Robustness Analysis of the European Air Traffic Network

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Abstract

The European Air Traffic Network (ATN), comprising of a set of airports and Area Control Centres (ACCs), is highly complex. The current indicator of its performance, *air traffic flow management* (ATFM) delays, is insufficient for planning and management purposes. Topological analysis of air traffic networks of this kind has highlighted *Betweeness Centrality* (*BC*) as an indicator of network robustness, although such an indicator assumes no knowledge of actual traffic flows and the network's operational characteristics. This paper conducts topological and operational analyses of the European ATN in order to derive a more relevant and appropriate indicator of robustness. By applying a flow maximisation model to the network influenced by a range of capacity reductions at the local level, we propose a new index called the *Relative Area Index* (*RAI*). The *RAI* quantifies the importance of an individual node to the performance of the entire network when it suffers from capacity reduction at a local scale. Air traffic data from three typical busy days in Europe are utilised to shown that the *RAI* is more flexible and capable than *BC* in capturing the network impact of local capacity degradation. This index can be used to assess network robustness and provide a valuable tool for airspace managers and planners.

Keywords

Air traffic network; robustness; capacity; linear programming

1. Introduction

High air traffic demand in Europe in recent decades has resulted in the severe congestion experienced at both busy airports and en-route airspace. The latter is controlled by Area Control Centres (ACCs). Such congestion has not only caused severe delays and detrimental environmental impacts, but also posed threats to the safety of air travel. Furthermore, with annual air traffic demand in Europe forecast to increase by 2.5 % between 2015 and 2021(EUROCONTROL, 2015), it is expected that the current capacity of the European air transport network will be simply insufficient to cope with this increase. The capacities of ACCs and airports can be defined in terms of maximum number of flights that can be handled in a given period of time.

Compounding this problem of a shortage of capacity on the network is the uneven distribution of air traffic in Europe. According to the Network Operations Report 2013, the top twenty busy airports and congested ACCs were responsible for 67% of all Air Traffic Flow Management (ATFM) delays in 2013 (EUROCONTROL, 2014).

In order to ameliorate these negative effects and to improve airspace capacity, the Single European Sky (SES) Air Traffic Management (ATM) Research (SESAR) program was launched and one of its features is to change the ATM of Europe from a local to a network level. SESAR envisages that European airspace will be managed as a continuum and as a consequence, network capacity becomes one of the most important Key Performance Areas (KPA). This move to a network level operation and management means that it is essential to understand the fundamental characteristics of the Air Traffic Network (ATN), especially the connections (i.e. the "connectivity") between elements of the network. The mathematical science of topology, which is concerned with network characteristics, provides a viable method for assessing the characteristics of the European ATN. In addition to the topological characteristics, the importance of each constituent node relative to the operation of the entire ATN, in terms of capacity, flow, and bottleneck, needs to be investigated in order to understand their roles and impact in the events of network deterioration or expansion. This paper applies complex network theory, robustness analysis, and network optimization to offer insights on the topological and operational characteristics of the European ATN and provide a quantifiable measure of the importance of its constituent nodes when the network suffers from local distress.

An ATN can be represented by a set of nodes and links. Conventionally, these nodes can be waypoints, en-route airspace or airports, and the links are the flight routes, between these nodes, e.g. airways in en-route airspace. The European ATN is a nonlinear, dynamic and complex system that comprises of numerous heterogeneous components and stakeholders such as airports, Air Traffic Control (ATC) and airspace users. In addition to the heterogeneity of the components, the operational concepts and interactions between different components make ATNs difficult to analyse. Initial studies on the ATNs tend to focus on their topological characteristics, through complex network theory (Holmes, 2004). Comprehensive reviews of existing studies on the application of complex network theory to air transport networks are provided by Sun et al. (2014) and Sun and Wandelt (2014); they found that these studies focus on airports and cities but failed to consider en-route airspace (Lordan et al., 2014; Wei et al., 2014; Zhao et al., 2014). Therefore, Sun et al. (2014) conducted topological analysis on the ATNs in 15 countries, including the USA and major European nations, in which the constituent nodes are both airports and en-route waypoints and the links are flight routes between the nodes. Five topological indices namely: degree, distance strength, Weighted Betweenness Centrality (hereafter referred to as betweenness centrality or *BC*), weighted closeness centrality and edge length distribution were calculated. The authors suggest that BC, originally proposed by Freeman (1979), can serve as an index of network robustness and indicates the number of shortest paths passing through a given node. A node with high BC is used by more flights and the capacity of it is consequently saturated earlier. Therefore, a network is considered more robust against capacity-reduction at nodes when the network contains nodes with a smaller *BC* compared to other nodes. However, this conclusion is solely based on topological characteristics and is not validated by using the relevant data of air traffic.

In response to the lack of a suitable robustness index that captures the operational aspect of an ATN, this paper proposes a new index, namely the Relative Area Index (*RAI*), to assess network robustness based on the actual flight data from three of Europe's busiest days and the published capacity of airports and ACCs. The *RAI* is developed based on the change in the maximum network flows, calculated through a linear programming (LP) approach, caused by a range of capacity reductions at a given node. Such capacity degradation may be due to local disruptions such as meteorological influence and industrial action. The LP approach is significant in that it provides a theoretical upper bound on the network flow, while taking into account the capacity reduction that occurs at its constituent nodes. Moreover, the estimated maximum flow on each node is significantly correlated with the empirical maximum flow on congested days. Therefore, the LP-based maximum network flows can reasonably reflect the network capacity of the European ATN.

This LP approach, along with the European ATN data that it relies on, are developed using flight profile data provided by the European Organisation for the Safety of Air Navigation (EUROCONTROL). The data were collected on a typical busy day in 2012. This paper extends the analyses of Sun et al. (2014) and Pien et al. (2014) by comparing *BC* with the new network robustness index (RAI) for the European ATN. The latter index takes into account a range of capacity reduction and its impact on the network operation, rather than simply removing the node, and thus is more realistic in characterizing the robustness of the ATN.

This paper not only calculates the topological index (betweenness) of the entire European ATN for the first time in the literature, but also provides a validated tool (RAI) for Europe's airspace managers and planners to assess network robustness in the event of any local deterioration of nodal capacity. The RAI is compared with the actual traffic demand and published capacity at each node and shown to have the potential to identify the important nodes in the network.

The rest of this paper is organised as follows. Section 2 introduces an overview of robustness analysis in transport networks and relevant literature. The development of the RAI is also introduced. The European ATN flight profiles, capacity constraints, and network structures are described in detail in Section 3. Section 4 presents the linear programming approach for traffic flow maximisation. Section 5 presents the results and analysis on the European ATN. The findings are discussed in Section 6 prior to the conclusions in Section 7.

2. Robustness Analysis

Since the definition of robustness varies in different fields, it is pertinent to review the definitions and their context in the literature to define the robustness of an ATN. Furthermore, the current *Key Performance Indicator* (KPI) of network capacity in the European ATN is introduced. Based on previous research, the conventional index of BC is described. Finally, we develop a new index, the RAI, to assess the robustness of the European ATN.

2.1. Literature review

The robustness of transport networks has been a central focus of network planning and management. It is often investigated in different performance areas such as stability, resilience and permanence to assess the capability of handling worsened or perturbed conditions of the network.

Since there are numerous categories of networks, there is no universal definition of network robustness, though the following three are highly relevant for this paper:

- "*The degree to which a system or component can function correctly in the presence of invalid inputs or stressful environmental conditions*" (Geraci et al., 1991).
- *''The degree to which a system is capable of functioning according to its design specifications in the case of serious disruptions''* (Immers et al., 2004).
- The robustness of an electrical network is defined as *the capability of maintaining its structure and function when the network is exposed to perturbations* (Holmgren, 2007)

Given these definitions, the robustness of a given system or component is therefore the capability of maintaining its function or performance in order to cope with disruptions, perturbations and stressful conditions. Whilst useful, these studies focus on either an individual system or a component. In order to cope with the robustness at network level, it is illustrative to consider the experience of research on transport networks as outlined below.

- Sakakibara et al. (2004) proposed a topological index to evaluate the depressiveness and concentration of road networks in the presence of disasters. They suggested that a network is considered robust when it is able to minimize the isolation of districts when catastrophic disasters occur.
- Scott et al. (2005) proposed the Network Robust Index (NRI) to identify the critical links of a highway network. This index was calculated by comparing the changes in travel time cost of the network when a given highway segment (i.e., network link) is removed from it. Compared to the conventional method of using the ratio of volume to capacity which can only reflect the congestion at local level, the NRI provides better planning solutions to enable the identification of critical links at the network level.
- Nagurney and Qiang (2007a) proposed a network efficiency measure to assess the efficiency of congested networks. Their approach is used to rank the importance of a given link by comparing the change of total travelling costs when the link is removed from the network. In their later work Nagurney and Qiang (2007b) use the relative change of network efficiency as an index to assess network performance when the capacities of all links are reduced by the same percentage. The authors therefore developed the Relative Total Cost Index (RTCI) to assess the robustness of networks against a global decrease of link capacities (Nagurney and Qiang, 2009). Compared to removing links from the network, this approach provides a more realistic method of assessing network robustness when disturbance occurs on any of its constituent components.

Based on these studies, the robustness of a network can be defined as the capability of maintaining network performance while its functioning components, namely nodes and links, are under stress. With this definition in mind, we conduct robustness analysis by taking into account the topological and operational characteristics of the European ATN, and treating network capacity as the main *key performance area* (KPA). In particular, we consider the *maximum network flow*, which is obtained through an optimization procedure, as the key indicator of network capacity. Accordingly, the robustness of the European ATN in this paper is related to the capability of delivering the maximum traffic flows against degradation of nodal capacities. In the following sections, the current KPIs of network capacity and the conventional index of network robustness are reviewed.

2.2. KPI of network capacity: ATFM delays

Currently, ATFM delays are used as the KPI to monitor network capacity (EUROCONTROL, 2007). This is the duration between the last take-off times requested by the aircraft operator and the take-off slots allocated by the central flow management unit namely the Network Manager Operations Centre. This duration follows an air traffic flow regulation, which is subsequently communicated by the flow management positions to an airport or en-route centre.

However, there is a major deficiency with this measure since ATFM delay is not a direct measure of capacity but rather is a proxy that reflects the extra time caused by capacity shortages, which are in turn caused by various factors at airports and in en-route airspace. Since it is an indirect measure, there is an inherent inaccuracy in identifying the important nodes in a network. For instance, Maastricht ACC highlights the limitation of ATFM delays as an indicator. Although the air traffic demand in Maastricht ACC is amongst the highest in Europe, the ATFM delays are relatively low. However, since the nodes with high ATFM delays are considered as bottlenecks in the European ATN, we use them to conduct an intuitive verification of the other indices.

2.3. Topological Index: Betweenness Centrality (BC)

In complex network theory, nodes within a network may be ranked by using different centrality measures (Wasserman, 1994). The rank of a given node reflects the measurement of some particular structural property in the network. Several centrality measures, such as degree, betweenness, and closeness, can be used to rank the importance of nodes, among which betweenness is the most widely used as an index of network resilience and robustness (Holme et al., 2002; Newman, 2001). The betweenness centrality (*BC*) of a node is defined as the probability that it lies in the shortest path(s) between all origin-destination (OD) pairs (Dehmer, 2011; Di Paolo et al., 2011). Mathematically, the conventional formulation of *BC* can be presented as:

$$BC_i = \sum_{m,n \in v} \frac{S_{mn}^i}{S_{mn}} \tag{1}$$

where BC_i is the betweenness of the node i. S_{mn} is the total number of shortest paths between any pair of nodes *(m, n)* and S_{mn}^i is the number of shortest paths passing through the node *i*.

Newman (2001) states that the higher the *BC*, the more influential is the node. The largest increase in the travel distance among nodes occurs when the node with the highest *BC* is removed. This explains intuitively the importance of high-*BC* nodes relevant to the overall performance of the network. In addition, a network with many low-*BC* nodes is more robust than that with many high-*BC* nodes. Brandes (2001) notes that *BC* is the most frequently employed centrality index in the analysis of social networks; and it is mostly based on shortest paths. Barrat et al. (2004) claim that nodes in the inner network are more likely to be used by shortest paths than those in the outer network. Therefore, in an ATN, it can be intuitively assumed that the airspace nodes are more likely to be passed by the shortest paths and the nodes with high *BC* are more likely to handle more traffic, when the traffic demands are uniformly distributed. Travellers within a network tend to choose the shortest paths and as a result, the nodes with high *BC* tend to be used by more travellers.

However, in the 'real world' of transport operations, nodes with high *BC* are not necessarily busy (or heavily loaded) due to the fact that the traffic network flows are jointly determined by a number of factors such as travel demand distribution, complex decision factors, and route choice patterns that are not based on shortest paths (i.e., not all-or-nothing assignment). For example, Cats and Jenelius (2014) developed a dynamic-stochastic model to evaluate the impacts of disruptions, and demonstrated that *BC* may not be a good indicator of link importance in a road network. Guimera and Amaral (2004) modelled a world-wide airport network and showed that the airports with high *BC* are not necessarily hubs. They argued that geo-political constraints play an important role in the growth of airport networks and other critical infrastructure. In addition, although both are modelled as nodes in an ATN, the roles of airports and ACCs are different. An airport not only acts as an origin/destination but also as a transfer node (hub), which means that it serves as the en-route node of a complete trip from the origin to the destination. However, the conventional formulation shown in (1) treats the airports and ACCs equally without considering their heterogeneity. Therefore, this conventional BC needs to be tailored and improved to accommodate the unique characteristics of the ATN. All these aforementioned factors contribute to the consensus that BC may no longer be sufficient to assess the robustness of a complex network such as the ATN. This paper contributes to this line of research by proposing the relative area index (RAI), which serves as an alternative robustness index that captures the ATN's flow capacity and certain aspects of its operational features.

2.4. Relative Area Index (RAI)

Built upon the definition of robustness, the *RAI* assesses and quantifies the impact on the maximum network flow of a wide range of capacity reductions that occur at a local (nodal) level. Its derivation is outlined below.

The potential reduction of nodal capacity in the ATN is parameterised with u, which is termed the *degradation parameter* (*DP*). We employ the notation $F_{max}^{i}(u)$ to represent the maximum network flow when a capacity reduction expressed by u is applied to node i. The procedure of finding the maximum network flow with a given degradation parameter amounts to a linear program, as we describe in detail later in Section 4.

In this study, two types of *DPs* are considered: (1) percentage based DP; and (2) absolute value based *DPs*. The Percentage-*DP* (u^P) ranges from 0% to 100%, and represents the reduction of the subject node's capacity in percentage. The Absolute-*DP* (u^A) ranges from 0 to 50 (in flights per hour). The value 50 is chosen since it is about half of the capacity of the node with the smallest capacity (LA PALMA airport). Anything significantly larger than 50 may cause the reduced capacity at some nodes to be negative, which is clearly infeasible. In order to distinguish these two methods of capacity reduction, we denote the *RAI* based on Percentage-*DPs* by RAI_P, and the RAI based on Absolute-*DPs* by RAI_A.

Since the network maximum flow problem is formulated as an optimization problem constrained by nodal capacities, we deduce that $F_{max}^{i}(u)$ is a monotonically decreasing function of u, where a larger u represents greater capacity degradation at the relevant node. **Error! Reference source not found.** illustrates the rationale behind *RAI*. As shown in **Error! Reference source not found.**(a), two functions, $F_{max}^{i}(u)$ and $F_{max}^{j}(u)$ corresponding to nodes i and j respectively, indicate that node i is the more important one as far as maximum network flow is concerned. This is because the same level of degradation yields a smaller network flow when applied to node i than node j. In general, the lower the function $F_{max}^{i}(u)$, the more detrimental it is to reduce the capacity at node i. In order to further quantify such an observation, we consider the area formed by the graph of $F_{max}^{i}(u)$, the vertical line passing through u_T, and the horizontal line passing through $F_{max}^{i}(u_0)$. Such an area is illustrated as the shaded part in **Error! Reference source not found.**(b). It is understood that the large the area, the more critical is the node. For an ATN, the more critical nodes there are, the less robust is the network against capacity reductions. Finally, we note that two distinct functions may yield the same area, as shown in **Error! Reference source not found.**(c). Thus, in order to distinguish such circumstances, we introduce the *weighting parameters* (WP) $w(u)$. The WPs assigns different priorities to different range of capacity reductions, and may depend on the node of interest, type and nature of capacity reduction, and application scenarios. For example, if the main cause of capacity reduction is scheduled maintenance, which has a mild effect on airport capacity, then lower values of u will be assigned higher weights. Therefore, in **Error! Reference source not found.**(c) the node i (solid line) corresponds to larger weighted area than node j (dashed line), and thus is more critical.

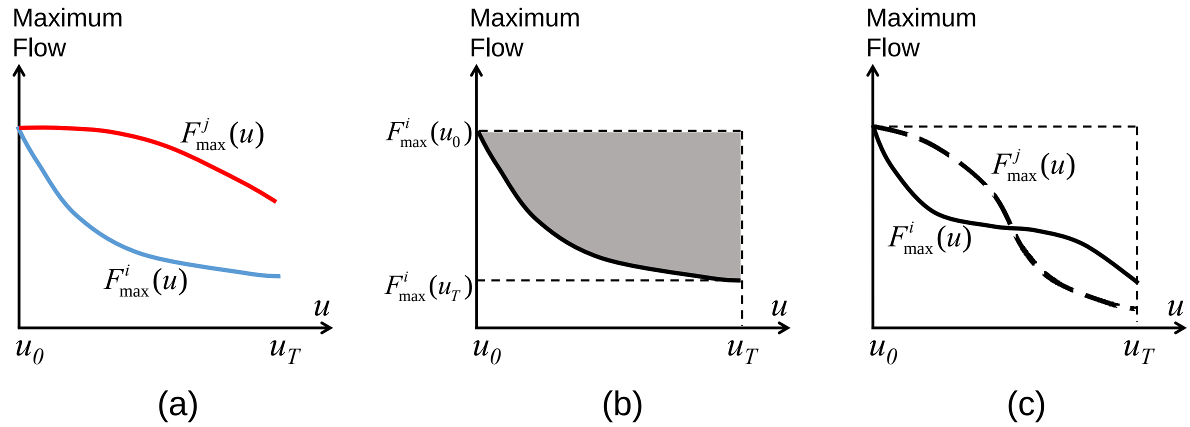

Figure 1. Illustration of the relative area index.

In this study, we select three types of WPs. The first type assumes equal weights for all values of the degradation parameter $u \in [u_0, u_T]$. The second type of WP assigns higher weights to small values of u, in contrast to the third type, which assigns lower weights to the small values. By using these three sets of WPs, it is possible to gain insights into the (global) influence of the local capacity-reductions. With this in mind, we formulate the *relative area index* (*RAI)* for a given node *i* as:

$$RAI^i = \frac{\int_{u_0}^{u_T} w(u)(F^i_{max}(u_0) - F^i_{max}(u))du}{\int_{u_0}^{u_T} w(u)F^i_{max}(u_0)du} \qquad (2)$$

We note that a normalisation factor (denominator) is applied to the aforementioned weighted area (numerator). As we previously mentioned, percentage-based and absolute-value-based degradation parameters (u^P and u^A) are considered; namely, $u^P \in [0, 1]$ and $u^A \in [0, 50]$.

In general, the *RAI* defined for a given node encapsulates the global impacts of capacity reductions at this node, which depend not only on the network topology, but also on the configuration of OD pairs, flight routes, and nodal capacities. The *RAI* is defined in terms of a flow maximisation problem, and is not available in a closed form. Thus, it is difficult to predict the distribution of *RAI*s using simple topological indicators such as *BC*. There are, however, a few simple interpretations of the *RAI*. In particular, it is reasonable to expect that nodes with higher capacities should in general have larger (percentage-based) RAI_P values than those with lower capacities based on the following observation: the percentage-based capacity reduction at a high-capacity node results in greater absolute reductions. However, as we subsequently show in Section 5.2, some nodes in the European ATN possess RAI_P that are quite counter-intuitive, as suggested by their size, capacity, and significance to the network.

3. The European Air Traffic Network

The key elements required to conduct the robustness analysis are the weighted adjacency matrix and the maximum network flow estimation method. The former is used to calculate betweenness, while the latter is used to calculate the *RAI*s for the network against the capacity reductions at every node. This section introduces the European ATN and the required data for calculating BC and the maximum network flows.

3.1. Data processing

According to the latest European Network Operations Plan (EUROCONTROL, 2014), the European ATN comprises of 41 countries, and the en-route airspace of Europe is controlled by 64 ACCs. In order to monitor the air traffic at the network level, EUROCONTROL records detailed daily profiles of all flights in Europe at the levels of the ACCs, en-route sectors and waypoints – the latter of which are often associated with navigation aids, in particular radars. Each flight profile represents a flight route that uses a sequence of nodes. This dataset provides the information on the times and coordinates of each flight at every node. In order to investigate the robustness at the network level, airports and ACCs are employed as the constituent nodes of the European ATN.

Flight profiles recorded on 1st July 2012 were used to extract the required information. On this day, a total of 28,904 flights were scheduled, among which 28,885 flight profiles were recorded by the radars. In total, 28,753 flight profiles are used for this study, with the remainder excluded since they either used unrecognized airports or passed through unrecognized airspace.

The average ATFM delays per flight on this particular day were among the highest in 2012. This date also falls within the European summer, which is the season with the highest traffic demand throughout a year. The advantage of using the flight profiles for this day is that the high ATFM delays enable us to capture the spatial configuration of traffic congestion, while the high traffic demands (flight routes) provide sufficient information on the connectivity between nodes. The ATFM delay data are provided by EUROCONTROL.

3.2. Network topology

The European ATN can be represented as a directed graph, in which the nodes represent airports and ACCs. A critical notion is connectivity, which can be defined as a binary state that exists between any two nodes in the network, taking a value one if the two nodes are connected by a link and zero otherwise. Unlike many traditional transport networks, the capacity constraints in an ATN are imposed at the nodes (airports and ACCs) rather than on the links. The declared capacities at airports and en-route airspace are applied to prevent the relevant node from overload through the mechanism of air traffic flow management (ATFM), which includes re-routing and the imposition of flow regulation. Air traffic at European airports and in en-route airspace is required to comply with the declared capacities. Based on these characteristics, the European ATN can be regarded as a capacitated transport network and, consequently the traffic flows within it cannot exceed the theoretical maximum. The fundamental components of a capacitated transport network are the constituent nodes and links, as detailed below. These components can be updated and re-selected by using different techniques with the latest operational reports when they are available.

3.2.1. Constituent nodes

Based on the flight profiles, this network consists of 850 nodes, which include 784 airports, 64 ACCs and two external nodes. The external nodes are used to represent airports and airspaces that are external to the European region. According to Pien et al. (2014), the airports can be categorized as ‘busy airports’ and ‘less busy airports’. A total of 67 busy airports were selected based on the top airports listed in EUROCONTROL (2013c) and EUROCONTROL/FAA (2009). These busy airports carry about 60% of the overall flights in Europe, while the less busy airports carry the remaining 40%.

The capacity data at less busy airports are not publically available. In order to overcome this difficulty, we treat the less busy airports collectively as one or several aggregate nodes. Figure 2 shows the method of creating an aggregate node that represents a group of less busy airports that are of interest to a particular ACC. The connectivity among these less busy airports and relevant ACC is identified through the flight profile data. It is worth noting that some less busy airports may be adjacent to more than one ACC. In this case, we assign this airport to the ACC that contains the most number of flights originating from it. Following this rule, we are able to assign each less busy airport to a unique ACC; and less busy airports assigned to the same ACC are aggregated to form an aggregate node. By using the aggregate nodes, the issue of unknown capacities of the less busy airports is circumvented since active bottlenecks can occur only at the level of ACCs that watch over the less busy airports, rather than at these airports themselves; in other words, the capacities of individual less busy airports are not explicitly needed for the flow maximization problem. Applying the aggregate nodes also reduces the network size while maintaining the connectivity between the less busy airports and their adjacent ACCs. The reduced network contains 197 nodes, including 67 busy airports, 64 aggregate airports, 64 ACCs and 2 external nodes.

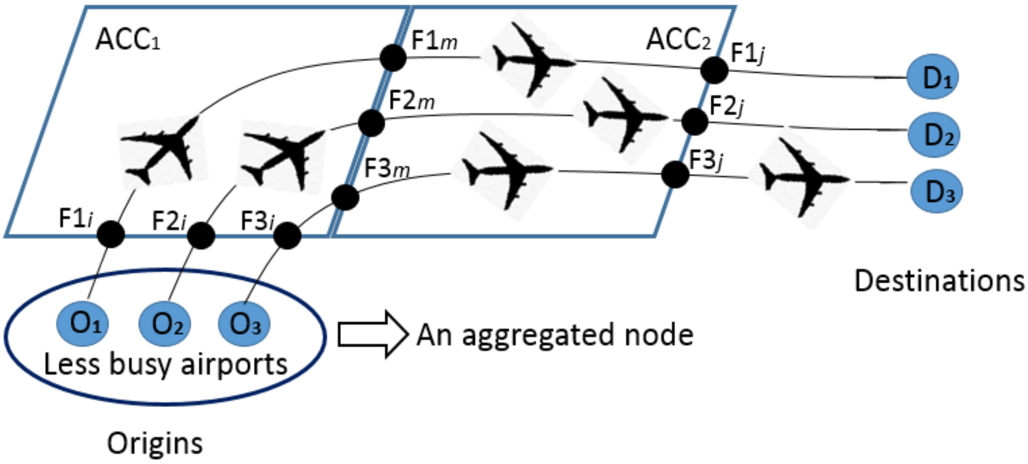

Figure 2. Illustration of the aggregate airports.

The flow maximization problem we shall consider later employs a static flow modelling approach; that is, we consider the stationary flows in the network on a daily basis, without explicitly considering the within-day dynamics of various variables. Thus, the capacities of the airports and ACCs (in flights per day) as we consider in this paper are calculated from the declared capacity (in flights per hour) by a multiplication factor of 16 and 24, respectively, meaning that the operational hours at the airports and ACCs are 16 hours and 24 hours per day. Although the air traffic demand and traffic intensity are unevenly distributed over the duration of operational hours, the static modelling approach for ACCs and airports enables the calculation of the theoretical maximum network flows, which can serve as a theoretical upper bound of network capacity.

3.2.2. Constituent links

A directional link in the network is defined as the connectivity between an ordered pair of nodes. The link between any pair of nodes is established if the flight profile data suggests the consecutive passing of the two nodes by at least one flight. The weight of the link is defined to be the average flying distance from its tail node to its head node, which is obtained using the actual flight data.

Issues arise from the different spatial characteristics of airports and ACCs. In particular, an airport can be regarded as a single point with negligible size while an ACC usually covers a significantly larger area. The distance between an airport (or an aggregated airport) to its adjacent ACC can be calculated by averaging the distance between the airport and the entry point to its adjacent ACC, and such entry points are recorded in the dataset of flight profiles.

However, the flying distance between two ACCs cannot be directly calculated by using the distance between their centres, due to their relatively large areas and irregular shapes (ACC boundaries in Europe follow national borders). Assuming that N flights are flying through node i and j, the flying distances of these N flights between the node i and j are $D_{(i,j)}^{N}$. The weight of the link between ACC$_i$ and its adjacent ACC$_j$ ($L_{(i,j)}$) can be formulated as:

$$L_{(\mathrm{i,j})} = \frac{1}{2\mathrm{N}} \sum_{N=1}^{N} D_{(i,j)}^{N} \tag{3}$$

3.2.3. Validity of the network data

As mentioned earlier, the network topology and the flow maximization problem are based on the flight data on 1st July 2012. Datasets collected on two additional busy days, namely 28th and 29th July 2012, are used to validate the network data. More specifically, we use correlation coefficients to assess the similarity of the network adjacency matrices calculated from the data on these three days. On the other hand, the Mean Absolute Percentage Error (MAPE) and correlation coefficients are used to compare the maximum flows (details to follow in Section 4) calculated by using data on these three days.

The correlation coefficients for the network adjacency matrices on the three days are above 0.85. In addition, the correlation coefficients among the maximum flows on these three days are above 0.99 and the MAPEs are less than 0.5%. This shows consistency of our data-processing method and the validity of the resulting network topological information and maximum flow data.

4. Traffic Flow Maximization

The problem of finding the theoretical maximum network flow subject to the network topology and capacity constraints is formulated as a linear program. In this section, we first recap the LP approach for maximising network flows originally proposed by Pien et al. (2014). This is followed by an interpretation of the *RAI* in relation to the Lagrange Multipliers (LM), which is relevant to the linear program and the marginal costs of local capacity reduction.

4.1. Network flow maximisation formulated as a linear program

As mentioned earlier, the European ATN is considered as a capacitated transport network in which the operations at airports and in ACCs are subject to their individual capacity limits. The maximum network flows are estimated by using a LP approach. The estimated maximum flow on each node is significantly correlated to the empirical maximum flow on congested days. Therefore, the estimated maximum network flows can reasonably reflect the network capacity of the European ATN.

Figure 3 depicts the structure of the network and the flight paths therein. The network comprises of busy airports, aggregated airports and ACCs. An aircraft departs from its origin airport and flies along its flight path in en-route airspace to its destination airport. The flights that use European airspace can be categorised into three groups: intra-Europe flights, inter-continental flights, and over-flights. The flights flying along the flight paths p_1 and p_2 are intra-Europe flights that fly between two European airports. The flights along paths p_3 and p_4 are inter-continental flights that fly from an European airport to an airport outside Europe or vice versa. The group of over-flights p_5 represents the flights passing European airspace without using any European airports.

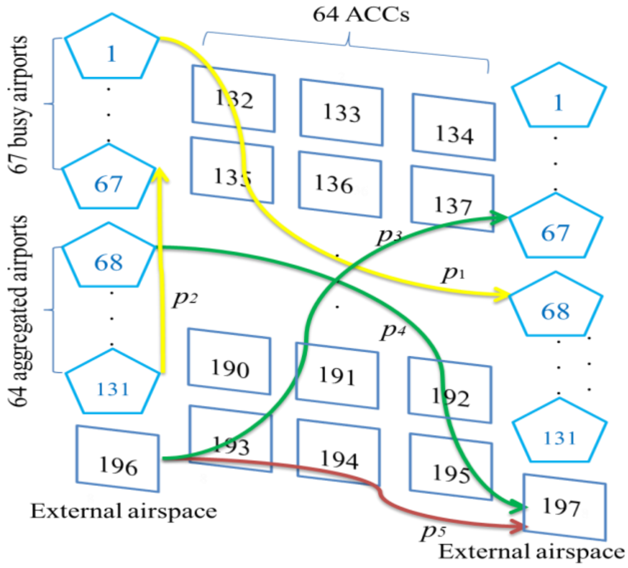

Figure 3. Structure of the European ATN.

We consider a network with given sets of paths (P) and origin-destination (OD) pairs (W). For any $(i,j) \in W$ where i denotes origin and j denotes destination, let P_{ij} be the set of flight paths connecting i to j. Each flight path is represented as a set of nodes (airports and ACCs) it traverses. The relationship between paths and nodes is encapsulated by the path-node incidence matrix (δ_{pv}):

$$\delta_{pv} = \begin{cases} 1 & \text{if } v \in p \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where v denotes a node, and p denotes a path. In addition, each node v in the network has a flow capacity C_v.

The maximum network flow problem is formulated as follows. The objective is to maximize the path-based flows in the entire network:

$$\max \sum_{p \in P} f_p \tag{5}$$

where f_p denotes the flow along path p. The constraints include:

Flow capacity constraint: $\forall k \in \boldsymbol{AN}$ (the set of nodes)

$$\sum_{p \in P} \delta_{pk} f_p \le C_k \tag{6}$$

Nonnegativity: $\forall\, p \in P$(the set of flight paths)

$$f_p \geq 0 \tag{7}$$

The influence of capacity reduction at a given node on the maximum network flows can be obtained by solving a family of such linear programs, each with a decreased flow capacity at a given node. This procedure is the basis for calculating the *RAI*.

4.2. Lagrange multiplier and *RAI*

The use of the Lagrange multiplier is a common approach to solving optimization problems (Jahn, 2007). The Lagrange multiplier is the rate of change of a quantity being optimized as a function of the constraint variable. In the case of maximizing network flows, the Lagrange multiplier is the sensitivity of the maximum network flow with respect to the change in the capacities. It can be interpreted simply as the marginal cost (gain) of the network maximum flow with respect to an infinitesimal reduction (increase) in the nodal capacity. Lagrange multipliers are zero at non-bottleneck nodes, which corresponds to the complementarity conditions arising from duality; this means that small changes in capacity at these non-bottleneck nodes have no effect on the maximum network flows. The higher the Lagrange multiplier, the more critical the node is to the overall throughput of the network.

The Lagrange multiplier is related to the *RAI*, since the former is precisely the derivative of the function $F_{max}^{i}(u)$ evaluated at 0, with a negative sign. However, the *RAI* presents knowledge of the rest of the function for $u \in (0\%, 100\%]\ or\ (0, 50]$ whereas a Lagrange multiplier only shows the initial trend of the curve when the capacity reduction is small; see Figure *4* for an example. In Figure *4*(a), the initial decrease of the curve $F_{max}^{i}(u)$ is small, indicating a small Lagrange multiplier. However, as the capacity reduces further, the maximum network flow drops drastically. In comparison, Figure *4*(b) shows a curve with steeper initial decrease, but which then stabilises for larger u. From this figure, we see that the Lagrange multiplier and the RAI may provide very different information regarding the importance of the subject node, despite their relationship illustrated above.

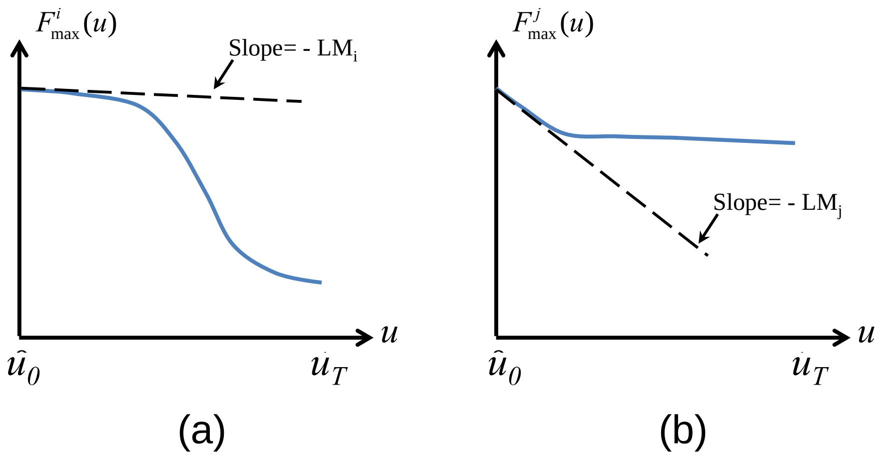

Figure 4. Relationship between the Lagrange multiplier and the RAI

We see from these simple examples that the *RAI* is a more comprehensive performance indicator for a node subject to capacity reductions than the Lagrange multiplier, as the former captures a whole range of capacity reductions. Moreover, the use of appropriately defined weighting parameters, shown in Eq. (2), makes the *RAI* quite flexible in addressing a target range of capacity reductions, which can be user defined.

5. Results and Analysis

In this section, we present results related to the *BC* and *RAI*, and provide some discussions on their managerial insights in the context of air traffic management on a network level. Since the focus and the underlying assumptions of *BC* and *RAI* are different, we compare *BC* to traffic load and *RAI* to nodal capacity. The rationale behind these comparisons is that nodes with higher BC tend to carry more traffic load since most flight routes follow the shortest paths; on the other hand, nodes with higher capacity are intuitively more important to the maximum flow of the entire network. In order to simplify our analysis and to distinguish between airports and airspaces, we extract the top ten airports and airspaces in each category (*BC* & *RAI*) and highlighting nodes that are significant according to both the indicators. Since the aggregated nodes are used to maintain the network structure and do not represent solid locations, we exclude them from the ranking list. The rankings of the airports and airspaces are then compared with the empirical data on air traffic load, nodal capacities, and ATFM delays.

In subsequent presentation, unless otherwise specified, we assign ID numbers 1- 67 to the 67 airport nodes, 68 - 131 to the 64 aggregate airport nodes, and 132 - 195 to the 64 ACC nodes.

5.1. Betweenness centrality (BC)

Figure *5* displays information of the *BC* in the entire network. Compared to the airport nodes (ID 1 - 131), the ACC nodes (ID 132 - 195) overall have larger *BCs*. An intuitive explanation is that the ACC nodes in the network can be considered as inner nodes while the airport nodes can be regarded as outer nodes (see Figure *3* for an illustration). Thus the inner nodes tend to have higher BCs, an observation consistent with the work of Barrat et al. (2004).

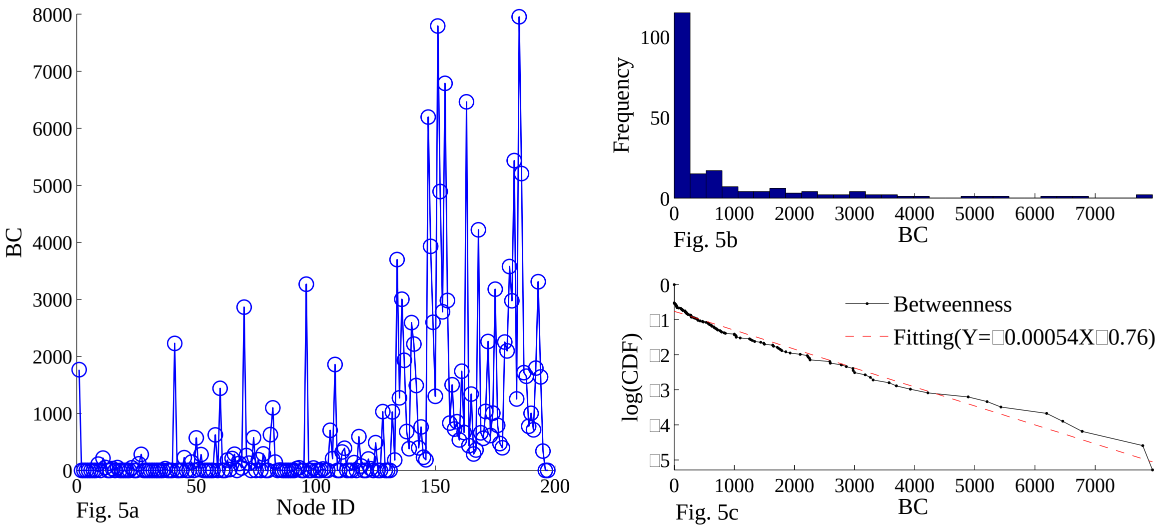

Figure 5. BC in the European air traffic network.

Figure *5*b and Figure *5*c show the histogram and the cumulative distribution function (CDF) of *BCs*, respectively. The CDF of the weighted *BC* can be fitted by an exponential function: $P(\geq b) \sim e^{-0.00054b}$. Such a fitting is a common approach in network science to quantify the robustness of a network based on topological indices such as the BC; the reader is referred to Sun et al. (2014) for a more elaborated discussion and more examples of such fitting for a variety of other air traffic networks.

The ten airports and airspace with the highest air traffic loads and BCs are listed in Table *1*. It is notable that airport nodes with high *BCs* do not necessarily have the highest traffic loads, and vice versa. The airports handling high traffic demands all locate on the capitals or economic centres rather than the high-*BC* airports. Therefore, the *BC* is not capable to capture the high-traffic airports and this results is consistent with Cats and Jenelius (2014) and Guimera and Amaral (2004).

Rank	Airport		Airspace (ACC)	
	BC	Traffic load	*BC*	Traffic load
1	VALENCIA	FRANKFURT	GENEVA	LONDON
2	BRUSSELS	PARIS CDG	BREMEN	MAASTRICHT
3	GENEVE	LONDON HEATHROW	**MUNICH**	KARLSRUHE
4	WIEN SCHWECHAT	SCHIPHOL AMSTERDAM	**ROME**	**MUNICH**
5	MAKEDONIA	MADRID BARAJAS	**MARSEILLE**	**MARSEILLE**
6	ISTANBUL SABIHA	MUENCHEN	MALMO	**ROME**
7	TRONDHEIM VAEMES	ISTANBUL ATATURK	ZURICH	LONDON TC
8	CATANIA FONTANAROSSA	ROME/FIUMICINO	**LANGEN**	**PARIS**
9	NICE	BARCELONA	AMSTERDAM	**LANGEN**
10	BIRMINGHAM	PALMA-DE-MALLORCA	**PARIS**	BREST

Table 1. Top ten airports and airspaces that have the highest *BCs* and traffic loads.

In terms of the *BCs* of airspace (ACC) nodes, five high-*BC* ACCs, namely Munich, Rome, Marsellie, Langen and Paris, also handle high air traffic. This result indicates that the *BC* is relatively more capable of capturing the important nodes with high traffic demands in airspace than at airports. However, the corresponding *BC*-rankings of the top three high-traffic ACCs, namely London, Maastricht and Karlsruhe are extremely low. Therefore, *BCs* cannot fully capture the traffic demands in the real world of operational traffic.

5.2. Relative area index (*RAI*)

According to our discussion of the *RAI*, the capacity of a node may be reduced by a certain percentage or by an absolute value. The resulting RAI_P and RAI_A, respectively, are presented and analysed in this section to examine the influence of capacity reductions on the network capacity. We use Spearman's rank correlation coefficient (r) and p-value (p) to measure the statistical dependence and the statistical significance between different sets of results. The detailed results of the *RAIs* and the *BCs*, as well as the empirical data, of a selection of nodes are presented in the Appendix.

5.2.1. Relative area index with capacity reduction by percentage (RAI_P)

In this section, we illustrate three sets of RAI_P over the entire European ATN in Figure *6*. These three sets of RAI_P are calculated by using three different weighting parameters that emphasize the capacity reduction at different levels. The first weighting parameter treats the importance of capacity reductions at all levels equally, and is indicated as 'Constant' in Figure *6*. The second and third weighting parameters assign a low (high) weight to low capacity reduction and a high (low) weight to high capacity reduction; in particular, they vary the weight from 0 to 10 and 10 to 0, respectively. Therefore, the second and third weight parameters emphasize the importance of higher and lower capacity reductions at each node, respectively.

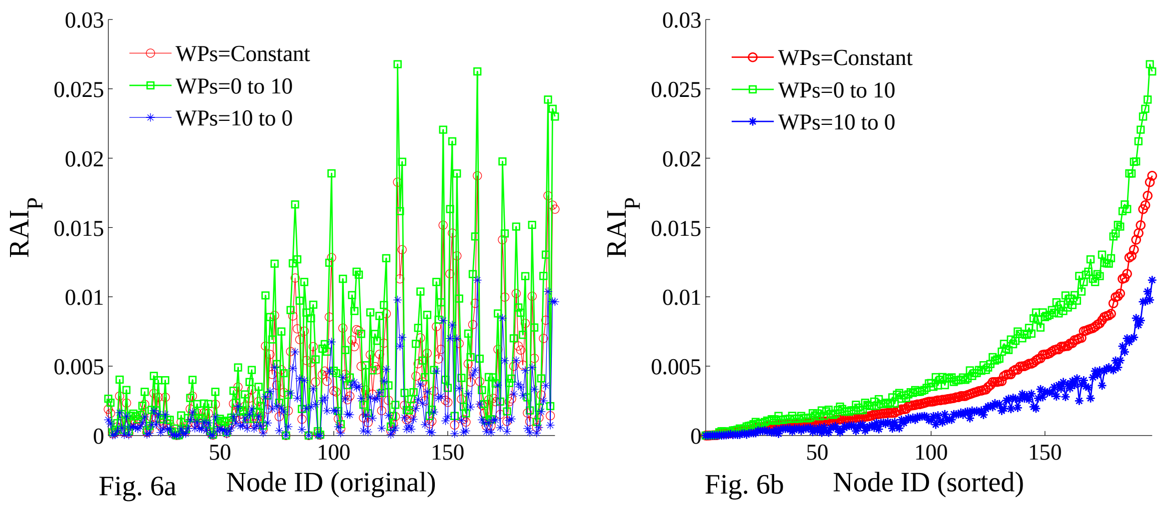

Figure 6. RAI_P in the European ATN. Fig. 6b is using new node IDs based on nodes sorted by their RAI_P (with constant weighting parameters).

Both the correlation coefficients and Spearman's rank correlation coefficients among these three sets of RAI_P shown in Figure *6* are above 0.97 and the *p*-values are all close to zero, which indicate that our results attain statistical significance. Moreover, the high Spearman's rank correlation coefficients show that the ranking of nodes based on RAI_P is insensitive to the change of the weighting parameters.

Figure *6*a also shows that the RAI_P of airports (ID 1 - 67) are generally smaller than the RAI_P of aggregated nodes and ACCs (ID 68 - 195). In view of the fact that airport nodes tend to have lower capacities, this result is in line with the anticipation that the influence of nodes with higher capacities is, in general, larger than those with lower capacities, since the capacity reduction is based on percentages.

Table *2* shows that there are five airports and five ACCs with both high RAI_P and high capacities, as appeared in the top ten; they are highlighted in bold. However, the RAI_P and capacity of some ACCs are counterintuitive. For instance, both the traffic load and capacity of London, Maastricht, Karlsruhe and Marseille are high but their RAI_P values are relatively low. In contrast, the RAI_P of Bucharest, Bremen, Madrid, Ankara/Istanbul, Belgrade are high while their capacities are comparatively low. These results imply that the importance of a given node in the presence of capacity reduction is not necessarily in line with its capacity.

Ranking	Airport		Airspace (ACC)	
	RAI_P	Capacity	RAI_P	Capacity
1	WIEN SCHWECHAT	PARIS CDG	**ROME**	LONDON
2	ISTANBUL ATATURK	**SCHIPHOL AMSTERDAM**	**PRESTWICK**	MAASTRICHT
3	**SCHIPHOL AMSTERDAM**	**KIEV BORISPOL**	**PARIS**	KARLSRUHE
4	**MADRID BARAJAS**	**MADRID BARAJAS**	**LANGEN**	**MUNICH**
5	**MUENCHEN 2**	FRANKFURT	BUCHAREST	LONDON TC
6	**COPENHAGEN KASTRUP**	**MUENCHEN 2**	**MUNICH**	**PARIS**
7	OSLO GARDERMOEN	ROME/FIUMICINO	BREMEN	**LANGEN**
8	ANTALYA	LONDON HEATHROW	MADRID	**ROME**
9	**KIEV BORISPOL**	STOCKHOLM ARLANDA	ANKARA/ISTANBUL	MARSEILLE
10	HELSINKI-VANTAA	**COPENHAGEN KASTRUP**	BELGRADE	**PRESTWICK**

Table 2. Top ten airports and airspaces that have the highest RAI_P and capacities.

5.2.2. Relative area index with capacity reduction by absolute value (RAI_A)

The RAI_A is calculated by applying capacity reductions to each node by a certain value (in flight per hour). We set the capacity reductions from 0 to 50 (flight per hour) to examine the influences of absolute-value capacity reductions on network capacity. Similar to the RAI_P case, three sets of weighting parameters are considered. The results of the RAI_A are shown in Figure 7. A high correlation among the three sets of RAI_A is observed, with all the correlation coefficients and Spearman's rank correlation coefficients above 0.99, and the p-values all close to zero. However, in contrast to RAI_P, no significant difference in the RAI_A exists between the airport nodes and the ACC nodes (see Figure 7a). This is partially due to the relatively small capacity reduction (by up to 50 flights per hour) such that most of the airports and ACCs are far from being bottlenecked. Thus the global effects of flow reduction induced by local degradation at airports or ACCs cannot be differentiated.

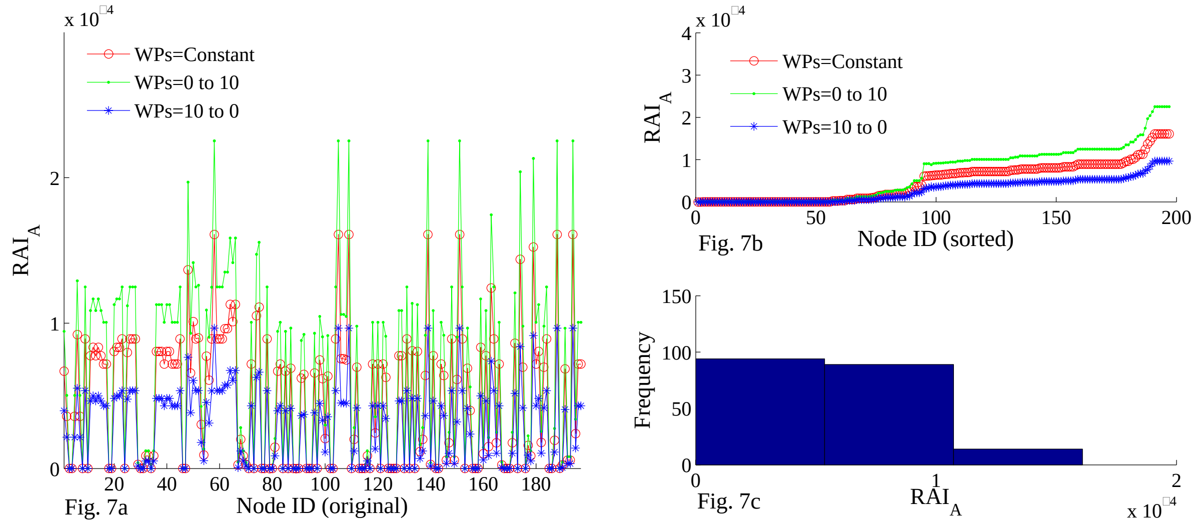

Figure 7. RAI_A in the European ATN. Fig. 7b is using new node IDs based on nodes sorted by their RAI_A (with constant weighting parameters).

An interesting result of the RAI_A is shown when we sort all the nodes in an ascending order with respect to RAI_A with constant weighting parameter (Figure 7b): The nodes can be intuitively clustered into three groups according to their RAI_A, namely low (ID 1 - 95), medium (ID 96 -180), and high (ID 181 - 195); see Figure 7b and Figure 7c.

Table 3 shows that the correlation between RAI_A and capacity is comparatively weak, with only a few nodes in common in the top ten. The influence of a minor absolute capacity reduction on the entire network is not in line with the magnitudes of the nodal capacities.

Ranking	Airport		Airspace (ACC)	
	RAI_A	Capacity	RAI_P	Capacity
1	WIEN SCHWECHAT	PARIS CDG	KYIV	LONDON
2	ATHINAI-E-VENIZELOS	SCHIPHOL AMSTERDAM	**PRESTWICK**	MAASTRICHT
3	ISTANBUL ATATURK	KIEV BORISPOL	NICOSIA	KARLSRUHE
4	ISTANBUL SABIHA	MADRID BARAJAS	BREMEN	MUNICH
5	MAKEDONIA	FRANKFURT	CANARIAS	LONDON TC
6	IZMIR ADNAN MENDERES	**MUENCHEN 2**	BUCHAREST	**PARIS**
7	ANTALYA	ROME/FIUMICINO	**ROME**	LANGEN
8	ANKARA ESENBOGA	LONDON HEATHROW	RIGA	**ROME**

9	**MUENCHEN 2**	STOCKHOLM ARLANDA	**PARIS**	MARSEILLE
10	CATANIA FONTANAROSSA	COPENHAGEN KASTRUP	STOCKHOLM	**PRESTWICK**

Table 3. Top ten airports and airspaces that have the highest RAI_A and capacities.

5.3. Comparison between *BC* and *RAI*

Given that the focus and the underlying assumptions of *BC* and *RAI* are different, it is difficult to conduct a direct comparison between these two robustness indices. Instead, we use relevant empirical data (ATFM delays) to examine the practical relevance of these two indices, and identify their strength and weakness.

Spearman's ranking correlation coefficient is used to reveal the relationships between the ranking of nodes based on the empirical data and the relevant robustness indices. Table 4 shows the correlation coefficients (r) among the six measures: the empirical ATFM delay, capacity, traffic load, RAI_P, RAI_A, and *BC*.

Measures		ATFM delay	Capacity	Traffic load	RAI_P	RAI_A	BC
ATFM delay	r	1.00	-0.44	0.23	-0.31	0.31	-0.12
	p	1.00	0.00	0.00	0.00	0.00	0.10
Capacity	r	-0.44	1.00	**0.47**	**0.72**	-0.24	0.37
	p	0.00	1.00	0.00	0.00	0.00	0.00
Traffic load	r	0.23	**0.47**	1.00	**0.51**	0.19	**0.51**
	p	0.00	0.00	1.00	0.00	0.01	0.00
RAI_P	r	-0.31	**0.72**	**0.51**	1.00	0.31	0.37
	p	0.00	0.00	0.00	1.00	0.00	0.00
RAI_A	r	0.31	-0.24	0.19	0.31	1.00	0.13
	p	0.00	0.00	0.01	0.00	1.00	0.06
BC	r	-0.12	0.37	0.51	0.37	0.13	1.00
	p	0.10	0.00	0.00	0.00	0.06	1.00

Table 4. Spearman's ranking correlation coefficients and the p-values among relevant indices and empirical data (ATFM delay)

Spearman's ranking correlation coefficient r between RAI_P and the traffic load is comparable with that between *BC* and the traffic load (0.51). Secondly, the ranking of the RAI_P is strongly correlated to the ranking of the nodal capacity (0.72), showing RAI_P to be an index more promising than *BC* to capture the nodal capacity. Thirdly, RAI_A has no meaningful correlation with any of the other indices. This indicates that the influence of a minor, absolute capacity reduction of a given node is not significantly related to traffic load, nodal capacity, or ATFM delays. However, we expect that when the absolute capacity reduction gets larger, say for a subset of the nodes that have large capacities, RAI_A is likely to provide a more meaningful characterization of the importance of nodes.

In Table 4, neither the *RAI* nor the *BC* captures the ATFM delays. This is explained by the fact that the ATFM delay is a result of a complex operational environment, involving multiple sectors and stakeholders; thus more sophisticated models are required to capture the ATFM delays in their entirety.

In order to further compare *RAI* and *BC*, we select the top 25 nodes with the highest ATFM delays [1], which are subsequently referred to as *bottlenecks*, and conduct a similar analysis restricted to these 25 bottlenecks. The findings are presented in Table 5. Here, BC again provides a poor performance with low Spearman's ranking correlation coefficient and high p-values. An important finding is that Spearman's ranking correlation coefficient

[1] These 25 nodes include 21 airports and 4 ACCs, with ATFM delays greater than the mean (3.9 minutes); see the Appendix for more information.

between RAI_A and RAI_P increases from 0.31 to 0.72, which means that the capacity reductions by percentage or absolute value have similar effects for these 25 nodes. In addition, both RAI_A and RAI_P better capture the ATFM delays at these bottlenecks, with $r = 0.44$ and $r = 0.40$, respectively.

Measures		ATFM delay	Capacity	Traffic load	RAI_P	RAI_A	BC
ATFM delay	r	1.00	0.06	0.23	**0.44**	**0.40**	0.19
	p	1.00	0.78	0.28	**0.03**	**0.05**	0.36
Capacity	r	0.06	1.00	0.82	**0.48**	-0.03	0.22
	p	0.78	1.00	0.00	**0.02**	0.90	0.28
Traffic load	r	0.23	0.82	1.00	**0.61**	0.16	0.30
	p	0.28	0.00	1.00	**0.00**	0.45	0.15
RAI_P	r	**0.44**	**0.48**	**0.61**	1.00	**0.72**	0.38
	p	**0.03**	**0.02**	**0.00**	1.00	**0.00**	0.06
RAI_A	r	**0.40**	-0.03	0.16	**0.72**	1.00	0.23
	p	**0.05**	0.90	0.45	**0.00**	1.00	0.27
BC	r	0.19	0.22	0.30	0.38	0.23	1.00
	p	0.36	0.28	0.15	0.06	0.27	1.00

Table 5. Spearman's ranking correlation coefficients and the p-values among relevant indices and empirical data (ATFM delay), among the 25 high-ATFM-delay nodes.

6. Discussion

The commonly employed network robustness index, namely betweenness centrality (*BC*), reflects only the topological characteristics of the network, without taking into account traffic demand and nodal capacities. As a result, it can only capture traffic loads rather than capacity or ATFM delays, as we show in our results for the European air traffic network. Using *BC* as the robustness index of an ATN is quite limited in capturing the influence of local disruption on the network level, especially when the operational characteristics are within the purview of network operators.

Our finding that the *BC* is capable of capturing the actual traffic load at a particular node differs from that of Cats and Jenelius (2014), who found limited correlation between the passenger loads and *BCs* in a road network. We argue that this difference is caused by the different nature of road and air traffic. Compared to road users who are free to minimize their cost of travel by selecting alternative routes in a networks, aircrafts do not have the freedom to select alternative routes; instead, they fly along the routes in given flight plans and follow the guidance of air traffic control (ATC). In addition, the delay of air traffic often occurs at the departing airport as a result of air traffic management, while congestion and delays of road traffic take place en route. These differences imply that the *BC* may be an adequate indicator for air traffic volume in an ATN, since the shortest distance is an important factor in the design of flight plans.

The proposed robustness index (*RAI*) is more capable than the *BC* of capturing the importance of a given node in the event of capacity reduction, by considering traffic demands, actual flight paths, and nodal capacities, in addition to the topological features. It also encapsulates a range of scenarios involving different levels of capacity reductions, instead of simply removing a node or a link, which is typical in topological analysis leading up to BC and other indices. The concept and formulation of *RAI* is flexible enough to accommodate a wide range of cases involving different nature and severity of the capacity reduction. We first adopt the percentage-based capacity reduction in combination with a network flow maximisation technique for assessing the theoretical network capacity. We find that the RAI_P ranking at the aggregated nodes and ACCs are generally higher than those at the airport nodes. Although the use of three sets of weighting parameters results in different rankings of nodes, the difference is extremely small. However, we note that the importance of the weighting parameters should be re-evaluated by introducing more sophisticated modelling elements, such as dynamic network modelling and routing and scheduling.

In addition to the percentage-based capacity reductions, we also applied the absolute capacity reduction to calculate RAI_A. This approach enables us to assess the influence of a certain event that causes absolute capacity reductions at given nodes. Unlike the RAI_P ranking, the result shows that the RAI_A ranking do not favour the high-capacity nodes such as the ACCs and aggregated nodes. In addition to the ranking, the RAI_A of the nodes in the European ATN can be intuitively categorized into three groups: High-, Medium- and Low-RAI_A nodes. A particular event such as sector- or runway-closure at the nodes in the High-RAI_A group will cause a greater impact on network capacity than if that event happens at the nodes in the Medium- and Low-RAI_A groups. This functionality provides the network management unit with a powerful tool to group and rank the critical nodes, not solely using the empirical delay data that may contain considerable inaccuracies. Compared to the BC, using the *RAI* is more flexible to assess network robustness (see RAI_P in Table *4* and RAI_A in Figure *7*). Therefore, the proposed new index has the potential to be used to reflect the ranking of the constituent nodes in an ATN and to assess network robustness. Table *5* shows the superiority of *RAI* over *BC* at the 25 main bottlenecks.

There are potentially four extensions of this paper for future research. First, the formulation of the conventional index, *BC* can be improved to accommodate the features of air traffic, including traffic demands, flight routes and the heterogeneity between the airport and airspace nodes. Second, since the European ATN is not saturated and the capacity of each node varies dynamically, there is a need to capture network flows dynamically by introducing dynamic capacity constraints and flight times. Third, both the $RAI_A$and RAI_P require real data on the influence of capacity fluctuation on network capacity for their validation. This validation would enable the superiority of the *RAI* when compared to the current KPI of ATFM delays to be evaluated. Finally, unexpected events that occur in real-time, such as large-scale meteorological events and industrial action, may reduce the capacity at multiple airports and en-route airspace. Hence, any future analysis would benefit from an evaluation of the influence of capacity-reductions at multiple nodes in the network simultaneously, rather than at just a single node.

7. Conclusion

This paper proposes a new index, the *RAI*, to assess the robustness of the European ATN by calculating the influence of nodal capacity-reductions on network capacity. Using data from three of the busiest air traffic days in Europe in 2012, the *RAI* was assessed along topological index, the *BC*, as potential indicators for robustness. The results indicate that the *RAI* is better able to capture the importance of each node by taking into account not only the topological features, but also the traffic demands and the nodal capacities.

There are several potential operational applications of the *RAI* to air traffic management. Compared to ATFM delay that is the current indicator of network capacity and of any bottlenecks in the network, the *RAI* provides a detailed ranking of the nodes in the European ATN from the standpoint of any potential local degradation of capacity and its consequent impact on the overall network. Such a consideration has not been addressed by the ATFM delay, the *BC* or any other existing network performance indices for air traffic networks. *RAI* therefore provides a potentially powerful tool for the European ATM unit to identify and categorize the critical nodes in the network. This in turn can aid in improving network management and resource allocations, by identifying nodes with higher ‘marginal benefits’.

Given the expected rise in air traffic demand in Europe in the coming years, SESAR is effectively revolutionising the nature of air traffic operations and their management in Europe. However, as EUROCONTROL noted in their "Challenges for growth 2013" (EUROCONTROL, 2013a), there is an urgent need to understand network performance in the future European air traffic network and then to have appropriate metrics for this performance, to a far greater degree of sophistication than the current ATFM delays. Given this need, we recommend the use of the *RAI* methodology for the development of the future of European network performance indicators.

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Appendix

The table below is concerned with the top 25 nodes in the European Air Traffic Network in terms of ATFM delays. Other performance indices of interest, including capacity, traffic load, *RAI* and *BC* are computed and presented for these 25 nodes, along with their rankings in each of the categories.

Nodes		Type	ID	ATFM delays		Capacity		Traffic		RAI				Betweenness	
				min/flt	**ranking**	flt/day	**ranking**	flt/day	**ranking**	RAI_P	**ranking**	RAI_A	**ranking**	BC	**ranking**
1	BARCELONA	Airport	LEBL	18.74	1	1056	9	926	6	0.00191	6	0.00008	8	30	9
2	MUENCHEN 2	Airport	EDDM	12.32	2	1440	4	1060	4	0.00283	3	0.000092	2	0	11
3	PALMA-DE-MALLORCA	Airport	LEPA	11.78	3	992	10	836	7	0.00162	8	0.00008	9	0	11
4	WARSZAWA OKECIE	Airport	EPWA	8.74	4	640	18	397	17	0.00016	24	0.000003	21	0	11
5	ALICANTE	Airport	LEAL	8.48	5	480	22	228	24	0.00088	14	0.00008	10	0	11
6	VALENCIA	Airport	LEVC	8.24	6	480	23	174	25	0.00088	15	0.00008	11	2226	3
7	BARCELONA	ACC	LECBCTA	7.78	7	3288	2	2893	2	0.00264	4	0.000009	19	397	5
8	NICOSIA	ACC	LCCCCTA	7.66	8	1200	6	976	5	0.00386	2	0.000161	1	380	6
9	ZURICH	Airport	LSZH	6.51	9	1152	7	788	9	0.00211	5	0.000089	3	0	11
10	GENEVE COINTRIN	Airport	LSGG	6.35	10	640	19	524	13	0.00126	12	0.000089	4	1439	4
11	NICE	Airport	LFMN	5.52	11	832	12	625	10	0.00159	9	0.000089	5	222	7
12	HERAKLION	Airport	LGIR	5.52	12	352	25	261	21	0.0006	21	0.000066	17	0	11
13	DUESSELDORF	Airport	EDDL	5.39	13	720	17	587	11	0.00061	20	0.000036	18	0	11
14	PALMA	ACC	LECPCTA	5.17	14	2208	3	1356	3	0.00646	1	0.00008	12	3575	1
15	BIRMINGHAM	Airport	EGBB	4.88	15	640	20	248	23	0.00101	13	0.000078	14	215	8
16	LONDON STANSTED	Airport	EGSS	4.68	16	800	14	411	16	0.00132	11	0.00008	13	0	11
17	MALAGA	Airport	LEMG	4.59	17	560	21	416	15	0.00088	16	0.000072	15	0	11
18	KARLSRUHE	ACC	EDMMCTA	4.5	18	7176	1	4301	1	0.00076	17	0	22	2782	2
19	PRAHA RUZYNE	Airport	LKPR	4.41	19	736	16	376	18	0.00163	7	0.000089	6	0	11
20	KOELN-BONN	Airport	EDDK	4.3	20	1280	5	288	20	0.0002	23	0	22	0	11
21	MILANO MALPENSA	Airport	LIMC	4.27	21	1120	8	562	12	0.00068	19	0.000009	20	0	11
22	HAMBURG	Airport	EDDH	4.18	22	768	15	372	19	0.00013	25	0	22	0	11
23	FERIHEGY BUDAPEST	Airport	LHBP	4.17	23	384	24	259	22	0.00074	18	0.000089	7	15	10
24	LONDON GATWICK	Airport	EGKK	4.06	24	960	11	797	8	0.00152	10	0.000072	16	0	11
25	TEGEL-BERLIN	Airport	EDDT	4.04	25	832	13	427	14	0.00023	22	0	22	0	11