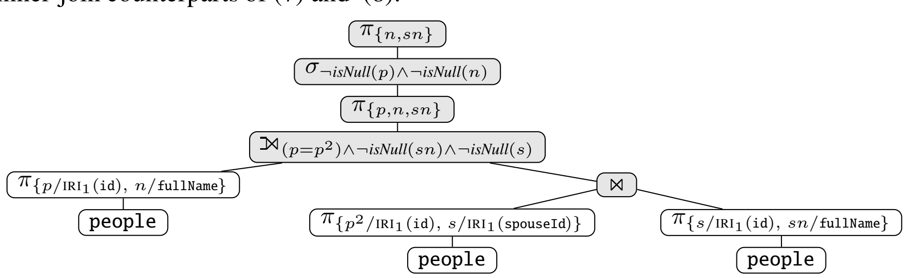
which is translated and unfolded into the following RA query:



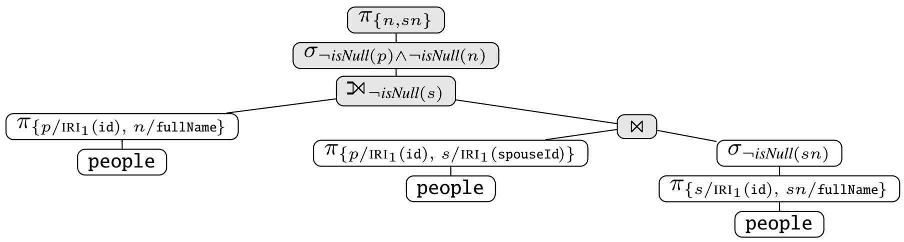
By pulling up the filters, removing *coalesce* and eliminating $\neg isNull(s^3)$ and $\neg isNull(p^2)$ from the filters of left joins (as described in Section 4.1), we obtain



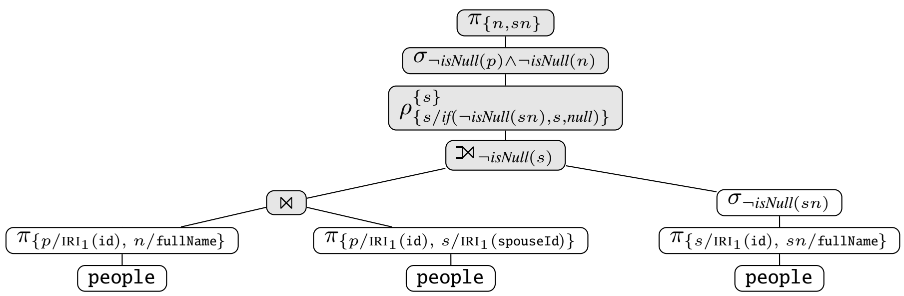
We then turn the inner join into a natural inner join by renaming s^3 into s using the inner-join counterparts of (7) and (8):



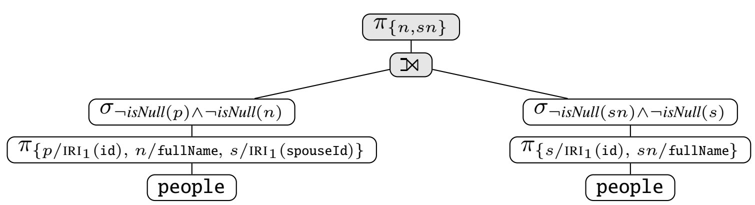
Then we similarly turn the left join into a natural left join and push down the filter on $\neg isNull(sn)$ using (3) as well as its trivial inner-join counterpart:



Observe that the inner join cannot be eliminated using the standard self-join elimination techniques because it is not on a primary (or alternate) key. However, by Proposition 7, we take sn as the non-nullable attribute w and obtain the following:



Now, the inner self-join can be eliminated (because `id` is the primary key of `people`); we can also remove the ρ operation (because its result is projected away) and, by (1) and (3), push down the filters $\neg isNull(s)$ and $\neg isNull(p) \land \neg isNull(n)$ to the right- and left-hand side arguments of the left join, respectively, to obtain



Now, the filters can all be removed because all the respective attributes are non-nullable, which gives us the following SQL:

```sql
SELECT p1.fullName AS n, p2.fullName AS sn
FROM people p1 LEFT JOIN people p2 ON p1.spouseId = p2.id
```

C.5 Example in Section 4.6

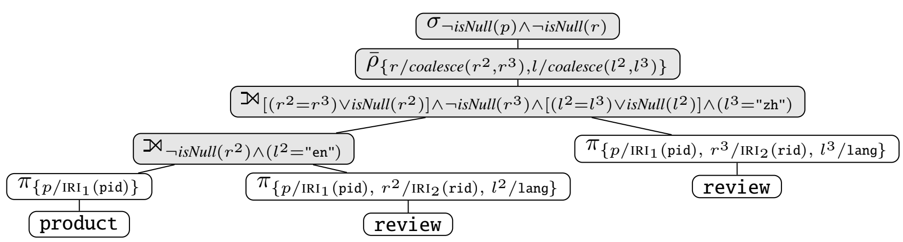
We consider the following SPARQL query:

FILTER(OPT(OPT(`?p a :Product`,
 FILTER(`{ ?p :hasReview ?r . ?r :hasLang ?l }`, `?l` = `"en"`), $\top$),
 FILTER(`{ ?p :hasReview ?r . ?r :hasLang ?l }`, `?l` = `"zh"`), $\top$), *bound*(`?r`)).

Note that both filters, `?l = "en"` and `?l = "zh"`, could in fact be moved to the third argument of OPT, thus replacing $\top$. In the database, we assume that attribute `pid` of relation `review` is a foreign key referencing the primary key `pid` in relation `product`. The following mapping connects the database to the ontology:

$$\begin{aligned}
\text{IRI}_1(\texttt{pid})\ \texttt{a}\ \text{IRI}_1(\texttt{:Product}) &\leftarrow \sigma_{\neg isNull(\texttt{pid})}\texttt{product},\\
\text{IRI}_1(\texttt{pid})\ \texttt{:hasReview}\ \text{IRI}_2(\texttt{rid}) &\leftarrow \sigma_{\neg isNull(\texttt{pid}) \wedge \neg isNull(\texttt{rid})}\texttt{review},\\
\text{IRI}_2(\texttt{rid})\ \texttt{:hasLang}\ \texttt{lang} &\leftarrow \sigma_{\neg isNull(\texttt{rid}) \wedge \neg isNull(\texttt{lang})}\texttt{review}.
\end{aligned}$$

We first translate the SPARQL query into SQL and apply the transformations in Section 4.1 to pull up and simplify the filters, then eliminate equalities from the filters as in Section 4.2 and eliminate inner self-joins (on the primary key `rid` of relation `review`), which results in the following (by (5), we can remove $\neg isNull(l^3)$ from the condition of the outermost left join):



Observe that Proposition 4 is not applicable to the innermost left join because each product can have many (in particular, many English) reviews.

By (20), $\sigma_{isNull(l^2) \vee (l^2 = \texttt{"en"})}$ could be added above the innermost left join and subsequently lifted to the filter of the outermost left join by using (2). The resulting join condition will then contain

$$[(l^2 = l^3) \vee isNull(l^2)] \quad \wedge \quad (l^3 = \texttt{"zh"}) \quad \wedge \quad [isNull(l^2) \vee (l^2 = \texttt{"en"})],$$

where the freshly lifted-up fragment is indicated in blue. By using distributivity of $\wedge$ over $\vee$ and absorption ($F \vee (F \wedge F') \equiv^+ F$), we obtain

$$\begin{aligned}
&[(l^2 = l^3) \vee isNull(l^2)] \wedge (l^3 = \texttt{"zh"}) \wedge [isNull(l^2) \vee (l^2 = \texttt{"en"})]\\
&\quad \equiv^+ \ \big([(l^2 = l^3) \wedge isNull(l^2)] \vee [isNull(l^2) \wedge isNull(l^2)] \vee\\
&\qquad\qquad [(l^2 = l^3) \wedge (l^2 = \texttt{"en"})] \vee [isNull(l^2) \wedge (l^2 = \texttt{"en"})]\big) \quad \wedge \quad (l^3 = \texttt{"zh"})\\
&\quad \equiv^+ \ [isNull(l^2) \wedge (l^3 = \texttt{"zh"})] \quad \vee \quad [(l^2 = l^3) \wedge (l^2 = \texttt{"en"}) \wedge (l^3 = \texttt{"zh"})],
\end{aligned}$$

which is clearly equivalent to $\mathit{isNull}(l^2) \land (l^3 = \texttt{"zh"})$. Next, we can use (21) to pull the weakening of the filter $\neg\mathit{isNull}(l^2) \land \neg\mathit{isNull}(r^2)$ through the innermost left join and attach the resulting disjunction, $[\neg\mathit{isNull}(l^2) \land \neg\mathit{isNull}(r^2)] \lor [\mathit{isNull}(l^2) \land \mathit{isNull}(r^2)]$, to the outermost left join. The resulting join condition will then contain

$$[(r^2 = r^3) \lor \mathit{isNull}(r^2)] \;\land\; \neg\mathit{isNull}(r^3) \;\land\; \mathit{isNull}(l^2) \;\land (l^3 = \texttt{"zh"}) \;\land \\ \big([\neg\mathit{isNull}(l^2) \land \neg\mathit{isNull}(r^2)] \lor [\mathit{isNull}(l^2) \land \mathit{isNull}(r^2)]\big),$$

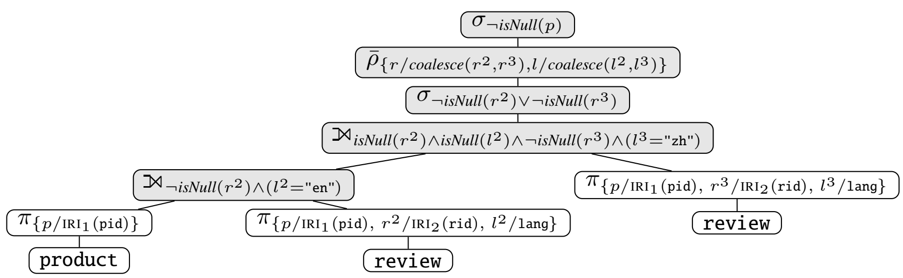
where the freshly lifted-up fragment is indicated in blue. This allows us to simplify the condition of the outermost left join to:

$$\mathit{isNull}(r^2) \land \mathit{isNull}(l^2) \land \neg\mathit{isNull}(r^3) \land (l^3 = \texttt{"zh"}).$$

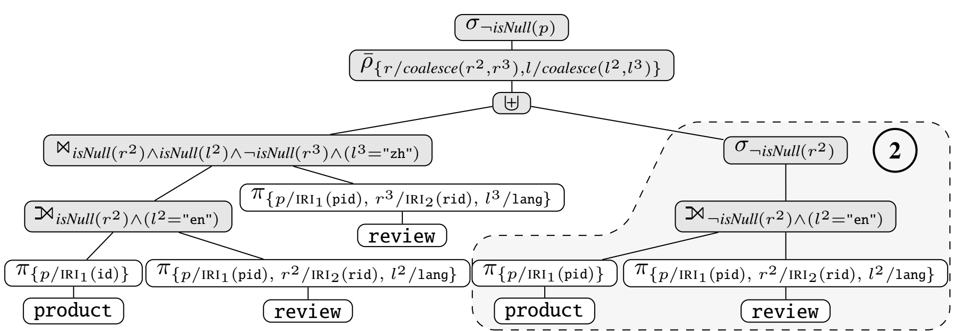
On the other hand, we can push $\neg\mathit{isNull}(r)$ through the renaming with *coalesce* using the following equivalence

$$\sigma_{\neg\mathit{isNull}(v)}\rho^{\{v^1,v^2\}}_{v/\mathit{coalesce}(v^1,v^2)}R \;\equiv\; \rho^{\{v^1,v^2\}}_{v/\mathit{coalesce}(v^1,v^2)}\sigma_{\neg\mathit{isNull}(v^1)\lor\neg\mathit{isNull}(v^2)}R. \qquad (29)$$

We thus obtain

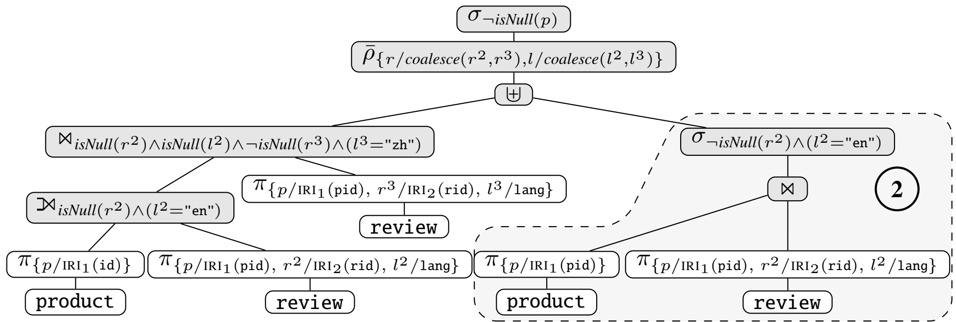


Now, since $\mathit{nullify}_{\{r^3,l^3\}}(\neg\mathit{isNull}(r^2) \lor \neg\mathit{isNull}(r^3))$ is inconsistent with $\mathit{isNull}(r^2)$ in the join condition of the outermost left join, by using (18), we replace the outermost left join by an outer union, thus resulting in

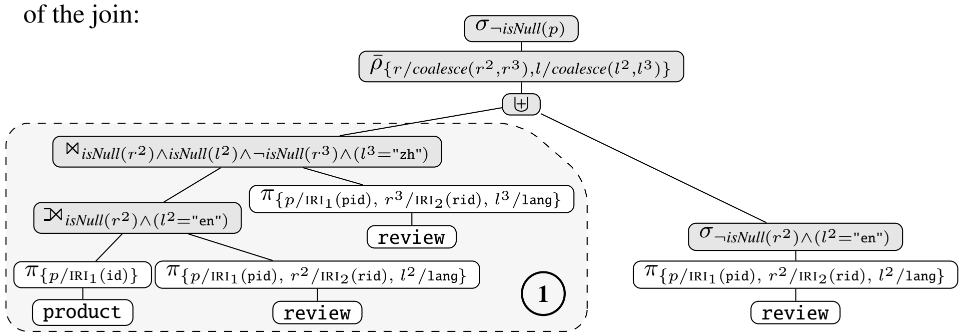


Let us first focus on the second component of the outer union, which is depicted in the shaded area above. Since $\mathit{nullify}_{\{r^2,l^2\}}(\neg\mathit{isNull}(r^2))$ is false, by applying (18) to the left join in the area shaded in the diagram above, we replace the left join by another outer union, the second component of which, however, is trivially empty because

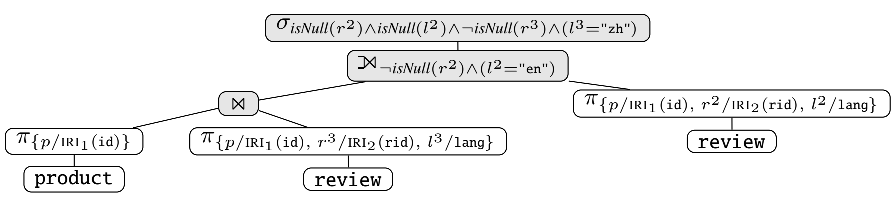
$\mathit{nullify}_{\{r^2,l^2\}}(\neg \mathit{isNull}(r^2)) \equiv^+ \bot$. Therefore, the left join is effectively replaced by an inner join, whose filter can be pulled up, resulting in



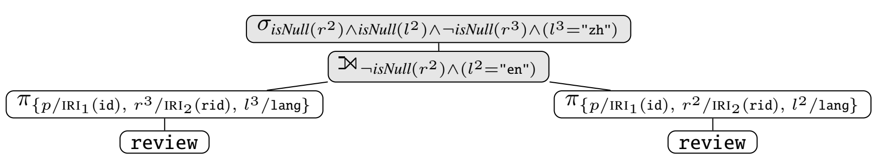
Next, the inner join can be eliminated because it is over a foreign key (`pid` in `review` references `pid` in `product`) and no attribute occurs only in the left-hand side argument of the join:



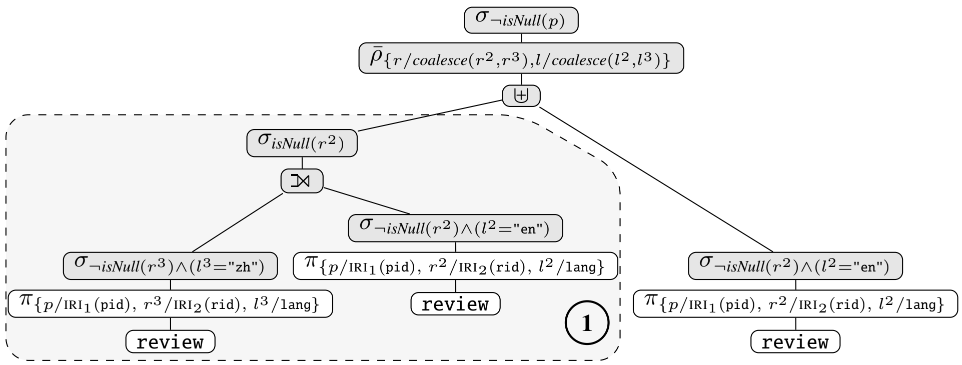
We now return to the first component of the outer union, which is in the shaded area in the diagram above. First, observe that the join condition of the inner join can be pulled up and the order of the left join and join changed using the well-known equivalence $(R_1 ⟕_F R_2) \bowtie R_3 \equiv (R_1 \bowtie R_3) ⟕_F R_2$ provided that $U_2 \cap U_3 \subseteq U_1$, where U_i are the attributes of R_i (see, e.g., (6) in [12]). Thus, we obtain



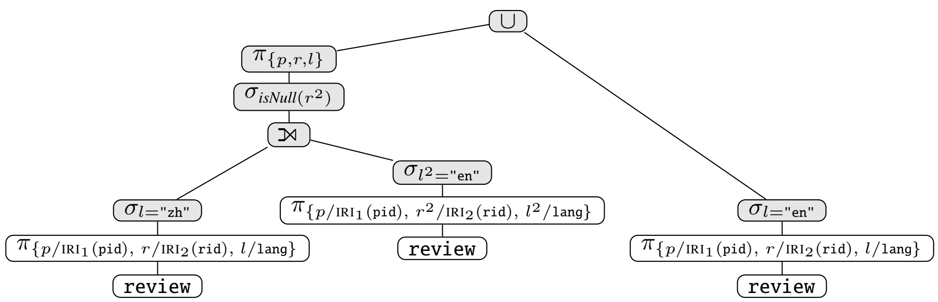
Now, the inner join can again be eliminated because it is over a foreign key (`pid`) and no attribute occurs only on the left:



Next, by (1), we can push the $\neg isNull(r^3) \wedge (l^3 = \texttt{"zh"})$ of the filter to the first component of the left join. By (19), since the remaining part of the filter, that is, $isNull(r^2) \wedge isNull(l^2)$, is inconsistent with the left join condition $\neg isNull(r^2) \wedge (l^2 = \texttt{"en"})$, we can simplify the filter above the left join to $isNull(r^2)$ because r^2 is not nullable in the right-hand side argument of the left join (and can be chosen as the w). Finally, by (3), we push the left join condition and obtain a natural left join:



We can now push the two *coalesce* through $\uplus$, simplify them, push down the projections and remove unnecessary padding μ in the second argument of the union. Finally, we remove $\neg isNull()$ for all non-nullable attributes and obtain



which corresponds to the following SQL:

```
SELECT CONCAT("IRI1", pid) AS p, CONCAT("IRI2", rid) AS r, lang AS l
FROM review WHERE l = "en"
UNION ALL
SELECT CONCAT("IRI1", r1.pid) AS p,
       CONCAT("IRI2", r1.rid) AS r, r1.lang AS l
FROM review r1 LEFT JOIN review r2
                    ON (r1.pid = r2.pid) AND r2.lang = "en"
WHERE r1.lang = "zh" AND r2.lang IS NULL
```

where the `CONCAT` functions construct IRIs. The second component of the union can also be expressed in SQL using `NOT IN`:

```
SELECT CONCAT("IRI1", pid) AS p, CONCAT("IRI2", rid) AS r, lang AS l
FROM review
WHERE lang = "zh" AND
      pid NOT IN (SELECT pid FROM review WHERE lang = "en")
```