Official URL:

https://doi.org/10.1103/PhysRevFluids.2.012601

To cite this version:

Luchini, Paolo and Charru, François. Quasilaminar regime in the linear response of a turbulent flow to wall waviness. (2017) Physical Review Fluids, 2 (1). 012601. ISSN 2469-990X

RAPID COMMUNICATIONS

PHYSICAL REVIEW FLUIDS **2**, 012601(R) (2017)

Quasilaminar regime in the linear response of a turbulent flow to wall waviness

Paolo Luchini*
Dipartimento di Ingegneria Industriale, Università di Salerno, 84084 Fisciano (SA), Italy

François Charru†
Institut de Mécanique des Fluides de Toulouse, CNRS and Université de Toulouse, 31400 Toulouse, France
(Received 28 October 2016; published 3 January 2017)

The linear response of the wall-shear stress of a turbulent flow to wall waviness is analyzed in the context of a comparison between existing experiments, direct numerical simulations, and analytical approximations. The spectral region where the response is largest is found to be amenable to a simplified quasilaminar analysis. The end result is a parameterless description of this phenomenon that completely captures its physics in a single analytical formula, a Padé approximation of the response function.

DOI: 10.1103/PhysRevFluids.2.012601

I. BACKGROUND AND INTRODUCTION

The linear response of a turbulent flow to a smooth variation in bottom topography is an important topic for many geophysical applications, including the flow over hills and dunes and the evolutionary growth of the dunes themselves. A vast amount of literature is devoted to the subject, with many references available in the recent review [1]. When the topography modulation is shallow enough that the response is linear, the problem can be addressed in Fourier space: The Fourier-transformed dimensionless shear-stress perturbation $\delta\tau^+$ is then proportional to the Fourier-transformed wall-height perturbation δh^+ through a complex transfer function $T(k^+)$,

$$\delta\tau^+ = T(k^+)\delta h^+.$$

Here $\delta\tau^+$ is defined as $\delta\tau/\tau_0$, with $\tau_0 = \rho u_\tau^2$ the unperturbed shear stress exerted on a flat wall and u_τ the consequently defined characteristic shear velocity; the dimensionless wave number k^+ is defined as $k\nu/u_\tau$, with ν the kinematic viscosity of the fluid, and the height δh^+ as $\delta h u_\tau/\nu$ (with a plus superscript denoting what are usually called wall units in turbulence literature; wall units will be understood everywhere in what follows).

In laminar flow the transfer function $T(k)$ was characterized by Benjamin [2] using asymptotic techniques. Today it can be easily found numerically as the solution of the steady Orr-Sommerfeld problem

$$-ik[(f'' - k^2 f)U - U'' f] = f'''' - 2k^2 f'' + k^4 f \tag{1}$$

with boundary conditions

$$f(0) = 0, \quad f'(0) = -U'(0) = -1, \quad f(H) = 0, \quad f''(H) = 0. \tag{2}$$

Equation (1) derives from linearizing the two-dimensional Navier-Stokes equations for an incompressible fluid around a base flow represented by $U(z)$ and a bottom wall located at $z = \delta h(x) = e^{-ikx}$, together with zero-flow and zero-stress conditions at a flat free surface with vanishing Froude number located at $z = H$. The channel's height H in wall-unit nondimensionalization becomes the

*luchini@unisa.it
†charru@imft.fr

shear-stress Reynolds number $\mathrm{Re}_\tau = Hu_\tau/\nu$; therefore, the last two boundary conditions in Eq. (2) are in fact imposed at $z = \mathrm{Re}_\tau$. In the same units the nondimensional shear stress and velocity gradient at the wall, and the viscosity, all take the value of 1. The shear-stress perturbation at the wall is finally expressed from the solution of (1) in the form of $T = f''(0) + U''(0)$.

In wall units the Poiseuille solution for laminar flow is given as

$$U(z) = z - z^2/2\,\mathrm{Re}_\tau, \tag{3}$$

whence it follows that $U_{\mathrm{max}} = \mathrm{Re}_\tau/2$, $U_{\mathrm{mean}} = \mathrm{Re}_\tau/3$, and $\mathrm{Re}_{\mathrm{bulk}} \equiv HU_{\mathrm{mean}}/\nu = \mathrm{Re}_\tau^2/3$. A transfer function obtained from (1) with $U(z)$ given by (3) was exhibited by Luchini [3] in Fig. 4 therein, as a digression before examining direct numerical simulations (DNSs) for the turbulent regime, for the purpose of identifying ranges were the long-wave and short-wave approximations conceived by Benjamin numerically appeared to apply. The conclusion was that the transition from one to the other regime occurs for $kH\,\mathrm{Re}_{\mathrm{bulk}} \approx 6$, or 2π, i.e., $\lambda/H \approx \mathrm{Re}_{\mathrm{bulk}}$, a very constraining limit for the applicability of the long-wave approximation, which might naively have been assumed to only require $\lambda/H \gg 1$. The same threshold in the present notation can be written as $k^+ \ll 20/\mathrm{Re}_\tau^3$. For $20/\mathrm{Re}_\tau^3 \ll k^+ \ll 1$, Benjamin's short-wave solution [for $U(z) = z$] gives

$$T(k) \sim 1.065 e^{-i\pi/6} k^{1/3}. \tag{4}$$

[The analytical expression of the numerical coefficient involves an integral of Airy's function and may be found in Eq. (12) of [4].]

For $k^+ \gg 1$ (where viscous Stokes flow prevails) the asymptotic behavior was found by Charru and Hinch [4] to be

$$T(k) \sim 2k - 0.5ik^{-1}. \tag{5}$$

In turbulent flow just as in laminar flow, a linear transfer function from bottom waviness to (the time-averaged component of) bottom shear stress can be defined in the limit of vanishing δh and is a quantity of great practical interest. Its determination, however, is much more delicate, requiring a separation of the average component from turbulent fluctuations that might instantaneously be larger than its externally induced variation. This turbulent transfer function was the subject of a long history of experimentation and modeling, and a compilation of results is illustrated by Charru *et al.* [1] in their Fig. 3. [Their $\mathcal{A}$ and $\mathcal{B}$ coefficients are related to T as $\mathcal{A} - i\mathcal{B} = k^{-1}T(k)$ and their reference length z_0 is such that $z_0 = 0.11\nu/u_\tau$, which implies $kz_0 = 0.11k^+$ for the purpose of comparison with the present results.] As may be seen there, a large oscillation in $\mathcal{A}$ and $\mathcal{B}$ occurs in a region where $10^{-4} \lesssim kz_0 \lesssim 10^{-3}$ ($10^{-3} \lesssim k^+ \lesssim 10^{-2}$), the region corresponding to the observed sand ripples and where also most of the quoted experimental measurements (for instance, those of Abrams and Hanratty [5]) reside.

This is, in fact, as well the same range of wave numbers where singular and unexpected behavior was observed in Ref. [3] (see Fig. 10 therein), although the origin of this behavior may still need to be elucidated. Luchini [3] performed immersed-boundary direct numerical simulations of the instantaneous turbulent flow and extracted the transfer function from an averaged response (much in the same way as would be done in an experiment). The goal was to confirm or disprove a previous result by Luchini and Russo [6] (which was in fact confirmed), that the infinitely slow turbulent response at $k^+ \to 0$ is opposite in sign to the laminar response, and to the prediction of frequently used eddy-viscosity models. This goal was achieved at the cost of an exacting numerical computation, because the unexpected behavior occurs at a wavelength $\lambda \gtrsim 100H$ ($k^+ \lesssim 10^{-3}$) at the smallest Reynolds number of $\mathrm{Re}_\tau = 100$ where DNS can be performed and this wavelength is much longer than the 4π-long periodic computational box traditionally used in such simulations. In fact, a box up to 256π long was adopted, with up to 12 288 discretization points in the longitudinal direction. Although the emphasis was on very long wavelengths, some shorter wavelengths were also explored, and to these shorter wavelengths attention will now be turned.

The region of $k^+ \gtrsim 10^{-2}$ is labeled "Laminar" by Charru *et al.* [1] in the transfer-function plot of their Fig. 3, with the understanding that at these large wave numbers the perturbation extends so little into the turbulent flow that it mostly resides in the viscous sublayer, and the shear stress is the same that would occur in a laminar flow of the same u_τ. Yet there may be doubt that a wave number of $k^+ \approx 10^{-2}$ can be considered large and therefore some confirmation is needed. To know in which range the response is laminar can be a useful simplification indeed, not only in the applications to geophysical flow but also, for instance, for the use of undulating or oscillating surfaces in drag reduction [7]. On the other hand, the argument given by Charru *et al.* [1], although essentially confirmed by what will be seen below, is qualitative and based upon orders of magnitudes; a quantitative evaluation of the laminar transfer function and its validity region can be useful. The availability of the simulations performed by Luchini [3] offers the occasion for such an evaluation.

II. LAMINAR RESPONSE OF A TURBULENT FLOW

Figure 1 offers a comparison between the asymptotic laminar response (4) and (5) for short waves, the numerical laminar response as obtained from (1) for infinite Couette flow [the $\mathrm{Re}_\tau \to \infty$ limit of (3)], and numerical turbulent DNS from [3] (together with more curves to be discussed below). As may be seen, the laminar response in the wave-number range $10^{-3} \leqslant k^+ \leqslant 10^{-1}$ is very close to its asymptotic behavior as given by (4); on the other hand, both the real (in phase) and imaginary (in quadrature) parts of the turbulent response function are initially much lower than predicted by (4), but conceivably point towards laminar behavior with increasing k^+. It can also be observed that the laminar results show hardly any dependence on channel height and its related Reynolds number (in fact, the curves for $\mathrm{Re}_\tau = 180$, 400, and ∞ were completely superposed and only the latter is displayed), whereas the turbulent DNS results available at $\mathrm{Re}_\tau = 180$ and 400, albeit clearly distinct from one another, exhibit a consistent trend and can be reasonably assumed to share a common increasing-wave-number behavior with even higher Reynolds numbers.

The distance between laminar and turbulent results can be ascribed to a combination of two different reasons: On the one hand, turbulent dissipation mediated by fluctuations is clearly different from laminar dissipation; on the other hand, the DNS result is being compared to the solution of (1) around a straight-line Couette velocity profile [(3) for $\mathrm{Re}_\tau \to \infty$], whereas it is known that the mean turbulent velocity profile quickly deviates from a straight line and becomes logarithmic through a universal behavior known as the law of the wall (see, for instance, Ref. [8]). In order to separate these two effects, we have performed the solution of (1) about the mean turbulent velocity profile. (A similar procedure was already given some early consideration by Benjamin [2], but only in the context of a long-wave first approximation.)

For this purpose we have used the turbulent mean velocity profiles calculated in Ref. [3] for a flat wall, the same for which the average turbulent response had been calculated by DNS. The response function obtained from (1) with each of these velocity profiles is additionally displayed in Fig. 1 for two values of the Reynolds number. As may be seen, it captures most of the difference between the DNS data and the fully laminar result down to $k \approx 10^{-2}$ for the real (in phase) component and down to $k \approx 2 \times 10^{-3}$ for the imaginary (quadrature) component of the response. We refer to the response obtained from a laminar solution of (1) around the turbulent mean flow as quasilaminar (not to be confused with the usage of the same term in Ref. [5] to denote the viscous $k^+ \approx 1$ region). In fact, in Fig. 1 the quasilaminar solution nearly perfectly matches the numerical simulations on one end of the spectrum and the fully laminar result on the other, thus giving confidence that the aforementioned laminar-flow conjecture by Charru *et al.* [1] was correct.

The turbulent response for wave numbers $k^+ \lesssim 10^{-3}$ exhibits unexpected behavior in both the experiments of Abrams and Hanratty [5] (whose results were also reproduced in Ref. [1]) and the simulations of Luchini [3], and a sign reversal in the latter, for the details of which the reader is referred to the respective original papers and to [9]. Despite this behavior still awaiting interpretation, as a matter of fact in both experiments and simulations the shear-stress response for $k^+ \lesssim 10^{-3}$ is of much smaller amplitude than in the quasilaminar range; for applications involving the latter it can just be assumed to be zero. It then becomes useful to have a compact expression of the quasilaminar response in its range of validity and for this purpose a rational polynomial interpolation can be used.

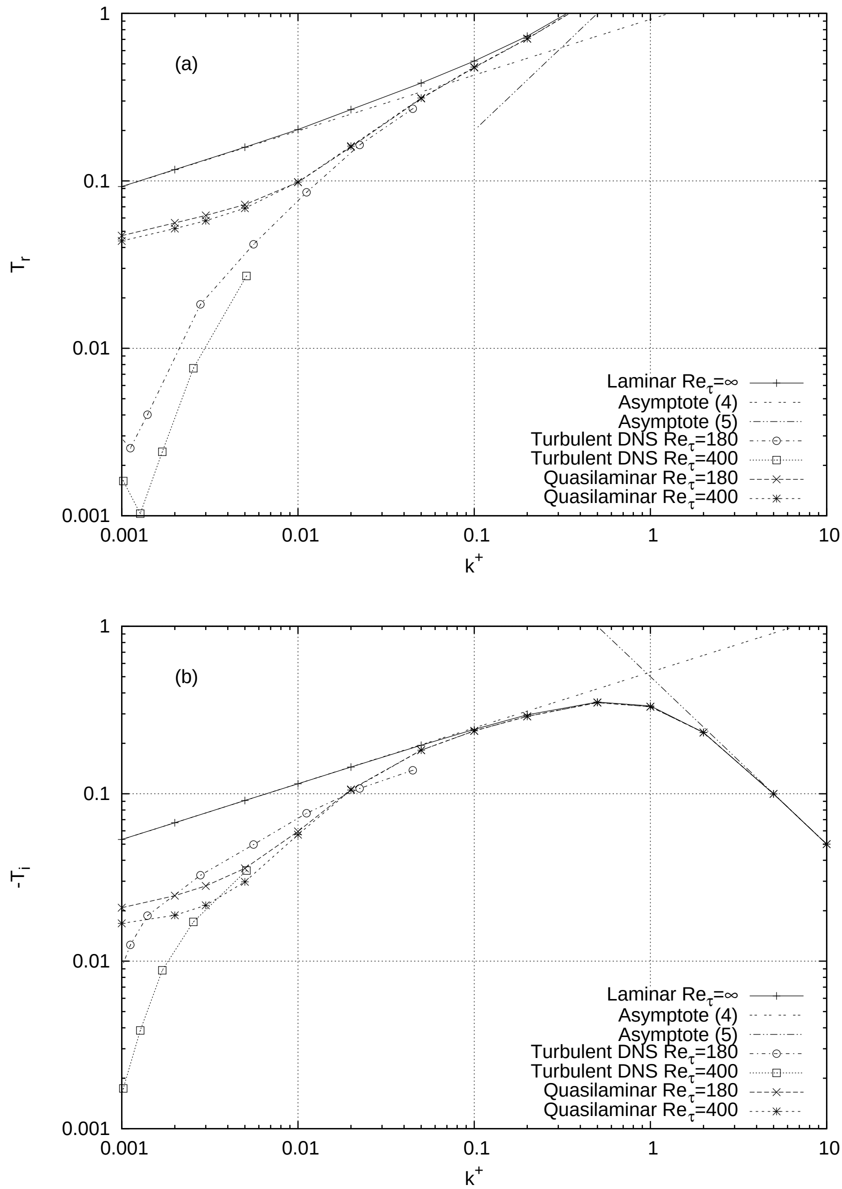

FIG. 1. (a) Real and (b) imaginary parts of the shear-stress response to wall waviness.

One such interpolation is the second-degree Padé approximant

$$T(k) - 2k = \frac{k - (0.002\,087 + 0.000\,928i)}{0.052\,20 + 0.038\,37i + (1.6592 + 1.2380i)k + (-0.7009 + 1.2051i)k^2}, \qquad (6)$$

where the asymptotic viscous-region behavior $T(k) \sim 2k$, as in Eq. (5), was subtracted in order to make the curve converge at both ends. The result of this interpolation is plotted in Fig. 2 together with the numerical simulations of Luchini [3] (at $\mathrm{Re}_\tau = 400$), the experimental data of Abrams and Hanratty [5] (having a variable Reynolds number $\mathrm{Re}_\tau = 2\pi/k^+$ in the range $600 \leqslant \mathrm{Re}_\tau \leqslant 6000$), and the quasilaminar solution of (1) at $\mathrm{Re}_\tau = 400$. As can be seen, the agreement is more than satisfactory (as to the interpolation as well as between simulations and experiments) and shows very

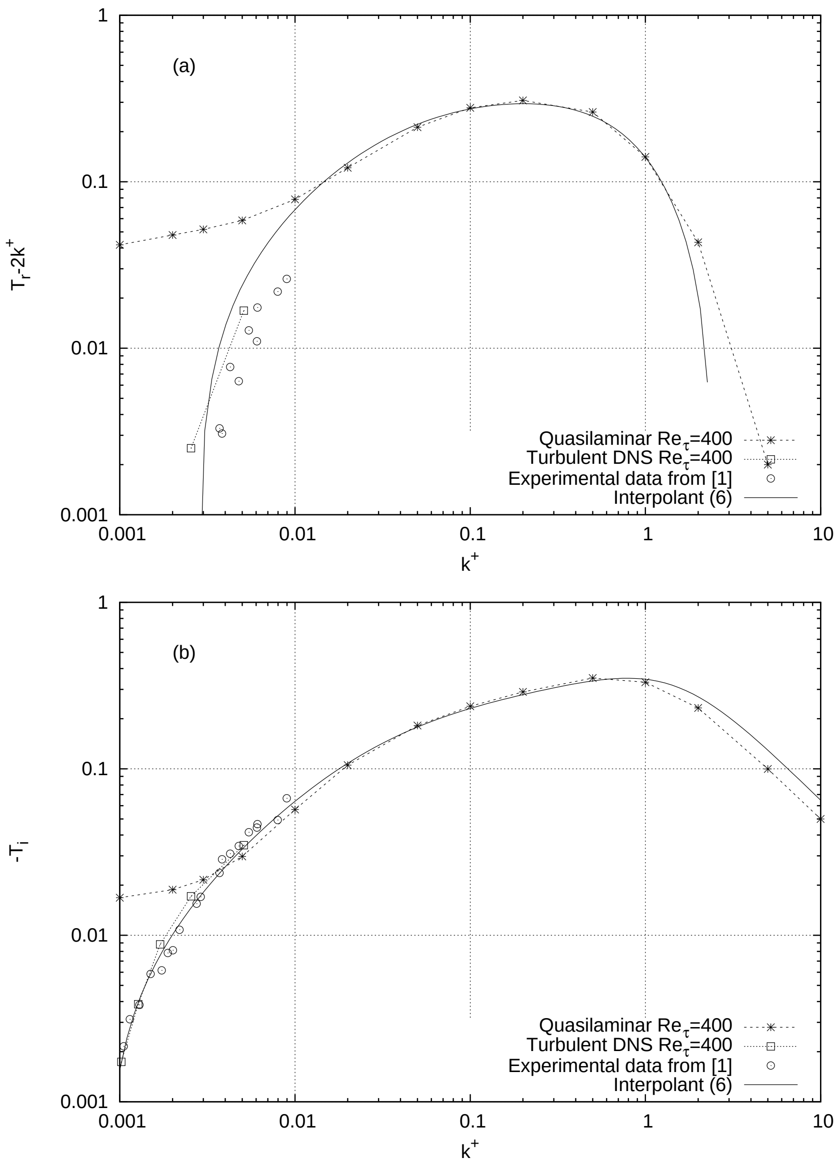

FIG. 2. Interpolation of the response function through a Padé approximant.

little Reynolds-number dependence, especially so in the quadrature component, which for stability analyses is the critical element.

III. CONCLUSION

The existence of a laminar response of the shear stress exerted by flow over a wavy surface has been known since the work of Benjamin [2]; this response is characterized by a boundary-layer scaling that produces the analytical formula (4) and occurs in a range of wave numbers bounded on the right (for $k^+ \gtrsim 1$) by the viscous limit (5) and on the left (for $k^+ \lesssim 20/\mathrm{Re}_\tau^3$) by interaction with the free surface or opposing wall. The ensuing phase shift between shear stress and wall geometry plays an important role, for instance, in the growth of sand ripples and dunes [1].

However, the flow over ripples and dunes and other geometry variations is most often turbulent. In turbulent flow the existence of a laminar region of the spectrum for $10^{-2} \lesssim k^+ \lesssim 1$ was conjectured by Charru *et al.* [1]; the present results support this conjecture, but on the condition that the laminar perturbation equation (1) is solved around the mean turbulent flow. The response may thus be denominated quasilaminar.

Because the quadrature response decays on both sides of the wave-number spectrum, for short waves because of viscous effects and for long waves as an empirical observation in both experiments and direct numerical simulations, the quasilaminar region acquires a dominant role. The quadratic rational approximation (6) allows the dominant response to be interpolated accurately in a dimensionless form independent of the Reynolds number Re_τ or any other parameter and is available as a reference for further studies.

ACKNOWLEDGMENTS

We are thankful to the late professor Thomas J. Hanratty, who visited IMFT in Toulouse many times, for the unique precise measurements performed during his career which have been essential to the present work.

[1] F. Charru, B. Andreotti, and P. Claudin, Sand ripples and dunes, Annu. Rev. Fluid Mech. **45**, 469 (2013).
[2] T. B. Benjamin, Shearing flow over a wavy boundary, J. Fluid Mech. **6**, 161 (1959).
[3] P. Luchini, Immersed-boundary simulations of turbulent flow past a sinusoidally undulated river bottom, Eur. J. Mech. B **55**, 340 (2016).
[4] F. Charru and E. J. Hinch, Phase diagram of interfacial instabilities in a two-layer Couette flow and mechanism of the long-wave instability, J. Fluid Mech. **414**, 195 (2000).
[5] J. Abrams and T. J. Hanratty, Relaxation effects observed for turbulent flow over a wavy surface, J. Fluid Mech. **151**, 443 (1985).
[6] P. Luchini and S. Russo, The linear response of turbulent flow to a volume force: Comparison between eddy-viscosity model and DNS, J. Fluid Mech. **790**, 104 (2016).
[7] C. Viotti, M. Quadrio, and P. Luchini, Streamwise oscillation of spanwise velocity at the wall of a channel for turbulent drag reduction, Phys. Fluids **21**, 115109 (2009)
[8] F. M. White, *Viscous Fluid Flow* (McGraw-Hill, New York, 1991).
[9] P. Luchini, Addendum to “Immersed-boundary simulations of turbulent flow past a sinusoidally undulated river bottom” [Eur. J. Mech. B **55**, 340 (2016)], Eur. J. Mech. B **62**, 57 (2017).