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Research Article

Structures and Low Dimensional Classifications of Hom-Poisson Superalgebras

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Hom-Poisson superalgebras can be considered as a deformation of Poisson superalgebras. We prove that Hom-Poisson superalgebras are closed under tensor products. Moreover, we show that Hom-Poisson superalgebras can be described using only the twisting map and one binary operation. Finally, all algebra endomorphisms on 2-dimensional complex Poisson superalgebras are computed, and their associated Hom-Poisson superalgebras are described explicitly.

1. Introduction

Poisson algebras are used in many fields in mathematics and physics. In mathematics, Poisson algebras play a fundamental role in Poisson geometry [1], quantum groups [2], and deformation of commutative associative algebras. In physics, Poisson algebras are a major part of deformation quantization, Hamiltonian mechanics [3], and topological field theories [4]. Poisson-like structures are also used in the study of vertex operator algebras [5]. Poisson superalgebras can be seen as a direct generalization of Poisson algebras. Remm show that a Poisson superalgebra can be described using only one binary operation in [6, 7].

Recently, a twisted generalization of noncommutative Poisson algebras, called Hom-noncommutative Poisson algebras, are studied in [8]. In a noncommutative Hom-Poisson algebra, there exists a twisted map, a Hom-Lie bracket and a Hom-associative product. The associativity, the Jacobi identity, and the Leibniz identity are considered as their Hom-type analogues in a noncommutative Hom-Poisson algebra. The purpose of this paper is to introduce and study a twisted generalization of Poisson superalgebras, called Hom-Poisson superalgebras.

This paper is organized as follows. In Section 2, we recall the construction of the Hom-Lie superalgebras. In Section 3, we give the definition of Hom-Poisson superalgebras. We show that starting with a Poisson superalgebra and an even Poisson superalgebra endomorphism, a Hom-Poisson superalgebra can be constructed. Moreover, we prove that Hom-Poisson superalgebras are closed under tensor products. In Section 4, we show that a Hom-Poisson superalgebra can be described using only the twisting map and one binary operation. Section 5 is devoted to classifying all the algebra endomorphisms α on all the 2-dimensional complex Poisson superalgebras and 2-dimensional Hom-Poisson superalgebras.

Throughout the paper $\mathbb{C}$ is the field of complex numbers. All algebras and vector spaces are considered over $\mathbb{C}$.

2. Hom-Lie and Hom-Associative Superalgebras

In this section, we first recall the notion of Hom-Lie superalgebras and then give some construction of the Hom-Lie superalgebras.

Definition 1 (see [9]). A Hom-associative superalgebra is a triple $(A, \circ, \alpha)$ consisting of a $\mathbb{Z}_2$-graded vector space A, an even bilinear map $\circ : A \times A \to A$, and an even homomorphism of algebras $\alpha : A \to A$ satisfying

$$\alpha(x) \circ (y \circ z) = (x \circ y) \circ \alpha(z) \tag{1}$$

for all homogeneous elements $x, y, z \in A$.

Definition 2 (see [9]). A Hom-Lie superalgebra is a triple $(A, [\cdot,], \alpha)$ consisting of a $\mathbb{Z}_2$-graded vector space A, an even bilinear map $[\cdot,\cdot] : A \times A \to A$ and an even homomorphism of algebras $\alpha : A \to A$ satisfying

$$[x, y] = -(-1)^{|x||y|}[y, x], \tag{2}$$

$$\begin{aligned}(-1)^{|x||z|}[\alpha(x), [y, z]] + (-1)^{|z||y|}[\alpha(z), [x, y]] \\ + (-1)^{|x||y|}[\alpha(y), [z, x]] = 0\end{aligned} \tag{3}$$

for all homogeneous elements $x, y, z \in A$.

Let $(V, [\cdot,\cdot], \alpha)$ and $(V', [\cdot,\cdot]', \alpha')$ be two Hom-Lie superalgebras. An even homomorphism $f : V \to V'$ is said to be a morphism of Hom-Lie superalgebras if

$$[f(x), f(y)]' = f([x, y]), \quad \forall x, y \in V, \tag{4}$$

$$f \circ \alpha = \alpha' \circ f. \tag{5}$$

Morphisms of Hom-associative superalgebras are defined similarly.

The following theorem provides a method to construct a Hom-Lie superalgebra by a Lie superalgebra and an even homomorphism of Lie superalgebras.

Proposition 3 (see [9]). *Let $(V, [\cdot,\cdot])$ be a Lie superalgebra and let $\alpha : V \to V$ be an even endomorphism of Lie superalgebras. Then $(V, [\cdot,\cdot]_\alpha, \alpha)$ is a Hom-Lie superalgebra, where $[x, y]_\alpha = \alpha([x, y])$.*

Moreover, suppose that $(V', [\cdot,\cdot]')$ is another Lie superalgebra and $\alpha' : V' \to V'$ is an even endomorphism of Lie superalgebras. If $f : V \to V'$ is a morphism of Lie superalgebras satisfying $f \circ \alpha = \alpha' \circ f$, then

$$f : (V, [\cdot,\cdot]_\alpha, \alpha) \longrightarrow (V', [\cdot,\cdot]'_{\alpha'}, \alpha') \tag{6}$$

is a morphism of Hom-Lie superalgebras.

Example 4 (see [9]). From the orthosymplectic Lie superalgebra $\mathrm{osp}(1, 2) = V_0 \oplus V_1$, where V_0 is spanned by

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{7}$$

and V_1 is spanned by

$$F = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}. \tag{8}$$

The defining nonzero relations are

$$\begin{aligned}[H, X] &= 2X, \\ [H, Y] &= -2Y, \\ [X, Y] &= H, \\ [Y, G] &= F, \\ [X, F] &= G, \\ [H, F] &= -F, \\ [H, G] &= G, \\ [G, F] &= H, \\ [G, G] &= -2X, \\ [F, F] &= 2Y.\end{aligned} \tag{9}$$

Let $\lambda \in \mathbb{R}^*$ define a linear map $\alpha_\lambda : \mathrm{osp}(1, 2) \to \mathrm{osp}(1, 2)$ by

$$\begin{aligned}\alpha_\lambda(X) &= \lambda^2 X, \\ \alpha_\lambda(Y) &= \frac{1}{\lambda^2}(Y), \\ \alpha_\lambda(H) &= H, \\ \alpha_\lambda(F) &= \frac{1}{\lambda}(F), \\ \alpha_\lambda(G) &= \lambda G;\end{aligned} \tag{10}$$

then α_λ is an even Lie superalgebra automorphism; by Proposition 3, we obtain a family of Hom-Lie superalgebras $(\mathrm{osp}(1,2), [\,,\,]_\lambda, \alpha_\lambda)$. These Hom-Lie superalgebras are not Lie superalgebras for $\lambda \neq 1$.

3. Hom-Poisson Superalgebras

Definition 5. A Hom-Poisson superalgebra is a tuple $(A, \cdot, [\cdot,\cdot], \alpha)$ consisting of a $\mathbb{Z}_2$-graded vector space V, two even bilinear maps $\cdot, [\cdot,\cdot] : V \times V \to V$, and an even homomorphism of algebras $\alpha : V \to V$, where $\alpha(x \cdot y) = \alpha(x) \cdot \alpha(y)$, $\alpha([x, y]) = [\alpha(x), \alpha(y)]$, satisfying the following axioms.

(1) $(A, \cdot, \alpha)$ is a supercommutative Hom-associative superalgebra.

(2) $(A, [\cdot,\cdot], \alpha)$ is a Hom-Lie superalgebra.

(3) The Hom-Leibniz superidentity $[x \cdot y, \alpha(z)] = \alpha(x) \cdot [y, z] + (-1)^{|y||z|}[x, z] \cdot \alpha(y)$ holds for all homogeneous elements $x, y, z \in A$.

Definition 6. Let $(A, \cdot, [\cdot,\cdot], \alpha)$ and $(A', \cdot', [\cdot,\cdot]', \alpha')$ be two Hom-Poisson superalgebras. An even homomorphism $f : A \to A'$ is said to be a morphism of Hom-Poisson superalgebras if

$$f(x) \cdot' f(y) = f(x \cdot y), \quad \forall x, y \in A,$$
$$[f(x), f(y)]' = f([x, y]), \quad \forall x, y \in A, \qquad (11)$$
$$f \cdot \alpha = \alpha' \cdot f.$$

Proposition 7. *Let $(A, \cdot, \alpha)$ be a Hom-associative superalgebra and $[\cdot,\cdot] : A \times A \to A$ be a binary operation on A defined by*

$$[x, y] = x \cdot y - (-1)^{|x||y|} y \cdot x, \quad \forall x, y \in A, \qquad (12)$$

then $(A, \cdot, [\cdot,\cdot], \alpha)$ is a Hom-Poisson superalgebra with the same twisting map α.

Proof. It is straightforward. □

From Proposition 7, there is the following construction of Hom-Poisson superalgebras by Poisson superalgebras and homomorphisms.

Theorem 8. *Let $(A, \cdot, [\cdot,\cdot])$ be a Poisson superalgebra and let $\alpha : A \to A$ be an even endomorphism of Poisson superalgebras. Then $(A, \cdot_\alpha, [\cdot,\cdot]_\alpha, \alpha)$ is a Hom-Poisson superalgebra, where $x \cdot_\alpha y = \alpha(x \cdot y)$ and $[x, y]_\alpha = \alpha([x, y])$.*

Moreover, suppose that $(A', \cdot', [\cdot,\cdot]')$ is another Poisson superalgebra and $\alpha' : A' \to A'$ is an even endomorphism of Poisson superalgebras. If $f : A \to A'$ is a morphism of Poisson superalgebras satisfying $f \cdot \alpha = \alpha' \cdot f$, then

$$f : (A, \cdot_\alpha, [\cdot,\cdot]_\alpha, \alpha) \longrightarrow (A', \cdot_{\alpha'}, [\cdot,\cdot]'_{\alpha'}, \alpha') \qquad (13)$$

is a morphism of Hom-Poisson superalgebras.

Proof. By Proposition 3, $(A, [\cdot,\cdot]_\alpha, \alpha)$ is a Hom-Lie superalgebra.

We will show that $(A, \cdot_\alpha, \alpha)$ satisfies the axioms (1) and (3) of Definition 5. In fact

$$\begin{aligned}
\alpha(x) \cdot_\alpha (y \cdot_\alpha z) &= \alpha(x) \cdot_\alpha \alpha(y \cdot z) = \alpha(\alpha(x) \cdot (y \cdot z)) \\
&= \alpha((x \cdot y) \cdot \alpha(z)) = \alpha(x \cdot y) \cdot_\alpha \alpha(z) \\
&= (x \cdot_\alpha y) \cdot_\alpha \alpha(z), \\
[x \cdot_\alpha y, \alpha(z)]_\alpha &= \alpha([x \cdot_\alpha y, \alpha(z)]) \\
&= \alpha([\alpha(x \cdot y), \alpha(z)]) \\
&= \alpha^2([x \cdot y, z]) \\
&= \alpha^2((-1)^{|y||z|}[x, z] \cdot y + x \cdot [y, z]) \\
&= (-1)^{|y||z|}\alpha(\alpha([x, z]) \cdot \alpha(y)) \\
&\quad + \alpha(\alpha(x) \cdot \alpha([y, z])) \\
&= (-1)^{|y||z|}\alpha([x, z]) \cdot_\alpha \alpha(y) \\
&\quad + \alpha(x) \cdot_\alpha \alpha([y, z]) \\
&= (-1)^{|y||z|}[x, z]_\alpha \cdot_\alpha \alpha(y) + \alpha(x) \\
&\quad \cdot_\alpha [y, z]_\alpha.
\end{aligned} \qquad (14)$$

Hence $(A, \cdot_\alpha, [\cdot,\cdot]_\alpha, \alpha)$ is a Hom-Poisson superalgebra. Then second assertion follows from

$$\begin{aligned}
f([x, y]_\alpha) &= f(\alpha([x, y])) = f \cdot \alpha([x, y]) \\
&= \alpha' \cdot f([x, y]) = \alpha'([f(x), f(y)]) \\
&= [f(x), f(y)]_{\alpha'}; \\
f(x \cdot_\alpha y) &= f(\alpha(x \cdot y)) = f \cdot \alpha(x \cdot y) \\
&= \alpha' \cdot f(x \cdot y) = \alpha'(f(x) \cdot f(y)) \\
&= f(x) \cdot_{\alpha'} f(y).
\end{aligned} \qquad (15)$$

□

This theorem provides a method to construct a Hom-Poisson superalgebra by a Poisson superalgebra and an even homomorphism of Poisson superalgebras.

Example 9. Let $A = A_{\bar{0}} \oplus A_{\bar{1}}$ be a 2-dimensional $\mathbb{Z}_2$-graded vector space, where $A_{\bar{0}}$ is generated by e_1 and $A_{\bar{1}}$ is generated by e_2 and nonzero products are given by

$$\begin{aligned}
e_1 \cdot e_1 &= e_1, \\
e_2 \cdot e_2 &= e_1, \\
e_1 \cdot e_2 &= e_2 \cdot e_1 = e_2, \\
[e_2, e_2] &= 2e_1;
\end{aligned} \qquad (16)$$

then $(A, \cdot, [\cdot,\cdot])$ is a Poisson superalgebra. For any $a \in \mathbb{C}$, we consider the homomorphism $\alpha : A \to A$ defined by $\alpha(e_1) = ae_1, \alpha(e_2) = ae_2$. By Theorem 8, for any $a \in \mathbb{C}$, there is the corresponding Hom-Poisson superalgebra $A_a = (A, \cdot_\alpha, [\cdot,\cdot]_\alpha, \alpha)$ with the nonzero products

$$\begin{aligned} e_1 \cdot_\alpha e_1 &= a e_1, \\ e_2 \cdot_\alpha e_2 &= a e_1, \\ e_1 \cdot_\alpha e_\alpha &= a e_2, \\ [e_2, e_2]_\alpha &= 2a e_1. \end{aligned} \tag{17}$$

It is not a Poisson superalgebra when $a \neq 0, 1$.

Example 10. Let $A = A_{\bar{0}} \oplus A_{\bar{1}}$ be a 3-dimensional $\mathbb{Z}_2$-graded vector space, where $A_{\bar{0}}$ is generated by e_1, e_2 and $A_{\bar{1}}$ is generated by e_3 and the nonzero products are given by

$$\begin{aligned} e_1 \cdot e_2 &= e_1, \\ e_2 \cdot e_2 &= e_2, \\ e_3 \cdot e_2 &= e_3, \\ [e_1, e_2] &= a e_1; \end{aligned} \tag{18}$$

then $(A, \cdot, [\cdot,\cdot])$ is a Poisson superalgebra. For any $a \in \mathbb{C}$, we consider the homomorphism $\alpha : A \to A$ defined by

$$\begin{aligned} \alpha(e_1) &= a e_1, \\ \alpha(e_2) &= e_1 + e_2. \end{aligned} \tag{19}$$

By Theorem 8, for any $a \in \mathbb{C}$, there is the corresponding Hom-Poisson superalgebra $A_\alpha = (A, \cdot_\alpha, [\cdot,\cdot]_\alpha, \alpha)$ with the nonzero products

$$\begin{aligned} e_1 \cdot_\alpha e_2 &= a e_1, \\ e_2 \cdot_\alpha e_2 &= e_1 + e_2, \\ [e_1, e_2]_\alpha &= a e_1. \end{aligned} \tag{20}$$

It is not a Poisson superalgebra when $a \neq 0, 1$.

We know Hom-Poisson algebras are closed under tensor products ([8] Theorem 2.9). Here we aim to consider it in detail in the superalgebra case.

Theorem 11. *Let $(A_i, \cdot_i, [\cdot,\cdot]_i, \alpha_i)$ be Hom-Poisson superalgebras for $i = 1, 2$, and let $A = A_1 \otimes A_2$. Define the operations $\alpha : A \to A$ and $\cdot, [\cdot,\cdot] : A^{\otimes 2} \to A$ by*

$$\begin{aligned} \alpha &= \alpha_1 \otimes \alpha_2, \\ (x_1 \otimes x_2) \cdot (y_1 \otimes y_2) &= (-1)^{|x_2||y_1|} (x_1 \cdot_1 y_1) \otimes (x_2 \cdot_2 y_2), \\ [x_1 \otimes x_2, y_1 \otimes y_2] &= (-1)^{|x_2||y_1|} [x_1, y_1]_1 \otimes (x_2 \cdot_2 y_2) + (-1)^{|x_2||y_1|} (x_1 \cdot_1 y_1) \otimes [x_2, y_2]_2, \end{aligned} \tag{21}$$

for $x_i, y_i \in A_i$. Then $(A, \cdot, [\cdot,\cdot], \alpha)$ is a Hom-Poisson superalgebra.

Proof. The $(A, \cdot, \alpha)$ is a supercommutative Hom-associative superalgebra following from the supercommutativity and Hom-associativity of both $\cdot_i$. Also, the supercommutativity of the $\cdot_i$ and the antisupersymmetry of the $[\cdot,\cdot]_i$ imply the antisupersymmetry of $[\cdot,\cdot]$. It remains to prove the Hom-Jacobi superidentity and the Hom-Leibniz superidentity in A.

To simplify the typography, we abbreviate $\cdot_1$, $\cdot_2$, and $\cdot$ using juxtaposition and drop the subscripts in $[\cdot,\cdot]_i$ and α_i. Pick $x = x_1 \otimes x_2$, $y = y_1 \otimes y_2$, $z = z_1 \otimes z_2 \in A$. Then

$$\begin{aligned} &(-1)^{|x||z|} [[x, y], \alpha(z)] \\ &= (-1)^{s_1} [[x_1, y_1] \otimes (x_2 y_2), \alpha_1(z_1) \otimes \alpha_2(z_2)] \\ &\quad + (-1)^{s_1} [(x_1 y_1) \otimes [x_2, y_2], \alpha_1(z_1) \otimes \alpha_2(z_2)] \\ &= (-1)^{s_2} [[x_1, y_1], \alpha_1(z_1)] \otimes (x_2 y_2) \alpha_2(z_2) \\ &\quad + (-1)^{s_2} [x_1, y_1] \alpha_1(z_1) \otimes [x_2 y_2, \alpha_2(z_2)] \\ &\quad + (-1)^{s_2} [x_1 y_1, \alpha_1(z_1)] \otimes [x_2, y_2] \alpha_2(z_2) \\ &\quad + (-1)^{s_2} (x_1 y_1) \alpha_1(z_1) \otimes [[x_2, y_2], \alpha_2(z_2)], \end{aligned} \tag{22}$$

where $s_1 = |x_1||z_1| + |x_1||z_2| + |x_2||z_1| + |x_2||z_2| + |x_2||y_1|$, $s_2 = |x_1||z_1| + |x_1||z_2| + |x_2||z_2| + |x_2||y_1| + |y_2||z_1|$. Consider

$$\begin{aligned} &(-1)^{|x||y|} [[y, z], \alpha(x)] \\ &= (-1)^{s_3} [[y_1, z_1], \alpha_1(x_1)] \otimes (y_2 z_2) \alpha_2(x_2) \\ &\quad + (-1)^{s_3} [y_1, z_1] \alpha_1(x_1) \otimes [y_2 z_2, \alpha_2(x_2)] \\ &\quad + (-1)^{s_3} [y_1 z_1, \alpha_1(x_1)] \otimes [y_2, z_2] \alpha_2(x_2) \\ &\quad + (-1)^{s_3} (y_1 z_1) \alpha_1(x_1) \otimes [[y_2, z_2], \alpha_2(x_2)], \end{aligned} \tag{23}$$

where $s_3 = |y_1||x_1| + |y_1||x_2| + |y_2||x_2| + |y_2||z_1| + |z_2||x_1|$. Consider

$$\begin{aligned} &(-1)^{|y||z|} [[z, x], \alpha(y)] \\ &= (-1)^{s_4} [[z_1, x_1], \alpha_1(y_1)] \otimes (z_2 x_2) \alpha_2(y_2) \\ &\quad + (-1)^{s_4} [z_1, x_1] \alpha_1(y_1) \otimes [z_2 x_2, \alpha_2(y_2)] \\ &\quad + (-1)^{s_4} [z_1 x_1, \alpha_1(y_1)] \otimes [z_2, x_2] \alpha_2(y_2) \\ &\quad + (-1)^{s_4} (z_1 x_1) \alpha_1(y_1) \otimes [[z_2, x_2], \alpha_2(y_2)], \end{aligned} \tag{24}$$

where $s_4 = |z_1||y_1| + |z_1||y_2| + |z_2||y_2| + |z_2||x_1| + |x_2||y_1|$. Using the Hom-Jacobi superidentity in A_1 and the supercommutativity and Hom-associativity in A_2, we have

$$\begin{aligned} &(-1)^{s_2} [[x_1, y_1], \alpha_1(z_1)] \otimes (x_2 y_2) \alpha_2(z_2) \\ &\quad + (-1)^{s_3} [[y_1, z_1], \alpha_1(x_1)] \otimes (y_2 z_2) \alpha_2(x_2) \\ &\quad + (-1)^{s_4} [[z_1, x_1], \alpha_1(y_1)] \otimes (z_2 x_2) \alpha_2(y_2) \\ &= 0. \end{aligned} \tag{25}$$

Likewise, using the supercommutativity and Hom-associativity in A_1 and the Hom-Jacobi superidentity in A_2, we obtain

$$\begin{aligned}(-1)^{s_2}&(x_1y_1)\alpha_1(z_1)\otimes[[x_2,y_2],\alpha_2(z_2)]\\&+(-1)^{s_3}(y_1z_1)\alpha_1(x_1)\otimes[[y_2,z_2],\alpha_2(x_2)]\\&+(-1)^{s_4}(z_1x_1)\alpha_1(y_1)\otimes[[z_2,x_2],\alpha_2(y_2)]\\&=0.\end{aligned}\tag{26}$$

Using the Hom-Leibniz superidentity in A_i, then we have

$$\begin{aligned}&(-1)^{s_2}[x_1,y_1]\alpha_1(z_1)\otimes[x_2y_2,\alpha_2(z_2)]+(-1)^{s_2}\\&\cdot[x_1y_1,\alpha_1(z_1)]\otimes[x_2,y_2]\alpha_2(z_2)+(-1)^{s_3}[y_1,z_1]\\&\cdot\alpha_1(x_1)\otimes[y_2z_2,\alpha_2(x_2)]+(-1)^{s_3}[y_1z_1,\alpha_1(x_1)]\\&\otimes[y_2,z_2]\alpha_2(x_2)+(-1)^{s_4}[z_1,x_1]\alpha_1(y_1)\\&\otimes[z_2x_2,\alpha_2(y_2)]+(-1)^{s_4}[z_1x_1,\alpha_1(y_1)]\\&\otimes[z_2,x_2]\alpha_2(y_2)=(-1)^{s_2+|y_2||z_2|}[x_1,y_1]\alpha_1(z_1)\\&\otimes[x_2,(z_2)]\alpha_2(y_2)+(-1)^{s_2}[x_1,y_1]\alpha_1(z_1)\\&\otimes\alpha_2(x_2)[y_2,z_2]+(-1)^{s_3+|z_2||x_2|}[y_1,z_1]\alpha_1(x_1)\\&\otimes[y_2,x_2]\alpha_2(z_2)+(-1)^{s_3}[y_1,z_1]\alpha_1(x_1)\\&\otimes\alpha_2(y_2)[z_2,x_2]+(-1)^{s_3+|z_1||x_1|}[y_1,x_1]\alpha_1(z_1)\\&\otimes[y_2,z_2]\alpha_2(x_2)+(-1)^{s_3}\alpha_1(y_1)[z_1,x_1]\\&\otimes[y_2,z_2]\alpha_2(x_2)+(-1)^{s_4+|y_2||z_2|}[z_1,x_1]\alpha_1(y_1)\\&\otimes[z_2x_2,\alpha_2(y_2)]+(-1)^{s_4}[z_1,x_1]\alpha_1(y_1)\\&\otimes[z_2,y_2]\alpha_2(x_2)+(-1)^{s_4}[z_1,x_1]\alpha_1(y_1)\\&\otimes\alpha_2(z_2)[x_2,y_2]+(-1)^{s_4+|x_1||y_1|}[z_1,y_1]\alpha_1(x_1)\\&\otimes[z_2,x_2]\alpha_2(y_2)+(-1)^{s_4}\alpha_1(z_1)[x_1,y_1]\\&\otimes[z_2,x_2]\alpha_2(y_2)=\left\{(-1)^{s_2+|y_2||z_2|}[x_1,y_1]\alpha_1(z_1)\right.\\&\otimes[x_2,z_2]\alpha_2(y_2)+(-1)^{s_4}\alpha_1(z_1)[x_1,y_1]\\&\left.\otimes[z_2,x_2]\alpha_2(y_2)\right\}+\left\{(-1)^{s_2}[x_1,y_1]\alpha_1(z_1)\right.\\&\otimes\alpha_2(x_2)[y_2,z_2]+(-1)^{s_3+|z_1||x_1|}[y_1,x_1]\alpha_1(z_1)\\&\left.\otimes[y_2,z_2]\alpha_2(x_2)\right\}+\left\{(-1)^{s_3}[y_1,z_1]\alpha_1(x_1)\right.\\&\otimes\alpha_2(y_2)[z_2,x_2]+(-1)^{s_3+|z_1||x_1|}[y_1,x_1]\alpha_1(z_1)\\&\left.\otimes[y_2,z_2]\alpha_2(x_2)\right\}+\left\{(-1)^{s_3}[y_1,z_1]\alpha_1(x_1)\right.\\&\otimes\alpha_2(y_2)[z_2,x_2]+(-1)^{s_4+|x_1||y_1|}[z_1,y_1]\alpha_1(x_1)\\&\left.\otimes[z_2,x_2]\alpha_2(y_2)\right\}+\left\{(-1)^{s_3}\alpha_1(y_1)[z_1,x_1]\right.\\&\otimes[y_2,z_2]\alpha_2(x_2)+(-1)^{s_4+|x_2||y_2|}[z_1,x_1]\alpha_1(y_1)\\&\left.\otimes[z_2,y_2]\alpha_2(x_2)\right\}+\left\{(-1)^{s_2+|y_1||z_1|}[x_1,z_1]\alpha_1(y_1)\right.\\&\otimes[x_2,y_2]\alpha_2(z_2)+(-1)^{s_4}[z_1,x_1]\alpha_1(y_1)\\&\left.\otimes\alpha_2(z_2)[x_2,y_2]\right\}=0+0+0+0+0+0=0.\end{aligned}\tag{27}$$

This shows that $(A,[\cdot,\cdot],\alpha)$ satisfies the Hom-Jacobi superidentity:

$$\begin{aligned}(-1)^{|x||z|}&[[x,y],\alpha(z)]+(-1)^{|x||y|}[[y,z],\alpha(x)]\\&+(-1)^{|y||z|}[[z,x],\alpha(y)]=0.\end{aligned}\tag{28}$$

Finally, we check the Hom-Leibniz superidentity in A. Using the Hom-associativity and the Hom-Leibniz superidentity in the A_i, we have

$$\begin{aligned}&[xy,\alpha(z)]=[(x_1\otimes y_1)(y_1\otimes y_2),\alpha_1(z_1)\otimes\alpha_2(z_2)]\\&\quad=(-1)^{|x_2||y_1|+|x_2||z_1|+|y_2||z_1|}[x_1y_1,\alpha_1(z_1)]\otimes(x_2y_2)\\&\quad\cdot\alpha_2(z_2)+(-1)^{|x_2||y_1|+|x_2||z_1|+|y_2||z_1|}(x_1y_1)\alpha_1(z_1)\\&\quad\otimes[x_2y_2,\alpha_2(z_2)]=(-1)^{|x_2||y_1|+|x_2||z_1|+|y_2||z_1|+|y_1||z_1|}\\&\quad\cdot[x_1,z_1]\alpha_1(y_1)\otimes(x_2y_2)\alpha_2(z_2)\\&\quad+(-1)^{|x_2||y_1|+|x_2||z_1|+|y_2||z_1|}\alpha_1(x_1)[y_1,z_1]\otimes(x_2y_2)\\&\quad\cdot\alpha_2(z_2)+(-1)^{|x_2||y_1|+|x_2||z_1|+|y_2||z_1|+|y_2||z_2|}(x_1y_1)\\&\quad\cdot\alpha_1(z_1)\otimes[x_2,z_2]\alpha_2(y_2)\\&\quad+(-1)^{|x_2||y_1|+|x_2||z_1|+|y_2||z_1|}(x_1y_1)\alpha_1(z_1)\otimes\alpha_2(x_2)\\&\quad\cdot[y_2,z_2],\\&(-1)^{|y||z|}[x,z]\alpha(y)+\alpha(x)[y,z]\\&\quad=(-1)^{|y_1||z_1|+|y_1||z_2|+|y_2||z_1|+|y_2||z_2|+|x_2||z_1|}\\&\quad\cdot([x_1,z_1]\otimes(x_2z_2))(\alpha_1(y_1)\otimes\alpha_2(y_2))\\&\quad+(-1)^{|y_1||z_1|+|y_1||z_2|+|y_2||z_1|+|y_2||z_2|+|x_2||z_1|}\\&\quad\cdot((x_1z_1)\otimes[x_2,z_2])(\alpha_1(y_1)\otimes\alpha_2(y_2))\\&\quad+(-1)^{|y_2||z_1|}\alpha_1(x_1\otimes\alpha_2(x_2))([y_1,z_1]\otimes(y_2z_2))\\&\quad+(-1)^{|y_2||z_1|}\alpha_1(x_1\otimes\alpha_2(x_2))((y_1z_1)\otimes[y_2,z_2])\\&\quad=(-1)^{|y_1||z_1|+|y_2||z_1|+|x_2||z_1|+|x_2||y_1|}[x_1,z_1]\alpha_1(y_1)\\&\quad\otimes(x_2z_2)\alpha_2(y_2)+(-1)^{|y_2||z_1|+|y_2||z_2|+|x_2||z_1|+|x_2||y_1|}\\&\quad\cdot(x_1z_1)\alpha_1(y_1)\otimes[x_2,z_2]\alpha_2(y_2)\\&\quad+(-1)^{|y_2||z_1|+|x_2||y_1|+|x_2||z_1|}\alpha_1(x_1)[y_1,z_1]\otimes\alpha_2(x_2)\end{aligned}$$

$$\cdot (y_2 z_2) + (-1)^{|y_2||z_1|+|x_2||y_1|+|x_2||z_1|}\alpha_1(x_1)(y_1 z_1) \otimes \alpha_2(x_2)[y_2, z_2]. \quad (29)$$

Therefore, we have

$$[xy, \alpha(z)] = (-1)^{|y||z|}[x,z]\alpha(y) + \alpha(x)[y,z]. \quad (30)$$

□

Setting $\alpha_i = Id_{A_i}$ in Theorem 11, we obtain the result about Poisson superalgebras.

Corollary 12. *Let $(A_i, \cdot_i, [\cdot,\cdot]_i,)$ be Poisson superalgebras for $i = 1, 2$, and let $A = A_1 \otimes A_2$. Define the operations $\cdot, [\cdot,\cdot] : A^{\otimes 2} \to A$ by*

$$\begin{aligned}(x_1 \otimes x_2)\cdot(y_1 \otimes y_2) &= (-1)^{|x_2||y_1|}(x_1 \cdot_1 y_1) \otimes (x_2 \cdot_2 y_2),\\ [x_1 \otimes x_2, y_1 \otimes y_2] &= (-1)^{|x_2||y_1|}[x_1, y_1]_1 \otimes (x_2 \cdot_2 y_2) + (-1)^{|x_2||y_1|}(x_1 \cdot_1 y_1) \otimes [x_2, y_2]_2,\end{aligned} \quad (31)$$

for $x_i, y_i \in A_i$. Then $(A, \cdot, [\cdot,\cdot])$ is a Poisson superalgebra.

4. Admissible Hom-Poisson Superalgebras

A Poisson algebra has two binary operations, the Lie bracket and the commutative associative product. It is shown in [10] that Poisson algebras can be described using only one binary operation via the polarization-depolarization process. Moreover, the result of Poisson algebras is extended to Hom-Poisson algebras in [8]. In other words, the paper shows that a Hom-Poisson algebra can be described using only the twisting map and one binary operation. The purpose of this section is to extend this alternative description of Poisson algebras to Hom-Poisson superalgebras.

Definition 13. An admissible Hom-Poisson superalgebra A is a Hom-superalgebra satisfying

$$\begin{aligned}3A(x,y,z) &+ (-1)^{|x||y|}(yx)\alpha(z)\\ &- (-1)^{|y||z|}(xz)\alpha(y)\\ &- (-1)^{|x||y|+|x||z|}(yz)\alpha(x)\\ &+ (-1)^{|x||z|+|y||z|}(zx)\alpha(y) = 0,\end{aligned} \quad (32)$$

where $A(x,y,z) = (xy)\alpha(z) - \alpha(x)(yz)$, for any homogeneous elements $x, y, z \in A$, the identity (32) is called the Hom-Remm identity.

Remark 14. In particular, taking $\alpha = Id_A$, we find the notion of admissible Poisson superalgebra presented in [8].

Theorem 15. *Let $(A, \cdot, [\cdot,\cdot], \alpha)$ be a double Hom-superalgebra. Then $(A, \cdot, [\cdot,\cdot], \alpha)$ is a Hom-Poisson superalgebra if and only if there exists on A a nonassociative product xy such that $(A, \cdot, \alpha)$ is an admissible Hom-Poisson superalgebra.*

Proof. Assume that $(A, \cdot, [\cdot,\cdot], \alpha)$ is a Hom-Poisson superalgebra. Consider the multiplication

$$xy = x \cdot y + [x, y]. \quad (33)$$

We deduce that

$$x \cdot y = \frac{1}{2}\left(xy + (-1)^{|x||y|} yx\right). \quad (34)$$

Thus the associativity condition can be denoted by

$$\begin{aligned}v_1(x,y,z) = A(x,y,z) &- (-1)^{|x||y|+|x||z|+|y||z|}A(z,y,x)\\ &+ (-1)^{|x||y|}(yx)\alpha(z)\\ &- (-1)^{|y||z|}\alpha(x)(zy)\\ &- (-1)^{|x||y|+|x||z|}(yz)\alpha(x)\\ &+ (-1)^{|x||z|+|y||z|}\alpha(z)(xy) = 0,\end{aligned} \quad (35)$$

where $A(x,y,z) = (xy)\alpha(z) - \alpha(x)(yz)$. Likewise, the Hom-Poisson bracket can be denoted by

$$[x, y] = \frac{1}{2}\left(xy - (-1)^{|x||y|} yx\right) \quad (36)$$

and the Hom-super Jacobi condition

$$\begin{aligned}v_2(x,y,z) = (-1)^{|x||z|}A(x,y,z) &- (-1)^{|x||y|+|x||z|}A(y,x,z)\\ &- (-1)^{|x||y|+|y||z|}A(z,y,x)\\ &- (-1)^{|x||z|+|y||z|}A(x,z,y)\\ &+ (-1)^{|x||y|}A(y,z,x)\\ &+ (-1)^{|y||z|}A(z,x,y) = 0.\end{aligned} \quad (37)$$

The Hom-Leibniz superidentity can be denoted by

$$\begin{aligned}v_3(x,y,z) = A(x,y,z) &- (-1)^{|x||y|}A(y,x,z)\\ &+ (-1)^{|x||y|+|x||z|+|y||z|}A(z,y,x)\\ &+ (-1)^{|y||z|}A(x,z,y)\\ &+ (-1)^{|x||y|+|x||z|}A(y,z,x)\\ &- (-1)^{|x||z|+|y||z|}A(z,x,y) = 0.\end{aligned} \quad (38)$$

Let us consider the vector

$$v(x,y,z) = \frac{1}{3}\{(-1)^{|x||y|}(yx)\alpha(z) - (-1)^{|y||z|}(xz)\alpha(y) - (-1)^{|x||y|+|x||z|}(yz)\alpha(x) + (-1)^{|x||z|+|y||z|}(zx)\alpha(y)\} + (xy)\alpha(z) - \alpha(x)\cdot(yz). \quad (39)$$

Then

$$v(x,y,z) = \frac{1}{6}\{2v_1(x,y,z) + (-1)^{|x||z|}v_2(x,y,z) + v_3(x,y,z) + 2(-1)^{|x||z|+|y||z|}v_3(z,x,y)\}. \quad (40)$$

We deduce that the product xy satisfies

$$v(x,y,z) = 0 \quad (41)$$

for any homogeneous elements $x, y, z \in A$.

Conversely, assume that the produce of the nonassociative product A satisfies $v(x,y,z) = 0$ for any homogeneous elements $x, y, z \in A$. Let $v_1(x,y,z)$, $v_2(x,y,z)$, $v_3(x,y,z)$ be the elements of A defined in the first part, respectively, in relation to the Hom-associativity, Hom-super Jacobi, and Hom-super Leibniz relations. We have

$$\begin{aligned} v_1(x,y,z) &= v(x,y,z) - (-1)^{|x||y|+|x||z|+|y||z|}v(z,y,x) + (-1)^{|y||z|}v(x,z,y) - (-1)^{|x||z|+|y||z|}v(z,x,y), \\ v_2(x,y,z) &= (-1)^{|x||z|}v(x,y,z) - (-1)^{|x||y|+|x||z|}v(y,x,z) - (-1)^{|x||y|+|y||z|}v(z,y,x) - (-1)^{|x||z|+|y||z|}v(x,z,y) + (-1)^{|x||y|}v(y,z,x) + (-1)^{|y||z|}v(z,x,y), \\ v_3(x,y,z) &= v(x,y,z) - (-1)^{|x||y|}v(y,x,z) + (-1)^{|x||y|+|x||z|+|y||z|}v(z,y,x) + (-1)^{|y||z|}v(x,z,y) + (-1)^{|x||y|+|x||z|}v(y,z,x) - (-1)^{|x||z|+|y||z|}v(z,x,y). \end{aligned} \quad (42)$$

□

Taking $\alpha = Id_A$ in Theorem 15, we obtain the following result, which is Theorem 1 in [7].

Corollary 16. *Let $(A,\cdot,[\cdot,\cdot])$ be a double superalgebra. Then $(A,\cdot,[\cdot,\cdot])$ is a Poisson superalgebra if and only if there exists on A a nonassociative product xy satisfying*

$$3A(x,y,z) + (-1)^{|x||y|}(yx)z - (-1)^{|y||z|}(xz)y - (-1)^{|x||y|+|x||z|}(yz)x + (-1)^{|x||z|+|y||z|}(zx)y = 0, \quad (43)$$

where $A(x,y,z) = (xy)z - x(yz)$, for any homogeneous elements $x, y, z \in A$.

Definition 17. A Hom-nonassociative superalgebra $(A,\cdot,\alpha)$ is called Hom-superflexive if the multiplication xy satisfies

$$A(x,y,z) + (-1)^{|x||y|+|x||z|+|y||z|}A(z,y,x) = 0 \quad (44)$$

for any homogeneous elements $x, y, z \in A$, where $A_\alpha(x,y,z) = (xy)\alpha(z) - \alpha(x)(yz)$ is called a Hom-associator of the multiplication.

Proposition 18. *Let $(A,\cdot,[\cdot,\cdot],\alpha)$ be a Hom-Poisson superalgebra. Then the Hom-Remm product defining the Hom-Poisson superalgebra structure is Hom-superflexive.*

Proof. Let

$$B(x,y,z) = 3\{A(x,y,z) + (-1)^{|x||y|+|x||z|+|y||z|}A(z,y,x)\}. \quad (45)$$

We have

$$\begin{aligned} B(x,y,z) &= -(-1)^{|x||y|}(yx)\alpha(z) + (-1)^{|y||z|}(xz)\cdot\alpha(y) + (-1)^{|x||y|+|x||z|}(yz)\alpha(x) - (-1)^{|x||z|+|y||z|}\cdot(zx)\alpha(y) + (-1)^{|x||y|+|x||z|+|y||z|} \\ &\cdot\{-(-1)^{|z||y|}(yz)\alpha(x) + (-1)^{|y||x|}(zx)\alpha(y) + (-1)^{|z||y|+|z||x|}(yx)\alpha(y) - (-1)^{|z||x|+|y||z|}(xz)\alpha(y)\} = \{-(-1)^{|x||y|} \\ &+ (-1)^{|x||y|}\}(yx)\alpha(z) + \{(-1)^{|y||z|} - (-1)^{|y||z|}\}\cdot(xz)\alpha(y) + \{(-1)^{|x||y|+|x||z|} - (-1)^{|x||y|+|x||z|}\} \\ &\cdot(yz)\alpha(x) + \{-(-1)^{|x||z|+|y||z|} + (-1)^{|x||z|+|y||z|}\}\cdot(zx)\alpha(y) = 0. \end{aligned} \quad (46)$$

□

Taking $\alpha = Id_A$ in Proposition 18, we obtain the following result, which is Proposition 3 in [7].

Corollary 19. *Let $(A,\cdot,[\cdot,\cdot])$ be a Poisson superalgebra. Then the Remm product defining Poisson superalgebra structure is superflexive.*

Remark 20. The deformation cohomology of Hom-Poisson superalgebras can be computed with the Hom-Remm identity, which is similar to the method in [11]. This content is not primary in the paper, we do not have a detailed discussion here.

5. A Classification of 2-Dimensional Hom-Poisson Superalgebras

In this section, we only consider that $A_{\bar{1}}$ is nontrivial. A_2^i denotes one of the 2-dimensional admissible Poisson superalgebra types. α_2^i denotes one of the homomorphism types corresponding to A_2^i. $\widetilde{A}_2^i$ denotes one of the 2-dimensional admissible Hom-Poisson superalgebra types corresponding to α_2^i. In the following, the products equal to zero are omitted.

Lemma 21. *Let $(A, ())$ be an admissible Poisson superalgebra and let $\alpha : A \to A$ be an even Poisson superalgebra endomorphism. Then $(A, ()_\alpha, \alpha)$ is an admissible Hom-Poisson superalgebra, where $(xy)_\alpha = \alpha(xy)$.*

Proof. It is straightforward by Definition 13. □

Lemma 22 (see [7]). *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with a basis $\{e_0, e_1\}$, where $e_0 \in A_{\bar{0}}, e_1 \in A_{\bar{1}}$. Then A is one of the following types:*

$A_2^1 : e_0e_0 = ae_0, e_0e_1 = ae_1, e_1e_0 = ae_1, e_1e_1 = de_0, d \neq 0$.

$A_2^2 : e_0e_0 = ae_0$.

$A_2^3 : e_0e_0 = ae_0, e_0e_1 = ae_1, e_1e_0 = ae_1, a \neq 0$.

$A_2^4 : e_0e_1 = be_1, e_1e_0 = -be_1, b \neq 0$.

Proof. Let

$$\begin{aligned} e_0e_0 &= ae_0, \\ e_0e_1 &= be_1, \\ e_1e_0 &= ce_1, \\ e_1e_1 &= de_0. \end{aligned} \tag{47}$$

By Corollary 16, we have

$$\begin{aligned} 3(a-b)b + ab - 2bc + c^2 &= 0, \\ d(b-a) &= 0, \\ 3(a-c)c + ab - 2bc + c^2 &= 0, \\ d(3c - b - 2a) &= 0, \\ d(b-c) &= 0. \end{aligned} \tag{48}$$

Now we consider the cases as follows.

Case 1. If $d \neq 0$, then $b = a = c$; hence we have A_2^1.

Case 2. If $d = 0$, then $(b-c)(a-b-c) = 0$.

Subcase 2.1. If $b = c$, then $b(a-b) = 0$.
If $b = 0$, then we have A_2^2.
If $b \neq 0$, then $a = b = c \neq 0$; hence we have A_2^3.

Subcase 2.2. If $b \neq c$, then $b = -c \neq 0$, $a = 0$; hence we have A_2^4. □

Remark 23. Lemma 5.2 in [6] has been obtained; however, there is one minor inaccuracy on the product operation in that proof.

Lemma 24. *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with dim $A_{\bar{0}} = 1$ and dim $A_{\bar{1}} = 1$. Then an even homomorphism α of type A_2^1 is as follows:*

$$\begin{aligned} &\alpha_{2(1)}^1 : \begin{cases} \alpha(e_0) = k^2 e_0 \\ \alpha(e_1) = ke_1; \end{cases} \\ &\alpha_{2(2)}^1 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases} \\ &\alpha_{2(3)}^1 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = \pm e_1. \end{cases} \end{aligned} \tag{49}$$

Proof. Let

$$\begin{aligned} \alpha(e_0) &= a_{10}e_0, \\ \alpha(e_1) &= a_{11}e_1. \end{aligned} \tag{50}$$

From α is an even homomorphism, we obtain

$$\begin{aligned} \alpha(e_0e_0) &= \alpha(e_0)\alpha(e_0), \\ \alpha(e_0e_1) &= \alpha(e_0)\alpha(e_1), \\ \alpha(e_1e_0) &= \alpha(e_1)\alpha(e_0), \\ \alpha(e_1e_1) &= \alpha(e_1)\alpha(e_1). \end{aligned} \tag{51}$$

By Lemma 22 and (50) and (51), we obtain

$$\begin{aligned} aa_{10}(a_{10} - 1) &= 0, \\ aa_{11}(a_{10} - 1) &= 0, \\ a_{10} &= a_{11}^2. \end{aligned} \tag{52}$$

Case 1. If $a = 0$, then we have $\alpha_{2(1)}^1$.

Case 2. If $a \neq 0$, then we consider two cases as follows.
If $a_{10} = 0$, then $a_{11} = 0$; hence we have $\alpha_{2(2)}^1$.
If $a_{10} \neq 0$, then $a_{10} = 1, a_{11} = \pm 1$; hence we have $\alpha_{2(3)}^1$. □

Corollary 25. *Let $(\widetilde{A}, ()_\alpha, \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\widetilde{A}$ with respect to α_2^1 is as follows:*

$\widetilde{A}_{2(1)}^1 : (e_1e_1)_\alpha = dk^2e_0, d \neq 0$.

$\widetilde{A}_{2(2)}^1 : (AA)_\alpha = 0$.

$\widetilde{A}^1_{2(3)}:(e_0e_0)_\alpha = ae_0$, $(e_0e_1)_\alpha = \pm ae_1$, $(e_1e_0)_\alpha = \pm ae_1$, $(e_1e_1)_\alpha = de_0$, $a \neq 0$, $d \neq 0$.

Proof. Apply Lemmas 21, 22, and 24. □

Lemma 26. *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with* $\dim A_{\bar{0}} = 1$ *and* $\dim A_{\bar{1}} = 1$. *Then an even homomorphism α of type A^2_2 is as follows:*

$$\alpha^2_{2(1)}: \begin{cases} \alpha(e_0) = k_0e_0 \\ \alpha(e_1) = ke_1; \end{cases}$$
$$\alpha^2_{2(2)}: \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = ke_1; \end{cases} \quad (53)$$
$$\alpha^2_{2(3)}: \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1. \end{cases}$$

Proof. By Lemma 22 and (20) and (21), we obtain $aa_{10}(a_{10}-1)=0$.
Case 1. Suppose that $a = 0$, we have $\alpha^2_{2(1)}$.

Case 2. Suppose that $a \neq 0$, we consider two cases as follows.
If $a_{10} = 0$, then we have $\alpha^2_{2(2)}$.
If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha^2_{2(3)}$. □

Corollary 27. *Let $(\widetilde{A}, ()_\alpha, \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\widetilde{A}$ with respect to α^2_2 is as follows:*

$\widetilde{A}^2_{2(1)}:(AA)_\alpha = 0$.

$\widetilde{A}^2_{2(2)}:(AA)_\alpha = 0$.

$\widetilde{A}^2_{2(3)}:(e_0e_0)_\alpha = ae_0$, $a \neq 0$.

Proof. Apply Lemmas 21, 22, and 26. □

Lemma 28. *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with dim $A_{\bar{0}} = 1$ and dim $A_{\bar{1}} = 1$. Then an even homomorphism α of type A^3_2 is as follows:*

$$\alpha^3_{2(1)}: \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$
$$\alpha^3_{2(2)}: \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1; \end{cases}$$
$$\alpha^3_{2(3)}: \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$
$$\alpha^3_{2(4)}: \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = 0; \end{cases}$$
$$\alpha^3_{2(5)}: \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1, \end{cases} \quad k \neq 0. \quad (54)$$

Proof. By Lemma 22 and (20) and (21), we obtain

$$a_{10}(a_{10}-1) = 0, \quad a_{11}(a_{10}-1) = 0. \quad (55)$$

Case 1. If $a_{10} = 0$, then $a_{11} = 0$; hence we have $\alpha^3_{2(1)}$.

Case 2. If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha^3_{2(2)}$.

Case 3. If $a_{11} = 0$, then we consider two cases as follows.
If $a_{10} = 0$, we have $\alpha^3_{2(3)}$.
If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha^3_{2(4)}$.

Case 4. If $a_{11} \neq 0$, then $a_{10} = 1$; hence we have $\alpha^3_{2(5)}$. □

Corollary 29. *Let $(\widetilde{A}, ()_\alpha, \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\widetilde{A}$ with respect to α^3_2 is as follows:*

$\widetilde{A}^3_{2(1)}:(AA)_\alpha = 0$.

$\widetilde{A}^3_{2(2)}:(e_0e_0)_\alpha = ae_0$, $(e_0e_1)_\alpha = kae_1$, $(e_1e_0)_\alpha = kae_1$, $a \neq 0$.

$\widetilde{A}^3_{2(3)}:(AA)_\alpha = 0$.

$\widetilde{A}^3_{2(4)}:(e_0e_0)_\alpha = ae_0$, $a \neq 0$.

$\widetilde{A}^3_{2(5)}:(e_0e_0)_\alpha = ae_0$, $(e_0e_1)_\alpha = kae_1$, $(e_1e_0)_\alpha = kae_1$, $a \neq 0$, $k \neq 0$.

Proof. Apply Lemmas 21, 22, and 28. □

Lemma 30. *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with* $\dim A_{\bar{0}} = 1$ *and* $\dim A_{\bar{1}} = 1$. *Then an even homomorphism α of type A^4_2 is as follows:*

$$\alpha^4_{2(1)}: \begin{cases} \alpha(e_0) = ke_0 \\ \alpha(e_1) = 0; \end{cases} \quad (56)$$
$$\alpha^4_{2(2)}: \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1, \end{cases} \quad k \neq 0.$$

Proof. By Lemma 22 and (20) and (21), we obtain $a_{11}(a_{10}-1) = 0$.

Case 1. If $a_{10} = 0$, then we have $\alpha^4_{2(1)}$.

Case 2. If $a_{10} \neq 0$, then $a_{10} = 1$; hence we have $\alpha^4_{2(2)}$. □

Corollary 31. *Let $(\widetilde{A}, ()_\alpha, \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\widetilde{A}$ with respect to α_2^4 is as follows:*

$$\widetilde{A}_{2(1)}^4 : (AA)_\alpha = 0.$$

$$\widetilde{A}_{2(2)}^4 : (e_0 e_1)_\alpha = kbe_1, (e_1 e_0)_\alpha = -kbe_1, b \neq 0, k \neq 0.$$

Proof. Apply Lemmas 21, 22, and 30. □

Theorem 32. *Let $(A, ())$ be a 2-dimensional admissible Poisson superalgebra with $\dim A_{\bar{0}} = 1$ and $\dim A_{\bar{1}} = 1$. Then an even homomorphism α of A is as follows:*

$$\alpha_{2(1)}^1 : \begin{cases} \alpha(e_0) = k^2 e_0 \\ \alpha(e_1) = ke_1; \end{cases}$$

$$\alpha_{2(2)}^1 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(3)}^1 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = \pm e_1; \end{cases}$$

$$\alpha_{2(1)}^2 : \begin{cases} \alpha(e_0) = k_0 e_0 \\ \alpha(e_1) = ke_1; \end{cases}$$

$$\alpha_{2(2)}^2 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = ke_1; \end{cases}$$

$$\alpha_{2(3)}^2 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1; \end{cases}$$

$$\alpha_{2(1)}^3 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(2)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1; \end{cases}$$

$$\alpha_{2(3)}^3 : \begin{cases} \alpha(e_0) = 0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(4)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\alpha_{2(5)}^3 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1, \end{cases} \quad k \neq 0;$$

$$\alpha_{2(1)}^4 : \begin{cases} \alpha(e_0) = ke_0 \\ \alpha(e_1) = 0; \end{cases}$$

$$\sigma_{2(2)}^4 : \begin{cases} \alpha(e_0) = e_0 \\ \alpha(e_1) = ke_1, \end{cases} \quad k \neq 0. \tag{57}$$

Proof. Apply Lemmas 24–30. □

Corollary 33. *Let $(\widetilde{A}, ()_\alpha, \alpha)$ be a 2-dimensional admissible Hom-Poisson superalgebra. Then $\widetilde{A}$ with respect to α_2^i is as follows:*

$\widetilde{A}_{2(1)}^1 : (e_1 e_1)_\alpha = dk^2 e_0$, $d \neq 0$.

$\widetilde{A}_{2(2)}^1 : (AA)_\alpha = 0$.

$\widetilde{A}_{2(3)}^1 : (e_0 e_0)_\alpha = ae_0$, $(e_0 e_1)_\alpha = \pm ae_1$, $(e_1 e_0)_\alpha = \pm ae_1$, $(e_1 e_1)_\alpha = de_0$, $a \neq 0$, $d \neq 0$.

$\widetilde{A}_{2(1)}^2 : (AA)_\alpha = 0$.

$\widetilde{A}_{2(2)}^2 : (AA)_\alpha = 0$.

$\widetilde{A}_{2(3)}^2 : (e_0 e_0)_\alpha = ae_0$, $a \neq 0$.

$\widetilde{A}_{2(1)}^3 : (AA)_\alpha = 0$.

$\widetilde{A}_{2(2)}^3 : (e_0 e_0)_\alpha = ae_0$, $(e_0 e_1)_\alpha = kae_1$, $(e_1 e_0)_\alpha = kae_1$, $a \neq 0$.

$\widetilde{A}_{2(3)}^3 : (AA)_\alpha = 0$.

$\widetilde{A}_{2(4)}^3 : (e_0 e_0)_\alpha = ae_0$, $a \neq 0$.

$\widetilde{A}_{2(5)}^3 : (e_0 e_0)_\alpha = ae_0$, $(e_0 e_1)_\alpha = kae_1$, $(e_1 e_0)_\alpha = kae_1$, $a \neq 0$, $k \neq 0$.

$\widetilde{A}_{2(1)}^4 : (AA)_\alpha = 0$.

$\widetilde{A}_{2(2)}^4 : (e_0 e_1)_\sigma = kbe_1$, $(e_1 e_0)_\alpha = -kbe_1$, $b \neq 0$, $k \neq 0$.

Proof. Apply Corollaries 25–31. □

Remark 34. (1) Some nonisomorphic 2-dimensional admissible Poisson superalgebras have isomorphic admissible Hom-Poisson deformations. For example, the admissible Poisson superalgebras A_2^2 (with $k = 0$ in its algebra homomorphism $\alpha_{2(3)}^2$) and A_2^3 can be deformed into isomorphic admissible Hom-Poisson superalgebras $\widetilde{A}_{2(3)}^2$ and $\widetilde{A}_{2(4)}^3$. There are several other such pairs in the cases above.

(2) We give a classification of 2-dimensional admissible Hom-Poisson superalgebras in Corollary 33. Using Theorem 15, a classification of 2-dimensional Hom-Poisson superalgebras will be obviously obtained.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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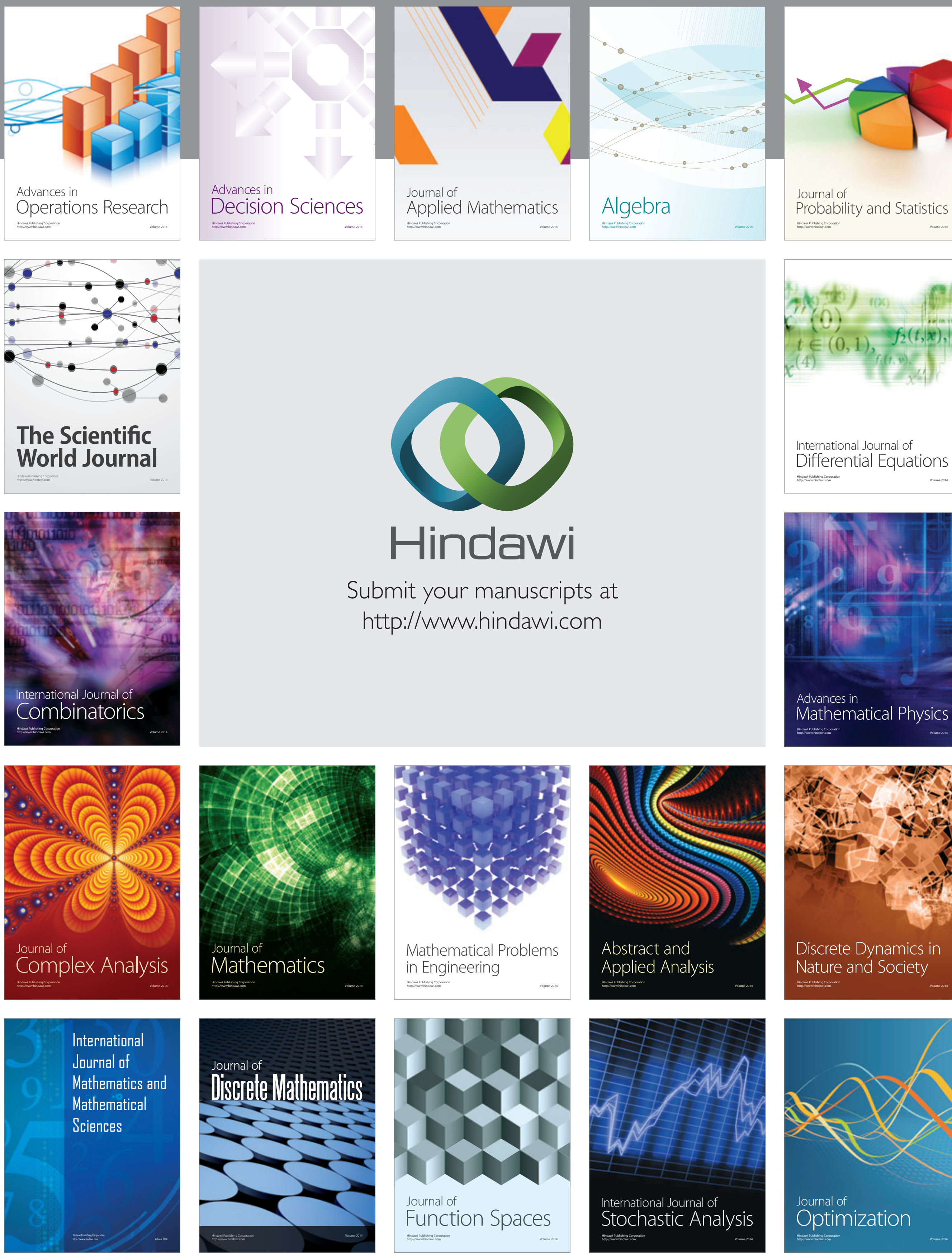
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