Opportunistic Multiuser Two-Way Amplify and Forward Relaying with a Multi Antenna Relay

Duckdong Hwang*, Bruno Clerckx**, Sung Sik Nam*** and Tae-Jin Lee*†

Abstract

We consider the opportunistic multiuser diversity in the multiuser two-way amplify and forward (AF) relay channel. The relay, equipped with multiple antennas and a simple zero-forcing beam-forming (ZFBF) scheme, selects a set of two way relaying user pairs to enhance the degree of freedom (DoF) and consequently the sum throughput of the system. The proposed channel aligned pair scheduling (CAPS) algorithm reduces the inter-pair interference and keeps the signal to interference plus noise power ratio (SINR) of user pairs interference free when the number of user pairs becomes very large. When the number of user pairs grows fast enough with the system signal to noise ratio (SNR), a DoF equal to the number of relay antennas can be achieved. For a realistic number of user pairs, we propose an adaptive CAPS and an adaptive semi-orthogonal CAPS (SCAPS) to improve the performance. Simulation results show that adaptive CAPS and adaptive SCAPS provide throughput gain in the low to mid SNR region.

Index Terms

Two-way relaying, opportunistic user pair selection, amplify and forward, degree of freedom.

I. INTRODUCTION

Two-way amplify-and-forward (AF) relaying [?], [?] is an attractive technique to enhance the spectral efficiency of AF relaying system, where a pair of users exchanges bidirectional

* Authors are with College of Information and Communication Engineering, Sungkyunkwan University, Suwon, Korea. Email: duckdonh@yahoo.com

** Author is with Electrical and Electronic Engineering, Imperial College, London, United Kingdom and with School of Electrical Engineering, Korea University, Korea. Email: b.clerckx@imperial.ac.uk

** Author is with Hanyang University, Seoul, Korea. Email: ssnam@hanyang.ac.kr

† The corresponding author. Email: tjlee@skku.edu

messages in two phases. The two transmissions from the users overlap in the first phase and the AF relay simply broadcasts the received signal toward the two users in the second phase. Each user subtracts out the reflected self-interference and can decode the message signal from the other user. Space division multiple access (SDMA) techniques at the multiple antenna AF relay enable a set of user pairs to exchange the two-way traffics using the same spectral resource [?], [?], [?], [?]. Since the reflected self-interference of a user can be subtracted out, the handling of the inter-pair interference is the key challenge. A base station and a set of users form a two-way traffic through a multi antenna relay in [?]. Here, the inter-pair interference is jointly handled by the base station and the relay and the achievable degree of freedom (DoF) is the minimum of the numbers of the base station antennas and the relay antennas. Multiple pairs of two-way users through a multi antenna relay are considered in [?], [?], [?], [?]. When users have a single antenna, the relay only handles the inter-pair interference [?], [?], [?] and a DoF up to the integer floor of $\frac{M+1}{2}$ is achievable. Alternatively, the relay and the users collaborate to suppress the inter-pair interference [?] with multi-antenna relay and users, where a DoF of M is achieved when M is the number of relay antennas, N is the number of user antennas and they satisfy $N \geq (M + 1)/2$.

When there are multiple users as in the cellular network, the independent fading of user channels can be exploited to provide the system with various performance gains [?], [?], [?]. This opportunistic multiuser diversity is utilized to schedule semi-orthogonal user channels in the conventional multiuser multiple input multiple output (Mu-MIMO) zero forcing beam-forming (ZFBF) system [?]. Also, it can be used for interference alignment in the cellular networks [?] or in the interference channels [?]. For the two-way relay channel with single antenna users and an AF relay with M antennas, we propose an opportunistic channel aligned pair scheduling (CAPS) scheme and the DoF is guaranteed to be M (M pairs of two-way traffic can be served in two phases) if the number of pair users (K) scales fast enough according to the signal to noise ratio (SNR). The DoF is improved compared to those of [?], [?] but remains the same as the one in [?]. Instead, the requirement for multiple antenna users is replaced by the opportunistic multiuser diversity from a large number of user pairs compared to the schemes in [?]. For realistic values of K, we propose an adaptive version of CAPS where the number of scheduled user pairs is adapted depending on the channel realizations. A semi-orthogonal scheduling as in [?] can be embedded into the CAPS and the resulting semi-orthogonal channel aligned scheduling (SCAPS) further

enhances the system sum rate performance in the low-to-mid SNR regime when the adaptive version is applied to finite K cases.

The paper is organized as follows. The system model appears in Section II. The presentation of CAPS algorithm and its properties appear in Section III. Two adaptive scheduling schemes, adaptive CAPS and adaptive SCAPS, are introduced in Section IV with numerical results. Section V concludes the paper. *Notations:* The bold lower case letter represents a vector and the bold upper case letter represents a matrix. $E[a]$ denotes the average of a random variable a. The notations $\mathbf{A}^T, \mathbf{A}^H, \mathbf{A}^\dagger$ and $Tr[\mathbf{A}]$ are the transpose, the Hermitian transpose, the pseudo inverse and the trace of a matrix $\mathbf{A}$, respectively. $\mathbf{A}^\perp$ and $\|\mathbf{a}\|$ denote the projection onto the space orthogonal to the columns of $\mathbf{A}$ and the norm of a vector $\mathbf{a}$, respectively. $|\mathcal{A}|$ denotes the cardinality of a set $\mathcal{A}$. $\mathbf{I}_k$ denotes the identity matrix with $k \times k$ dimensions. $\mathcal{CN}(\mathbf{0}, \mathbf{C})$ denotes the complex white Gaussian random vector with zero mean vector $\mathbf{0}$ and the covariance matrix $\mathbf{C}$. The integer floor function $\lfloor a \rfloor$ returns the largest integer less than or equal to a.

II. SYSTEM MODEL

In Fig. 1, the multiuser two-way relay channel is depicted, where the half-duplex AF relay has M $(M \geq 2)$ antennas and the $2K$ single antenna user terminals make K two way pairs, where two users in a pair exchange bidirectional information through the relay. The $M \times 1$ channel vector between the user i and the relay is denoted by $\mathbf{h}_i$. The elements of these channel vectors are independent and identically distributed (*i.i.d.*) $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. We assume that the i-th user $(i = 1, \ldots, K)$ is paired with the $(i+K)$-th user without loss of generality. Each user sends a pilot signal so that the relay can learn the channels for all users ($\mathbf{h}_i, \ i = 1, \ldots, 2K$), based on which m $(m \leq M)$ two way pairs are selected by the relay.

The transmission of two-way relaying is composed of two phases. The $2m$ users in the selected pairs transmit their messages toward the relay in the first phase and the relay broadcasts the beam-formed signal toward the $2m$ users in the second phase. All the channel vectors do not change during the two transmission phases. The i-th user sends the message symbol $x_i, \ (E[|x_i|^2] = P_s)$ through the antenna in the first phase. The received signal at the relay is given as

$$\mathbf{y}_r = \sum_{i=1}^{m} (\mathbf{h}_i x_i + \mathbf{h}_{i+K} x_{i+K}) + \mathbf{n}_r, \tag{1}$$

$$y_j = \mathbf{h}_j^T \mathbf{W}_r [\mathbf{h}_{j+K} x_{j+K} + \sum_{i \neq j}^{m} (\mathbf{h}_i x_i + \mathbf{h}_{i+K} x_{i+K}) + \mathbf{n}_r] + n_j$$

$$= \mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_{j+K} x_{j+K} + \mathcal{I}_j + n_{r,j} + n_j, \tag{3}$$

where the $M \times 1$ vector $\mathbf{n}_r$ is $\mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. The relay applies a $M \times M$ beam-former $\mathbf{W}_r$ to the received signal (1) and transmits the product vector $\mathbf{W}_r \mathbf{y}_r$ in the second phase. Then the signal received at the j-th user in the second phase is given as

$$y_j = \mathbf{h}_j^T \mathbf{W}_r \mathbf{y}_r + n_j$$

$$= \mathbf{h}_j^T \mathbf{W}_r [\sum_{i=1}^{m} (\mathbf{h}_i x_i + \mathbf{h}_{i+K} x_{i+K}) + \mathbf{n}_r] + n_j, \tag{2}$$

where n_j is $\mathcal{CN}(0,1)$ again. If $\mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_j$ is known to the j-th user from the embedded pilots, it can subtract out the self-interference term $\mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_j x_j$. Then, (2) becomes as (3), where $n_{r,j} = \mathbf{h}_j^T \mathbf{W}_r \mathbf{n}_r$ and the inter-pair interference term $\mathcal{I}_j = \mathbf{h}_j^T \mathbf{W}_r \sum_{i \neq j}^{m} (\mathbf{h}_i x_i + \mathbf{h}_{i+K} x_{i+K})$. The signal to interference plus noise power ratio (SINR) of the j-th user is given as

$$SINR_j = \frac{P_s |\mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_{j+K}|^2}{|\mathcal{I}_j|^2 + \|\mathbf{W}_r^H \mathbf{h}_j^*\|^2 + 1}. \tag{4}$$

A. Relay beamformer Design

To limit the transmit power at the relay, the relay beamformer $\mathbf{W}_r$ should meet

$$Tr[P_s \mathbf{W}_r \mathbf{H} \mathbf{H}^H \mathbf{W}_r^H + \mathbf{W}_r \mathbf{W}_r^H] = P_r, \tag{5}$$

where the columns of $\mathbf{H}$ are $2m$ channel vectors ($\mathbf{h}_j$) of the selected user pairs and P_r is the relay power constraint. Let β be the power control parameter so that Eq. (5) is to be satisfied, then we have $\mathbf{W}_r = \beta \tilde{\mathbf{W}}_r^H \tilde{\mathbf{W}}_r$, where the j-th row of the $m \times M$ matrix $\tilde{\mathbf{W}}_r$ is denoted as $1 \times M$ vector $\mathbf{w}_j$, ($\|\mathbf{w}_j\| = 1$) and has the property that the angular distance toward the user channels of the other pairs are bounded. Also, let the selected user pair set $\mathcal{S} = \{1, \ldots, m, K+1, \ldots, K+m\}$. Mathematically, we can write the property of the j-th row as

$$\frac{|\mathbf{w}_j \mathbf{h}_k|}{\|\mathbf{h}_k\|} \leq \delta, \ k \in \mathcal{S}_j, \tag{6}$$

where $\mathcal{S}_j = \mathcal{S} \setminus \{j, j+K\}$. When δ can be made equal to zero, the inter-pair interference can be forced to zero by the relay beamformer $\mathbf{W}_r$. It is well known that the beamformer with M antennas has the capability to suppress the interference among the M channel vectors by satisfying (6). If $m(> M/2)$ two-way pairs are scheduled, there are more than M channel vectors from these overloaded user pairs and $\mathbf{W}_r$ suffers from handling the inter-pair interferences (δ in (6) becomes large). By aligning the channels within a user pair, we can keep δ small enough and thus maintain the inter-pair interference within a certain level.

With the aid of the property in (6), the following Lemma 1, the proof of which is provided in Appendix-A, shows that the inter-pair interference power can be bounded if $m \leq M$.

Lemma 1: As long as $m \leq M$, the inter-pair interference power is bounded as $|\mathcal{I}_j|^2/P_s \leq (\beta m)^2 \delta^2 \|\mathbf{h}_j\|^2 \sum_{k \in \mathcal{S}_j} \|\mathbf{h}_k\|^2$.

Similarly, we can show that the relay noise power term delivered to the j-th user receiver $\|\mathbf{W}_r^H \mathbf{h}_j^*\|^2$ is bounded as $\|\mathbf{W}_r^H \mathbf{h}_j^*\|^2 \leq \beta^2 \|\mathbf{h}_j\|^2 (1 + (m-1)\delta)^2$. Once the j-th user pair beamformers ($\mathbf{w}_j$) satisfy the property in (6), the power of the two hop channel $\mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_{j+K}$ is lower bounded as $|\mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_{j+K}|^2 \geq \beta^2 |\mathbf{h}_j^T \mathbf{w}_j^H \mathbf{w}_j \mathbf{h}_{j+K}|^2$. Therefore, a reasonable choice of $\mathbf{w}_j$ is to maximize $|\mathbf{h}_j^T \mathbf{w}_j^H \mathbf{w}_j \mathbf{h}_{j+K}|$ within the constraint given in (6).

III. CHANNEL ALIGNED PAIR SCHEDULING

In this section, we utilize the opportunistic diversity from a large number of user channels to align the channels of users in a pair so that the well known beamformers like ZFBF, applied at the relay, can handle the inter-pair interference easily by satisfying (6). The CAPS algorithm in Table I picks up $m (\leq M)$ user pairs whose channels within a pair are mostly aligned. Starting from the largest correlation, reorder the selected pairs in decreasing order. Let $i = \varphi(k)$ denote this reordering, where i runs from 1 to m. Find the mean direction vector[1] between the two vectors in each selected pair and construct the relay precoder $\mathbf{W}_r$ based on these mean direction vectors. From the SINR expression in (4), it is easy to see that the inter-pair interference power $|\mathcal{I}_j|^2$ becomes the bottleneck to the j-th user throughput at high SNR if the alignment of the channels within a pair is imperfect so that the relay beamformer $\mathbf{W}_r$ fails to reduce this quantity.

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[1]The mean direction vector is the one that halves the angle between two vectors. Simple interference power analysis with trigonometry reveals that, if we pick vectors, each of them is a vector from a pair vectors, and form a ZFBF from these vectors, we have about twice more interference than the mean direction vector approach. Note that (6) is satisfied by ZFBF.

$$f_\mu(\mu) = m(M-1)\binom{K}{m}\sum_{n=0}^{K-m}\binom{K-m}{n}(1-\mu)^{(M-1)(K-n)-1}(-1)^{(K-m-n)}. \qquad (7)$$

Therefore, we will see in this section that the CAPS algorithm reduces this interference power and achieves a DoF of M through the opportunistic multiuser diversity if K goes to the infinity.

First, we are interested in the distribution of the worst case correlation $\nu_{\varphi^{-1}(m)}$ since we will use the upper bound of the interference power. Lemma 2, proved in Appendix-B, presents the probability density function (pdf) of $\nu_{\varphi^{-1}(m)}$ selected by the CAPS algorithm.

Lemma 2: The pdf of $\mu = \nu_{\varphi^{-1}(m)}^2$ is given as in (7).

Let us define $\theta_k = \cos^{-1}\nu_k$, the angle between the two channel vectors of the k-th pair. Since the relay beamformer $\tilde{\mathbf{W}}_r$ of CAPS is constructed to zero-force the mean channel vectors of other pairs, the δ in (6) is determined by $\delta^2 = \sin^2\theta_k/2$. Using the trigonometric identity, we can define $\varrho(K,M,m) = E[\delta^2] = \frac{1-E[\mu]}{2}$. Proposition 1 provides the convergence behavior of $\varrho(K,M,m)$ with a large K.

Proposition 1: In a large K, $\varrho(K,M,m)$ converges to

$$\lim_{K\to\infty}\varrho(K,M,m) = \lim_{K\to\infty}\frac{\Gamma\left(m+\frac{1}{M-1}\right)}{2\Gamma(m)}\frac{1}{\sqrt[M-1]{K+1}} = 0. \qquad (8)$$

Proof: See Appendix-C. ∎

Here, $\frac{1}{\sqrt[M-1]{K+1}}$ determines the convergence speed while $\frac{\Gamma\left(m+\frac{1}{M-1}\right)}{2\Gamma(m)}$ decides the overall scale of $\varrho(K,M,m)$. The convergence speed slows down as M increases while the scale increases with m. This results suggest that opportunistic multiuser diversity provides an opportunity to schedule more than $M/2$ pairs at the same time when K is sufficiently large. For a large M, the convergence of $\varrho(K,M,m)$ becomes slow. In this case, we can lower the scale by taking a small m (schedule less pairs) since the overall scale $\frac{\Gamma\left(m+\frac{1}{M-1}\right)}{2\Gamma(m)}$ decreases with a smaller m.

Second, the upper bound of the inter-pair interference power $|\mathcal{I}_j|^2$, the CAPS algorithm produces, is presented in Lemma 3.

Lemma 3: With the CAPS algorithm, the inter-pair interference power $|\mathcal{I}_j|^2$ can be upper bounded in large Ks as in (9).

$$|\mathcal{I}_j|^2 \le (\beta m)^2 P_s \frac{\Gamma\left(m+\frac{1}{M-1}\right)}{2\Gamma(m)}\frac{1}{\sqrt[M-1]{K+1}}\|\mathbf{h}_j\|^2\sum_{k\in\mathcal{S}_j}\|\mathbf{h}_k\|^2. \qquad (9)$$

Proof: Combining the results of Lemma 1 and Lemma 2 by inserting $\varrho(K, M, m)$ into δ^2 in (11), we can arrive at the upper bound in (9). ∎

Finally, Lemma 4 with the proof in Appendix-D gives the DoF convergence property of CAPS.

Lemma 4: The CAPS algorithm can achieve the DoF of M as K goes to the infinity.

However, the DoF result of Lemma 4 should not be over stressed since the property holds in the infinite K and the convergence speed of the interference power becomes slower as M increases. For a finite K, $|\mathcal{I}_j|^2$ cannot be nulled out so that the CAPS suffers from ceiling effect in the high SNR region though a smaller m or a large K raises the ceiling upward. Hence, the DoF values more than $M/2$ can be hardly achieved for a finite K. In the following sections, we provide adaptive scheduling approaches for realistic K, which enhance the low-to-mid SNR performance rather than the DoF.

IV. ADAPTIVE (S)CAPS ALGORITHMS AND NUMERICAL RESULTS

When K is finite and thus it is hard to reduce $\varrho(K, M, m)$. In this section, we will show how to implement the idea of CAPS in realistic K cases by, first, decreasing the scale of $\varrho(K, M, m)$ (choosing $m(< M)$ pairs) and, second, embedding the semi-orthogonal scheduling of [**?**]. If the CAPS chooses only $m(< M)$ pairs, the relay ZFBF has a room to handle additional interference of up to $M - m$ dimensions if we assume that the aligned pair channel vectors take up only m spatial dimensions. Therefore, we can modify the CAPS by scheduling additional J arbitrary pairs at the same time, where $J \leq \lfloor (M - m)/2 \rfloor$. The total number of channels in the system is given as $2(m + J)$. Then, the total spatial dimensions of the channel vectors that the relay ZFBF deals with becomes $m + 2J \ (\leq M)$. The ZFBF handles weaker inter-pair interference than the M pair scheduling case and a better sum rate is expected at low to mid SNR due to the additional J pair channels. The relay beam-former is constructed as follows

$$\mathbf{G} = [\mathbf{h}_1, \ldots, \mathbf{h}_J, \mathbf{h}_{K+1} \ldots, \mathbf{h}_{K+J}, \acute{\mathbf{h}}_{\varphi^{-1}(1)}, \ldots, \acute{\mathbf{h}}_{\varphi^{-1}(m)}]$$
$$\mathbf{W}_r = \beta \tilde{\mathbf{W}}_r^H \tilde{\mathbf{W}}_r, \ \tilde{\mathbf{W}}_r = \rho \mathbf{G}^\dagger. \tag{10}$$

Here, $\mathbf{h}_j, \mathbf{h}_{K+j}, \ (j = 1, \ldots, J)$ denote the channel vectors of additionally scheduled pairs and $\acute{\mathbf{h}}_{\varphi^{-1}(1)}, \ldots, \acute{\mathbf{h}}_{\varphi^{-1}(m)}$ are the mean vectors of the pair channels selected by CAPS. Now for each channel realization, the CAPS is given a choice between different combinations of m and J for the best sum rate performance. We call this scheduling approach as adaptive CAPS.

We define the system sum rate as

$$R = \sum_{k=1}^{m+J/2} \frac{1}{2} \left[\log_2(1 + SINR_k) + \log_2(1 + SINR_{k+K}) \right].$$

The *i.i.d.* $\mathcal{CN}(\mathbf{0}, \mathbf{I})$ distributed channel vectors of K user pairs are generated so that the CAPS algorithm can select m pairs and form $\mathbf{W}_r$. The user terminals are assumed to be the same distance apart from the relay and use the same power (P_s) while the relay power is $P_r = P_s$, which includes the path-loss effect. Figure 2 compares the system sum rate of adaptive CAPS scheme with that of ZFBF scheme without scheduling. Also, plotted are the system sum rates of three combinations (m, J) of modified CAPS. As we increases m the system sum rate saturates faster at high SNR while the gain in the low SNR region increases. The adaptive CAPS harvests the benefits of the modified CAPS schemes throughout the SNR region, though most of the gain is observed in the low SNR region.

For a finite K, the gain from opportunistic channel alignment is limited. Further improvement in this case is expected if we embed the semi-orthogonal channel selection of [?] into CAPS. The SCAPS algorithm summarized in Table II first selects a set of user pairs whose pair channel alignments are greater than a threshold. Then, it sequentially chooses pairs, the minimum magnitude of the pair channel vectors after the projection onto the space of $\mathbf{H}_S^\perp$ is the strongest, where the columns of $\mathbf{H}_S$ are composed of the already selected pair channel vectors.[2] It is better to make ϵ small to keep the channels of a pair well aligned, which forces the cardinality of the set $\mathcal{S}_0$ to be small as well. On the other hand, $|\mathcal{S}_0|$ needs to be large enough to reap the benefit of semi-orthogonal channel scheduling. Similarly with adaptive CAPS, the adaptive scheduling through the modification of SCAPS is implemented for a practical K and we name this scheduling as adaptive SCAPS. In Fig. 3(a) and Fig. 3(b), the system sum rates of SCAPS are compared for $M = 2$ and $M = 4$, respectively. In simulations, we control ϵ so that $2M$ user pairs are selected for *Step 2* of the SCAPS algorithm. It is shown that adaptive SCAPS, by introducing semi-orthogonal channel selection, further enhances the sum rate performance of adaptive CAPS in the low to mid SNR region.

[2]The virtually orthogonal channels of SCAPS allow us to consider the proportional fair scheduling similar to [?] as well. In this case, the pair selection in Table II can be changed to $k^* = \arg\max_{k \in \mathcal{S}_0} \min[w_k \log(1+\|\mathbf{h}_k\|^2 P_s), w_{k+K} \log(1+\|\mathbf{h}_{k+K}\|^2 P_s)]$, where w_k is the fairness weight for the user k.

$$\frac{|\mathcal{I}_j|^2}{P_s} = \beta^2 \|\mathbf{h}_j^t \tilde{\mathbf{W}}_r^H \tilde{\mathbf{W}}_r \mathbf{H}_j\|^2 \leq \beta^2 \|\mathbf{h}_j\|^2 \upsilon \mathbf{\Xi} \mathbf{\Lambda} \mathbf{\Xi}^t \upsilon^t = \beta^2 \left((m-2)\delta^2 + 2\delta \right)^2 \|\mathbf{h}_j\|^2 \sum_{k \in \mathcal{S}_j} \|\mathbf{h}_k\|^2. \tag{11}$$

V. Conclusion

We show that the opportunistic multiuser diversity can be utilized to enhance the sum rate performance of the multiuser two-way AF relay channel. Simple zero-forcing based beamforming and an efficient scheduling algorithm implemented at the multi antenna relay enhances the degree of freedom and the sum throughput of the system. To keep the SINR of user pairs interference free when the number of user pairs becomes very large, we propose the CAPS algorithm. The SCAPS algorithm not only aligns the pair channels but also forms the inter-pair channels semi-orthogonal to enhance CAPS. In practice where the number of pairs K is limited, adaptive CAPS and adaptive SCAPS provide scheduling gain in the low to mid SNR region.

Appendix

A. Proof of Lemma 1

Let $\mathbf{H}_j$ be the $M \times (M-2)$ matrix, where the columns corresponding to the j-th user pair are struck out from the matrix $\mathbf{H}$ and let the $1 \times m$ vector $\upsilon = [\delta, \ldots, \delta, 1, \delta, \ldots, \delta]$, where one is on the j-th entry of υ. Also, let the $m \times (2m-2)$ matrix $\mathbf{\Xi}$ be the matrix whose elements are all δ except for ones on the entries of (k,k), $k \neq j$ and $(k, k+K)$, where (i,k) denotes the entry on the i-th row and the j-th column. Applying the property in (6) repeatedly, we get (11), which is certainly less than or equal to $(\beta m)^2 \delta^2 \|\mathbf{h}_j\|^2 \sum_{k \in \mathcal{S}_j} \|\mathbf{h}_k\|^2$. Here, $\mathbf{\Lambda}$ is the $(2m-2) \times (2m-2)$ diagonal matrix with $\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2, \ldots, \|\mathbf{h}_{K+m}\|^2$ on its diagonal entries.

B. Proof of Lemma 2

The cumulative density function (cdf) of the angular distance (ν) of two complex random vectors is derived in [?] through a reinterpretation of Theorem 1 of [?]. It is given as

$$F_\nu(\nu) = 1 - (1 - \nu)^{M-1},$$

$$f_\mu(\mu) = m \binom{K}{m} f_t(1-\mu) F_t(1-\mu)^{(m-1)} (1 - F_t(1-\mu))^{K-m}. \tag{12}$$

$$E\left(\mu\right) = \int_0^1 \mu f_\mu\left(\mu\right) d\mu = m\left(M-1\right)\binom{K}{m}\int_0^1 \mu(1-\mu)^{M-2+(m-1)(M-1)}\left(1-(1-\mu)^{M-1}\right)^{K-m} d\mu. \tag{13}$$

over the set $\nu \in [0,1]$. Let us define a new random variable $t = 1 - \nu$, then it is easy to see that the pdf and the cdf of t are $f_t(t) = (M-1)t^{M-2}$ and $F_t(t) = t^{M-1}$, respectively. Note that choosing m largest members among K realizations of ν is statistically equivalent to choosing m smallest members among the same number of realizations of t. Using the property of order statistics [?], the pdf of μ can be found as in (12), which can be rewritten as (7).

C. Proof of Proposition 1

From a different form of $E\left(\mu\right)$ derivation, we can see the behavior of $\varrho(K, M, m)$ in a large K. Starting from (12), we have (13). In (13), let $x = (1-\mu)^{M-1}$, then $\mu = 1 - x^{\frac{1}{M-1}}$ and $d\mu = \frac{1}{(M-1)}(1-\mu)^{-M+2}dx$. As a result, we can re-write the integral expression in (13) as

$$\int_0^1 \mu(1-\mu)^{M-2+(m-1)(M-1)}\left(1-(1-\mu)^{M-1}\right)^{K-m} d\mu$$

$$= \frac{1}{(M-1)}\int_0^1 \left(1 - x^{\frac{1}{M-1}}\right) x^{m-1}(1-x)^{K-m} dx. \tag{14}$$

Now, (14) can be re-written as the two simple integral expressions in (15). By the definition of [?, (3.191.3)], the first and the second inner integrals in (15) can be re-written as the following closed-form expressions in (16,17), respectively where $B\left(\cdot, \cdot\right)$ is the beta function [?, (6.2)].

After substituting (16) and (17) into (15) and some manipulations, the desired closed-form expression of (13) can be obtained as (18), where $\Gamma(\cdot)$ is the Gamma function [?, (8.32)] with the property $\frac{\Gamma(t+1)}{\Gamma(t)} = t$. For infinite K, the property $\lim_{x\to\infty}\frac{\Gamma(x+\alpha)}{\Gamma(x)} = x^\alpha$ allows us

$$\lim_{K\to\infty} E(\mu) = \left[1 - \frac{\Gamma\left(m+\frac{1}{M-1}\right)}{\Gamma(m)}\left(\frac{1}{K+1}\right)^{\frac{1}{M-1}}\right],$$

$$\lim_{K\to\infty} \varrho(K, M, m) = \frac{\Gamma\left(m+\frac{1}{M-1}\right)}{2\Gamma(m)}\left(\frac{1}{K+1}\right)^{\frac{1}{M-1}}. \tag{19}$$

$$\frac{1}{(M-1)}\left[\int_0^1 x^{m-1}(1-x)^{K-m}dx \quad -\int_0^1 x^{m-1+\frac{1}{M-1}}(1-x)^{K-m}dx\right]. \tag{15}$$

$$\int_0^1 x^{m-1}(1-x)^{K-m}dx = B(K-m+1,m), \quad \text{for } K-m+1>0, \tag{16}$$

and

$$\int_0^1 x^{m-1+\frac{1}{M-1}}(1-x)^{K-m}dx = B\left(K-m+1,m+\frac{1}{M-1}\right), \quad \text{for } K-m+1>0, \tag{17}$$

In a very large K, $\varrho(K,M,m) = (1-E(\mu))/2$ converges to zero for all $M \geq 2$.

D. Proof of Lemma 4

Let $\rho_j = m^2\|\mathbf{h}_j\|^2 \sum_{k\in\mathcal{S}_j}\|\mathbf{h}_k\|^2$. The SINR expression in (4) can be lower bounded (invoking Jensen's Inequality) as in (20).

$$SINR_j \geq \frac{\beta^2 P_s|\mathbf{h}_j^T \mathbf{w}_j^H \mathbf{w}_j \mathbf{h}_{j+K}|^2}{\frac{2\beta^2 P_s \gamma_M}{M-\sqrt[3]{K+1}}\rho_j + \beta^2(\|\mathbf{h}_j\|^2 + \frac{2(M-1)\gamma_M}{M-\sqrt[3]{K+1}}) + 1}. \tag{20}$$

Assuming that K approaches infinity, we can ignore the inter-pair interference terms and have SINR expressions as

$$SINR_j \geq \frac{\beta^2 P_s|\mathbf{h}_j^T \mathbf{w}_j^H \mathbf{w}_j \mathbf{h}_{j+K}|^2}{\beta^2\|\mathbf{h}_j\|^2 + 1}. \tag{21}$$

Since $\beta^2 = (P_r - M)/Tr[\mathbf{H}^H \mathbf{W}_r^H \mathbf{W}_r \mathbf{H}P_s]$, it is clear that β scales with P_r. Hence with P_r increasing, the $SINR_j$ are dominated by $\frac{P_s|\mathbf{h}_j^T \mathbf{w}_j^H \mathbf{w}_j \mathbf{h}_{j+K}|^2}{\|\mathbf{h}_j\|^2}$. If P_s scales to infinity as well, DoF of 1 is achieved per user as long as the relay beam-former keeps the term $|\mathbf{h}_j^T \mathbf{w}_j^H \mathbf{w}_j \mathbf{h}_{j+K}|$ non-zero. Since (21) is satisfied for all m pairs and we can schedule up to M pairs, we can achieve $2M$ DoFs in two phases (DoF of M achieved).

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$$E(\mu) = m\binom{K}{m}\left[B(K-m+1,m) - B(K-m+1,m+\frac{1}{M-1})\right],$$

$$= m\frac{\Gamma(K+1)}{\Gamma(m+1)\Gamma(K-m+1)}\left[\frac{\Gamma(K-m+1)\Gamma(m)}{\Gamma(K+1)} - \frac{\Gamma(K-m+1)\Gamma(m+\frac{1}{M-1})}{\Gamma(K+1+\frac{1}{M-1})}\right],$$

$$= \left[1 - \frac{\Gamma(K+1)\Gamma(m+\frac{1}{M-1})}{\Gamma(m)\Gamma(K+1+\frac{1}{M-1})}\right], \tag{18}$$

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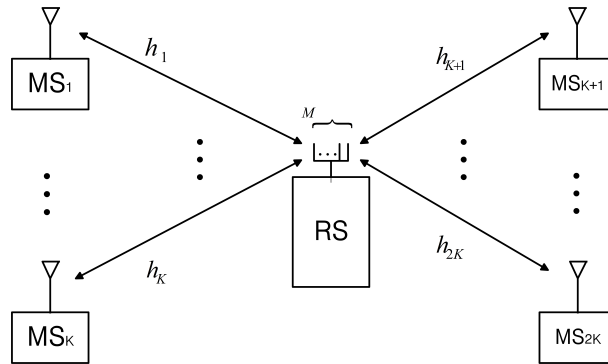
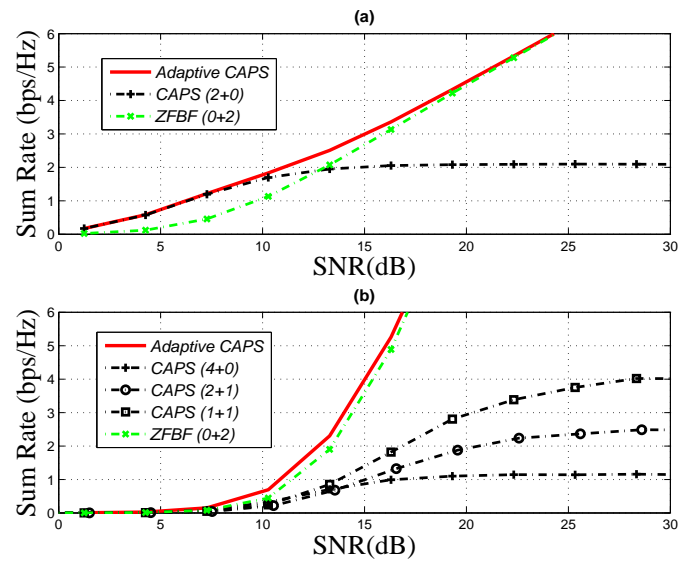Fig. 1. The Multiuser Two-Way MIMO relay channel with K user pairs.

Fig. 2. The average sum rates of the adaptive CAPS and modified CAPS with different $(m + J)$ values. Here, $M = 2$ in (a) and $M = 4$ in (b) with the same $K = 100$.

TABLE I

THE CAPS ALGORITHM

Step 1.	For $k = 1$ to K, calculate $\nu_k = \frac{|\mathbf{h}_k^H \mathbf{h}_{k+K}|}{\|\mathbf{h}_k\|\|\mathbf{h}_{k+K}\|}$.
Step 2.	Pick up m user pairs with the largest channel correlation values ν_k.
Step 3.	Starting from the largest correlation, order the selected pairs. Let $i = \varphi(k)$ denote this reordering.
Step 4.	Normalize the channel vectors of user j and user $j + K$ and take the mean vector of them as $\breve{\mathbf{h}}_{\varphi^{-1}(j)} = \mathbf{h}_{\varphi^{-1}(j)}/\|\mathbf{h}_{\varphi^{-1}(j)}\|$ $\qquad\qquad + \mathbf{h}_{\varphi^{-1}(j)+K}/\|\mathbf{h}_{\varphi^{-1}(j)+K}\|, j = 1, \ldots, m.$ $\acute{\mathbf{h}}_{\varphi^{-1}(j)} = \breve{\mathbf{h}}_{\varphi^{-1}(j)}/\|\breve{\mathbf{h}}_{\varphi^{-1}(j)}\|, j = 1, \ldots, m.$ Set $\mathbf{G} = [\acute{\mathbf{h}}_{\varphi^{-1}(1)}, \ldots, \acute{\mathbf{h}}_{\varphi^{-1}(m)}]$ and find $\tilde{\mathbf{W}}_r = \rho \mathbf{G}^\dagger$ to meet the property in (6). Here, ρ is set to make $Tr[\tilde{\mathbf{W}}_r^H \tilde{\mathbf{W}}_r] = M.$
Step 5.	Set $\mathbf{W}_r = \beta \tilde{\mathbf{W}}_r^H \tilde{\mathbf{W}}_r$ and find β using (5).
Step 6.	Inform the selected users of the scheduling grants.

TABLE II

The SCAPS algorithm

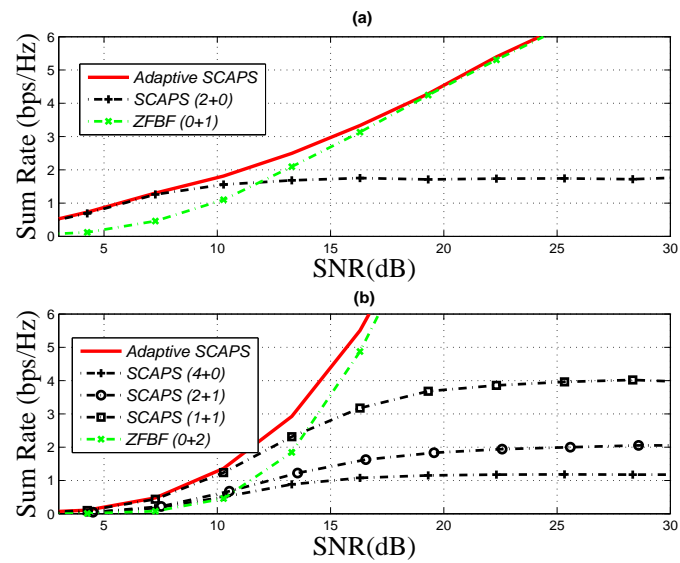|	
Step 1.	Take a small value ϵ $(0 < \epsilon \ll 1)$ and take an empty set $\mathcal{S} = \phi$.
Step 2.	For $k = 1$ to K, calculate $\nu_k = \frac{|\mathbf{h}_k^H \mathbf{h}_{k+K}|}{\|\mathbf{h}_k\|\|\mathbf{h}_{k+K}\|}$.
Step 3.	Let $\mathcal{S}_0$ be the set of user pairs with ν_k greater than $1 - \epsilon$.
Step 4.	Set $t = 1$. Among the user pairs in $\mathcal{S}_0$, pick up a pair $k^* = \arg\max_{k \in \mathcal{S}_0} \min[\|\mathbf{h}_k\|, \|\mathbf{h}_{k+K}\|]$. Set $\mathcal{S} = \mathcal{S} \cup \{k^*\}$, $\mathcal{S}_0 = \mathcal{S}_0 \setminus \{k^*\}$, $\varphi(k^*) = t$ and $\mathbf{H}_S = [\mathbf{h}_k, \mathbf{h}_{k+K}]$. Set $t = t + 1$.
Step 5.	Among the user pairs in $\mathcal{S}_0$, pick up a pair $k^* = \arg\max_{k \in \mathcal{S}_0} \min[\|\mathbf{H}_S^\perp \mathbf{h}_k\|, \|\mathbf{H}_S^\perp \mathbf{h}_{k+K}\|]$. Set $\mathcal{S} = \mathcal{S} \cup \{k^*\}$, $\mathcal{S}_0 = \mathcal{S}_0 \setminus \{k^*\}$, $\varphi(k^*) = t$ and append $\mathbf{h}_{k^*}$ and $\mathbf{h}_{k^*+K}$ to the last two columns of $\mathbf{H}_S$ $((2t - 1)$-th and $2t$-th columns of $\mathbf{H}_S$). Set $t = t + 1$ and repeat *Step 5* while $t \leq M$.
Step 6.	Set $\mathbf{G} = [\acute{\mathbf{h}}_{\varphi^{-1}(1)}, \ldots, \acute{\mathbf{h}}_{\varphi^{-1}(M)}]$ and find $\tilde{\mathbf{W}}_r = \rho \mathbf{G}^\dagger$. Here, ρ is set to make $Tr[\tilde{\mathbf{W}}_r^H \tilde{\mathbf{W}}_r] = M$.
Step 7.	Set $\mathbf{W}_r = \beta \tilde{\mathbf{W}}_r^H \tilde{\mathbf{W}}_r$ and find β using (5).
Step 8.	Inform the selected users of the scheduling grants.

Fig. 3. The average sum rates of the adaptive SCAPS and modified SCAPS with different $(m + J)$ values. Here, $M = 2$ in (a) and $M = 4$ in (b) with the same $K = 100$.